Processing Induced Correlation in FMRI Data

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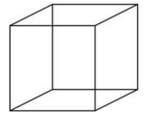
Outline

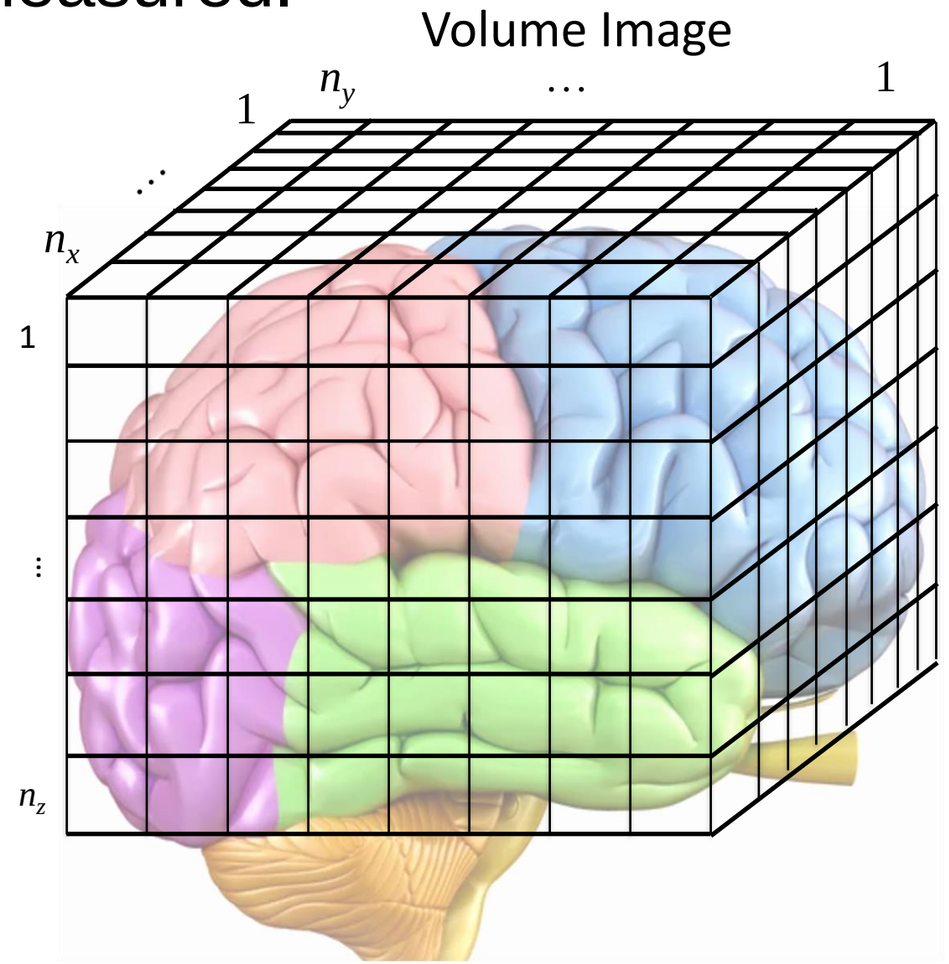
1. Introduction to FMRI
2. Temporal Processing Correlation
3. Spatial Processing Correlation
4. Reconstruction & Spatial Processing Correlation
5. Spatio-Temporal Processing Correlation
6. Discussion

1. Introduction to FMRI



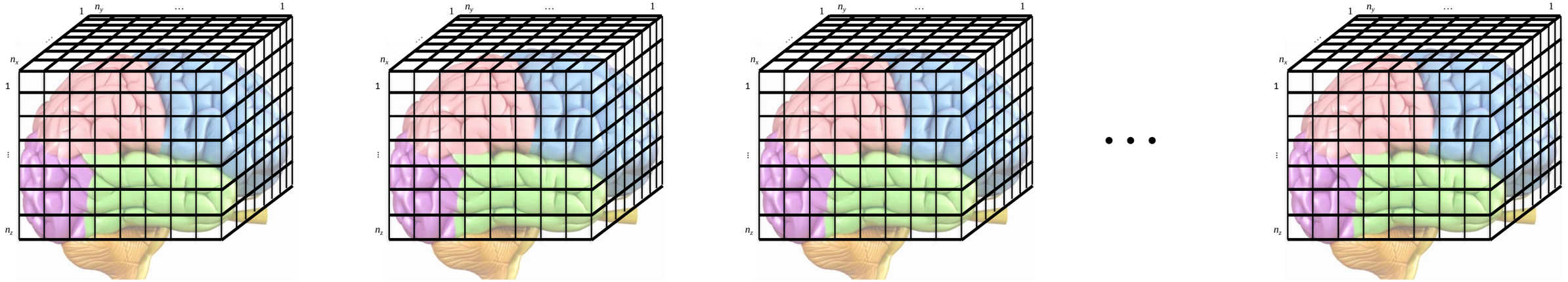
voxel
volume-element

In fMRI, a subject is placed in the MRI machine and slice-wise volume images of their brain are measured.



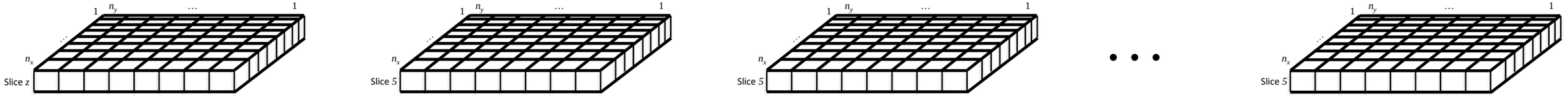
1. Introduction to FMRI

We observe volume images over time, $t=1, \ldots, n$.



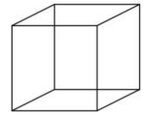
$t=1$ $t=2$ $t=3$ . . . $t=n$

Let's focus on a single slice over time. Slice 5.



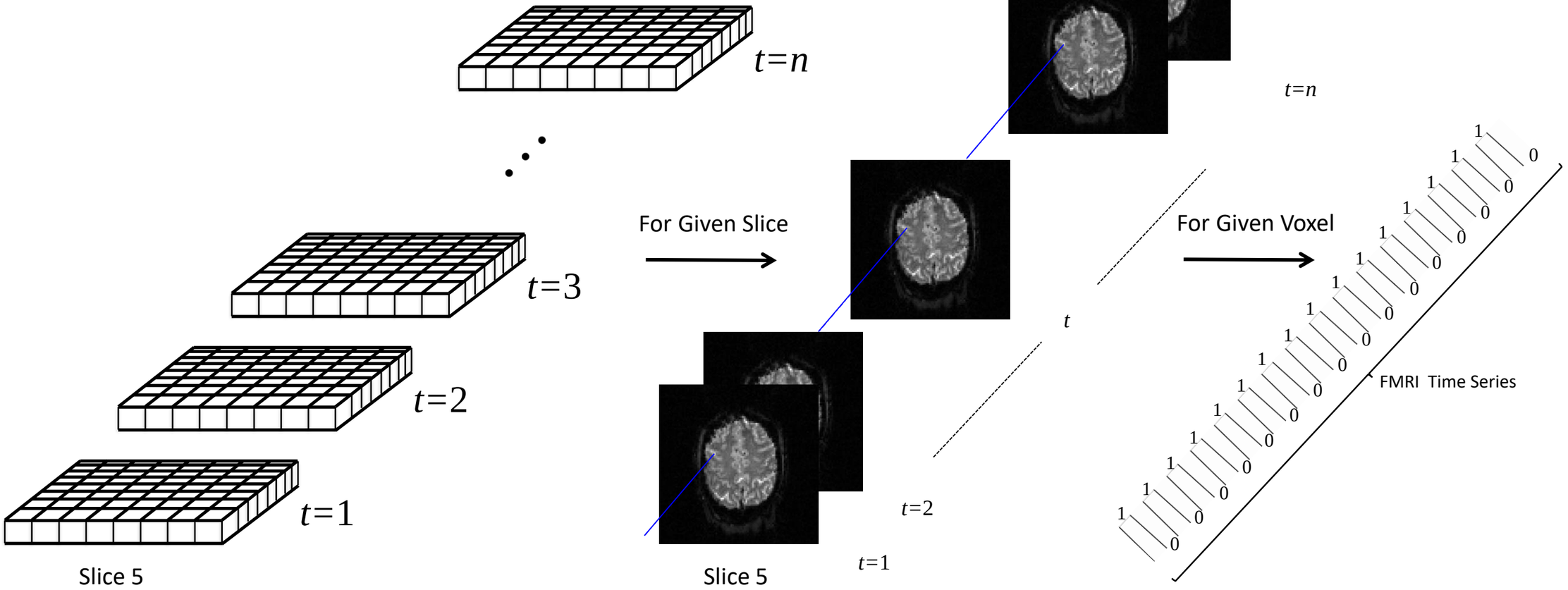
$t=1$ $t=2$ $t=3$ . . . $t=n$

1. Introduction to FMRI



voxel
volume-element

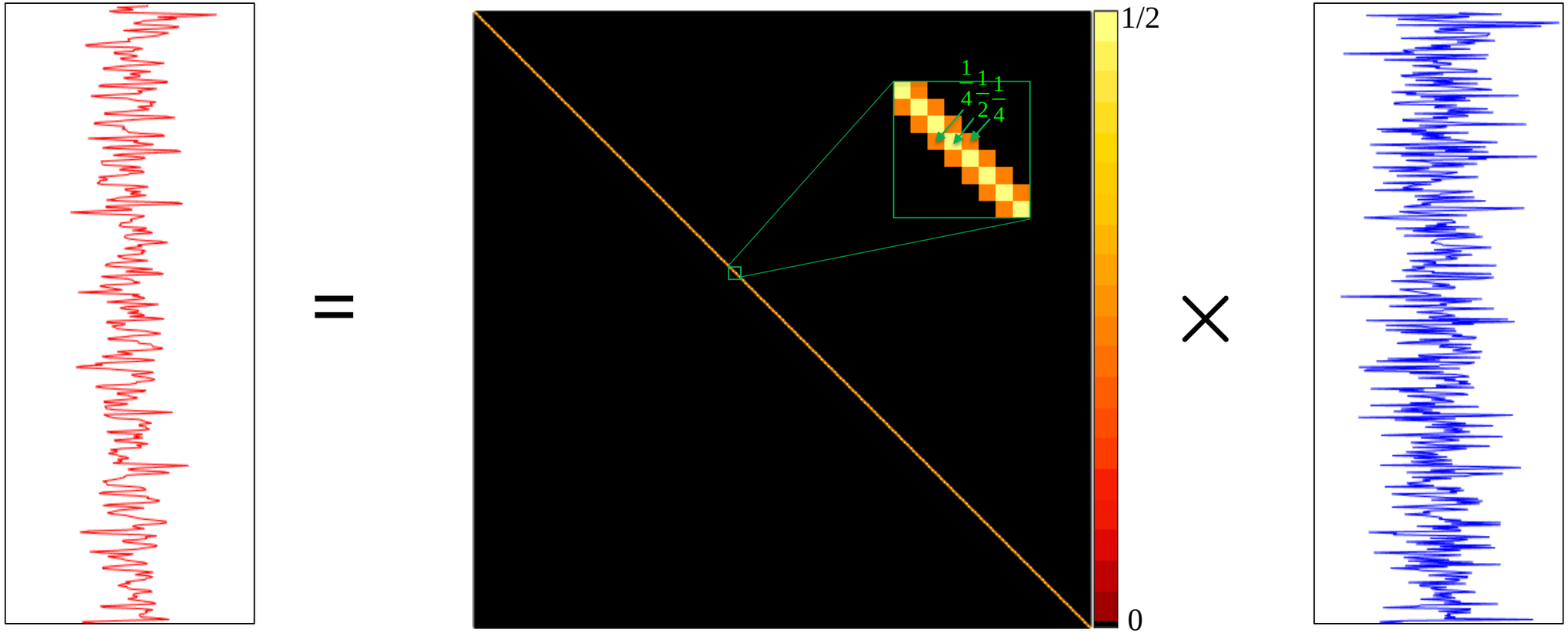
Subjects are generally performing a designed cognitive task through the n time points.



2. Temporal Processing Correlation

1/4	1/2	1/4

Time series processing is a weighting of the original voxel measurements to produce a new time series.



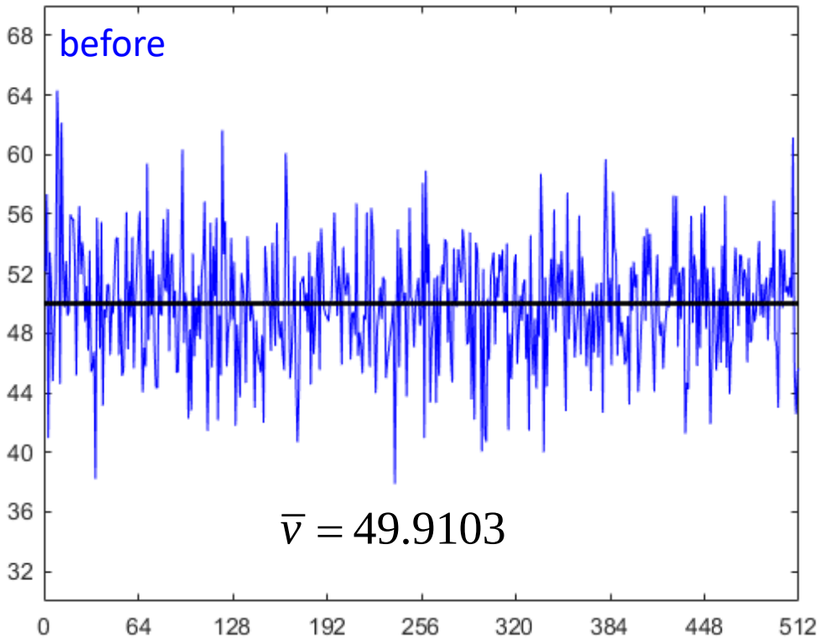
smoothed

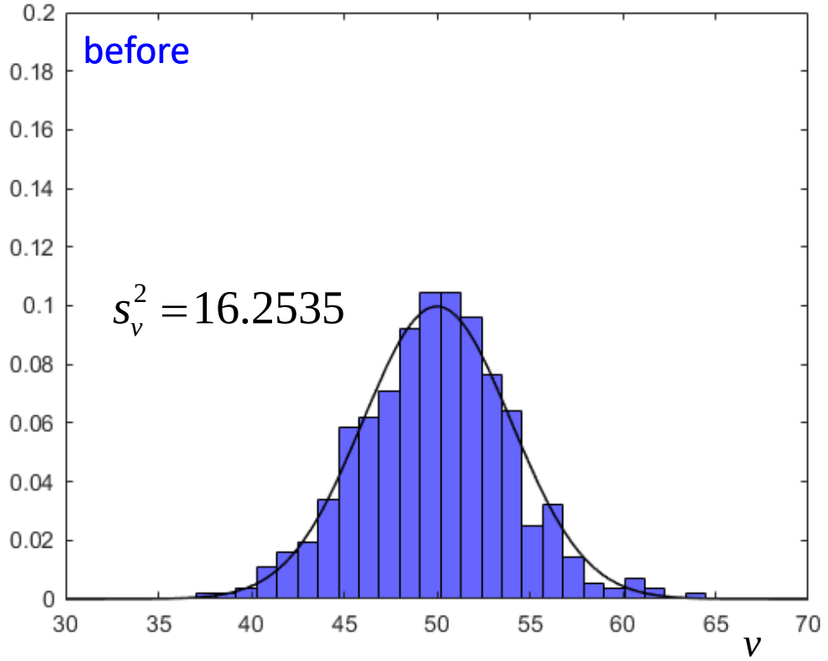
convolution matrix

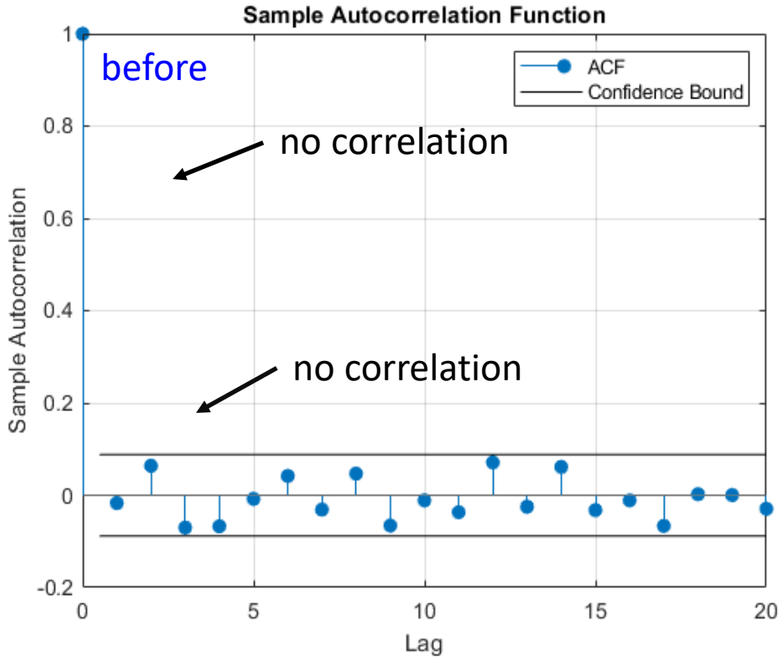
original

2. Temporal Processing Correlation

Imagine a voxel time series of measurements v_t, t=1,…n=512 iid N(50,16).



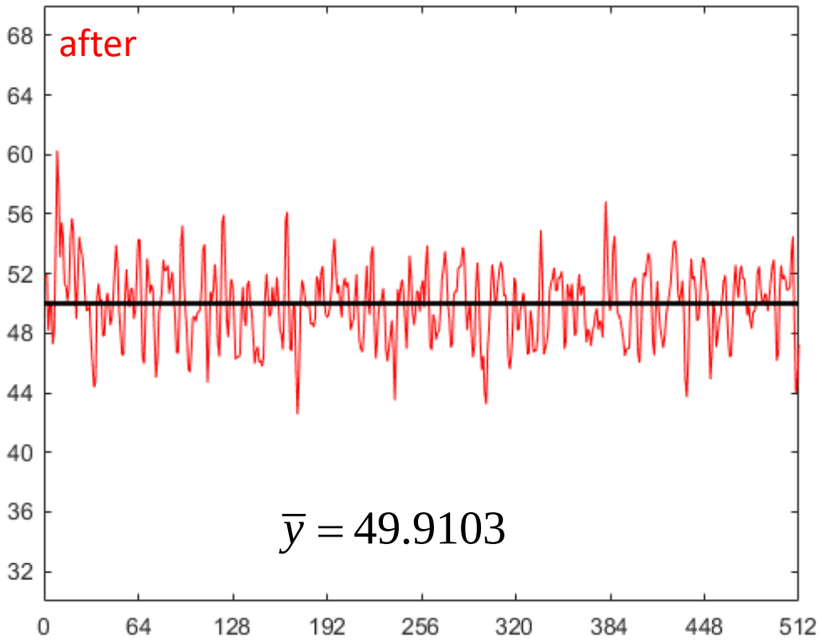


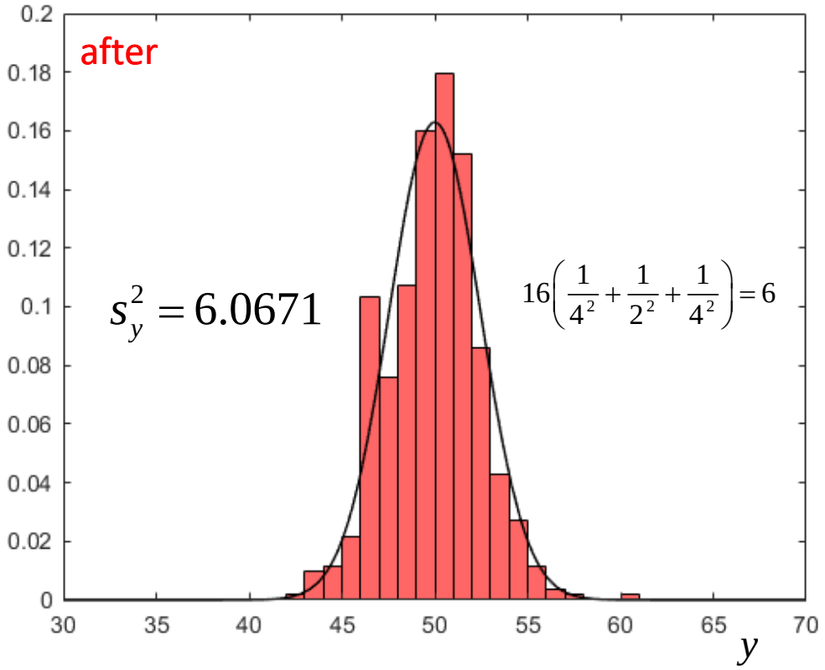


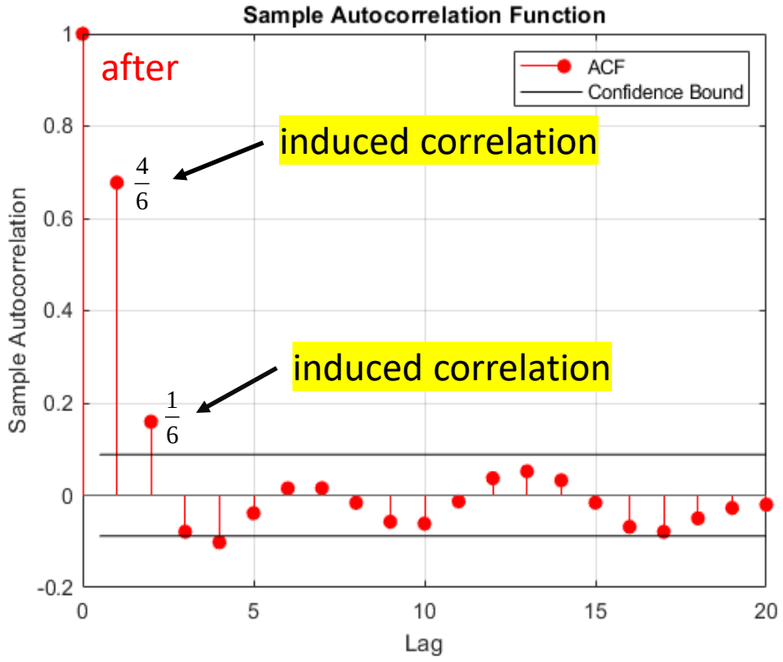
Statistical properties exactly as designed. Initially no temporal correlation.

2. Temporal Processing Correlation

Imagine a voxel time series of measurements v_t, t=1,…n=512 iid $N(50,16)$.
If we smooth the time series with a 3 point kernel | 1/4 | 1/2 | 1/4 |, then …







Same mean, reduced variance, but induced temporal correlation!
Need to account for or remove induced correlation!

2. Temporal Processing Correlation

In statistics, we know that if a vector v has a mean δ, and covariance Γ,

then $y=Ov$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

Then Σ can converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$,

where $D^{-1/2} = 1/\sqrt{diag(\Sigma)}$.

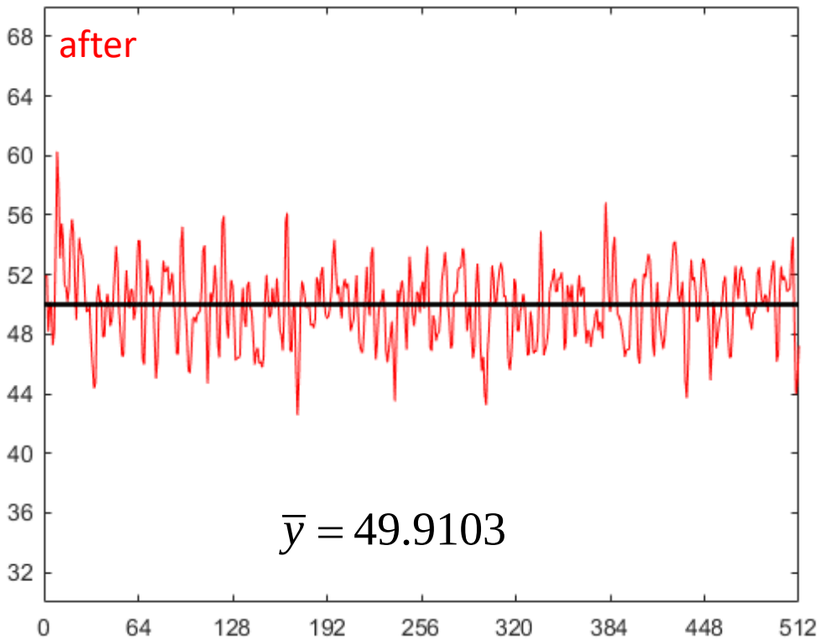
Assume voxel measurements are temporally independent, $\Gamma=I$.

We can calculate the effects of temporal processing.

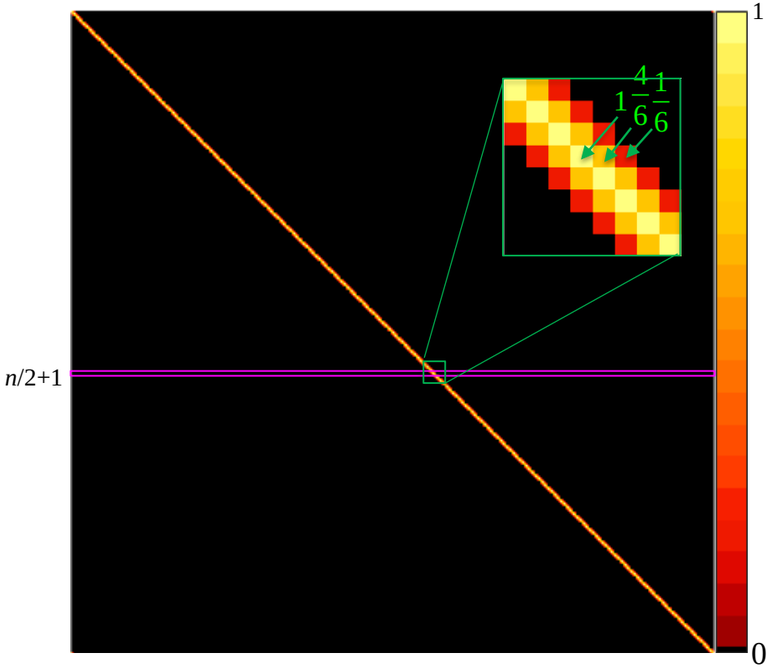
3. Spatial Processing Correlation

1/4	1/2	1/4

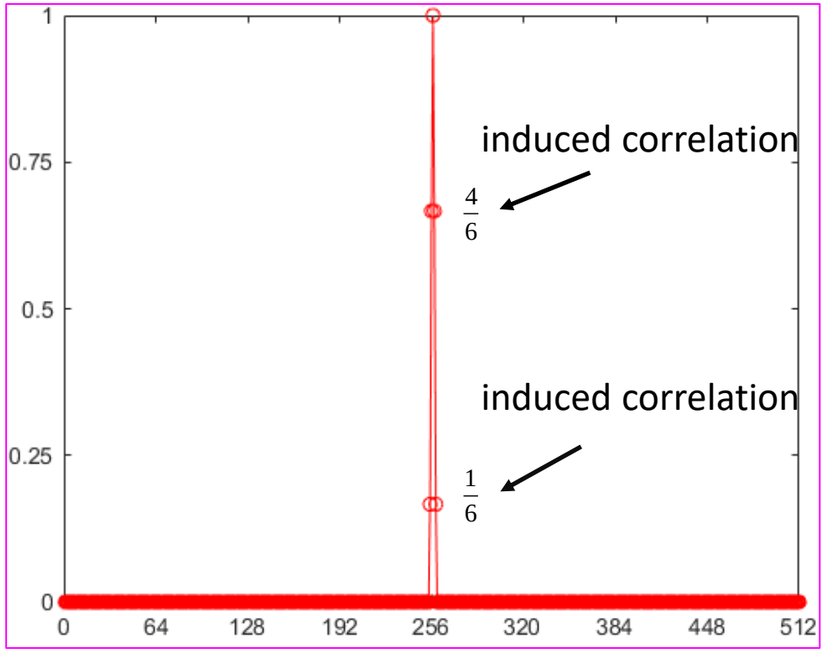
Image processing induces a local spatial correlation between voxels.



smoothed



correlation matrix



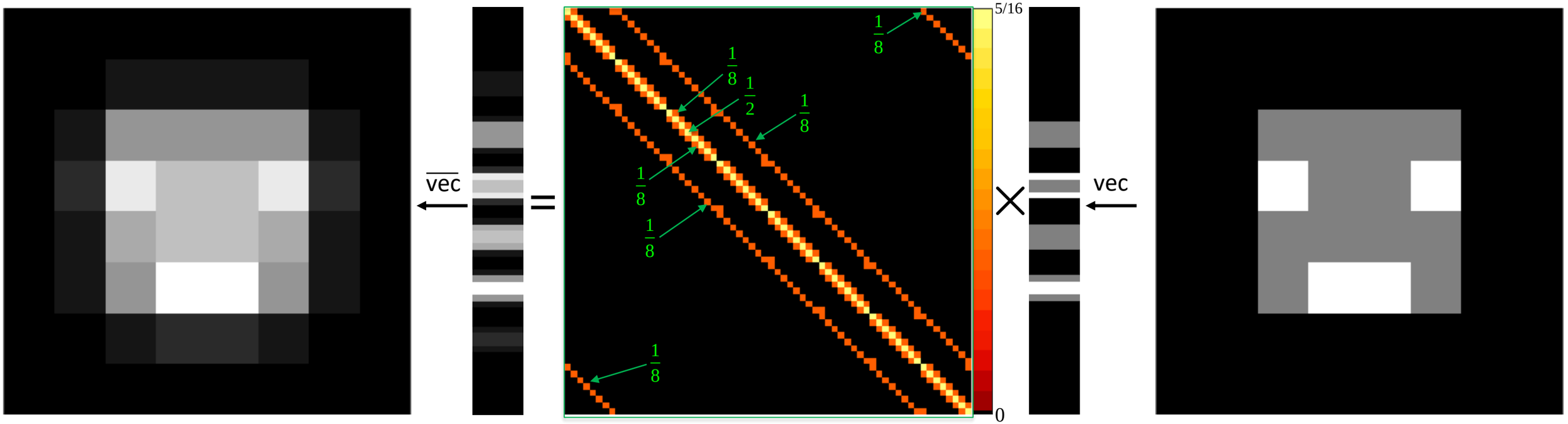
correlation image

covariance $\Sigma = O\Gamma O^T$.

3. Spatial Processing Correlation

0	1/8	0
1/8	1/2	1/8
0	1/8	0

Image processing is a weighting of the original voxel measurements to produce a new series of images.



3. Spatial Processing Correlation

Implications: Induced correlations.

$Y(x,y)$

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

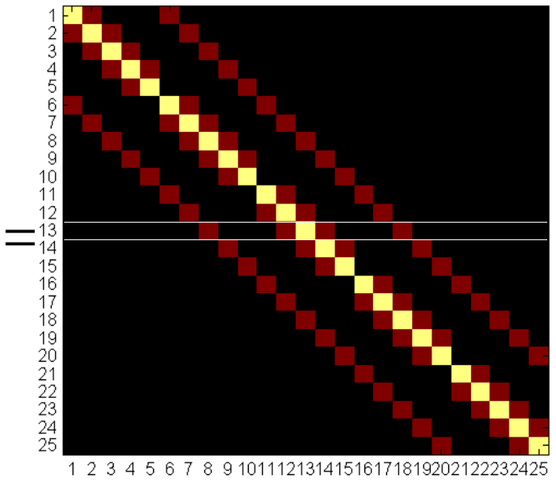
5×5 image

→ $y =$

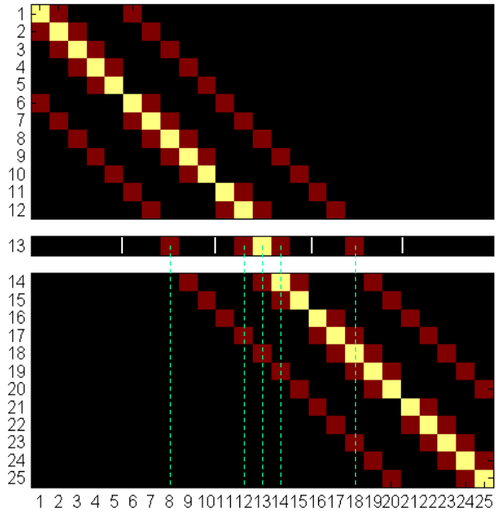
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25

$cor(y) =$

25×25 correlation matrix



→



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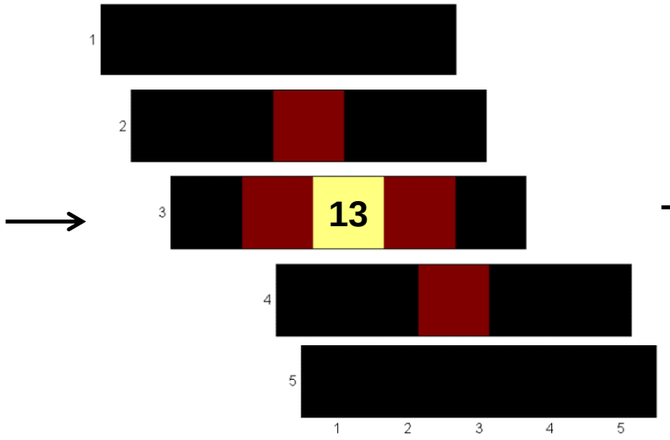

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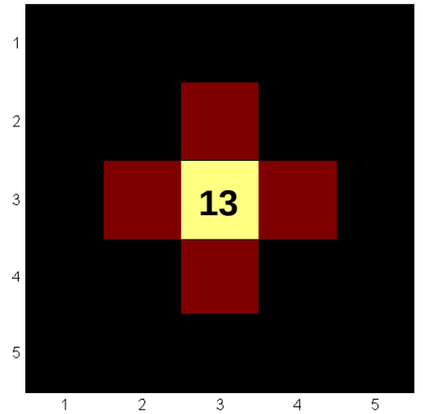
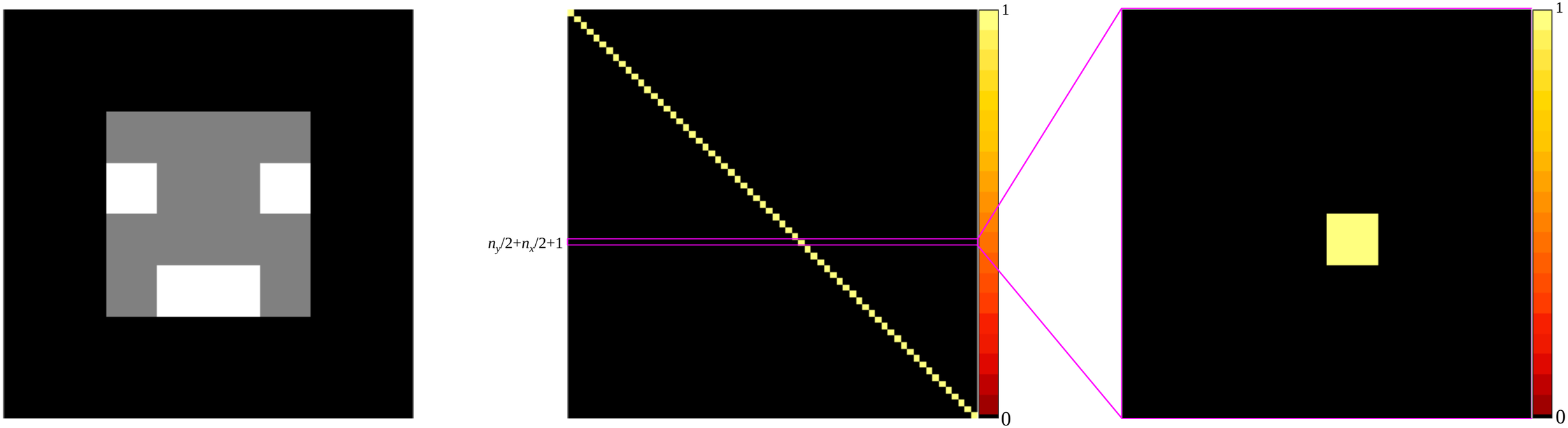


5×5 correlation image

3. Spatial Processing Correlation

Image processing induces a local spatial correlation between voxels. $\Sigma = O\Gamma O^T$



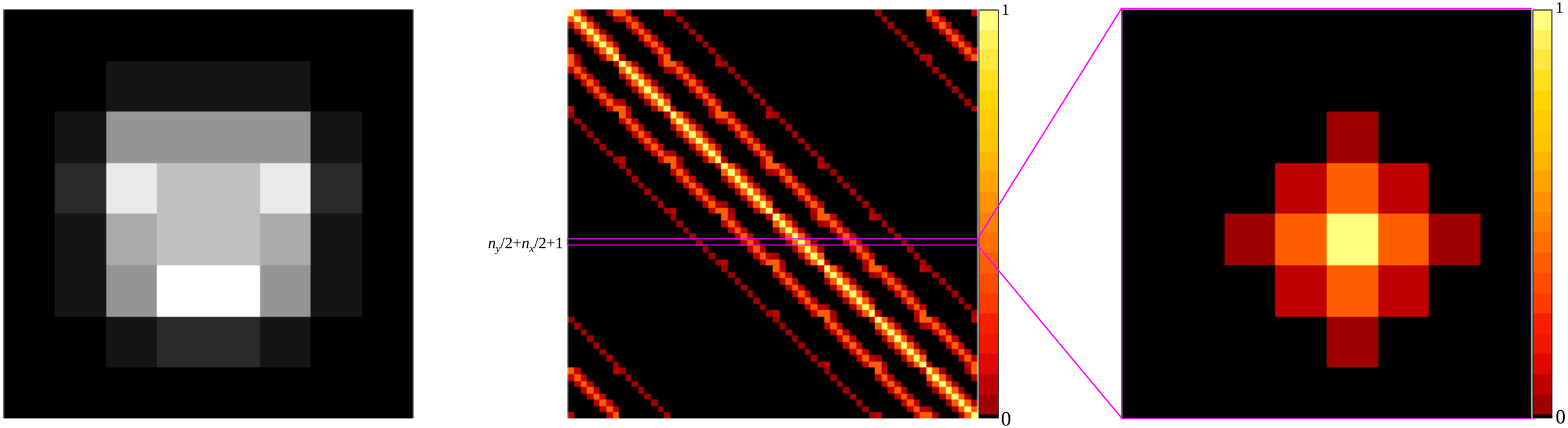
original correlation matrix correlation image

Initially no correlation between a voxel and it's neighbors.

3. Spatial Processing Correlation

0	1/8	0
1/8	1/2	1/8
0	1/8	0

Image processing induces a local spatial correlation between voxels. $\Sigma = O \Gamma O^T$

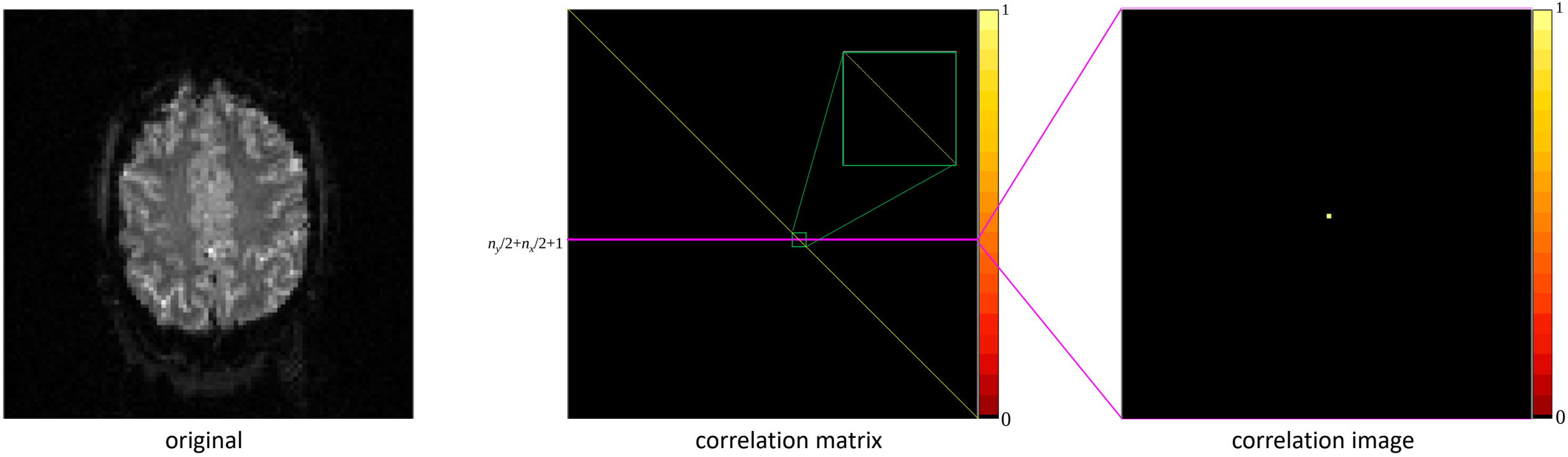


smoothed | correlation matrix | correlation image

We can see the induced correlation between a voxel and it's neighbors.

3. Spatial Processing Correlation

Image processing induces a local spatial correlation between voxels.



original

correlation matrix

correlation image

Initially no correlation between a voxel and it’s neighbors.

3. Spatial Processing Correlation

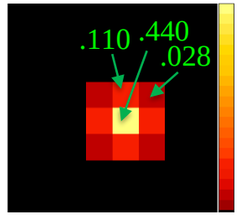
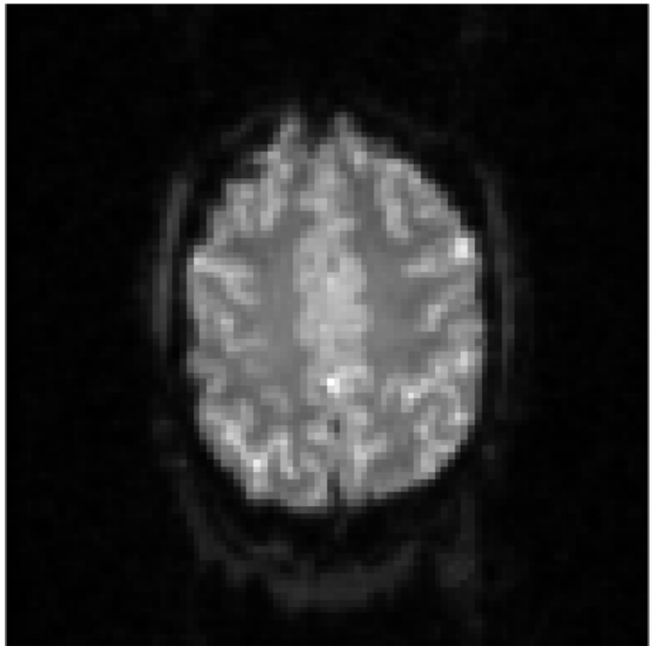
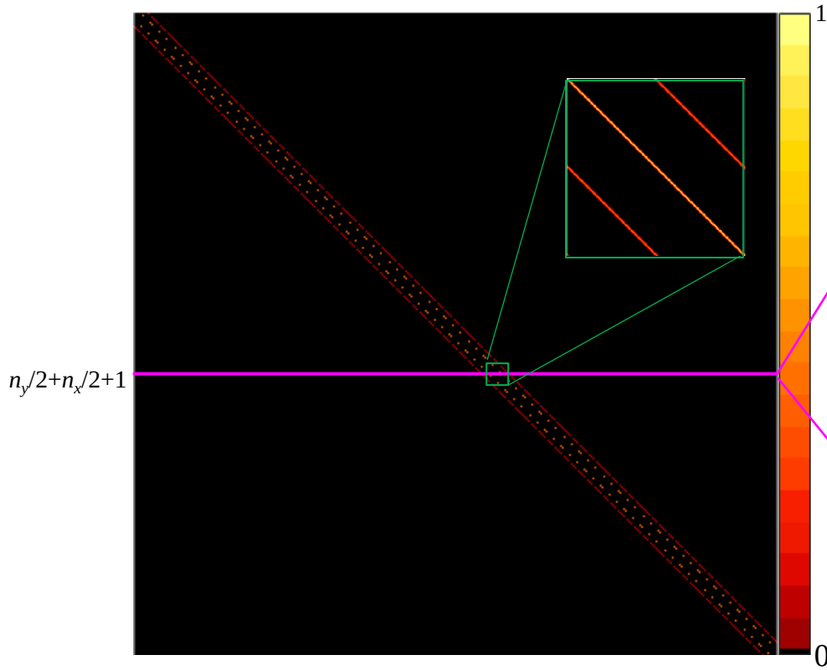


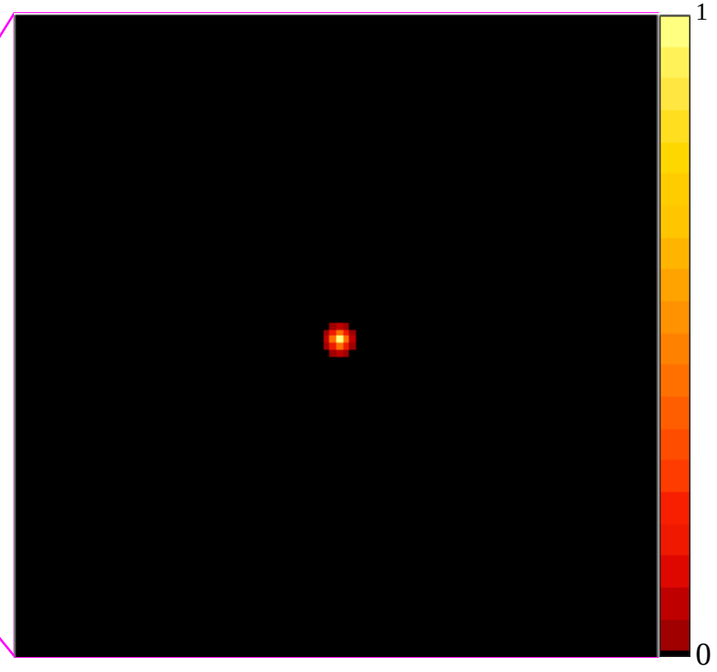
Image processing induces a local spatial correlation between voxels. $\Sigma = O\Gamma O^T$



smoothed



correlation matrix



correlation image

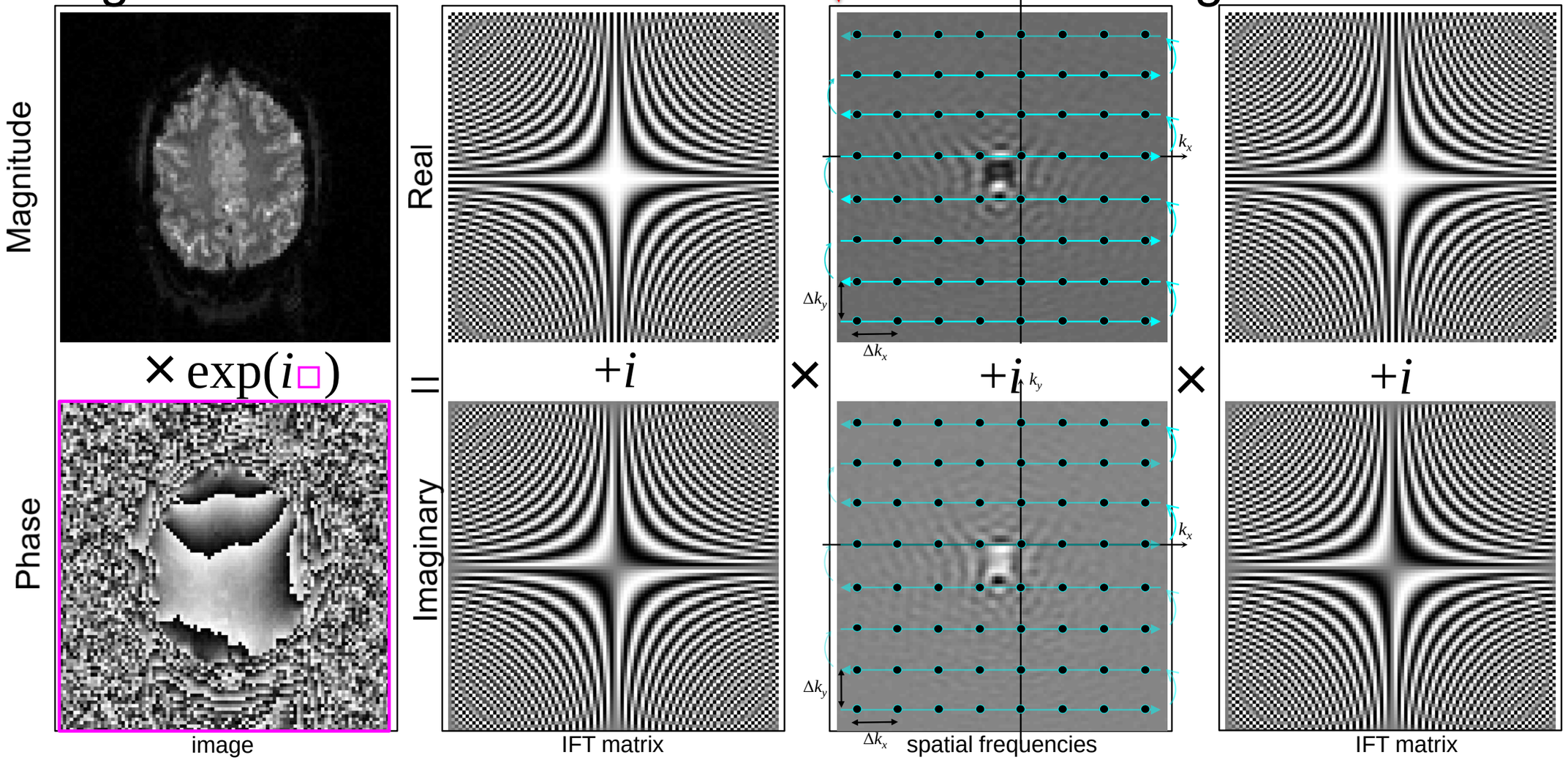
We can see the induced correlation between a voxel and it's neighbors.

4. Reconstruction & Spatial Processing Correlation

T_2* weighted images

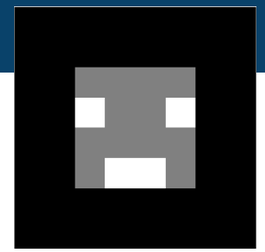
$S(k_x, k_y)$

Images aren't measured but are reconstructed using the IDFT.

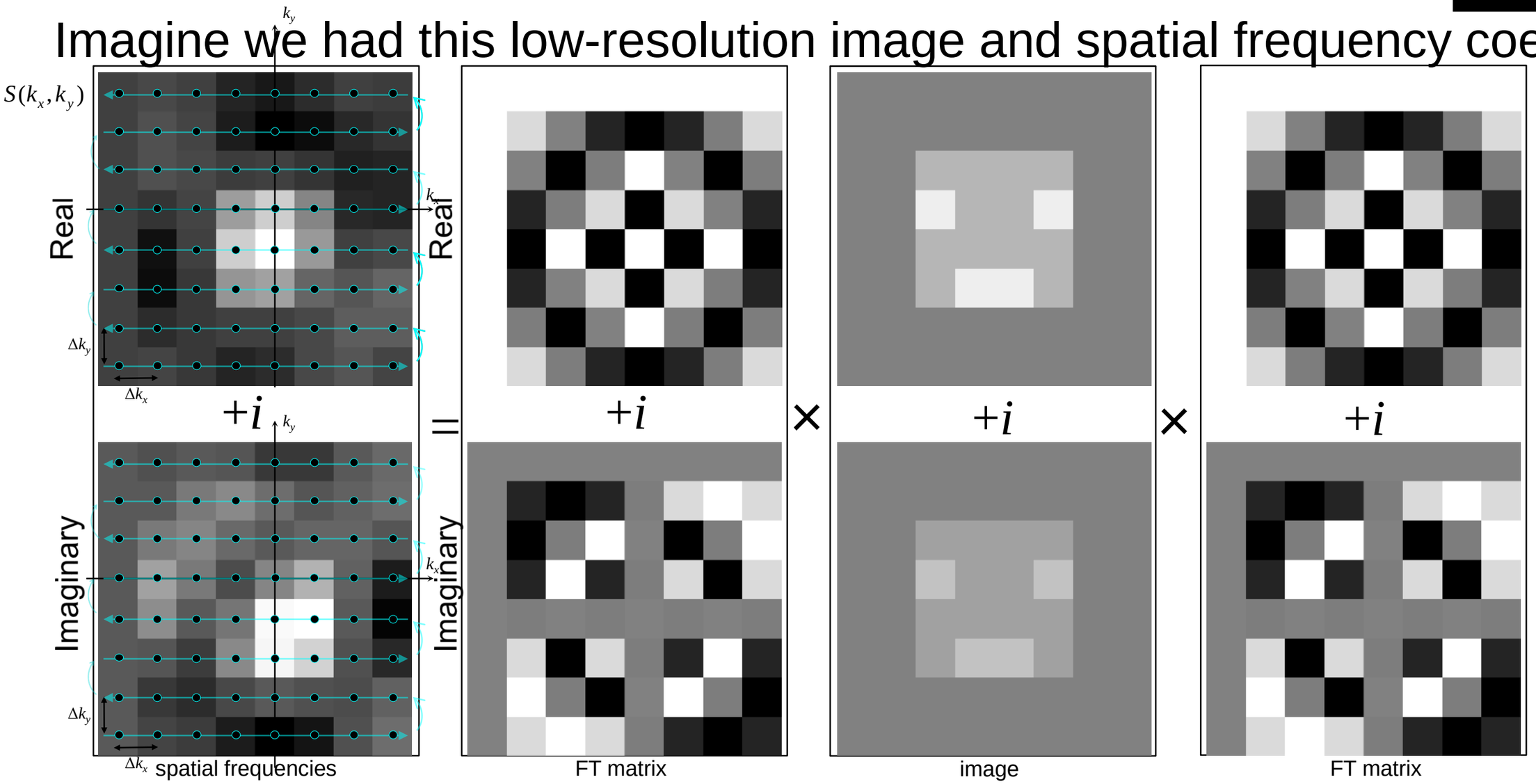


$n_x = n_y = 96$
$\Delta x = \Delta y = 2.5$ mm
FOV=240 mm

4. Reconstruction & Spatial Processing Correlation



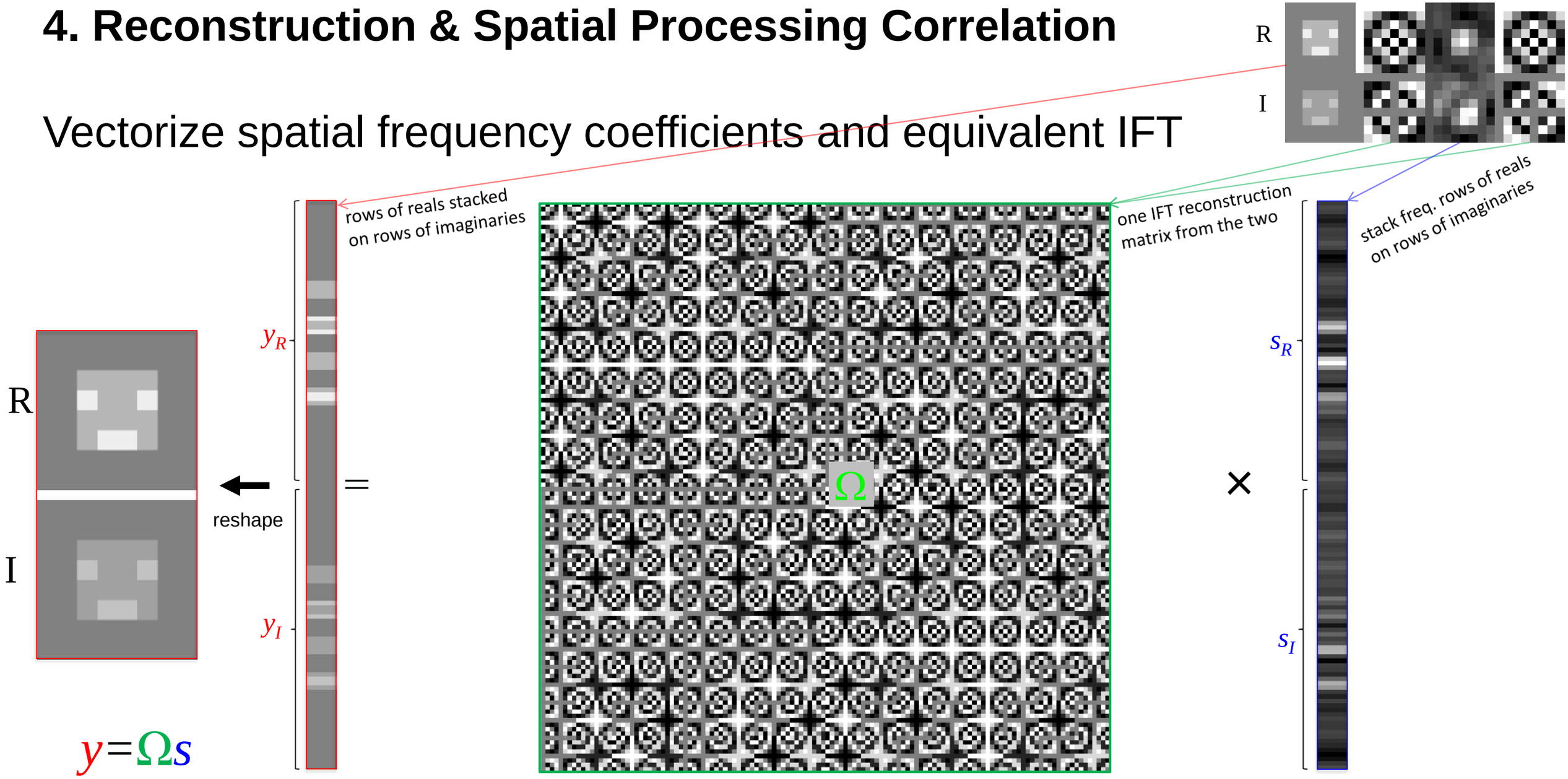
Imagine we had this low-resolution image and spatial frequency coefficients.



$n_x = n_y = 8$
$\Delta x = \Delta y = 30$ mm
FOV=240 mm

4. Reconstruction & Spatial Processing Correlation

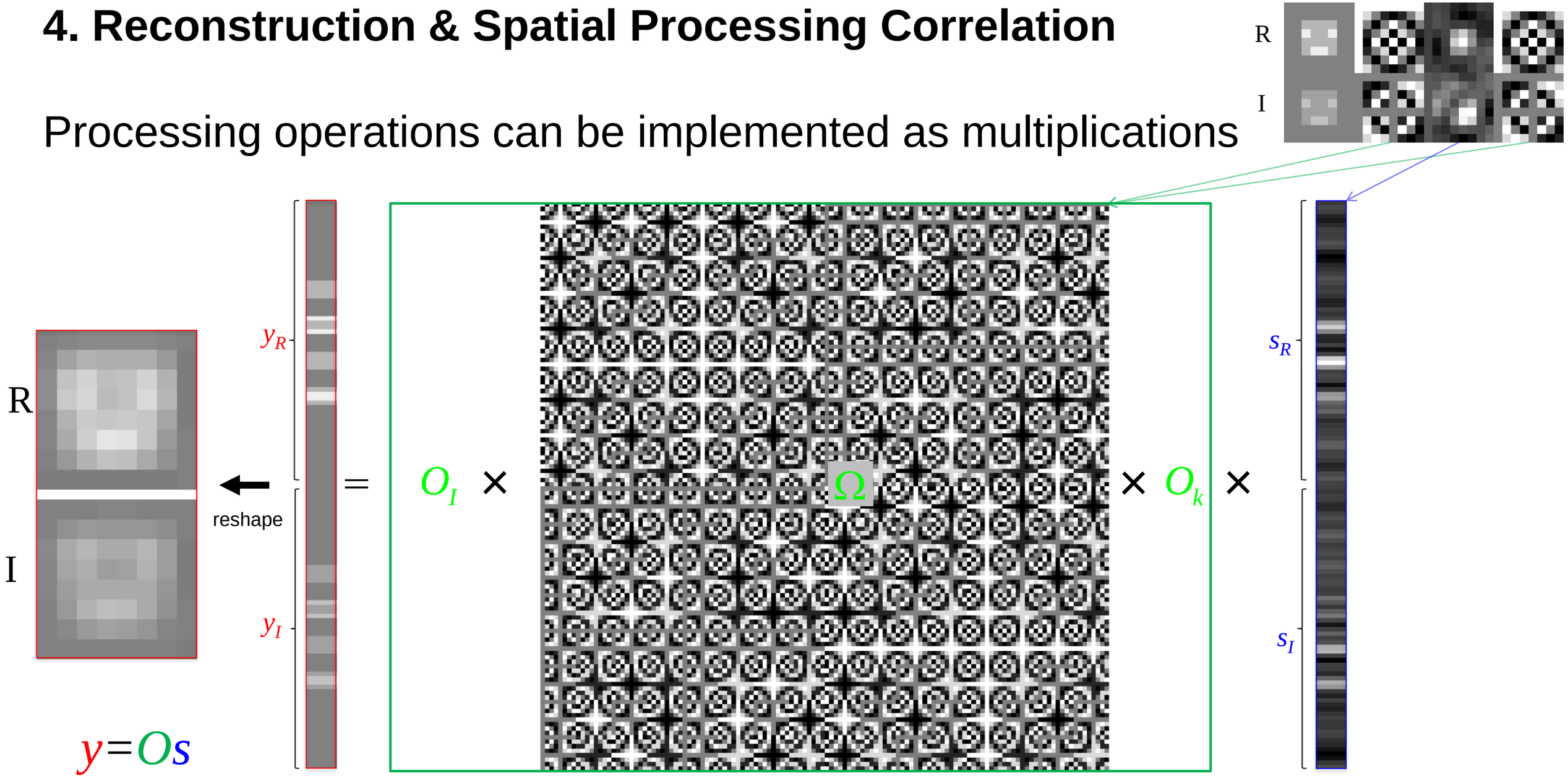
Vectorize spatial frequency coefficients and equivalent IFT



Rowe 2009; Nencka, Hahn, Rowe, 2009; Karaman, Nencka, Bruce, Rowe, 2014.

4. Reconstruction & Spatial Processing Correlation

Processing operations can be implemented as multiplications



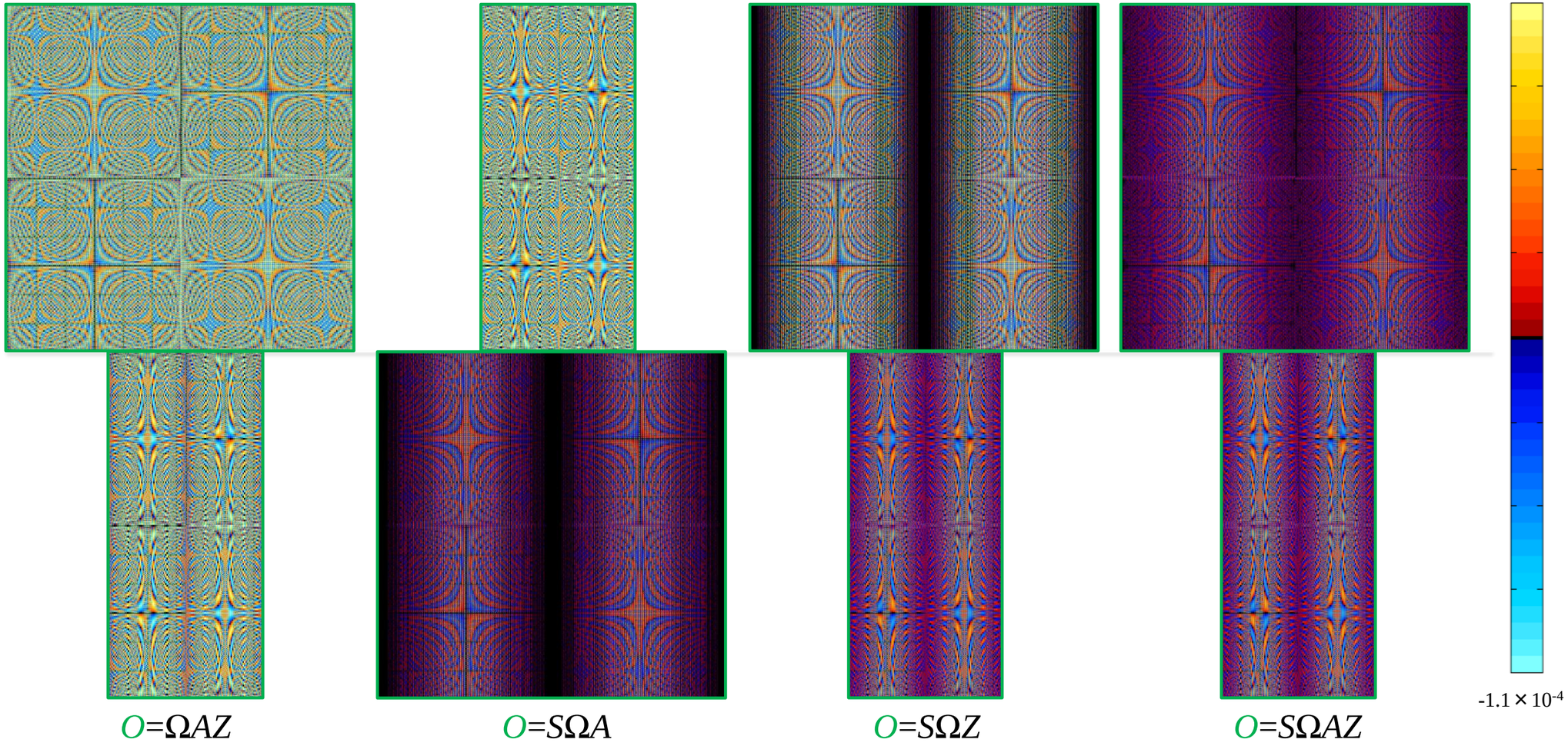
Rowe 2009; Nencka, Hahn, Rowe, 2009; Karaman, Nencka, Bruce, Rowe, 2014.

4. Reconstruction & Spatial Processing Correlation

Z , Zero Fill Interpolation
A , Apodization
Ω, IFT Reconstruction
S, Smoothing

Select reconstruction and processing matrices, O.

$O=\Omega$ $O=\Omega Z$ $O=\Omega A$ $O=S\Omega$



$O=\Omega AZ$ $O=S\Omega A$ $O=S\Omega Z$ $O=S\Omega AZ$

$$E(s)=\delta$$
$$cov(s)=\Gamma$$

$$y=Os$$

$$\mu=O\delta$$
$$\Sigma=O\Gamma O^T$$
$$R=D^{-1/2}\Sigma D^{-1/2}$$

Rowe: Handbook of Neuroimaging Data Analysis, 2016.

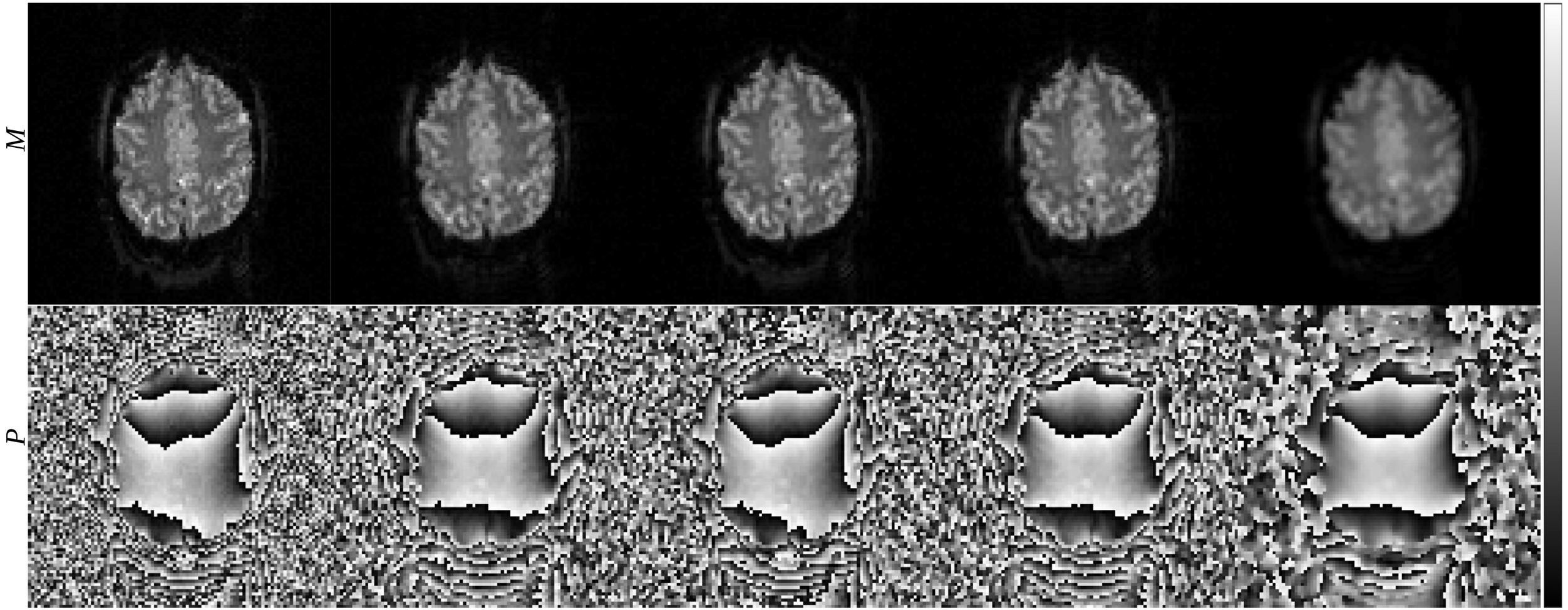
4. Reconstruction & Spatial Processing Correlation

$\mu = O\delta$

Z , Zero Fill Interpolation
A , Apodization
Ω, IFT Reconstruction
S, Smoothing

Implications: Changes in image mean.

$O=\Omega$ $O=\Omega Z$ $O=\Omega A$ $O=S\Omega$ $O=S\Omega AZ$

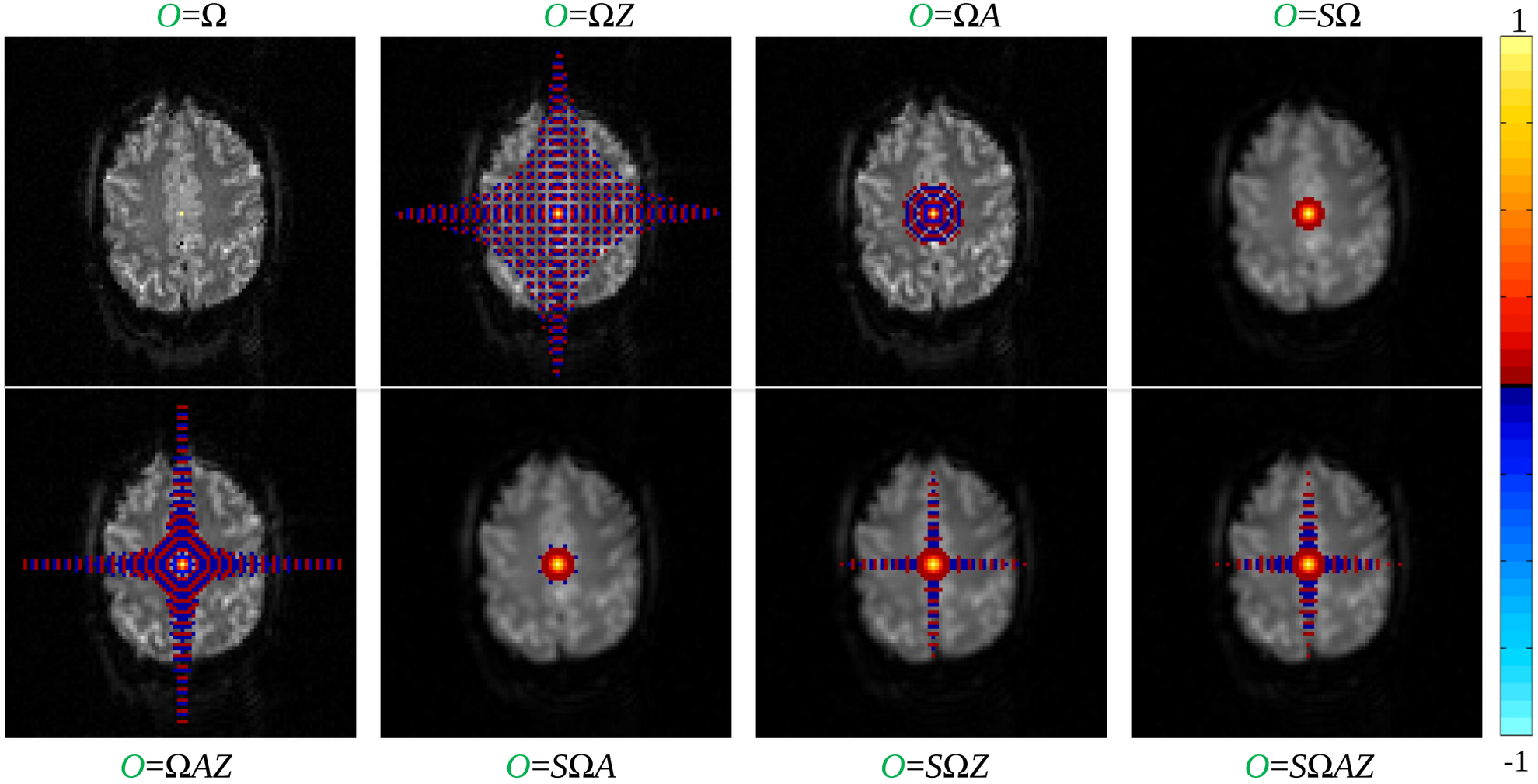


Rowe, Handbook of Neuroimaging Data Analysis, 2016.

4. Reconstruction & Spatial Processing Correlation

Z , Zero Fill Interpolation
A , Apodization
Ω, IFT Reconstruction
S, Smoothing

Implications: Induced correlations.



$$E(s)=\delta$$
$$cov(s)=\Gamma$$

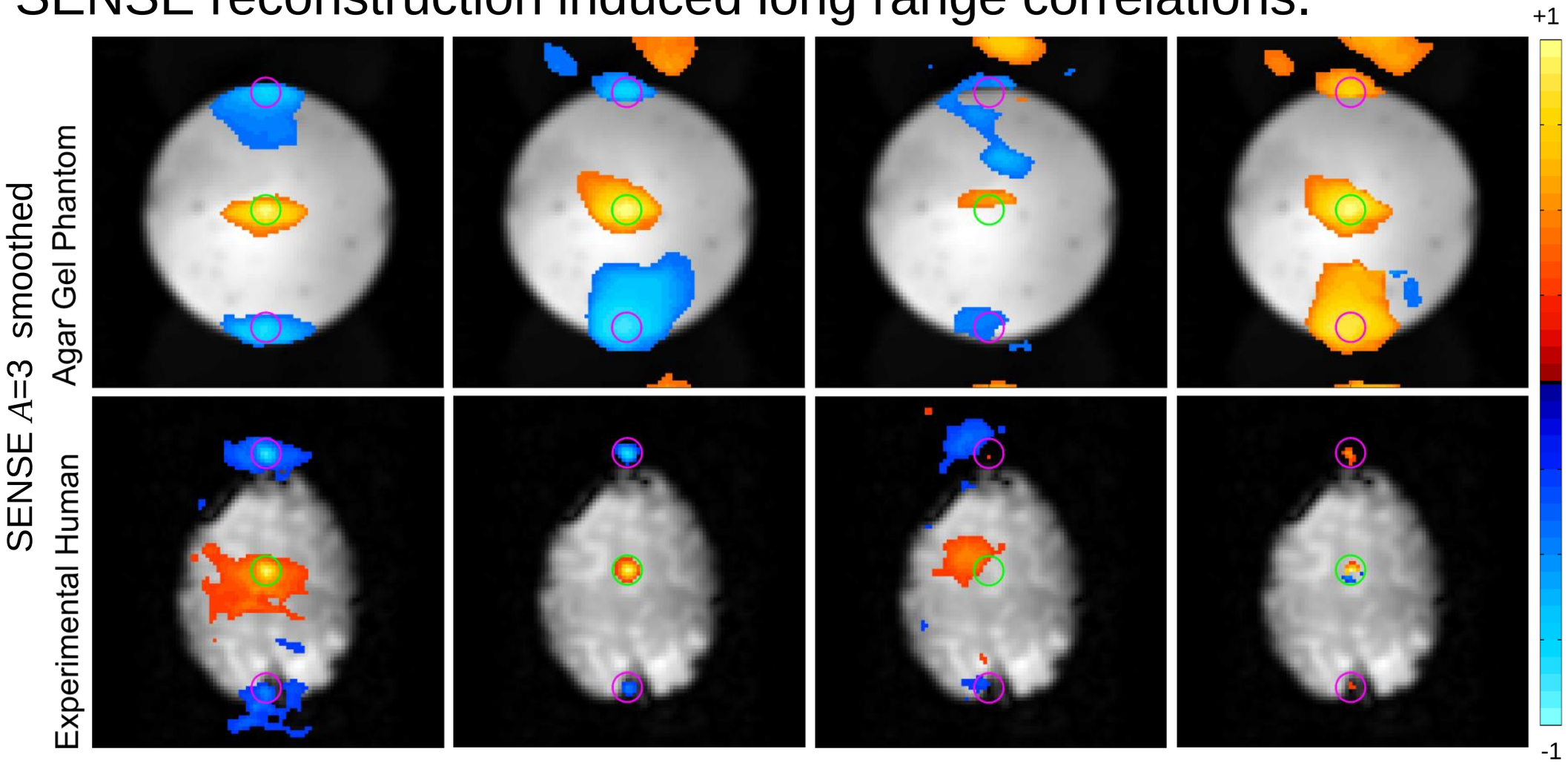
$$y=Os$$

$$\mu=O\delta$$
$$\Sigma=O\Gamma O^T$$
$$R=D^{-1/2}\Sigma D^{-1/2}$$

Rowe, Handbook of Neuroimaging Data Analysis, 2016.

4. Reconstruction & Spatial Processing Correlation

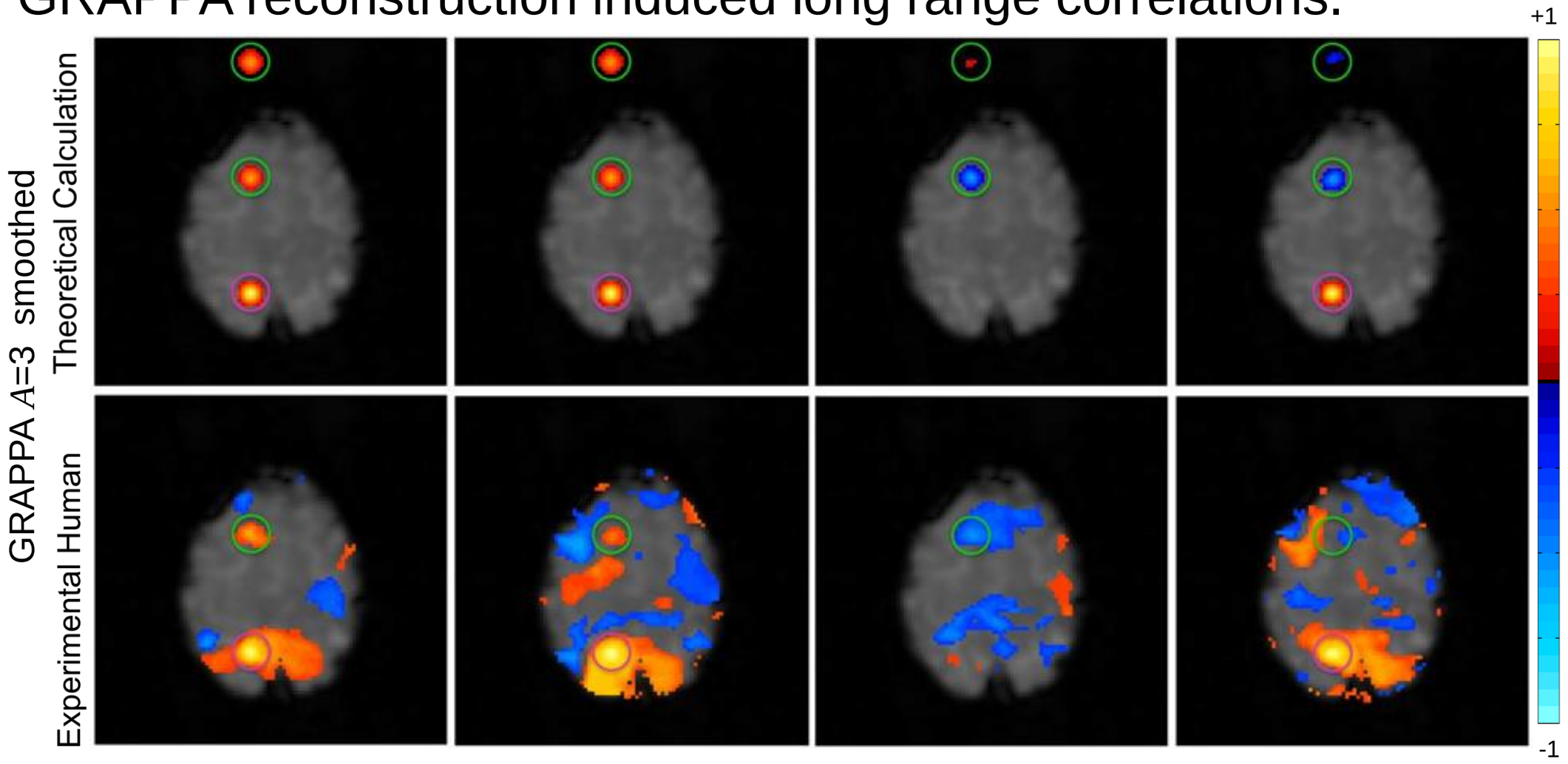
SENSE reconstruction induced long range correlations.



Bruce, Karaman, Rowe: MRI 2012.
Karaman, Nencka, Bruce, Rowe: Brain Connect,2014. Bruce, Karaman, Rowe: MRI, 2011.

4. Reconstruction & Spatial Processing Correlation

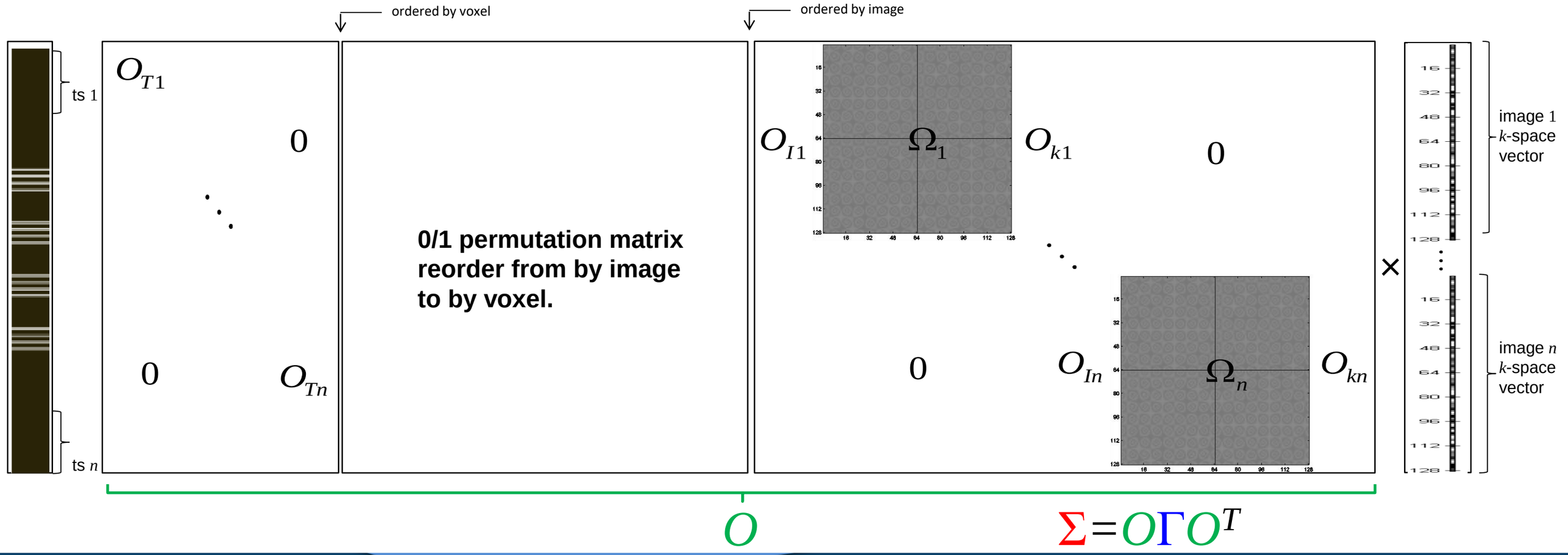
GRAPPA reconstruction induced long range correlations.



Risk, Kociuba Rowe: NIMG 2016
Karaman,Nencka,Bruce,Rowe: Brain Connect, 2014. Bruce, Rowe: IEEE-TMI, 2014.

5. Spatio-Temporal Processing Correlation

Image & time series processing weights of the original voxel measurements to produce a new series of images. Spatial and temporal correlation induced.



Rowe 2009; Nencka, Hahn, Rowe, 2009; Karaman, Nencka, Bruce, Rowe, 2014.

6. Discussion

Time Series Processing Induces Temporal Correlation

Image Processing Induces Spatial Correlation

SENSE & GRAPPA reconstructions induce long range correlations.

Image & Time Series Processing Induces Spatio-Temporal Correlation

We need to obtain rawest data and minimize processing.

We need to find ways to include effects of processing in our analysis.

Thank You

Questions?

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