Math 20340 Section 02: Statistics for Life Sciences
Mid-semester Exam 2 Solutions

1. A Coke machine is set to fill 8-ounce cups. It is known that the number of ounces it dispenses is normally distributed with mean 7.7 ounces and standard deviation 0.15 ounces.

   (a) (9 points) What is the probability of having an 8-ounce cup overflow?

   Solution: Let x be the amount of Coke the machine puts in a cup. Then x is a normal random variable with mean 7.7 and standard deviation 0.15. Therefore,

   $$P(x > 8) = P(z > \frac{8 - 7.7}{0.15}) = P(z > 2) = 1 - P(z \leq 2) = .0228$$

   (b) (6 points) If I fill up 20 cups (for me and my 19 friends), what is the probability that at least 1 of the 20 cups overflows?

   Solution: The since at least 1 cup overflows is the complement of no cups overflowing, we have

   $$P(\text{at least 1 overflow}) = 1 - P(\text{no cups overflow}) = 1 - (.9772)^{20} = .3695...$$

   Remark: I allowed a Poisson approximation to this problem since $np < 7$. While a Poisson approximation requires "n large," the book doesn't give a specific range of n for which this works. A binomial approximation should not be used here since $np < 5$, based on our rule of thumb (but it is actually somewhat close *if* you use the correct continuity correction).

2. Suppose that a cup of South Bend city water contains too many coliform bacteria with probability 0.001. I drink 3000 cups of water in a given year, and I can tolerate having at most 5 cups in a year with too many coliform bacteria before I get sick. What is the probability that I get sick in a given year? Justify any assumptions you make about the relevant probability distribution.

   Solution: Let x be the number of cups with coliform bacteria that I drink in a year. Then x is a binomial random variable; since $np = 3000(.001) = 3 < 7$, we can approximate x with a Poisson random variable that has $\mu = np = 3$. Then

   $$P(x > 5) = 1 - P(x \leq 5) = 1 - .916 = .084.$$

3. My bike ride to school takes me an average of 12 minutes, with a standard deviation of 2.5 minutes. I bike to school 35 times, and compute the average time it takes me to get to school.

(a) (8 points) What is the probability that my average time is between 11 and 11.5 minutes?

Solution: Since 35 is bigger than 30, by the Central Limit Theorem we have that $\bar{x}$ is approximately normally distributed with mean 12 and standard deviation $2.5/\sqrt{35} \approx .4226$. Therefore

$$P(11 < \bar{x} < 11.5) = P(-2.37 < z < -1.18) = .1190 - .0089 = .1101.$$

(b) (7 points) How many times should I bike (instead of 35) so that the probability of seeing an average of greater than 12.2 is 2.5%?

Solution: We have

$$.025 = P(\bar{x} > 12.2) = P(z > \frac{.2}{2.5/\sqrt{n}}) = 1 - P(z \leq \frac{.2}{2.5/\sqrt{n}}),$$

so

$$P(z \leq \frac{.2}{2.5/\sqrt{n}}) = .975.$$

From the table, we have $P(z \leq 1.96) = .9750$, so

$$\frac{.2}{2.5\sqrt{n}} = 1.96 \implies n = 600.25.$$

We take $n = 601$, making the probability just under 2.5%. I also accepted 600 here, since I didn't explicitly say that I wanted the probability to be *at most* 2.5%. You did need to round your answer to a whole number though, since these are the only values of n that make sense.

4. A recent study shows that 85% of children have the dreaded "Bieber fever." A random sample of 530 children are selected. How likely is it that we see a sample proportion of 89%? Justify any assumptions you make about the relevant probability distribution.

Solution: Upon further review, I think this problem is a bit misleading in the wording. You should read the remark on the next page; here is what I intended an answer to be: We have a binomial experiment where we want to calculate the probability of seeing a sample proportion. Since $np = 450.5 > 5$ and $nq = 79.5 > 5$, we can know that the sampling distribution of $\hat{p}$ is approximately normal with mean .85 and standard deviation $\sqrt{(.85)(.15)/530} = .01551....$ Therefore

$$P(\hat{p} > .89) = P(z > .04/.01551) = 1 - P(z \leq 2.58) = .0049$$

so it is quite unlikely to see a sample proportion that high (*Note*: you could also show that .89 is $.04/.01551 = 2.58$ standard deviations away from the mean, so it is considered unlikely).

Remark: I think this problem is worded poorly — it should have asked for the probability of seeing a sample proportion of *at least* 89%. Therefore, I mainly looked for you doing a correct assumption on the probability distribution, and if you used the actual binomial distribution (not the sample proportion) I looked for the correct use of the continuity correction, and then you should have done some attempt at a calculation to support or refute the claim (and not made mistakes on the way).

5. According to the NBC Nightly News[1], 39% of men ages 50-60 who lose their job don't find another job. We randomly find 45 men, aged 50-60, who recently lost their job. Let x be the number that find another job. What is the probability that the number of men who find another job is between 16 and 22 (inclusive)? Justify any assumptions you make about the relevant probability distribution.

   Solution: Note that x is a binomial random variable (with $p = .61$); since $nq = 45(.39) = 17.55$ and $np = 45(.61) = 27.45$, we assume that x is approximately normal with mean 27.45 and standard deviation $\sqrt{45(.61)(.39)} = 3.272$. Therefore, using the continuity correction (since we're approximating a binomial random variable with the normal random variable)

$$P(16 \leq x \leq 22) = P(15.5 \leq x_{normal} \leq 22.5) = P(-3.65 \leq z \leq -1.51) = .0655.$$

6. The second-hand store "New to μ" is run by a statistically-minded lady, Marge N. Avera. Marge realized that each customer that visits the store has a 25% chance of spending nothing, a 50% chance of spending \$1, and a 25% chance of spending \$2. On one day, Marge has 800 customers visit the store; let x be the total daily sales on that day.

   (a) (3 points) What is the approximate distribution of x? Explain.

   Solution: Since 800 is larger than 30 and the total daily sales is the sum of the sales of each individual person (meaning the sum sample values from the same distribution), the Central Limit Theorem tells us that x is approximately normal.

   (b) (9 points) What are the mean and standard deviation of x?

   Solution: First we calculate the mean μ and standard deviation σ for how much a single shopper spends:

$$\mu = 0(.25) + 1(.5) + 2(.25) = 1,$$

$$\sigma = \sqrt{(0-1)^2(.25) + (1-1)^2(.5) + (2-1)^2(.25)} = \sqrt{.5}$$

   Then the mean for x is $n\mu = (800)(1) = 800$ and the standard deviation for x is $\sqrt{n}\sigma = \sqrt{800}\sqrt{.5} = 20$.

[1]airing on Tuesday, March 1, 2011

(c) (3 points) What is the probability that the total daily sales is less than \$750?

Solution: We have

$$P(x < 750) = P(z < \frac{750 - 800}{20}) = P(z < -2.5) = .0062.$$

Remark: Note that x is, in fact, a discrete random variable which takes values in the integers (think about the problem to convince yourself of this), and through the CLT we are approximating this with a normal random variable. Technically, a continuity correction is in order here and the correct calculation is

$$P(x \leq 749.5) = P(z < -2.53) = .0057.$$