**Math 20340 Section 02: Statistics for Life Sciences**
**Spring 2011**
**Exam 3 Solutions**

1. A store predicts that the average number of miles that a person can hike in their Chaco sandals (before the strap breaks) is 5,200 miles. To test whether or not this really is the case, 200 people hike in their Chaco's and their miles are recorded.

   (a) (3 points) State the null and alternative hypotheses for this test.

   **Solution:** Let $\mu$ be the true average number of miles that a person can hike in their sandals before the strap breaks. Then

$$\begin{aligned} H_0 &: \mu = 5200 \\ H_a &: \mu \neq 5200 \end{aligned}$$

   (b) (5 points) If the average number of miles hiked (before the strap breaks) is 5,287 miles with a standard deviation of 421 miles, what is the test statistic?

   **Solution:** Test statistic is $\frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{5287-5200}{421/\sqrt{200}} = 2.92$.

   (c) (4 points) Calculate the $p$-value for this test.

   **Solution:** The $p$-value (for this *two*-sided test) is $P(z \geq 2.92) + P(z \leq -2.92) = .0017 + .0017 = .0034$.

   (d) (3 points) Do we have sufficient evidence to contradict store's claim at a 5% significance level? At 1%?

   **Solution:** Yes at 5%, since the $p$-value is less than .05. Yes at 1%, since the $p$-value is less than .01.

   (e) (5 points) Explain precisely what this $p$-value means.

   **Solution:** This $p$-value is the probability that we see a test statistic with absolute value 2.92 or greater *under the assumption that* $\mu = 5200$.

2. You head to a friends house and find 5 12-oz bottles of your favorite beverage in their fridge. You know that the the amount per bottle is normally distributed, and think that the factory actually fills each bottle with fewer than 12 oz on average. Your evidence? In the 5 bottles, you record the following amounts (in oz):

$$11.7,\ 11.8,\ 11.8,\ 12.0,\ 11.8.$$

   After your initial anger subsides, you drift into the sweet and calming realm of statistical hypothesis testing to see if your suspicion is correct or not.

(a) (3 points) State the null and alternative hypotheses.

**Solution:** Let $\mu$ be the average amount of liquid in a bottle. Then we have

$$\begin{aligned} H_0 &: \mu = 12 \\ H_a &: \mu < 12 \end{aligned}$$

(b) (6 points) Calculate the sample mean and sample standard deviation.

**Solution:** We have

$$\bar{x} = \frac{11.7 + 3 \cdot 11.8 + 12.0}{5} = 11.82,$$

$$s = \sqrt{\frac{(11.7 - 11.82)^2 + 3(11.8 - 11.82)^2 + (12 - 11.82)^2}{4}} = .1095.$$

(c) (11 points) Is there evidence that the average amount a bottle is filled is less than 12 oz? Test at 1% significance.

**Solution:** Since $n < 30$ and our population is normally distributed, $\bar{x}$ has a $t$-distribution with 4 degrees of freedom. Therefore, the critical value is $-t_{.01} = -3.747$. The test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{11.82 - 12}{.1095/\sqrt{5}} = -3.676.$$

Since the test statistic is not smaller than the critical value, we do not have enough evidence to reject $H_0$ at 1% significance (but we can reject at 5% and 2.5%, based on those critical values).

3. (a) (16 points) You want to know if there is a difference between ND freshman and ND seniors in the average number of hours that a student is awake (on the night before their first final exam of the semester) between 12AM and 8AM. You'd like your 95% margin of error to be at most $\pm 0.5$ (hours). If you use the same size sample for each class (ND freshman and ND seniors), how many people from each class should you select?

**Solution:** The experiment is asking about finding the difference in two means. You want $1.96 \cdot SE \leq 0.5$, so $1.96\sqrt{\frac{\sigma_1^2}{n} + \frac{\sigma_2^2}{n}} \leq 0.5$ (since $n_1 = n_2 = n$). To estimate $\sigma_1$ and $\sigma_2$, we use $\sigma_1 \approx \text{Range}/4 = 8/4 = 2$, and similarly $\sigma_2 \approx 2$. Therefore we want $1.96\sqrt{2^2/n + 2^2/n} \leq 0.5$ implies $\frac{1.96}{0.5}\sqrt{8} \leq \sqrt{n}$, or $122.93 \leq n$. We take $123 \leq n$.

(b) (4 points) You run the experiment above and get $\bar{x}_1 - \bar{x}_2 = 0.76$ with the 95% margin of error at $\pm 0.5$. Is it likely that there is a difference between the two means? Explain.

**Solution:** Yes, since 0 is more than 0.5 away from the point estimate of 0.76. This is because we know that 95% of the time that the procedure is performed (assuming these are done randomly), the true value of the difference is within the 95% margin of error of $\bar{x}_1 - \bar{x}_2$.

**Note:** Many people tried to do an actual test here by calculating a test statistic, and using the sample standard deviation of 2. This isn't quite right - the running of the experiment will give some value of $s_1$ and $s_2$. What you *should* realize is that I gave you the margin of error to be $\pm 0.5$, meaning that $1.96 \cdot SE = 0.5$. You can use this to get the actual standard error (which differs slightly from using $s_1 = 2, s_2 = 2$).

4. A March 2011 Gallup poll surveyed 1027 randomly selected US adults. Of these, 596 indicated that they think nuclear power plants in the United States are safe.

   (a) (16 points) Construct a 98% confidence interval for the true proportion of US adults who think that nuclear power plants in the United States are safe.

   **Solution:** We have $\hat{p} = 596/1027 = .5803$ and the standard error for $\hat{p}$ is approximately $\sqrt{.5803(.4197)/1027} = .0154$. Since $\alpha = .02$, we use $z_{.01} = 2.33$, and our 98% confidence interval is $.5803 \pm 2.33 \cdot SE = .5803 \pm 2.33(.0154) = .5803 \pm .0359$, or $(.5444, .6162)$.

   (b) (4 points) Suppose that you repeat this procedure 50 times (assume the procedures are done independently). What is the probability that *all* 50 intervals will contain $p$, the true number of adults who think that nuclear power plans in the United States are safe?

   **Solution:** Since the probability that any single interval contains $p$ is .98 (by definition of a 98% confidence interval), we have (by independence)

$$P(\text{all 50 contain } p) = (.98)^{50} = .364.$$

5. An experimental drug is supposed to reduce the occurrence of prostate cancer in men over the age of 65. To test this claim, a placebo-controlled study enrolls 1623 men over 65, of whom 816 are randomly selected to receive the drug, with the other 807 receiving a placebo. The following observations are made over the next 5 years:

   - 43 out of the 816 taking the drug develop prostate cancer,
   - 64 out of the 807 taking the placebo develop prostate cancer.

   Is the drug effective at reducing the occurrence of prostate cancer?

   Be sure to state clearly what your null and alternative hypotheses are, and at what significance level you are drawing your conclusion.

**Solution:** If we let the 1's be the population with the drug and the 2's be the population with the placebo, we have:

$$\begin{aligned} H_0 &: p_1 - p_2 = 0 \\ H_a &: p_1 - p_2 < 0 \end{aligned}$$

We have $\hat{p}_1 = 43/816 = .0527...$, $\hat{p}_2 = 64/807 = .0793...$, and our pooled estimator of the proportion (under the assumption that $H_0$ is true) is $\hat{p} = (43+64)/1623 = .0659...$. Our test statistic is:

$$z \approx \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}} = \frac{.0527 - .0793}{\sqrt{.0659(.9341)/816 + .0659(.9341)/807}} = -2.16$$

The $p$-value is $P(z \leq -2.16) = .0154$. Therefore we do not reject $H_0$ at 1% significance, but we do reject $H_0$ at 5% significance. Note that the $\alpha$ value is the significance level, not the $(1-\alpha)$ value.