A Bayesian Approach to Fused GRAPPA and SENSE MR Image Reconstruction

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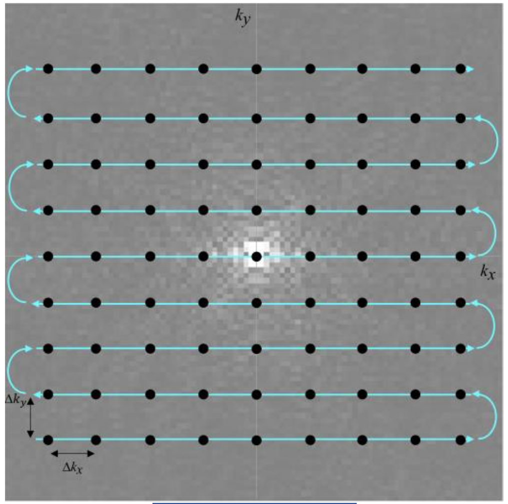
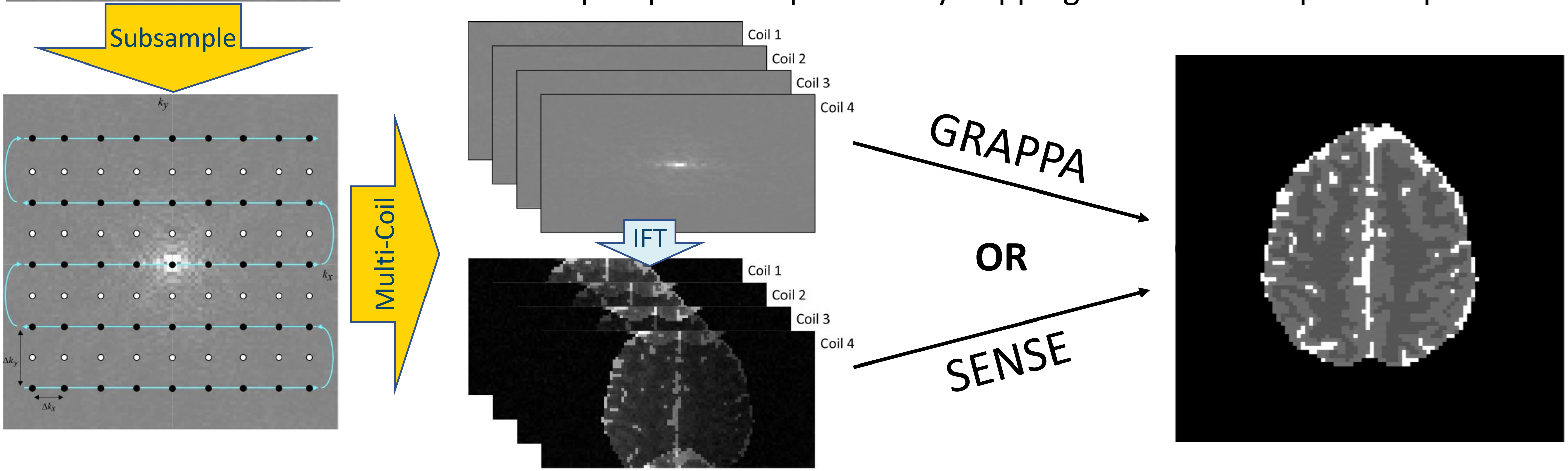
Outline

1. Introduction
2. Bayesian MUGS (BMUGS)
3. Simulated/Experimental Results
4. Discussion

1. Introduction



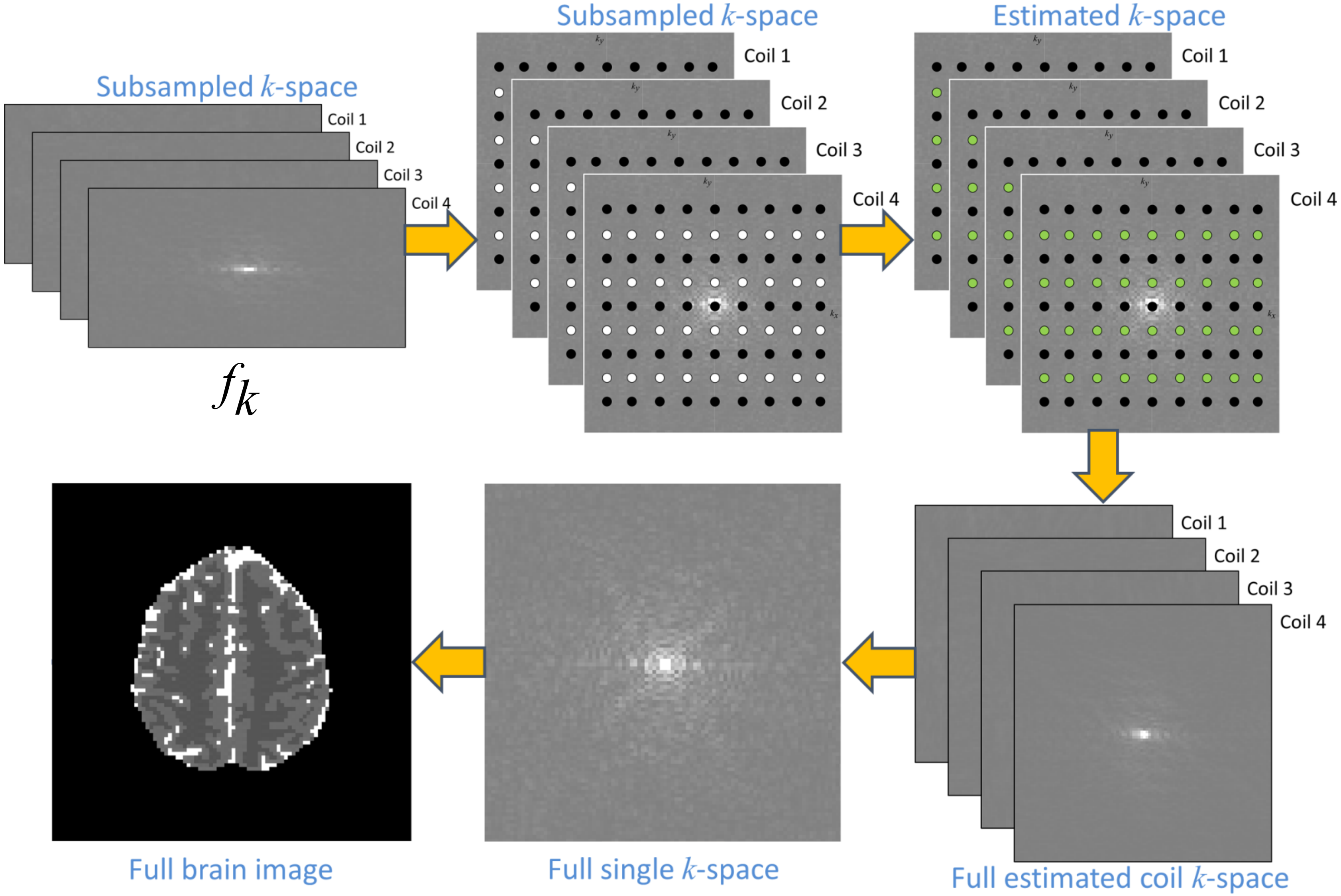
- Functional Magnetic Resonance Imaging (fMRI) is a noninvasive medical imaging technique that observes the brain in action
- Machine uses receiver coils to capture complex-valued arrays of spatial frequencies called k-space
    - Takes a considerable amount of time to fully sample k-space arrays
    - Can diminish effectively capturing brain activity
- Solution: Measure less data
    - Subsample spatial frequencies by skipping lines in the acquisition process



1. Introduction

● Acquired ○ Unacquired ● Estimated

GRAPPA - Introduction



$$f_e = W_C f_k + \eta$$

$$\eta \sim N(0, \tau^2 I_{2n_C})$$

f_k = Acquired spatial frequencies

W_C = Estimated localized weights

- Estimated using full FOV coil k-space arrays

f_e = Unacquired spatial frequencies

- White dots in the top middle of figure

**Interested in interpolating these points (green dots in top right of figure)

1. Introduction

BGRAPPA - Model

- ● Acquired
- ○ Unacquired
- ● Calibration

<u>Subsampled k-space array</u>

GRAPPA

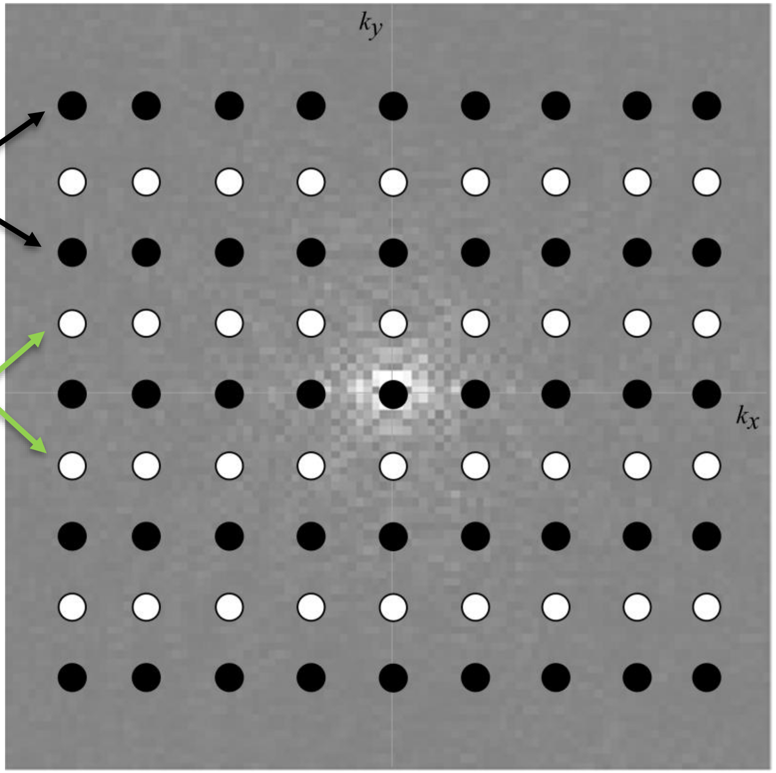
$$f_e = W_C f_k + \eta$$

f_k: acquired

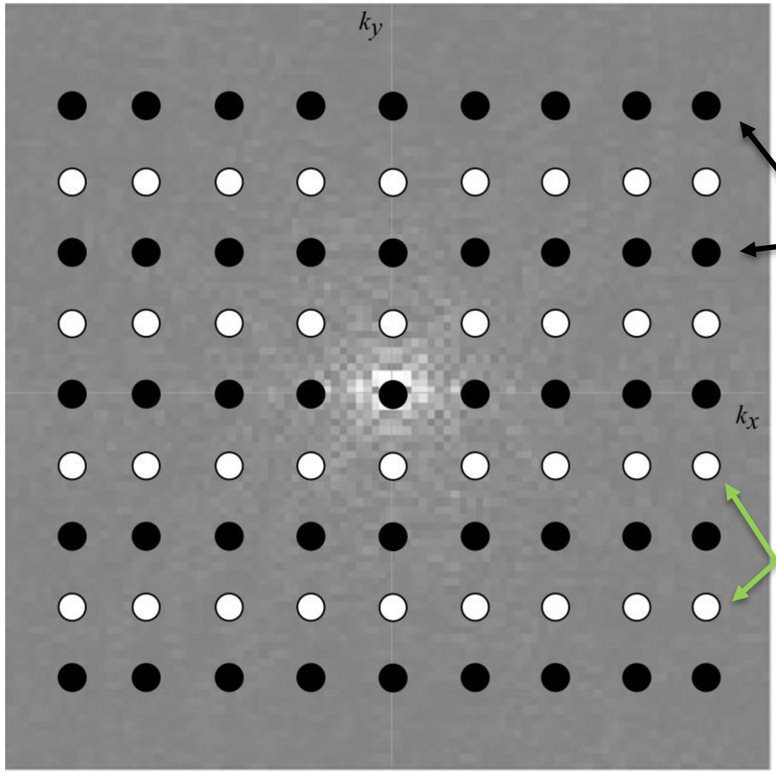
W_C: estimated

f_e : unacquired

**What we are interested in



BGRAPPA



$$f_e = W f_k + \eta$$

$$\eta \sim N(0, \tau^2 I_{2n_C})$$

f_e: acquired

W: unknown (prior)

τ^2: unknown (prior)

f_k: unknown (prior)

**What we are interested in

1. Introduction

$n_C = number\ of\ coils$
$n_A = acceleration\ factor$

BGRAPPA Likelihood and Prior Distributions

- Subsampled k-space measurements are observed with random error
  - $f_e = Wf_k + \eta$, where $\eta \sim N(0, \tau^2 I_{2n_C})$
- Data Likelihood
  - $P(f_e \mid W, f_k, \tau^2) \propto (\tau^2)^{-\frac{2n_C}{2}} \exp\left[-\frac{1}{2\tau^2}(f_e - Wf_k)'(f_e - Wf_k)\right]$ ← normal
- Priors
  - $P(f_k \mid n_k, f_{k0}, \tau^2) \propto (\tau^2)^{-\frac{2p}{2}} \exp\left[-\frac{n_k}{2\tau^2}(f_k - f_{k0})'(f_k - f_{k0})\right]$ ← normal
  - $P(D \mid n_w, D_0, \tau^2) \propto (\tau^2)^{-\frac{2n_C p}{2}} \exp\left[-\frac{n_w}{2\tau^2} tr[(D - D_0)'(D - D_0)]\right]$ ← normal
  - $P(\tau^2 \mid \alpha_k, \delta) \propto (\tau^2)^{-(\alpha_k+1)} \exp\left[-\frac{\delta}{\tau^2}\right]$ ← inverse gamma
- Assessed Hyperparameters: n_w, D_0, n_k, f_{k0}, α_k, and δ
- Posterior
  - $P(D, f_k, \tau^2 \mid f_e) \propto P(f_e \mid W, f_k, \tau^2)\, P(f_k \mid n_k, f_{k0}, \tau^2)\, P(D \mid n_w, D_0, \tau^2)\, P(\tau^2 \mid \alpha_k, \delta)$

$$\underset{2n_C\times 1}{f_e} = \begin{bmatrix} f_{eR} \\ f_{eI} \end{bmatrix}$$

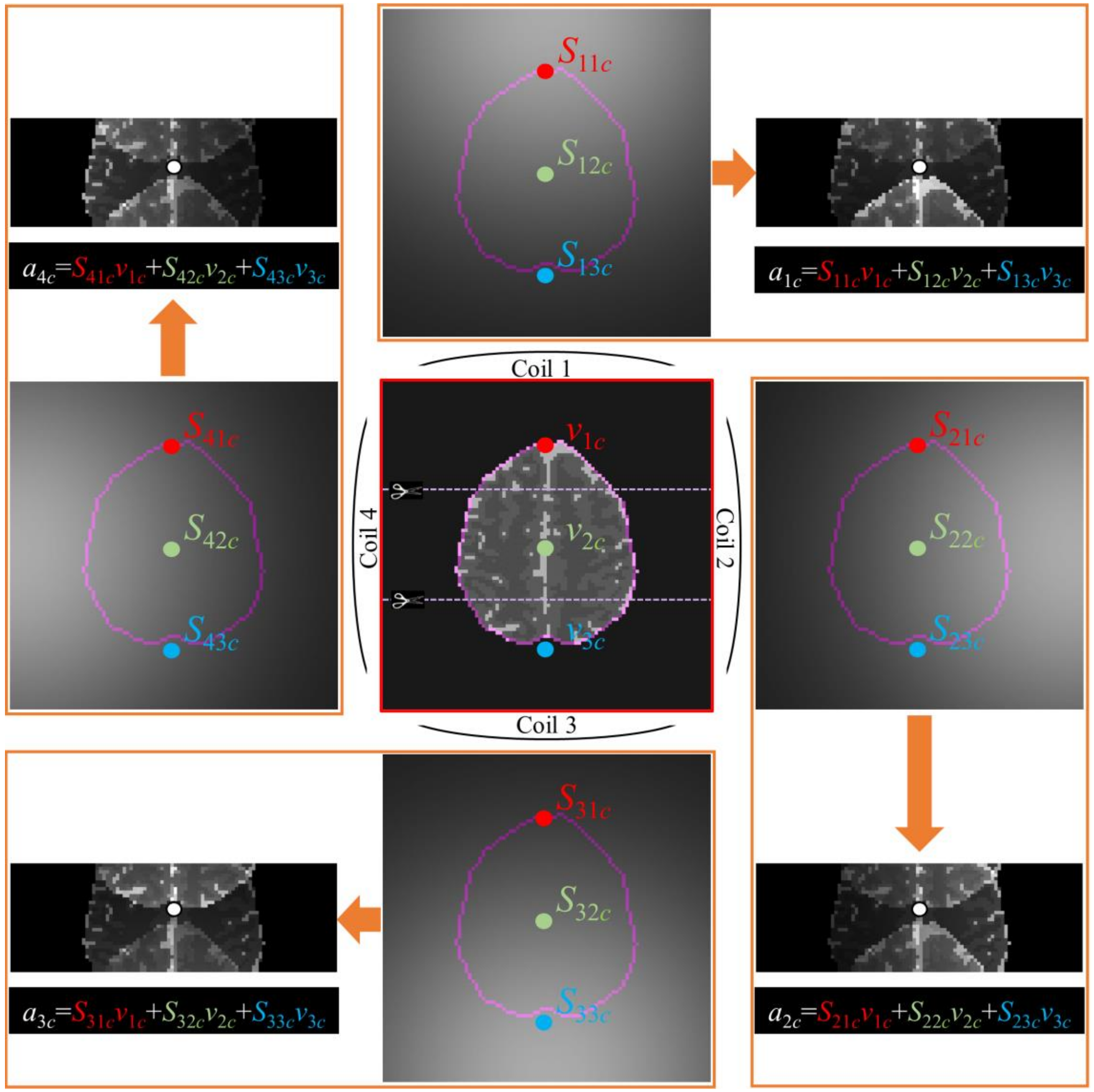
$$\underset{2n_C\times 2p}{W} = \begin{bmatrix} W_R & -W_I \\ W_I & W_R \end{bmatrix}$$

$$\underset{n_C\times 2p}{D} = \begin{bmatrix} W_R & W_I \end{bmatrix}$$

$$\underset{2p\times 1}{f_k} = \begin{bmatrix} f_{kR} \\ f_{kI} \end{bmatrix}$$

1. Introduction

SENSE - Introduction



**All Parameters are complex-valued

$n_C = number\ of\ coils(4)$
$n_A = acceleration\ factor(3)$

$$a = Sv \longrightarrow$$

Observed: a (aliased)
Unobserved: v, S

$$\begin{bmatrix} a_{1c} \\ a_{2c} \\ a_{3c} \\ a_{4c} \end{bmatrix} = \begin{bmatrix} S_{11c} & S_{12c} & S_{13c} \\ S_{21c} & S_{22c} & S_{23c} \\ S_{31c} & S_{32c} & S_{33c} \\ S_{41c} & S_{42c} & S_{43c} \end{bmatrix} \begin{bmatrix} v_{1c} \\ v_{2c} \\ v_{3c} \end{bmatrix}$$

$\hat{S}$ is estimated from calibration images

$$\hat{v} = \left(\hat{S}'\hat{S}\right)^{-1} \hat{S}'a$$

**$\hat{S}'\hat{S}$ is not generally positive definite

1. Introduction

$n_C = number\ of\ coils$
$n_A = acceleration\ factor$

BSENSE Model, Likelihood, and Prior Distributions

- Aliased voxel measurements are observed with random error
  - $a = Sv + \varepsilon$, where $\varepsilon \sim N(0, \sigma^2 I_{2n_C})$
- Data Likelihood
  - $P(a \mid S, v, \sigma^2) \propto (\sigma^2)^{-\frac{2n_C}{2}} \exp\left[-\frac{1}{2\sigma^2}(a - Sv)'(a - Sv)\right]$ ← normal
- Priors
  - $P(v \mid n_v, v_0, \sigma^2) \propto (\sigma^2)^{-\frac{2n_A}{2}} \exp\left[-\frac{n_v}{2\sigma^2}(v - v_0)'(v - v_0)\right]$ ← normal
  - $P(H \mid n_S, H_0, \sigma^2) \propto (\sigma^2)^{-\frac{2n_C n_A}{2}} \exp\left[-\frac{n_S}{2\sigma^2} tr[(H - H_0)'(H - H_0)]\right]$ ← normal
  - $P(\sigma^2 \mid \alpha, \beta) \propto (\sigma^2)^{-(\alpha+1)} \exp\left[-\frac{\beta}{\sigma^2}\right]$ ← inverse gamma
- Assessed Hyperparameters: n_S, H_0, n_v,v_0, α, and β
- Posterior
  - $P(H, v, \sigma^2 \mid a) \propto P(a \mid S, v, \sigma^2) P(H \mid n_S, H_0, \sigma^2) P(v \mid n_v, v_0, \sigma^2) P(\sigma^2 \mid \alpha, \beta)$

$$\underset{2n_C \times 1}{a} = \begin{bmatrix} a_R \\ a_I \end{bmatrix}$$

$$\underset{2n_c \times 2n_A}{S} = \begin{bmatrix} S_R & -S_I \\ S_I & S_R \end{bmatrix}$$

$$\underset{n_c \times 2n_A}{H} = \begin{bmatrix} S_R & S_I \end{bmatrix}$$

$$\underset{2n_A \times 1}{v} = \begin{bmatrix} v_R \\ v_I \end{bmatrix}$$

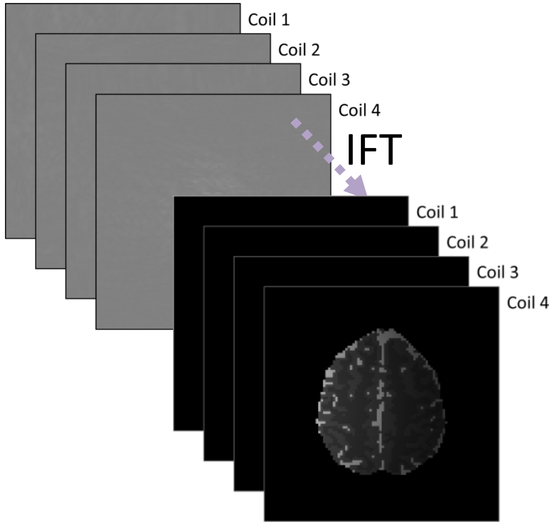
2. Bayesian MUGS

Reconstruction Process

Merged Utilization of GRAPPA and SENSE (MUGS)

Numbering Color Legend
Red = Pre-Scan/Calibration
Green = GRAPPA
Light Blue = SENSE

1.

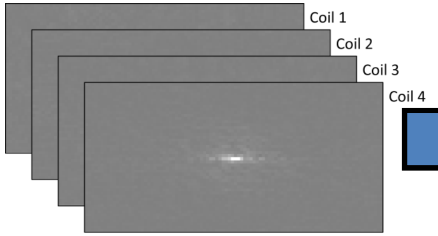


2.

Assess the weights for GRAPPA **AND** the coil sensitivities for SENSE

3.

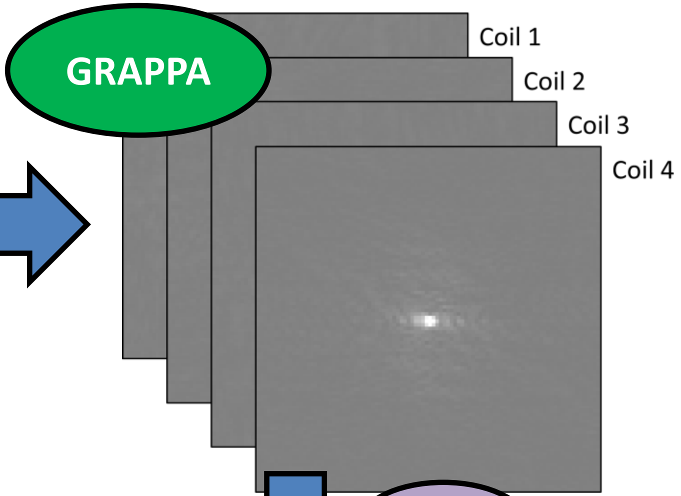
Obtain subsampled coil k-space arrays



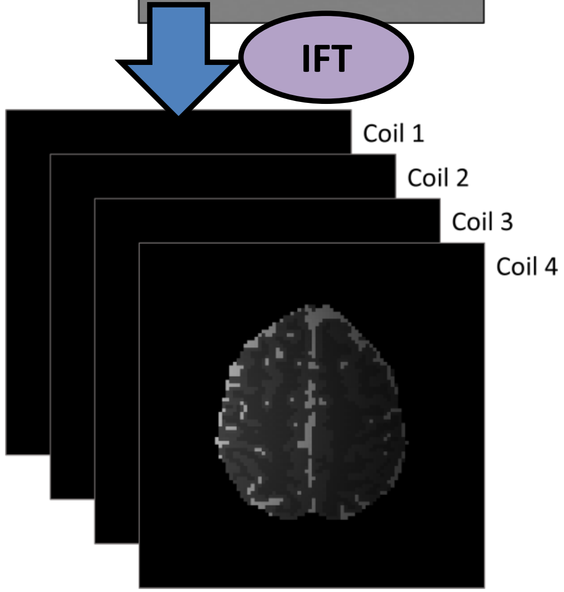
Then, initialize values for localized weights

**From Calibration Information (step 2)*

4.



5.



6.

Initialize values for coil sensitivities

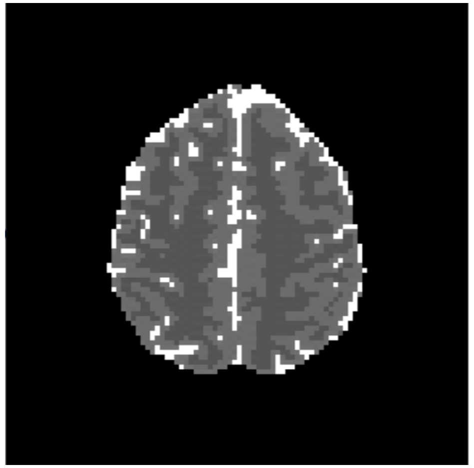
**From Calibration Information (step 2)*

7.

SENSE

Will unfold and combine the coil images from step 5

8.



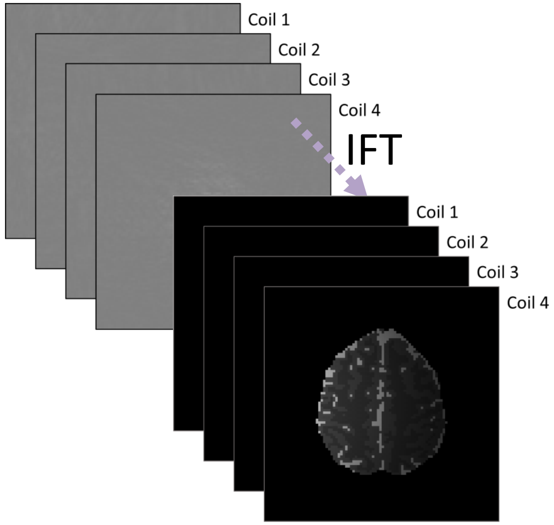
2. Bayesian MUGS

Bayesian approach to MUGS

Numbering Color Legend
Red = Pre-Scan/Calibration
Green = BGRAPPA
Light Blue = BSENSE

Bayesian Reconstruction Process

1.

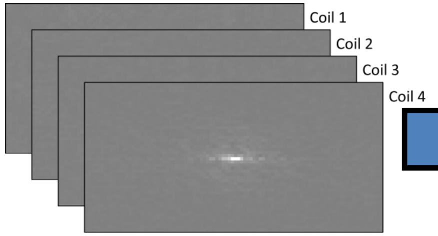


2.

Assess the hyperparameters for BGRAPPA using the full coil k-space arrays. Assess the hyperparameters for BSENSE using full coil images

3.

Obtain subsampled coil k-space arrays

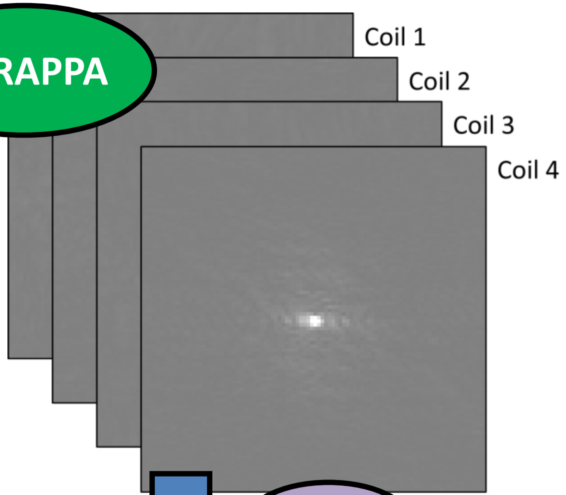


Then, initialize values for missing k-space values and weights

**From Calibration Information (step 2)*

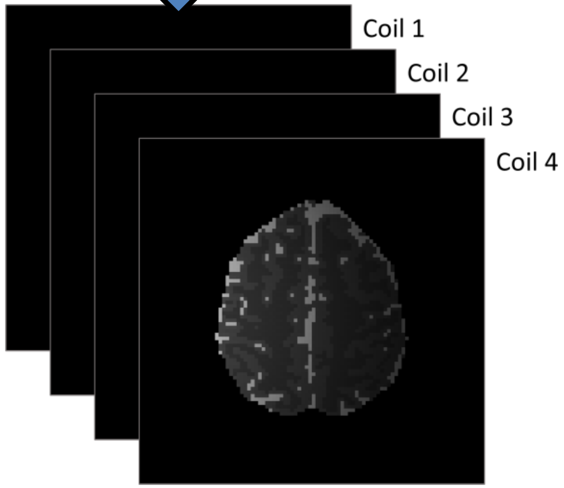
4.

BGRAPPA



IFT

5.



6.

Initialize values for unaliased voxel values and coil sensitivities

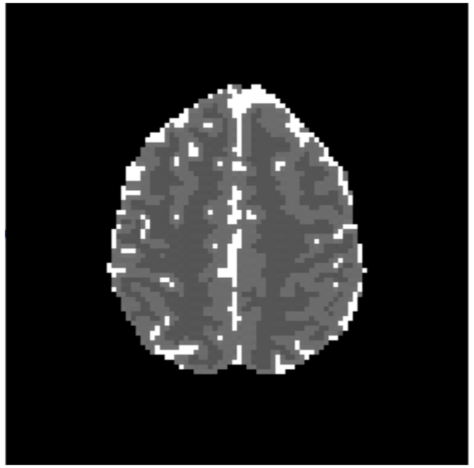
**From Calibration Information (step 2)*

7.

BSENSE

Will unfold and combine the coil images from step 5

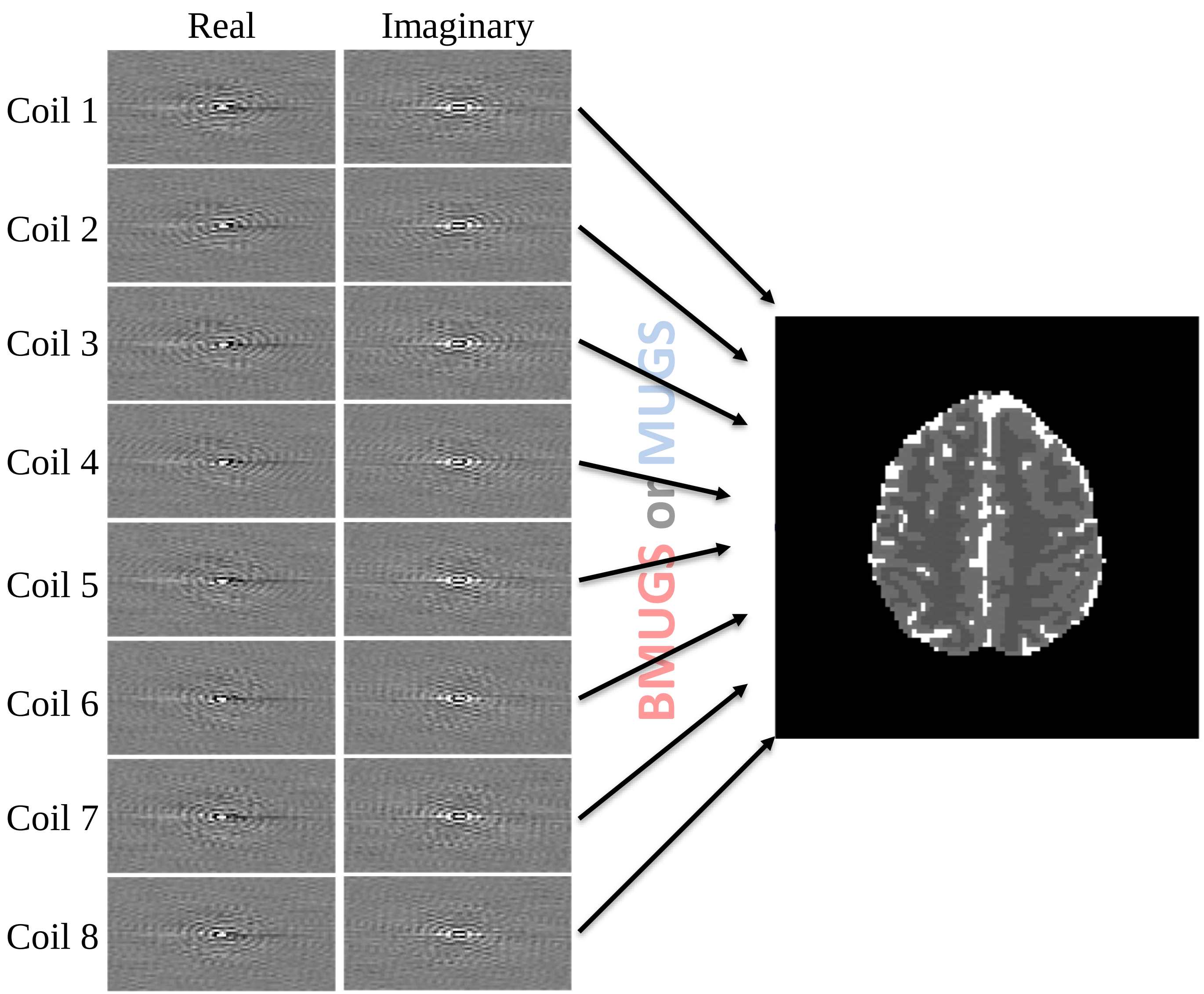
8.



3. Simulated/Experimental Results

Simulated Data

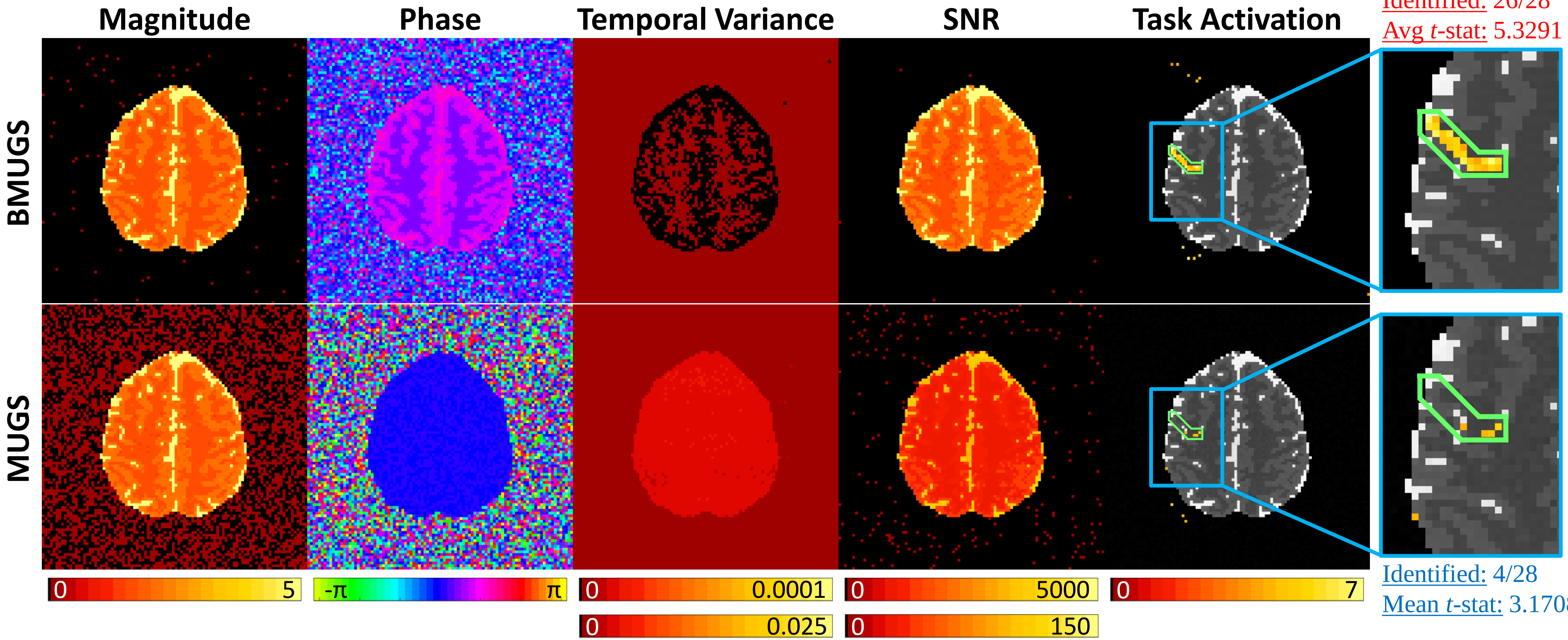
- 490 time points in the simulated fMRI time series
    - Started with 510 time points discarding first 20 to mimic experimental fMRI
- 30 calibration time points utilized for hyperparameter assessment
    - Calibration time points from a separate simulated series
- Number of coils used is 8 with an acceleration factor of 3
- 2x1 kernel size used for GRAPPA/BGRAPPA components hyperparameter assessment and parameter estimation
- Reconstruction Method: MAP estimate via ICM for BMUGS



3. Simulated/Experimental Results

Simulated Data Results

$n_C = number\ of\ coils (8)$
$n_A = acceleration\ factor (3)$



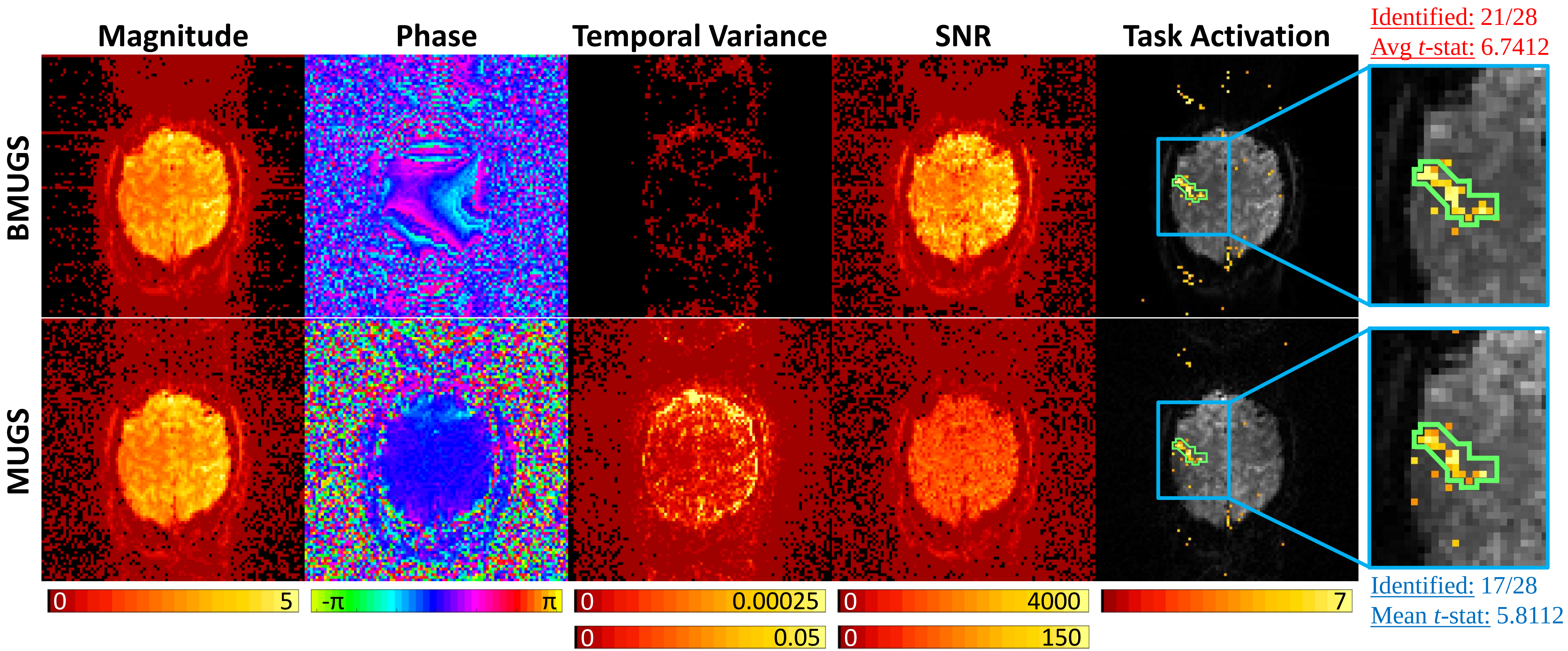
Identified: 26/28
Avg t-stat: 5.3291

Identified: 4/28
Mean t-stat: 3.1708

3. Simulated/Experimental Results

Experimental Data Results

$n_C = number\ of\ coils (8)$
$n_A = acceleration\ factor (3)$



4. Discussion

Conclusion and Future Work

- Acquiring all data points takes a considerable amount of time
  - Subsampling k-space reduces acquisition time but causes aliasing
- SENSE and GRAPPA reconstruct the subsampled data into full FOV brain images
- Here, we discuss the Merged Utilization of GRAPPA and SENSE (MUGS) technique
  - Operates in both the spatial frequency domain and the image domain
- BMUGS incorporates more valuable prior information in estimating the missing spatial frequency values and the unaliased voxels
  - BMUGS reconstructed images more accurately, decreased temporal variation, increased SNR, and improved task detection power
- Future work:
  - Analyze correlation between previously aliased voxels and all other voxels
  - Potential bootstrapping of the calibration images

Thank You

Questions?