

Bayesian Non-Conjugate Priors

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Outline

Prior Information

Deterministic Integration

Stochastic Integration

Non-Conjugate Prior for Binomial RVs
(Parabolic prior for p)

Discussion

Homework

Prior Information

When we expect observations from a particular distribution, $f(x|\theta)$, it may be the case that the information that we have about the parameter(s) θ do not fit into the conjugate prior framework. It is too constraining.

In such cases, we have non-conjugate prior distributions which when combined with the likelihood for the forthcoming observations does not necessarily form a “nice” $f(x,\theta)$ or “friendly” $f(\theta|x)$.

We may need to use advanced computational methods.

Prior Information

These advanced computational methods have the goal to compute the (marginal) posterior mean

$$E(\theta | x_1, \dots, x_n) = \int_{\theta} \theta f(\theta | x_1, \dots, x_n) d\theta \quad \leftarrow \text{Can't always be found with pencil and paper!}$$

where

$$f(\theta | x_1, \dots, x_n) = \frac{f(x_1, \dots, x_n | \theta) f(\theta)}{f(x_1, \dots, x_n)} .$$

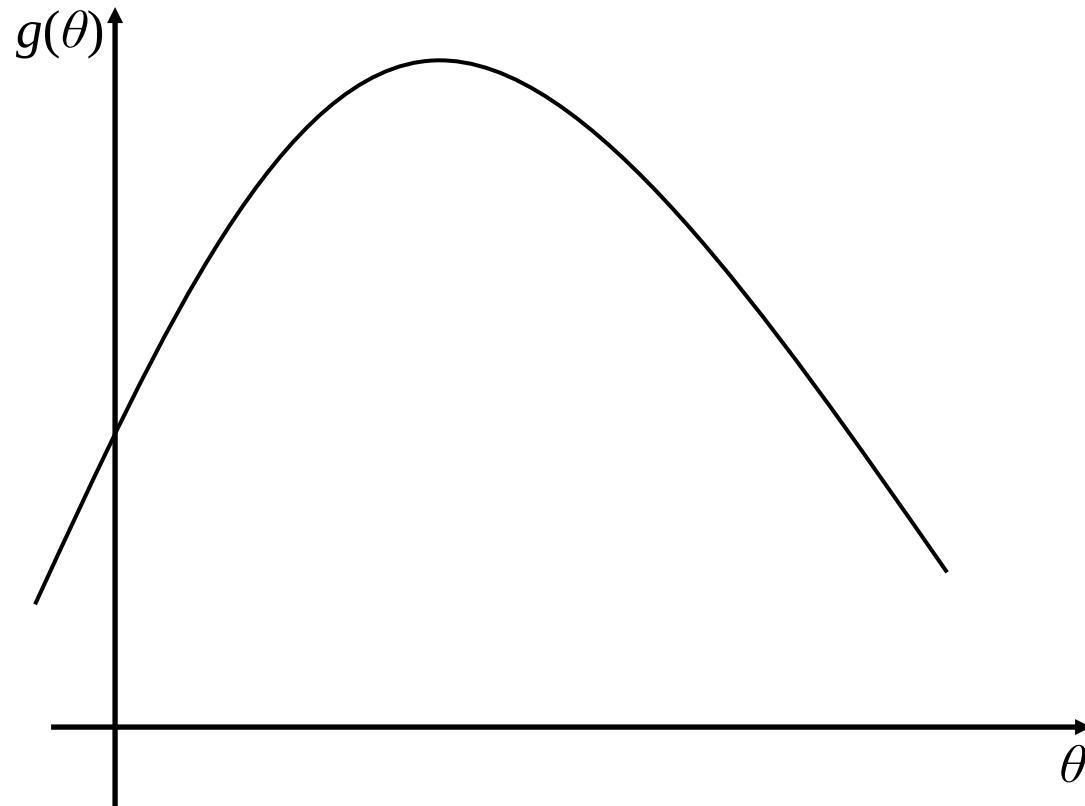
But to do this, don't forget that we also need

$$f(x_1, \dots, x_n) = \int_{\theta} f(x_1, \dots, x_n | \theta) f(\theta) d\theta . \quad \leftarrow \text{Can't always be found with pencil and paper!}$$

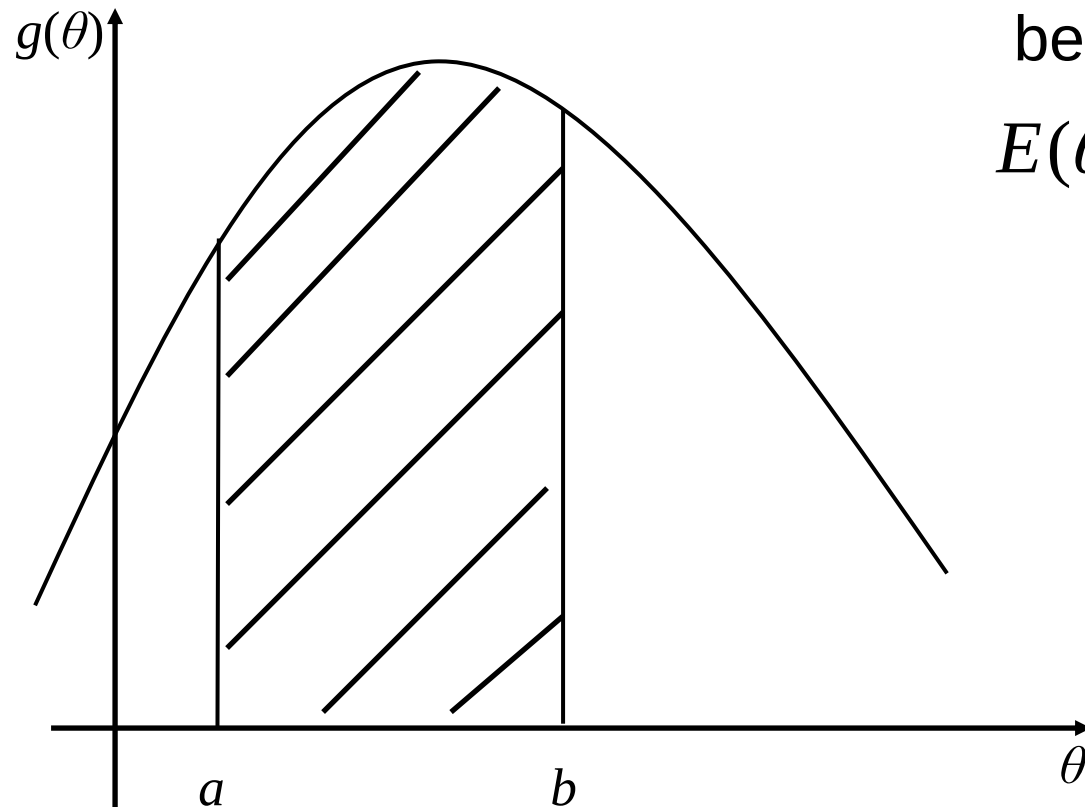
Deterministic Integration

goal

$$E(\theta \mid x_1, \dots, x_n) = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$



Deterministic Integration



Area under curve
between a and b .

$$E(\theta | \cdot) = \int_{\theta} g(\theta) d\theta$$

goal

$$E(\theta | x_1, \dots, x_n) = \int_{\theta} \underbrace{\theta f(\theta | x_1, \dots, x_n)}_{g(\theta)} d\theta$$

think of as

$$g(\theta) = \theta f(\theta)$$

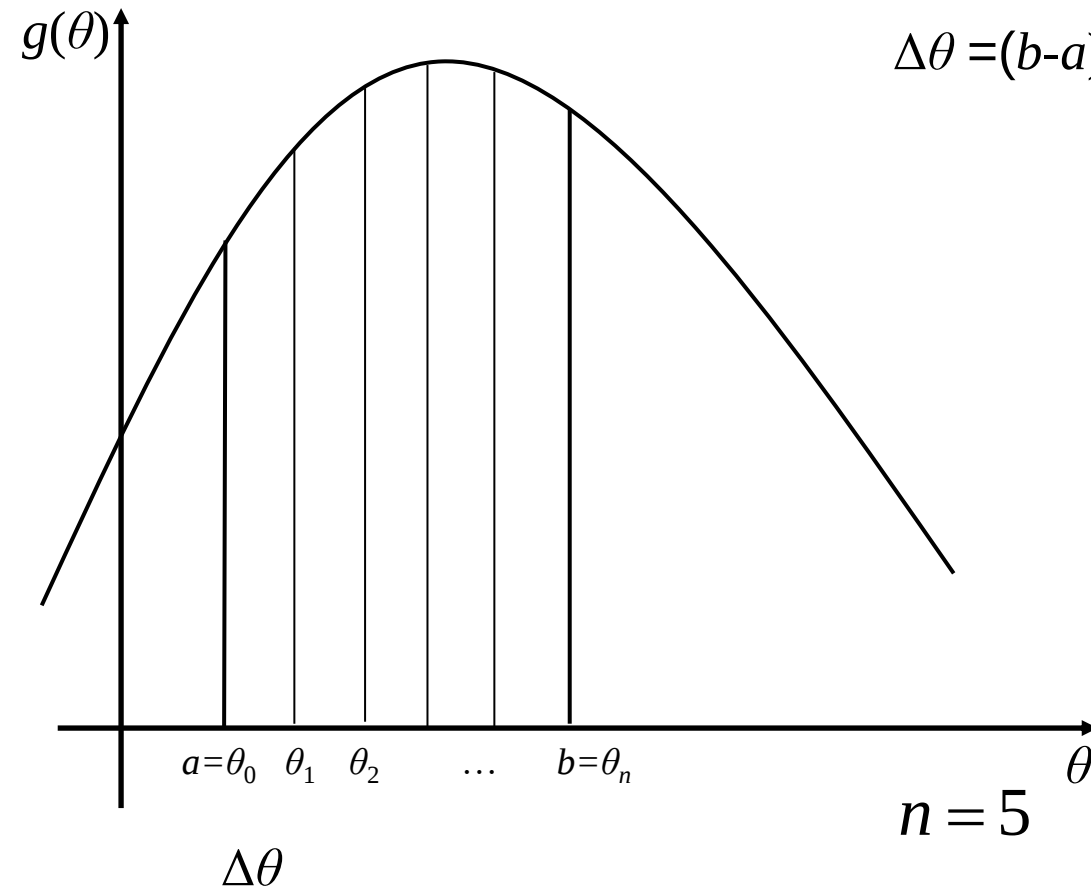
Deterministic Integration

goal

$$E(\theta \mid x_1, \dots, x_n) = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$

Divide into intervals: $\Delta\theta$ small

$$\Delta\theta = (b-a)/n \quad \Delta\theta = \theta_i - \theta_{i-1}$$



think of as

$$g(\theta) = \theta f(\theta)$$

Deterministic Integration

goal

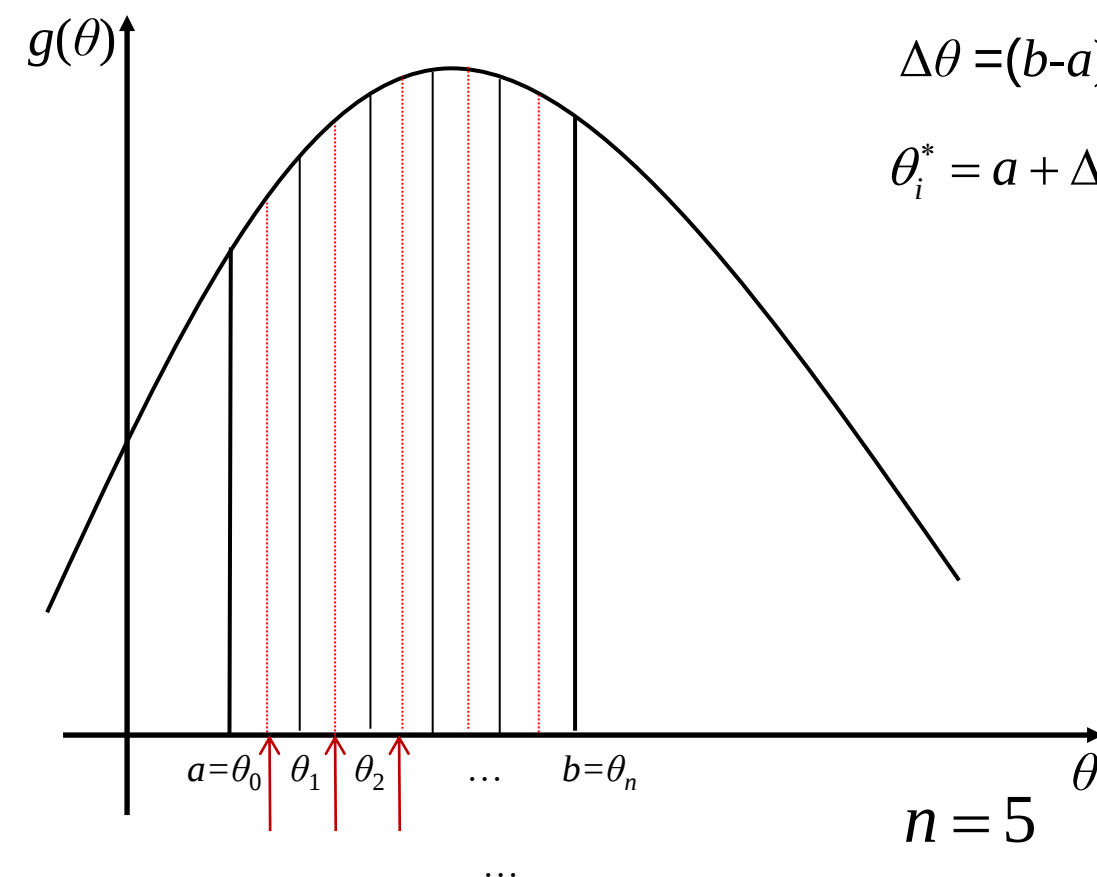
$$E(\theta \mid x_1, \dots, x_n) = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$

Divide into intervals: $\Delta\theta$ small

$$\Delta\theta = (b-a)/n \quad \Delta\theta = \theta_i - \theta_{i-1}$$

$$\theta_i^* = a + \Delta\theta / 2 + (i-1)\Delta\theta$$

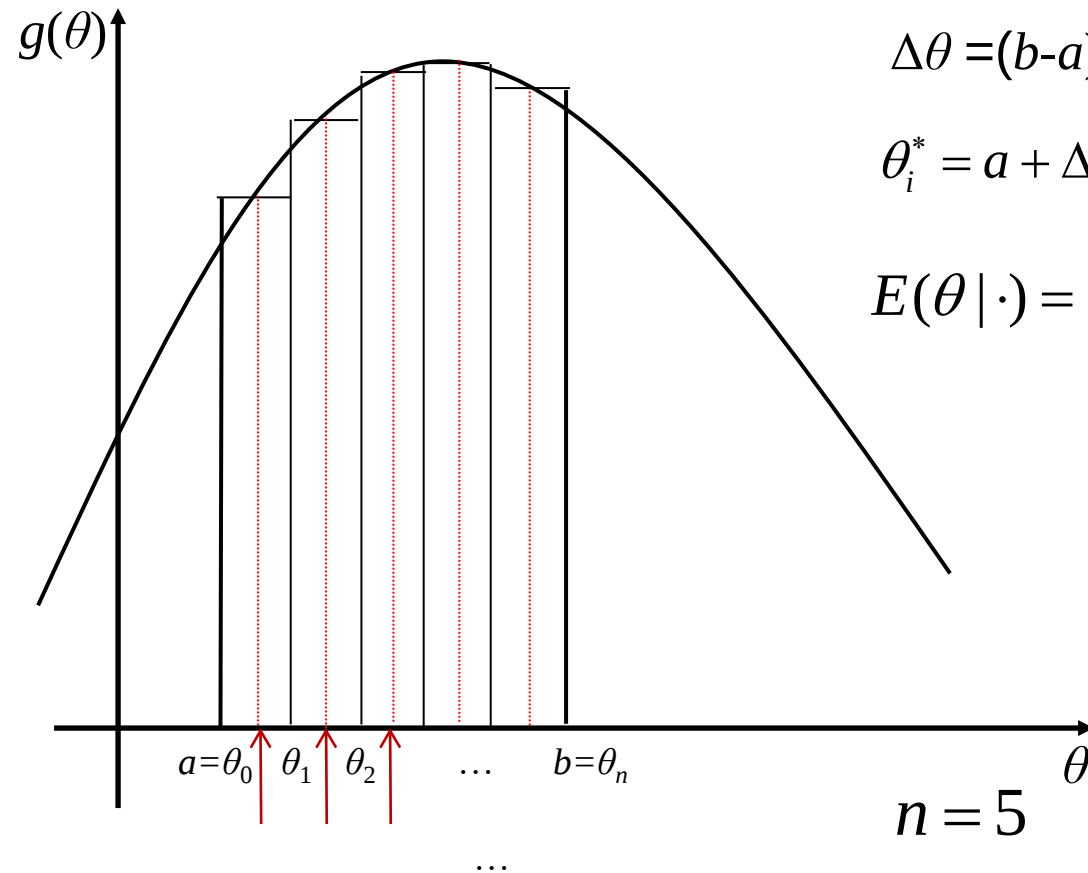
$i = 1, \dots, n$



Deterministic Integration

goal

$$E(\theta \mid x_1, \dots, x_n) = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$



Divide into intervals: $\Delta\theta$ small

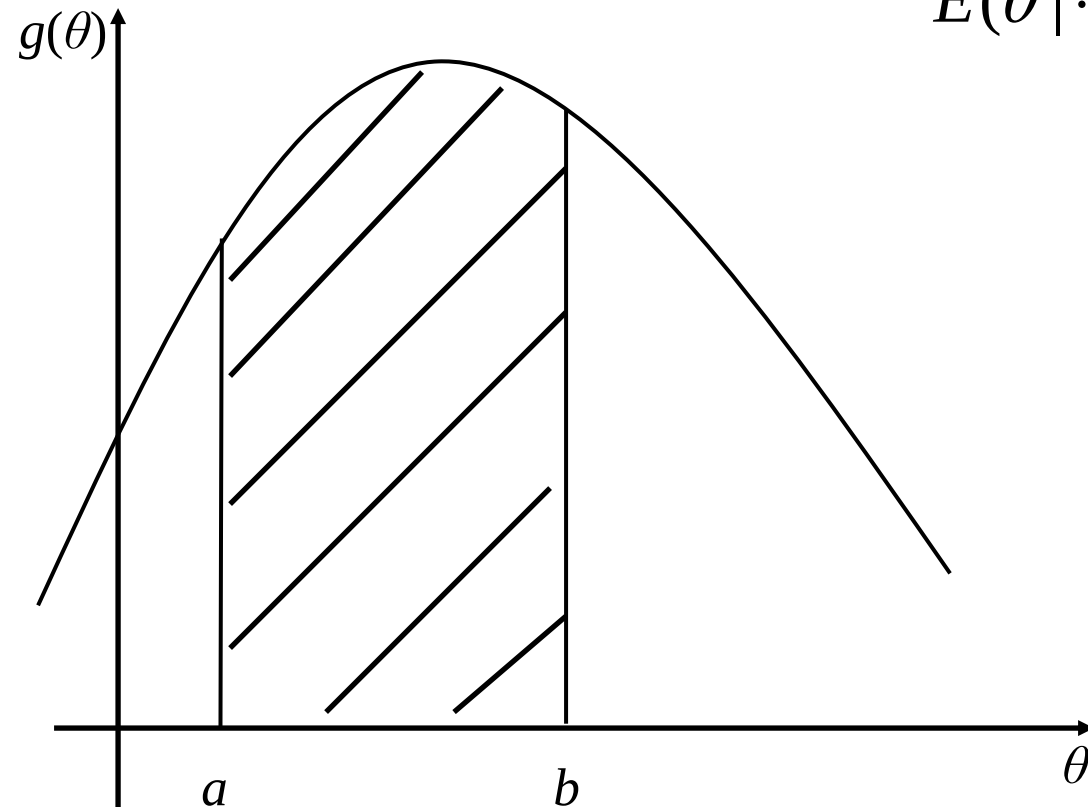
$$\Delta\theta = (b-a)/n \quad \Delta\theta = \theta_i - \theta_{i-1}$$

$$\theta_i^* = a + \Delta\theta / 2 + (i-1)\Delta\theta$$

$$E(\theta \mid \cdot) = \sum_{i=1}^n g(\theta_i^*) \Delta\theta \quad i = 1, \dots, n$$

Deterministic Integration

$$\overset{\text{goal}}{E(\theta \mid x_1, \dots, x_n)} = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$



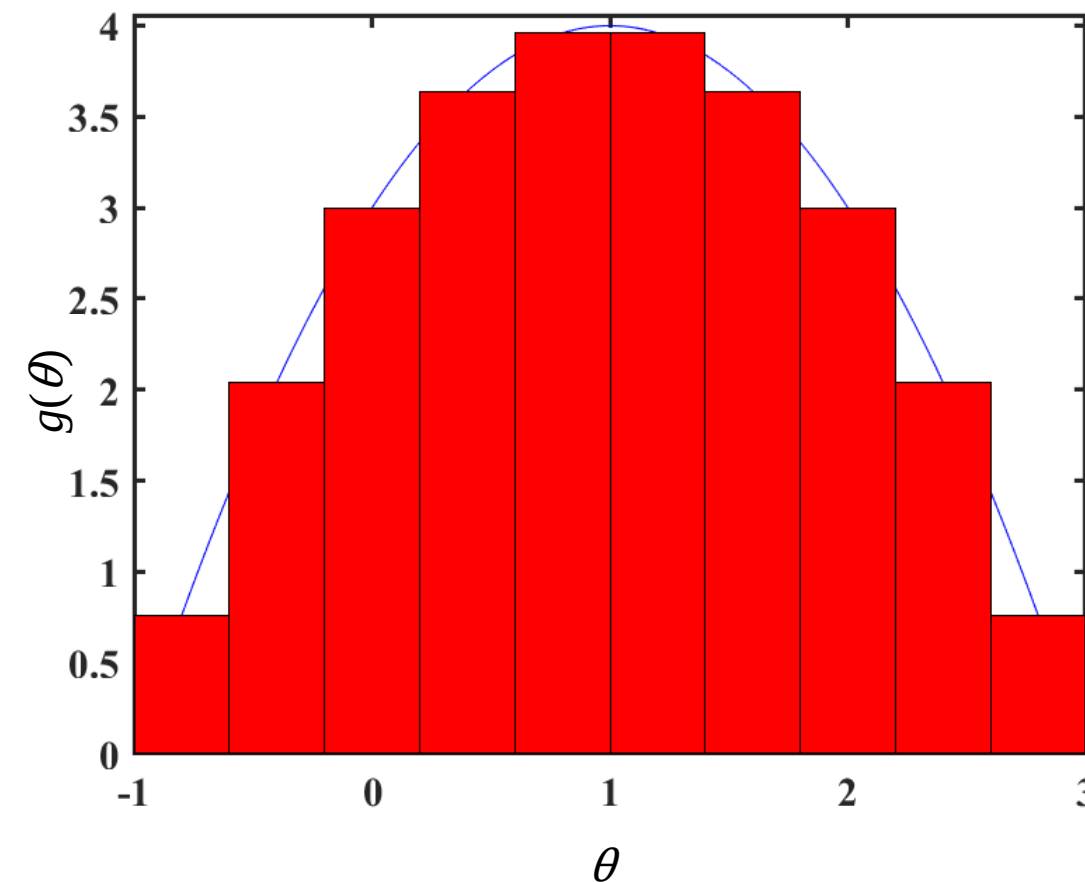
$$\begin{aligned} E(\theta \mid \cdot) &= \lim_{\Delta\theta \rightarrow 0} \sum_{i=1}^n g(\theta_i^*) \Delta\theta \\ &= \int_{\theta=a}^b g(\theta) d\theta \end{aligned}$$

$$\Delta\theta = (b - a) / n$$

$$\theta_i^* = a + \Delta\theta / 2 + (i - 1)\Delta\theta$$

Deterministic Integration

$$\overset{\text{goal}}{E(\theta | x_1, \dots, x_n)} = \int_{\theta} \underbrace{\theta f(\theta | x_1, \dots, x_n)}_{g(\theta)} d\theta$$



$$g(\theta) = 4 - (\theta - 1)^2$$

← numerical

$$n=10, \quad \Delta\theta=0.400$$

$$\hat{E}(\theta | \cdot) = \Delta\theta \sum_{i=1}^{10} g(\theta_i^*) = 10.7200$$

analytic

$$\int_{\theta=-1}^3 [4 - (\theta - 1)^2] d\theta = 10.6667$$

`% Numerical Integral`

`a=-1; b=3;`

`n=10; dt=(b-a)/n;`

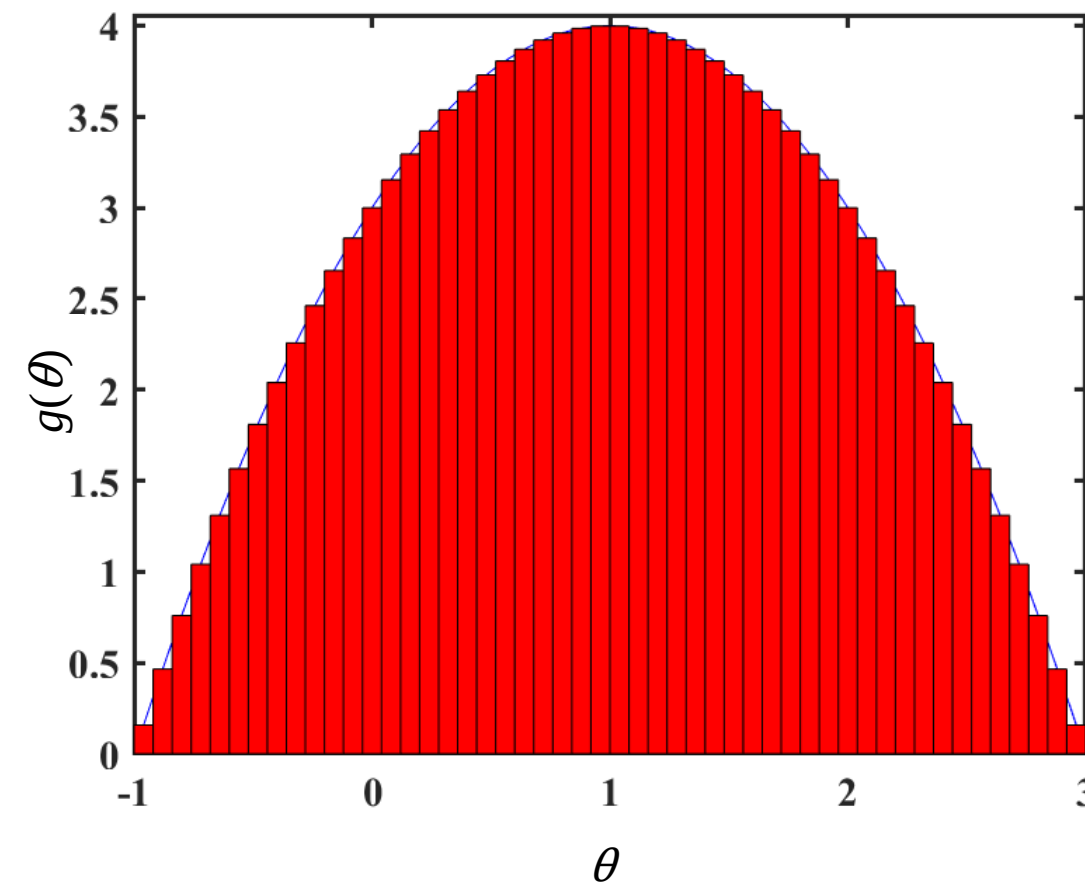
`tpts=(a+dt/2:dt:b)';`

`gpts=4-(tpts-1).^2;`

`Ethat=dt*sum(gpts)`

Deterministic Integration

$$\overset{\text{goal}}{E(\theta | x_1, \dots, x_n)} = \int_{\theta} \underbrace{\theta f(\theta | x_1, \dots, x_n)}_{g(\theta)} d\theta$$



$$g(\theta) = 4 - (\theta - 1)^2$$

← numerical

$$n=50, \quad \Delta\theta=0.080$$

$$\hat{E}(\theta | \cdot) = \Delta\theta \sum_{i=1}^{50} g(\theta_i^*) = 10.6688$$

analytic

$$\int_{\theta=-1}^3 [4 - (\theta - 1)^2] d\theta = 10.6667$$

% Numerical Integral

a=-1; b=3;

n=50; dt=(b-a)/n;

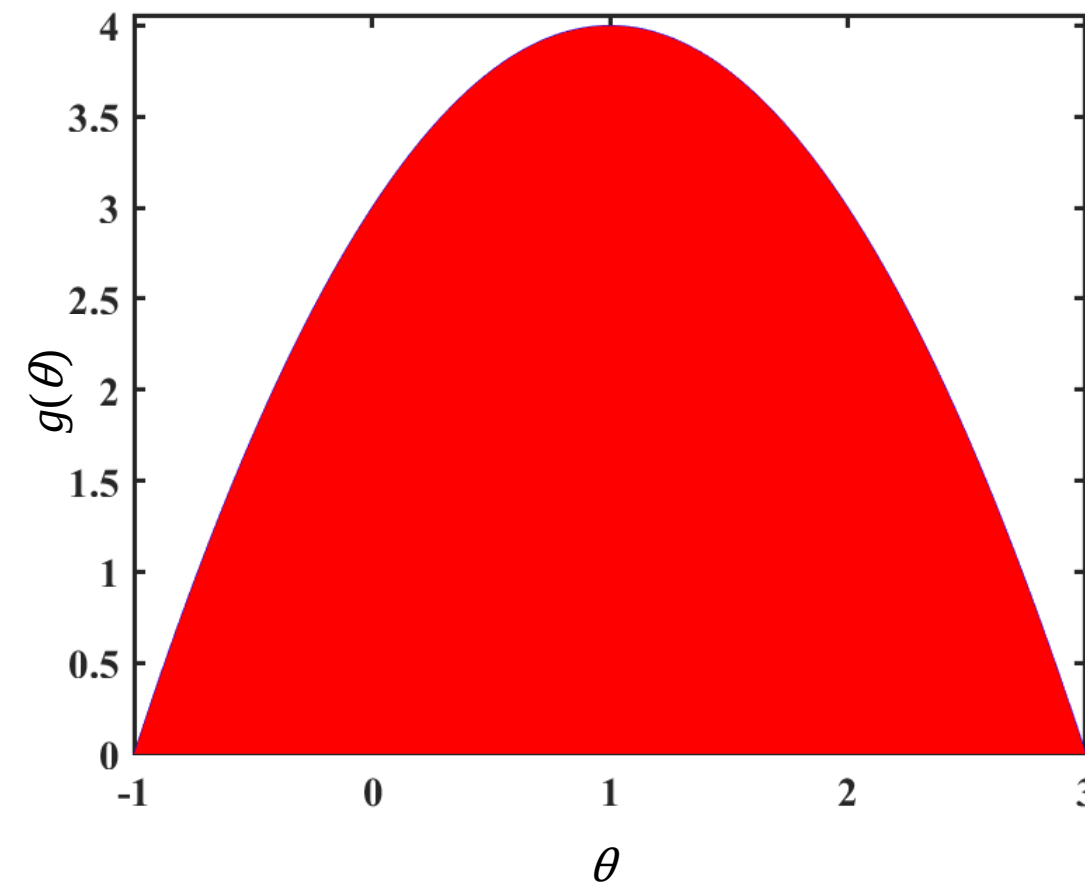
tpts=(a+dt/2:dt:b)';

gpts=4-(tpts-1).^2;

Ethat=dt*sum(gpts)

Deterministic Integration

$$\overset{\text{goal}}{E(\theta \mid x_1, \dots, x_n)} = \int_{\theta} \underbrace{\theta f(\theta \mid x_1, \dots, x_n)}_{g(\theta)} d\theta$$



$$g(\theta) = 4 - (\theta - 1)^2$$

← numerical

$$n=1000, \Delta\theta=0.004$$

$$\hat{E}(\theta \mid \cdot) = \Delta\theta \sum_{i=1}^{1000} g(\theta_i^*) = 10.6667$$

analytic

$$\int_{\theta=-1}^3 [4 - (\theta - 1)^2] d\theta = 10.6667$$

% Numerical Integral

a=-1; b=3;

n=1000; dt=(b-a)/n;

tpts=(a+dt/2:dt:b)';

gpts=4-(tpts-1).^2;

Ethat=dt*sum(gpts)

Stochastic Integration

We can also integrate a function $g(\theta)$ via random uniform numbers.

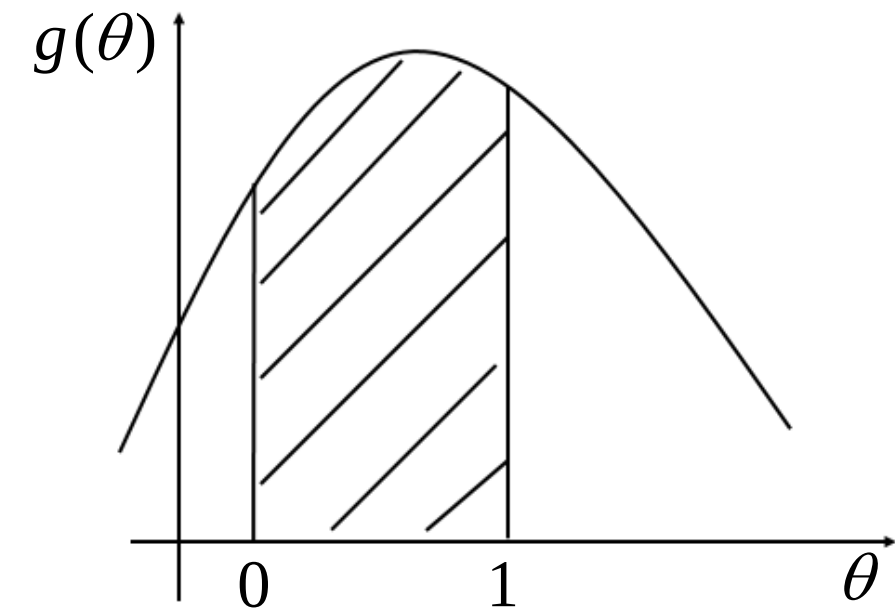
$$E(\theta | \cdot) = \int_0^1 g(u) du \quad \leftarrow \text{changed symbol of integration}$$

We can view this as there being a PDF $f(u)$ where u is uniformly distributed over $(0,1)$

$$f(u) = 1 \quad 0 \leq u \leq 1$$

and we are calculating the expected value

$$E(\theta | \cdot) = E(g(u)) = \int_0^1 g(u) f(u) du . \quad \leftarrow \text{key idea}$$



Stochastic Integration

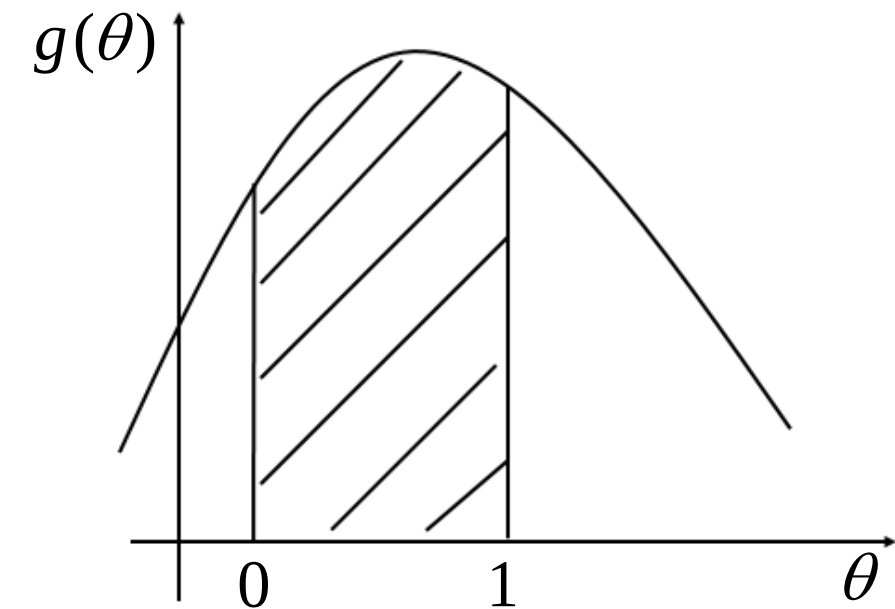
If we have a random sample $u_1, \dots, u_n$ from $f(u)$, then we can calculate a sample version of

$$E(\theta | \cdot) = E(g(u)) = \int_0^1 g(u) f(u) du$$

with the iid sample $u_1, \dots, u_n$, from $f(u)$.

The computed values $g(u_1), \dots, g(u_n)$ are an iid sample with mean $E(g(u))$.

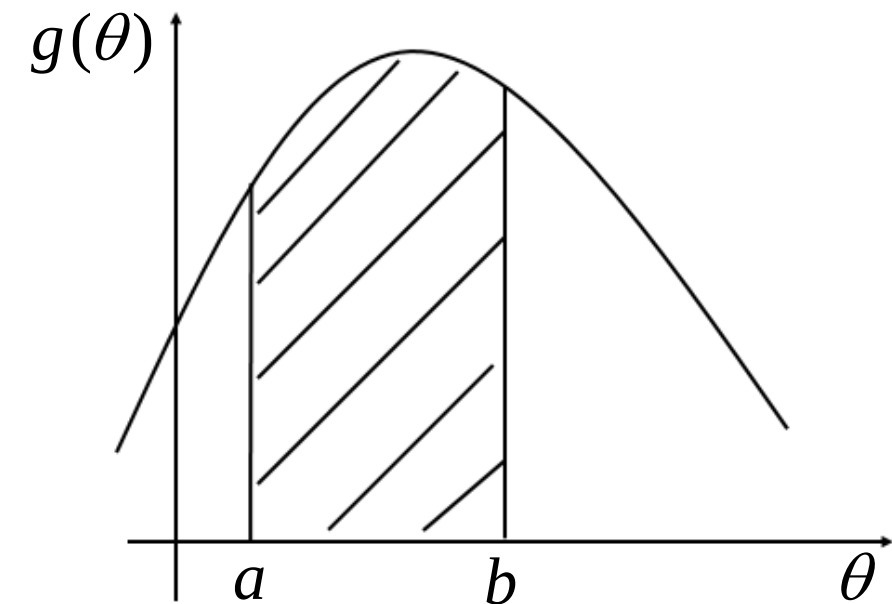
$$\frac{1}{n} \sum_{i=1}^n g(u_i) \rightarrow E[g(u)] = E(\theta | \cdot) \quad \text{as } n \rightarrow \infty$$



Stochastic Integration

If we want $E(\theta | \cdot) = \int_a^b g(\theta) d\theta$, then we can transform θ to u as $u = (\theta - a)/(b - a)$, with $du = d\theta / (b - a)$

$$E(\theta | \cdot) = \int_a^b g(\theta) d\theta \quad \begin{aligned} d\theta &= (b - a) du \\ \theta &= (b - a)u + a \end{aligned}$$



Then our integral becomes

$$E(\theta | \cdot) = \int_0^1 g(a + (b - a)u)(b - a) du$$

$$\begin{aligned} h(u) &= g(a + (b - a)u)(b - a) \\ f(u) &= 1 \quad 0 \leq u \leq 1 \end{aligned}$$

which is the same as

$$E(\theta | \cdot) = E(h(u)) = \int_0^1 h(u) f(u) du .$$

Stochastic Integration

Let's use this idea to evaluate the same integral.

Deterministic

$$E(\theta | \cdot) = \int_{\theta} g(\theta) d\theta$$

$$\hat{E}_n(\theta | \cdot) = \Delta\theta \sum_{i=1}^n g(\theta_i^*)$$

$$\hat{E}_{10}(\theta | \cdot) = 10.7200$$

Stochastic

$$h(u) = g(a + (b - a)u)(b - a)$$

$$\frac{1}{n} \sum_{i=1}^n h(u_i) = 10.6718$$

`% stochastic integral`

`rng('default')`

`a=-1; b=3; n=10^6;`

`u=rand(n,1);`

`t=(a+(b-a)*u);`

`hu=(4-(t-1).^2)*(b-a);`

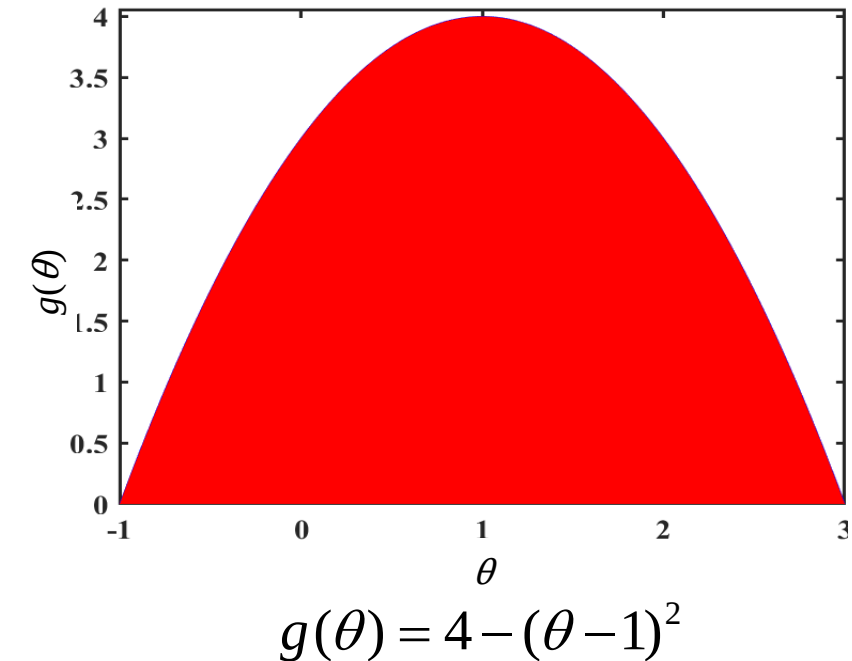
`Ethat=sum(hu)/n`

Theoretical

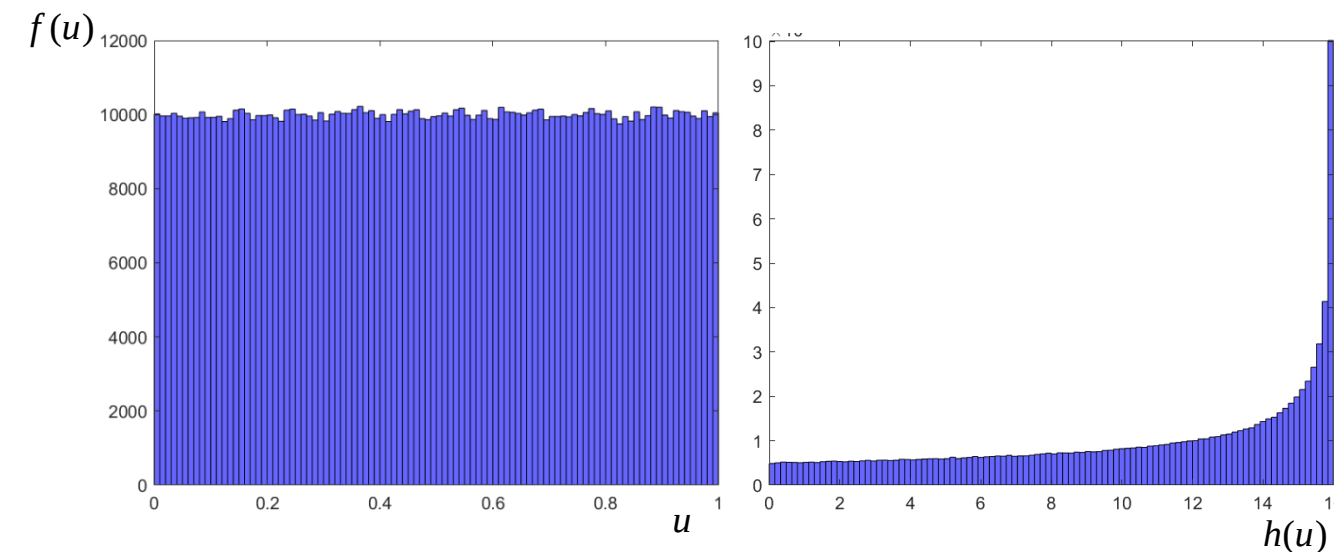
$$\int_{-1}^3 [4 - (\theta - 1)^2] d\theta = 10.6667$$

$$(b - a) = 4$$

Deterministic



Stochastic



Stochastic Integration

If we want $E(\theta | \cdot) = \int_0^\infty g(\theta) d\theta$, then we can transform θ to u as $u=1/(\theta +1)$, with $du=-1/(\theta +1)^2 d\theta = -u^2 d\theta$

$$E(\theta | \cdot) = \int_0^\infty g(\theta) d\theta \quad \begin{array}{l} d\theta = -du / u^2 \\ \theta = 1 / u - 1 \end{array} \quad \begin{array}{l} \text{limits} \\ \theta = 0 \rightarrow u = 1 \\ \theta = \infty \rightarrow u = 0 \end{array}$$

Then our integral becomes

$$E(\theta | \cdot) = \int_0^1 g(1 / u - 1) \frac{du}{u^2} \quad u = 1 / (\theta + 1)$$

$$E(\theta | \cdot) = \int_0^1 h(u) f(u) du \quad h(u) = g(1/u - 1) / u^2$$

and we can use the same technique.

$$\bar{E}(\theta | \cdot) = \frac{1}{n} \sum_{i=1}^n h(u_i)$$

Stochastic Integration

logistic function

If we want $E(\theta | \cdot) = \int_{-\infty}^{\infty} g(\theta) d\theta$, then we can transform θ to u as $u = e^{\theta} / (1 + e^{\theta})$, with $du = e^{\theta} / (1 + e^{\theta})^2 d\theta = u d\theta / (1 - u)$

$$E(\theta | \cdot) = \int_{-\infty}^{\infty} g(\theta) d\theta \quad \begin{array}{l} d\theta = u du / (1 - u) \text{ limits} \\ \theta = \ln\left(\frac{u}{1-u}\right) \end{array} \quad \begin{array}{l} \theta = -\infty \rightarrow u = 0 \\ \theta = +\infty \rightarrow u = 1 \end{array}$$

Then our integral becomes

$$E(\theta | \cdot) = \int_0^1 g\left(\ln\left(\frac{u}{1-u}\right)\right) \frac{du}{u(1-u)} \quad \begin{array}{l} u = e^{\theta} / (1 + e^{\theta}) \\ h(u) = g\left(\ln\left(\frac{u}{1-u}\right)\right) \frac{1}{u(1-u)} \end{array}$$

$$E(\theta | \cdot) = \int_0^1 h(u) f(u) du$$

and we can use the same technique.

$$\bar{E}(\theta | \cdot) = \frac{1}{n} \sum_{i=1}^n h(u_i)$$

Non-Conjugate Prior For Binomial RVs

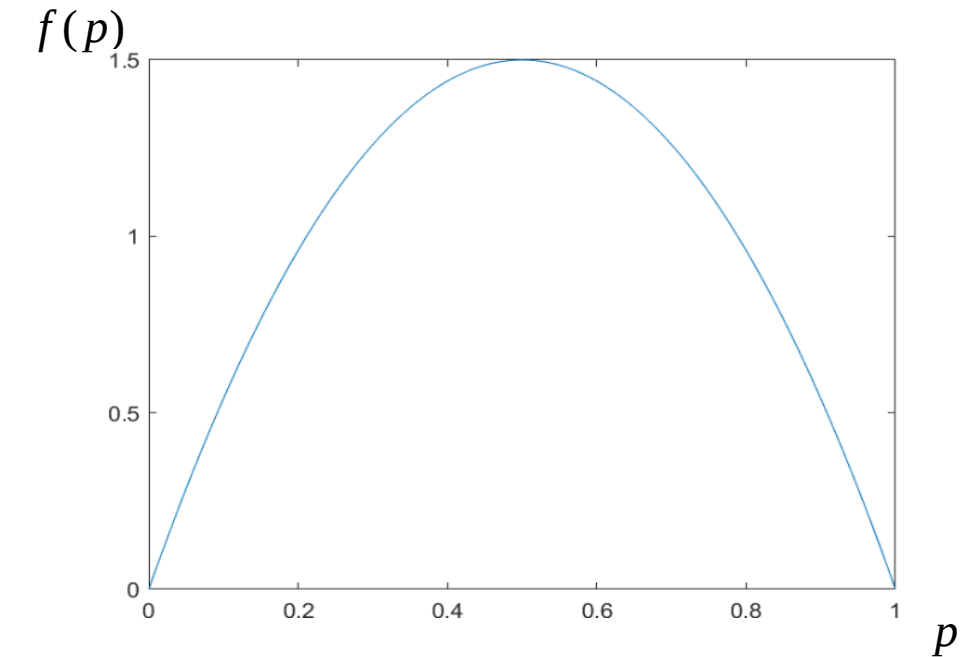
Binomial observation x :

Let's imagine that our prior information for p was consistent with the following parabolic prior for p ,

$$f(p) = \frac{3}{2} \left[1 - 4 \left(p - 1/2 \right)^2 \right] \quad p \in [0,1]$$

to go along with the binomial likelihood for x

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad \begin{array}{l} p \in [0,1] \\ x = 0,1,\dots,n \end{array}$$



Non-Conjugate Prior For Binomial RVs

Binomial observation x :

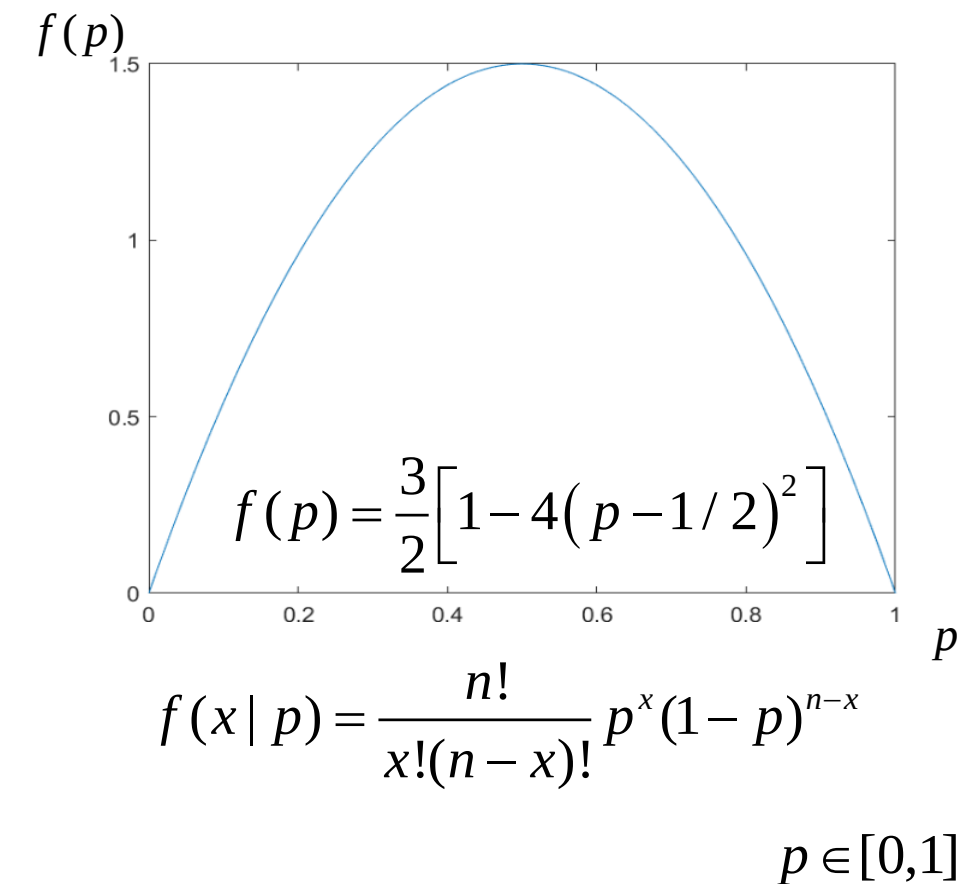
The parabolic prior for p and binomial likelihood for x do not combine to form a “nice” joint distribution $f(x,p)$

$$f(x, p) = \frac{3}{2} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right]$$

or yield a “friendly” posterior distribution $f(p|x)$ for p .

So we need to resort to more advanced methods.

i.e. Try harder for pencil & paper integral, deterministic integration (rectangles), stochastic integration (random u 's), MCMC.



Non-Conjugate Prior For Binomial RVs

Binomial observation x :

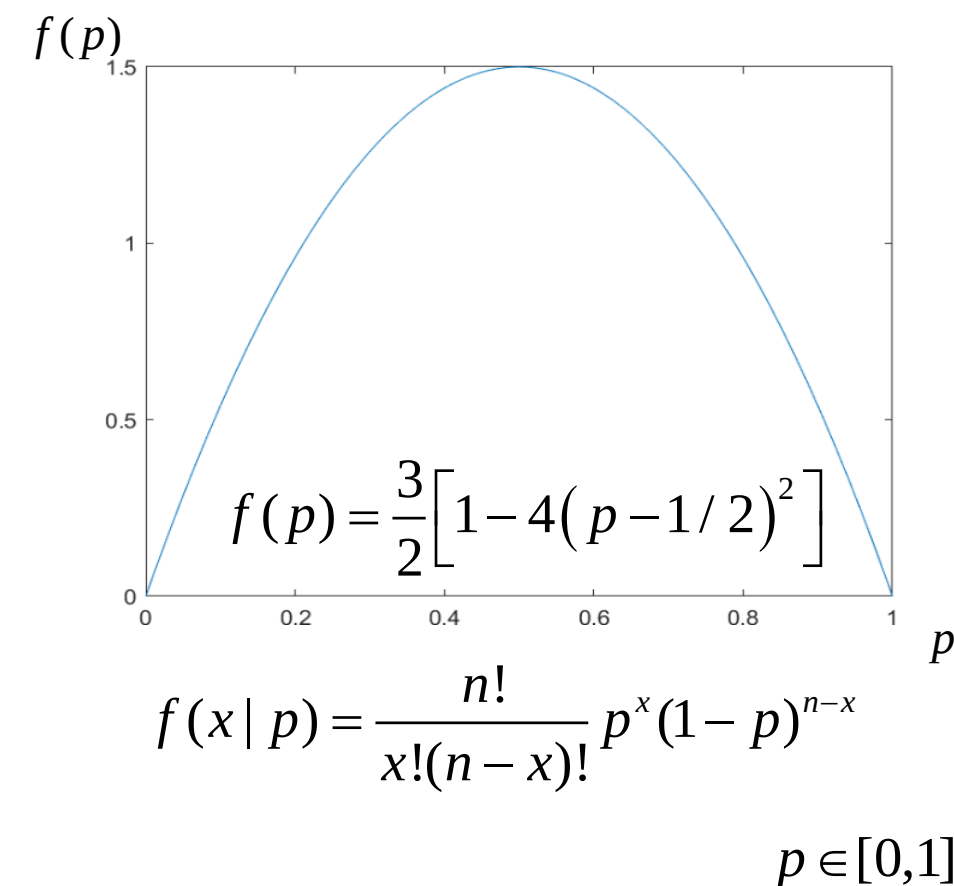
The $f(p)$ and $f(x|p)$ do not combine to form a “nice” $f(x,p)$ or yield a “friendly” posterior $f(p|x)$.

It can be shown (homework problem) that

$$f(x) = 6 \frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

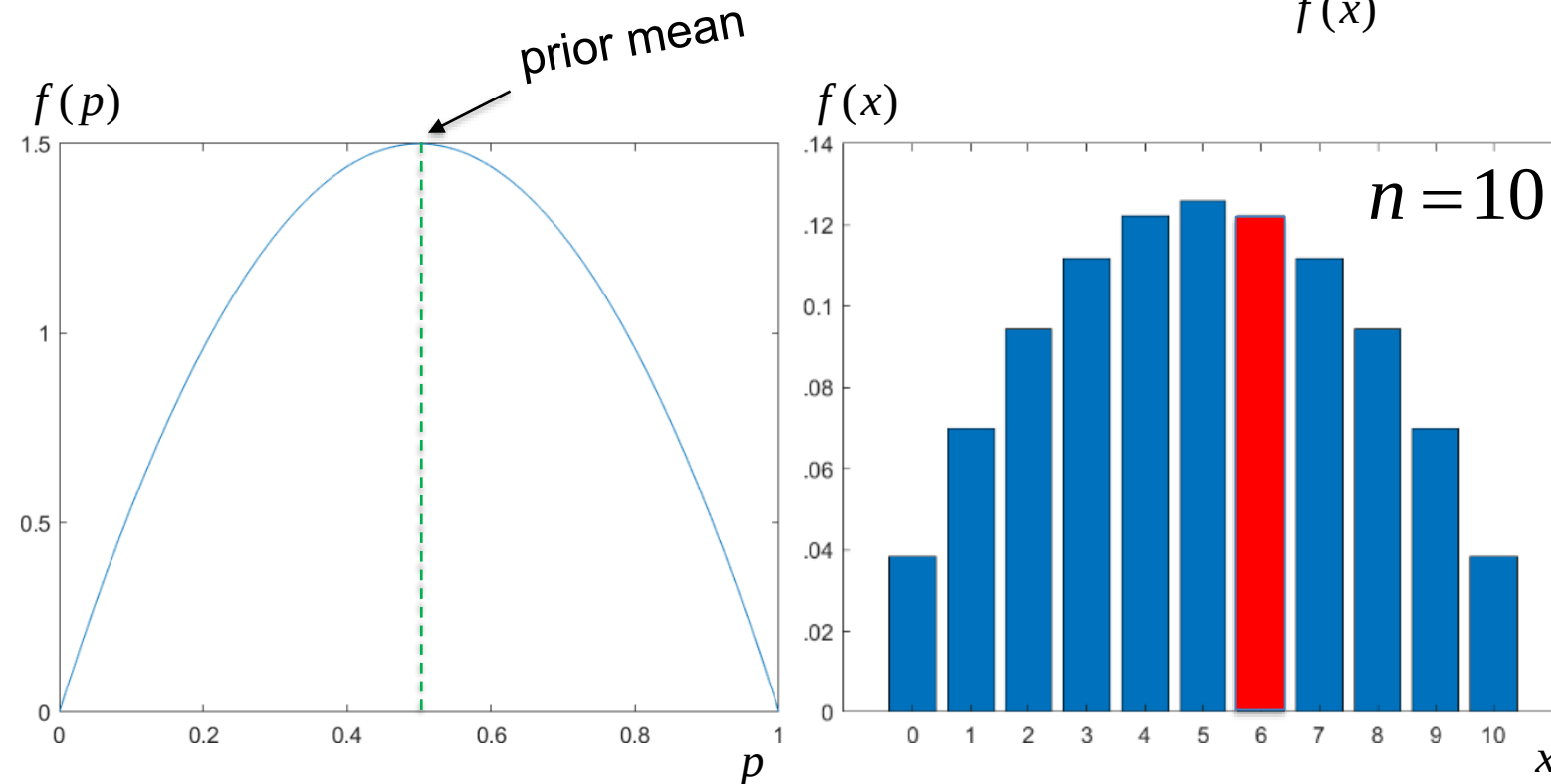
and thus

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4 \left(p - 1/2 \right)^2 \right]$$



Non-Conjugate Prior For Binomial RVs

Binomial observation x : $f(p|x) = \frac{f(x|p)f(p)}{f(x)}$

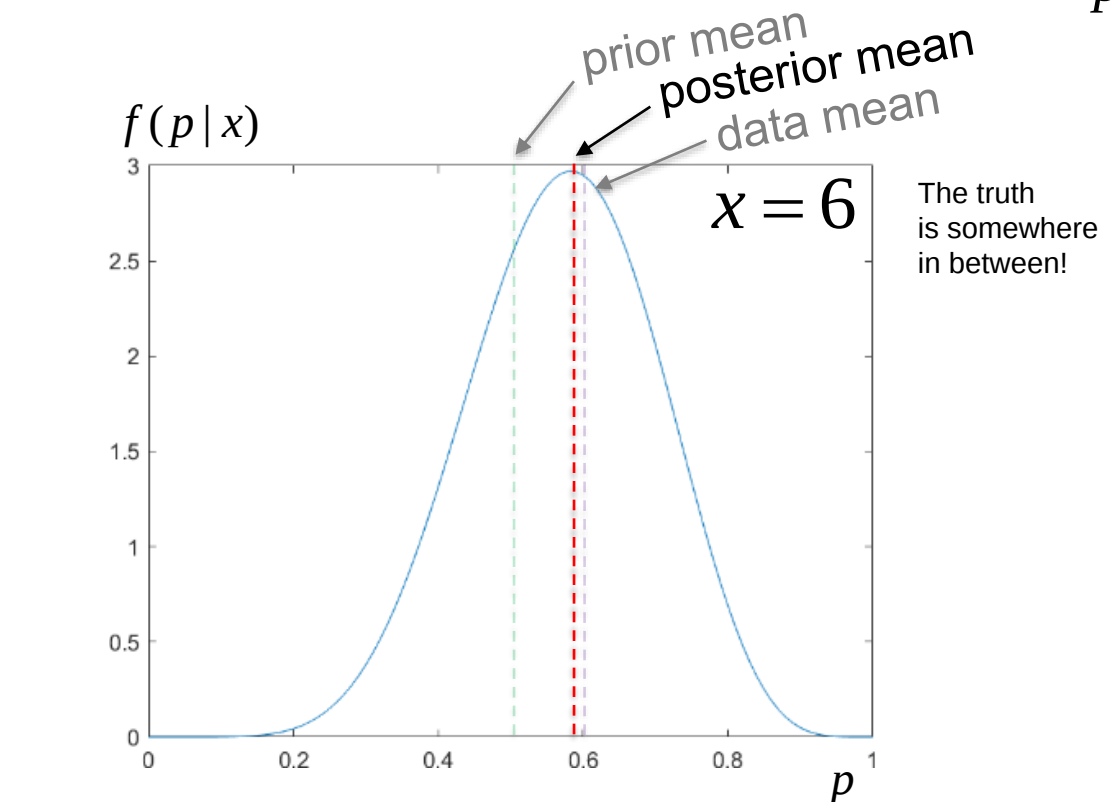


$$f(p) = \frac{3}{2} \left[1 - 4(p - 1/2)^2 \right]$$

$$f(x) = 6 \frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$



$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right]$$

Non-Conjugate Prior For Binomial RVs

Binomial observation x :

From this posterior PDF for p ,

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right]$$
$$p \in [0,1]$$

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$f(p) = \frac{3}{2} \left[1 - 4(p - 1/2)^2 \right]$$

$$n = 10$$
$$x = 6$$

we need to compute summary measures.

i.e. mode, mean, median, variance of $p|x$.

Similar to what we do when we have conjugate priors.

Non-Conjugate Prior For Binomial RVs

Binomial observation x :

We can differentiate to maximize the posterior

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right]$$

$$p \in [0,1]$$

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$f(p) = \frac{3}{2} \left[1 - 4(p - 1/2)^2 \right]$$

$$n = 10$$

$$x = 6$$

Or we can try every value of p that is $\Delta p = 0.0001$ apart and select the one that yields the maximum.

$$\underset{p}{\text{ArgMax}} f(p|x, n) = 0.5833$$

Non-Conjugate Prior For Binomial RVs

Binomial observation x :

To calculate the expected value $E(p|x)$ of

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right],$$

$$p \in [0,1]$$

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$f(p) = \frac{3}{2} \left[1 - 4(p - 1/2)^2 \right]$$

$$n = 10$$

$$x = 6$$

there are several techniques at our disposal

- exact integration (calculus)
- deterministic (*numerical analysis*)
- and stochastic (*statistical simulation*).

Non-Conjugate Prior For Binomial RVs

Binomial observation x :
Stochastic and numerical integration.

$$f(p | x) = \frac{1}{4} \frac{(n + 3)!}{(x + 1)!(n - x + 1)!} p^x (1 - p)^{n-x} \left[1 - 4 \left(p - 1/2 \right)^2 \right]$$

$p \in [0,1]$

$$f(x | p) = \frac{n!}{x!(n - x)!} p^x (1 - p)^{n-x}$$
$$f(p) = \frac{3}{2} \left[1 - 4 \left(p - 1/2 \right)^2 \right]$$

$n = 10$
 $x = 6$

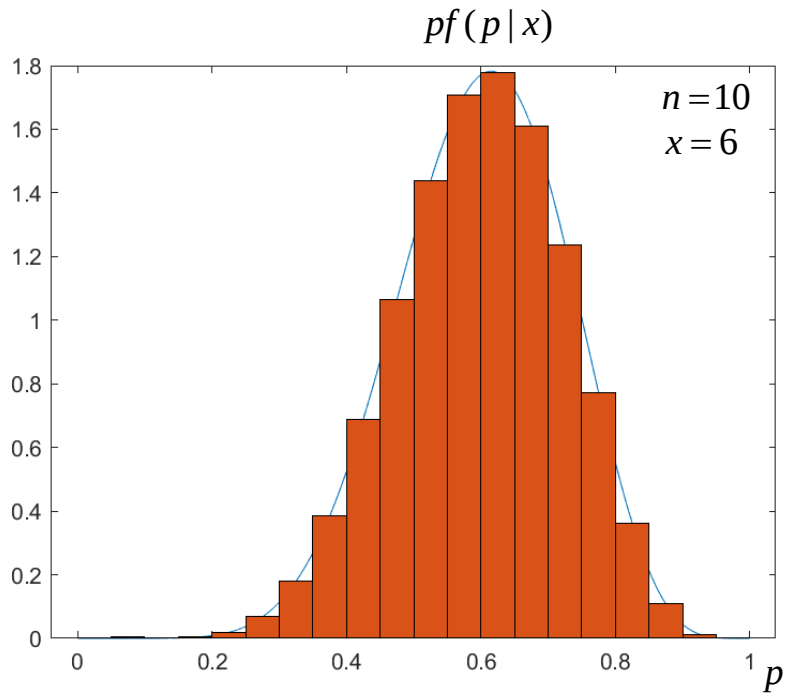
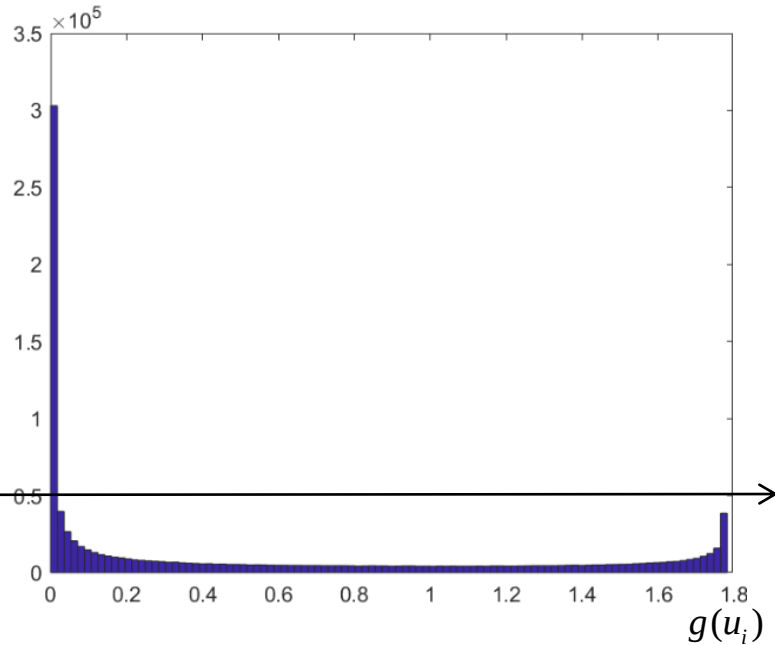
$$E_{10^6}(p | x, n) = 0.5721$$

$n = 10^6 \quad \frac{1}{n} \sum_{i=1}^n g(u_i)$

$$E_{20}(p | x, n) = 0.5714$$

$$\Delta p = 0.05 \quad \Delta p \sum f(p_{mid} | x)$$

same answer for 5 rectangles



Discussion

Life is more difficult when we have non-conjugate priors!

$$f(x_1, \dots, x_n) = \int_{\theta} f(x_1, \dots, x_n | \theta) f(\theta) d\theta$$

$$f(\theta | x_1, \dots, x_n) = \frac{f(x_1, \dots, x_n | \theta) f(\theta)}{f(x_1, \dots, x_n)}$$

$$E(\theta | x_1, \dots, x_n) = \int_{\theta} \theta f(\theta | x_1, \dots, x_n) d\theta$$

Deterministic and Stochastic integration requires $f(x_1, \dots, x_n)$.

For each n .

It may be extremely difficult to get. May need other methods!

Discussion

Questions?

Pencil and Paper Integration

$$E(\theta \mid x_1, \dots, x_n) = \int_{\theta} \theta f(\theta \mid x_1, \dots, x_n) d\theta$$

Deterministic Numerical Integration

$$\hat{E}(\theta \mid \cdot) = \Delta\theta \sum_{i=1}^n g(\theta_i^*)$$

$$\Delta\theta = (b - a) / n$$

$$\theta_i^* = a + \Delta\theta / 2 + (i - 1)\Delta\theta$$

Stochastic Simulation Integration

$$\bar{E}(\theta \mid \cdot) = \frac{1}{n} \sum_{i=1}^n h(u_i)$$

$$a < \theta < b$$

$$u = (\theta - a) / (b - a)$$

$$h(u) = g(a + (b - a)u)(b - a)$$

$$0 < \theta < \infty$$

$$u = 1 / (\theta + 1)$$

$$h(u) = g(1/u - 1) / u^2$$

$$-\infty < \theta < \infty$$

$$u = e^{\theta} / (1 + e^{\theta})$$

$$h(u) = g\left(\ln\left(\frac{u}{1-u}\right)\right) \frac{1}{u(1-u)}$$

Homework 9

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$

1. With the parabolic prior for p (prove integrates to 1)

$$f(p) = \frac{3}{2} \left[1 - 4(p - 1/2)^2 \right] \quad p \in [0,1]$$

and binomial likelihood for x ,

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \quad x = 0,1,\dots,n$$

a) prove that the marginal PDF for x is

$$f(x) = 6 \frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

← Want at least a page of calculus.
Hint: This is a Beta PDF integral.

b) and that the posterior for $p|x$ is

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4(p - 1/2)^2 \right]$$

Homework 9

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$

2. With the posterior PDF for p

$$f(p|x) = \frac{1}{4} \frac{(n+3)!}{(x+1)!(n-x+1)!} p^x (1-p)^{n-x} \left[1 - 4 \left(p - 1/2 \right)^2 \right], \quad x = 0, 1, \dots, n.$$

a) Calculate the mode and prove or disprove that it is 0.5833. $n=10$ and $x=6$
Use deterministic AND stochastic methods.

$$\leftarrow f'(p|x)=0?$$

b) Calculate the mean and prove or disprove that it is 0.5714.
Use both deterministic AND stochastic integration.

$$\leftarrow \int_{p=0}^1 p f(p|x) dp?$$

Homework 9

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$

3*. Use the following non-conjugate prior PDF for p

$$f(p) = \frac{\pi}{2} \cos[\pi(p - 1/2)] \quad p \in [0,1]$$

to combine with the binomial likelihood for $x|p$.

a) Try pencil & paper options to prove $f(p)$ integrates to 1.

b) Try pencil & paper options to calculate $f(x)$ and $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil and paper, calculate with deterministic and stochastic methods.

c) Try pencil & paper options to calculate the mode of $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil & paper, calculate with deterministic and stochastic methods.

d) Try pencil & paper options to calculate the mean of $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil & paper, calculate with deterministic and stochastic methods.

* For students in 5790.

Homework 9

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$
$$p \in [0,1]$$

4**. Make up your own *fun* non-conjugate prior PDF for p

$$f(p) = \text{?????} \quad p \in [0,1]$$

to combine with the binomial likelihood for $x|p$.

a) Try pencil & paper options to calculate $f(x)$ and hence $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

b) Try pencil & paper options to calculate the mode of $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

c) Try pencil & paper options to calculate the mean of $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

** For students that have had MSSC 6010 and 6020.

Homework 9

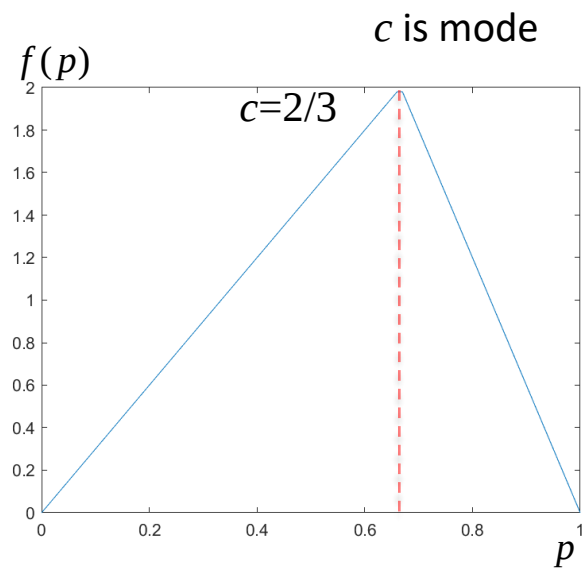
Potential *Fun* Non-Conjugate Priors

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$

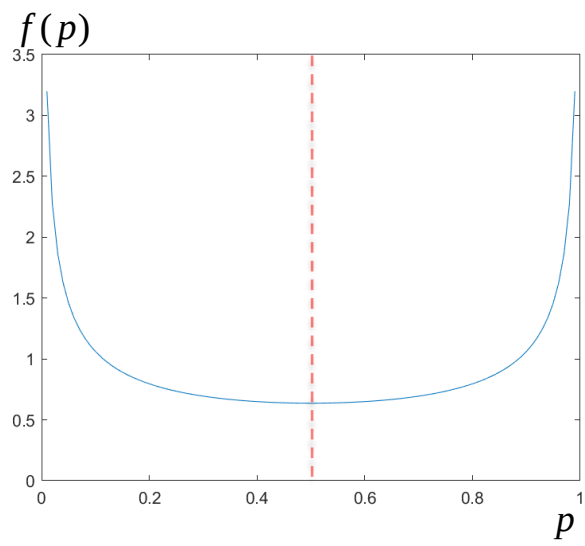
Triangular

$$f(p) = \begin{cases} \frac{2p}{c}, & 0 \leq p \leq c \\ \frac{2(1-p)}{1-c}, & c \leq p \leq 1 \end{cases}$$



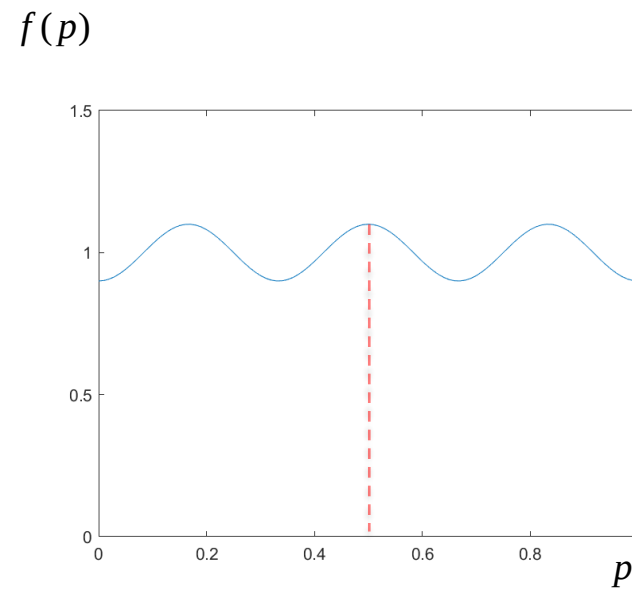
Arcsine U Shaped

$$f(p) = \frac{1}{\pi \sqrt{p(1-p)}}$$



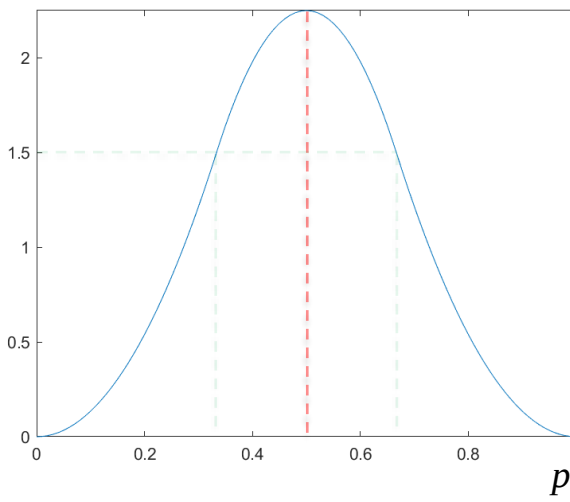
Roller Coaster

$$f(p) = \frac{1}{10} \cos(6\pi p + \pi) + 1$$



3-Piece Quadratic

$$f(p) = \begin{cases} \frac{27}{2} p^2 & 0 \leq p \leq \frac{1}{3} \\ -27(p - \frac{1}{2})^2 + \frac{9}{4} & \frac{1}{3} \leq p \leq \frac{2}{3} \\ \frac{27}{2} (p-1)^2 & \frac{2}{3} \leq p \leq 1 \end{cases}$$



Homework 9

$$f(x|p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

5**. Is there a difference between using conjugate vs non-conjugate priors? $p \in [0,1]$

- a) Are the two posterior PDFs and CDFs different? Make plots.
- b) Are the posterior means and variances different?
- c) Are the 95th percentiles different?

** For students that have had MSSC 6010 and 6020.