Formal Bayesian Approach to GRAPPA Image Reconstruction

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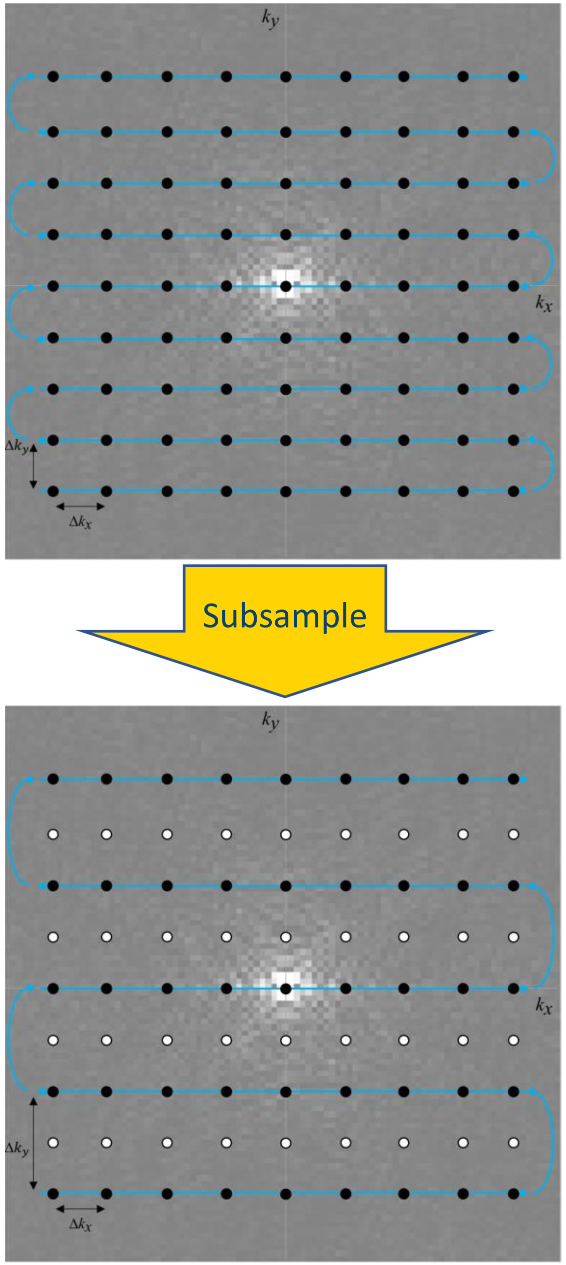
Outline

1. Introduction
2. GRAPPA
3. Bayesian GRAPPA (BGRAPPA)
4. Simulation Study
5. Discussion

1. Introduction

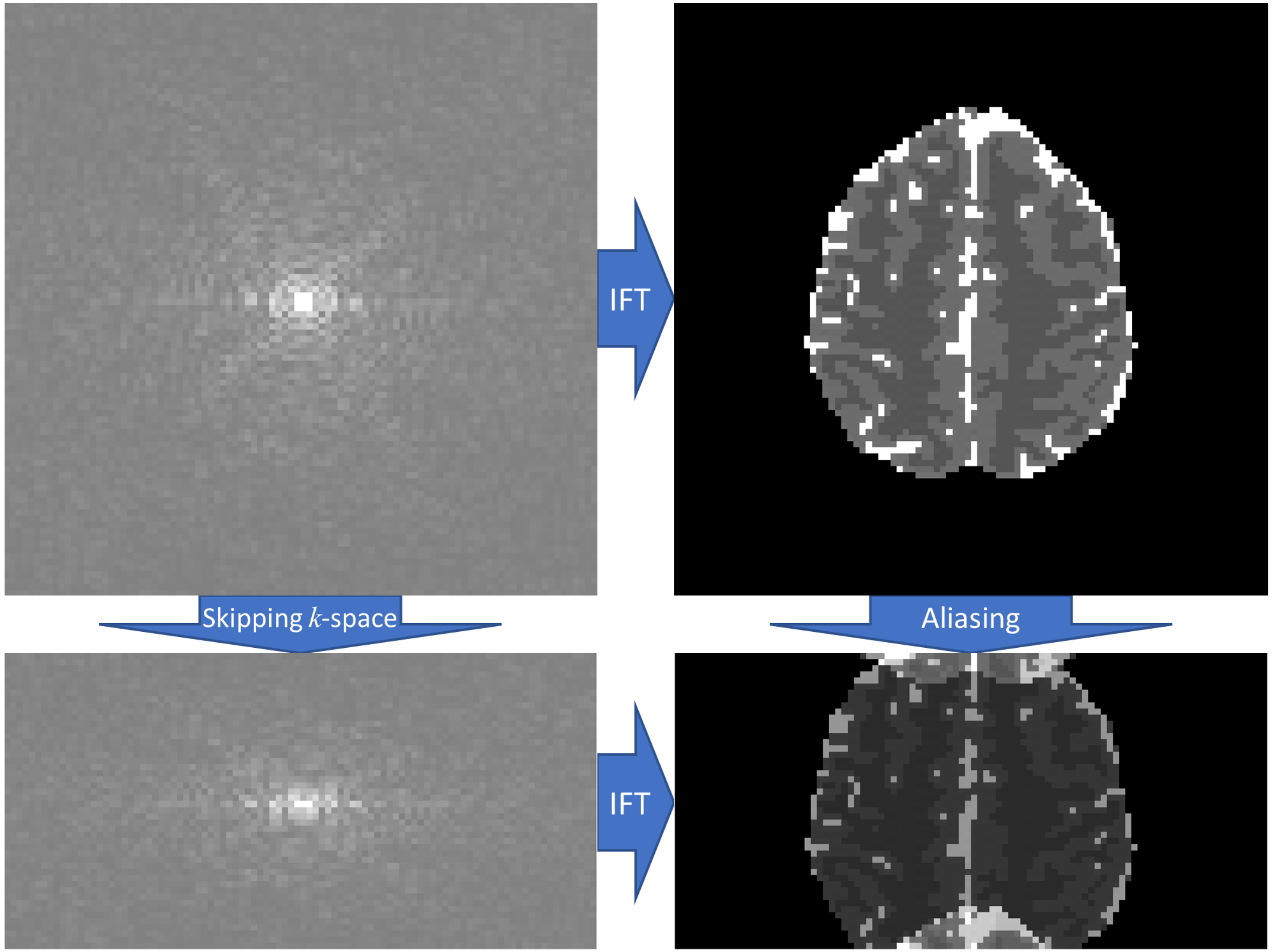
fMRI Background

- Functional Magnetic Resonance Imaging (fMRI) is a noninvasive medical imaging technique that observes the human brain in action
  - Primary goal: Detect brain activity
- Machine uses receiver coils to capture complex-valued arrays of spatial frequencies called k-space
  - Can take a considerable amount of time to fully sample k-space
  - Limits the temporal and spatial resolution of the acquired images which can diminish effectively capturing brain activity
- Solution: Measure less data
  - Subsample spatial frequencies by skipping lines in the sequential acquisition process
  - Causes reconstructed image to be aliased

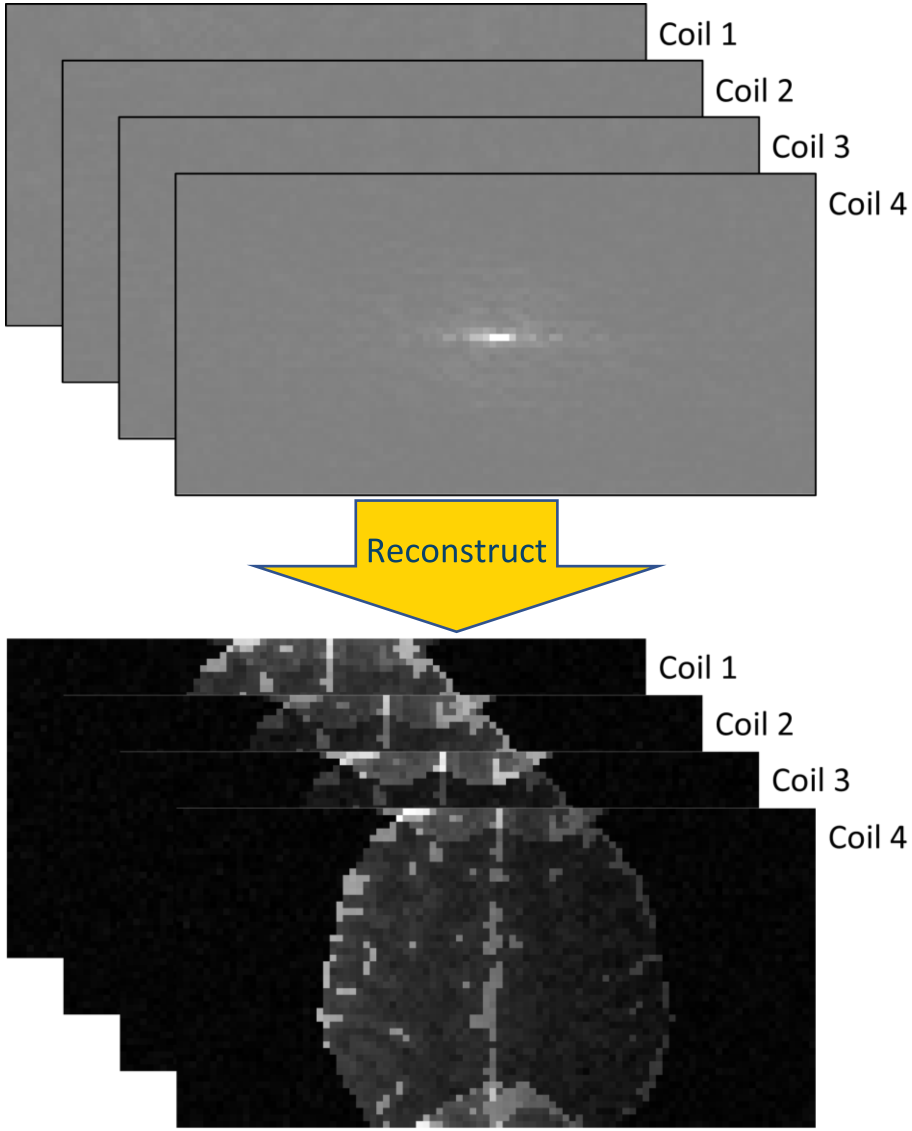


1. Introduction

Subsampling *k*-space



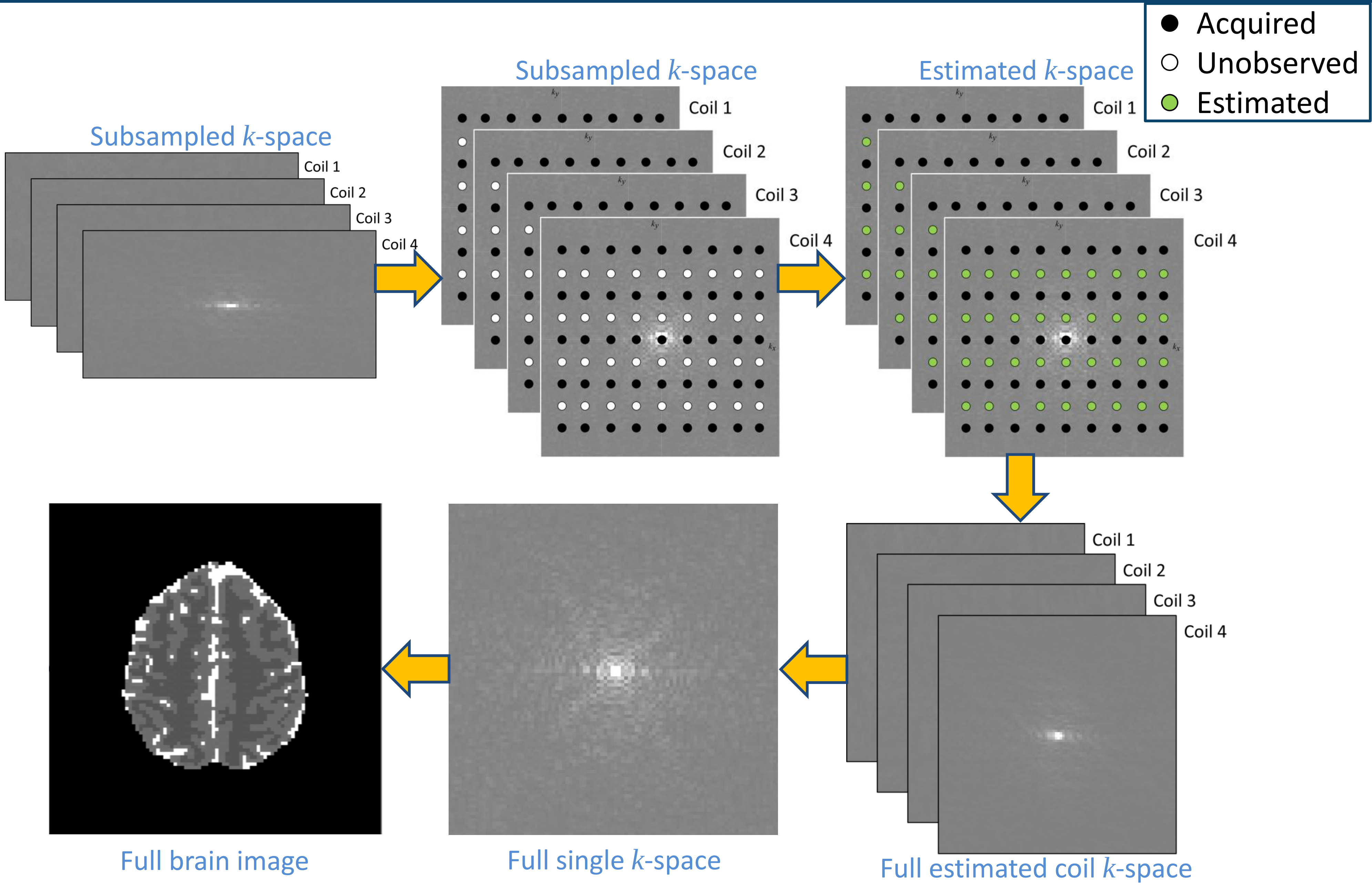
Multiple Coils



2. GRAPPA

Method

- GRAPPA is a parallel imaging technique that utilizes weights to interpolate the missing spatial frequencies
- Estimates weights using full FOV calibration coil k-space arrays acquired before the fMRI experiment

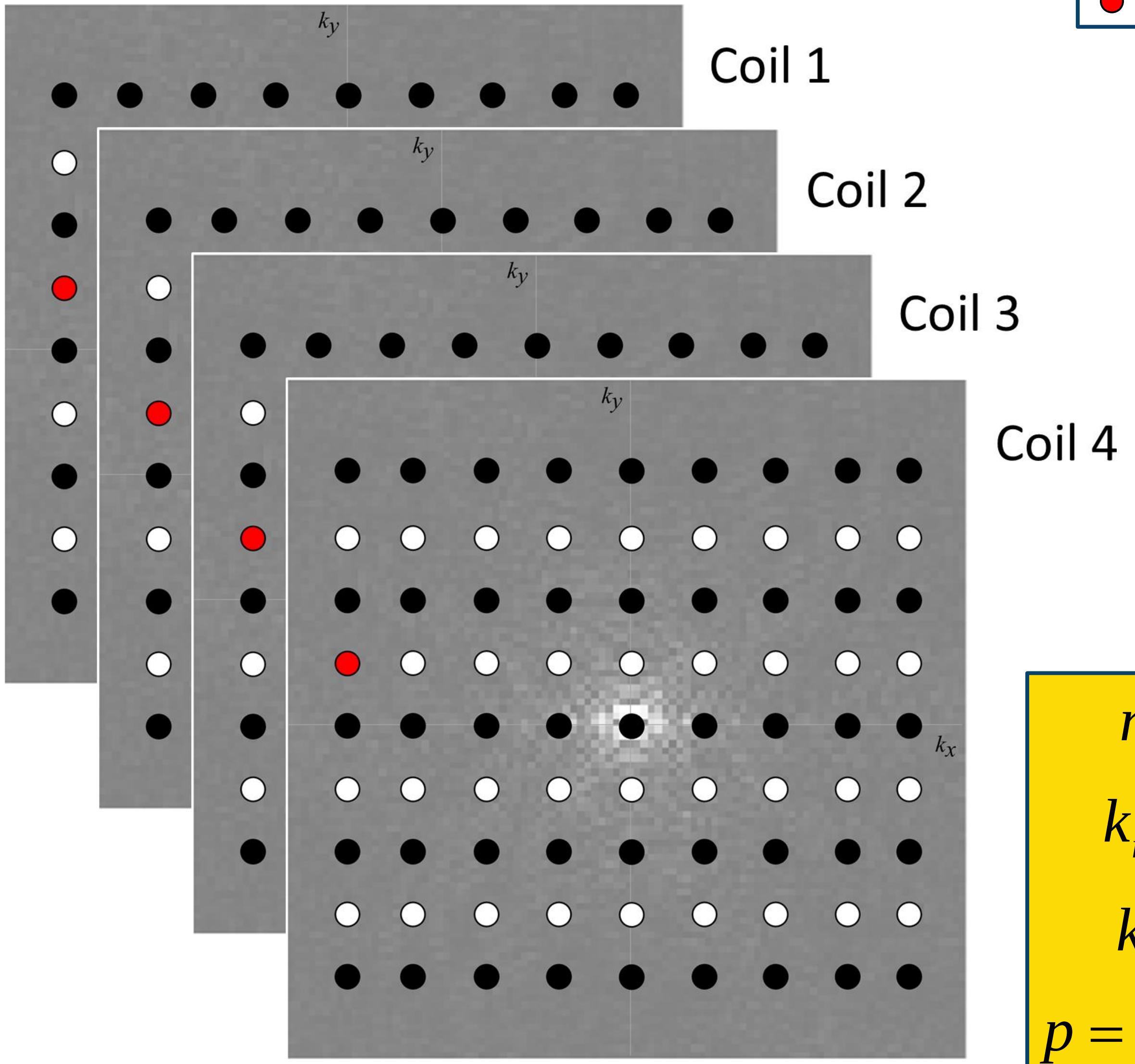


2. GRAPPA

Interpolating – Kernel Size: 2x1

Full Spatial Frequency Arrays

- ● Acquired
- ○ Omitted
- ● Calibration



$$n_C = 4$$

$$k_{rows} = 2$$

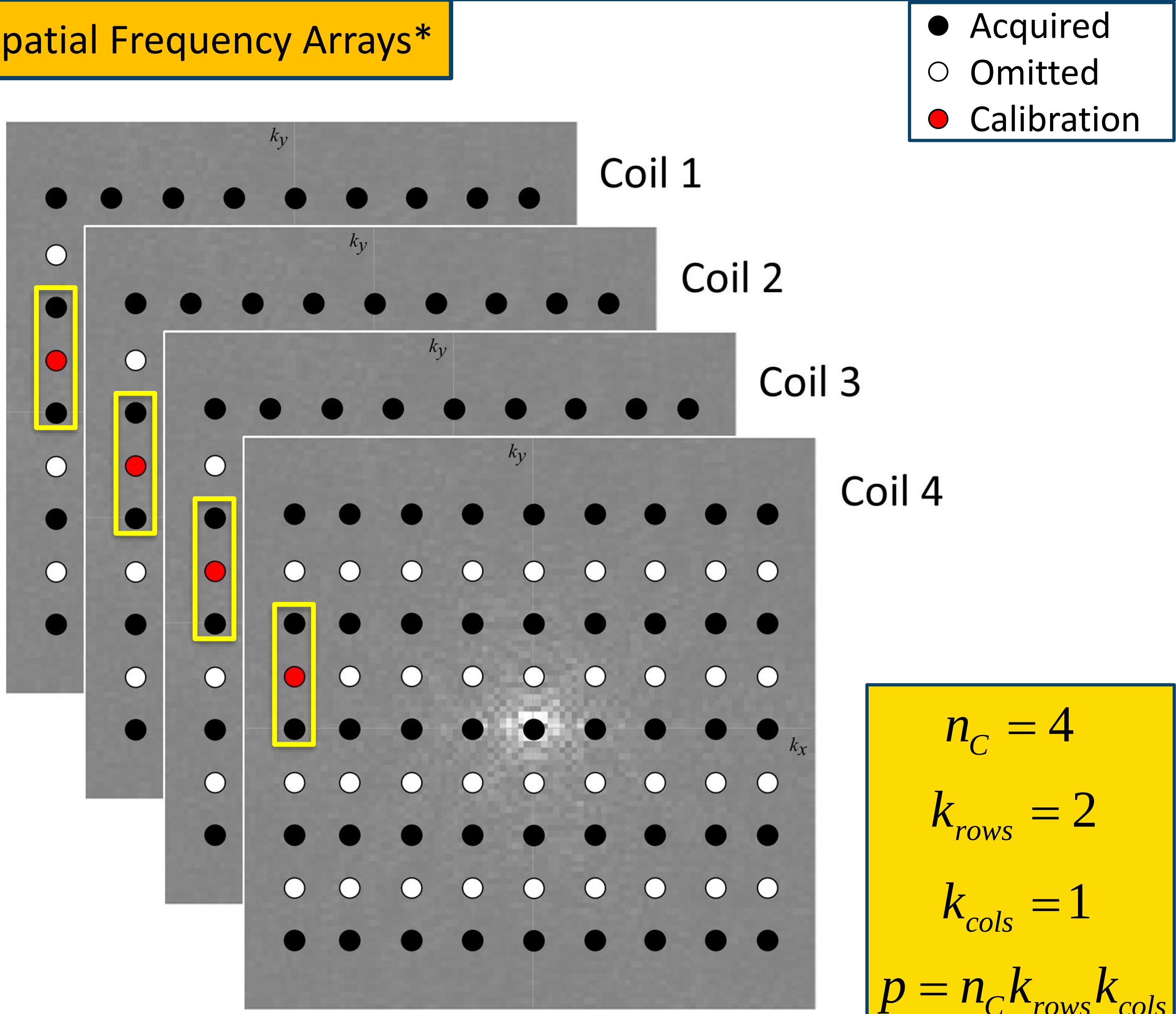
$$k_{cols} = 1$$

$$p = n_C k_{rows} k_{cols}$$

2. GRAPPA

Interpolating – Kernel Size: 2x1

Full Spatial Frequency Arrays



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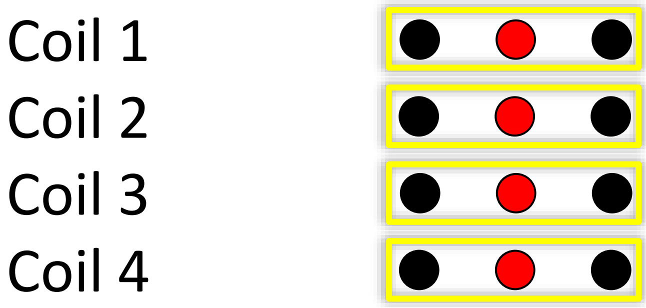
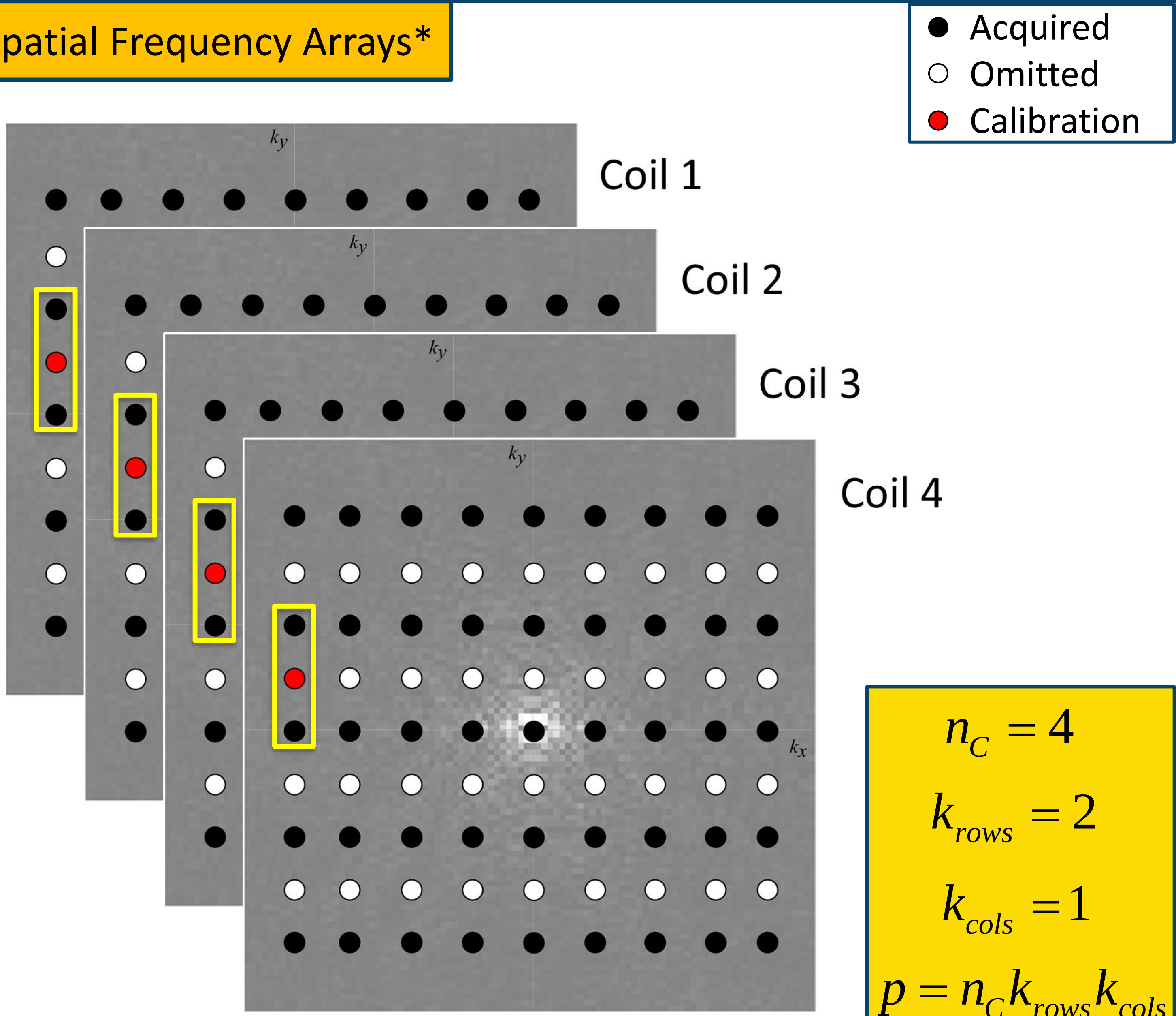
2. GRAPPA

Interpolating – Kernel Size: 2x1

Full Spatial Frequency Arrays





$$n_C = 4$$

$$k_{rows} = 2$$

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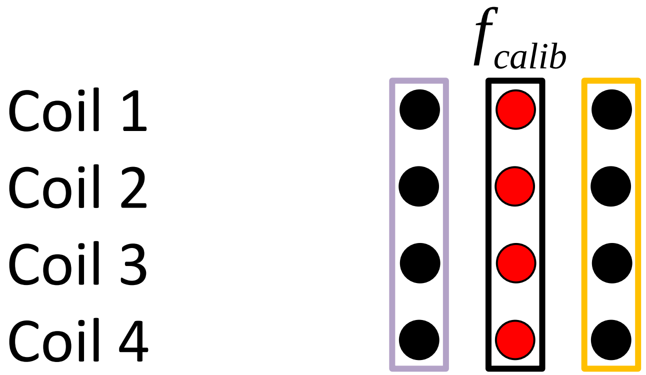
$$p = n_C k_{rows} k_{cols}$$

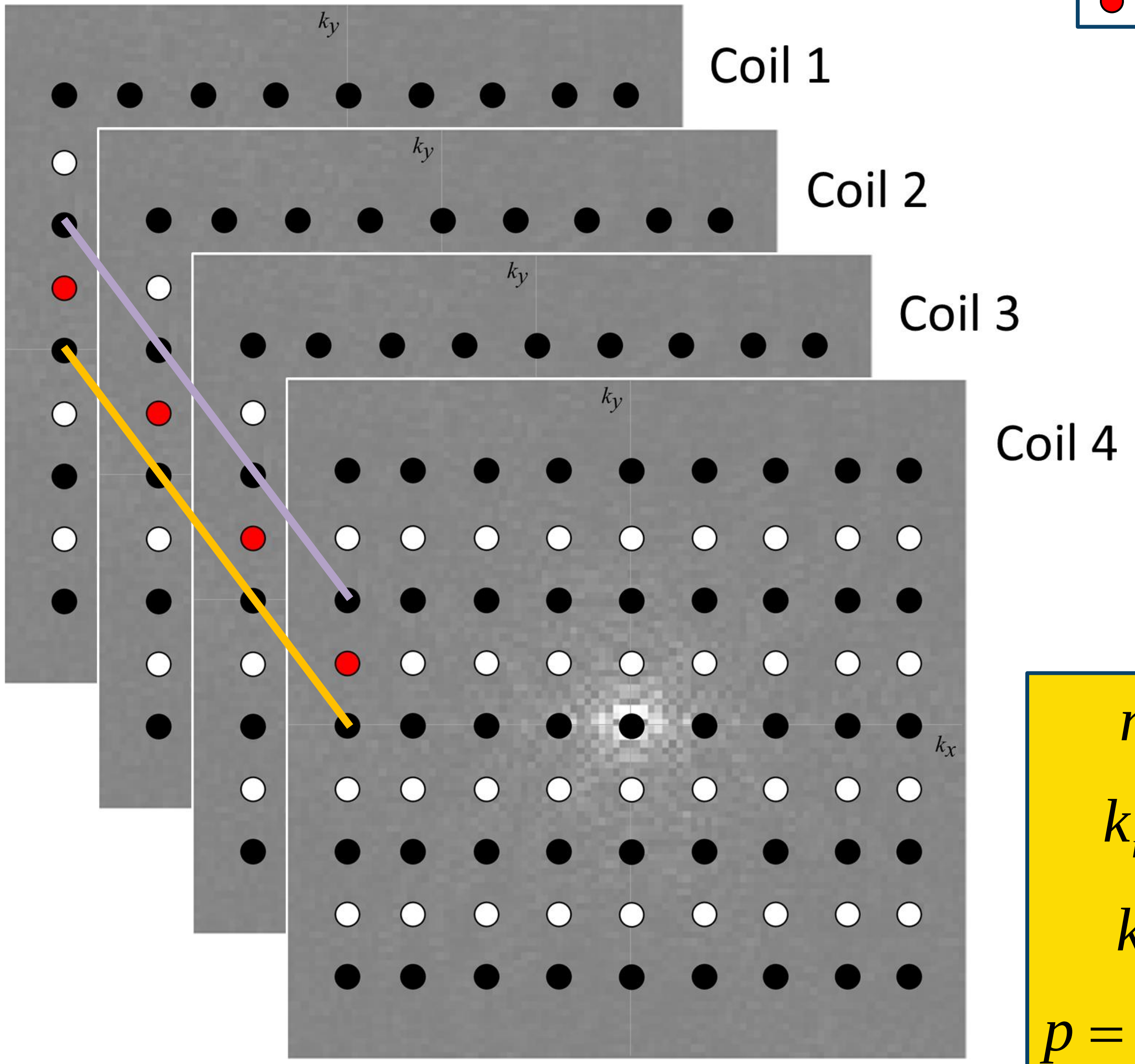
2. GRAPPA

Interpolating – Kernel Size: 2x1

Full Spatial Frequency Arrays

● Acquired
○ Omitted
● Calibration





$$f_{calib} = w f_l$$

$$w = f_{calib} f_l^H \left(f_l f_l^H \right)^{-1}$$

$$n_C = 4$$
$$k_{rows} = 2$$
$$k_{cols} = 1$$
$$p = n_C k_{rows} k_{cols}$$

$f_l = acquired$
$f_{calib} = calibration$

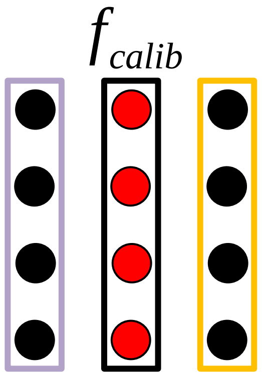
2. GRAPPA

Interpolating – Kernel Size: 2x1

Full Spatial Frequency Arrays

- ● Acquired
- ○ Omitted
- ● Calibration

Coil 1
Coil 2
Coil 3
Coil 4



$$f_{calib} = w f_l$$

$$w = f_{calib} f_l^H \left(f_l f_l^H \right)^{-1}$$

$$\underset{n_C \times 1}{f_{calib}} = \underset{n_C \times p}{w} \underset{p \times 1}{f_l}, \quad w = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1p} \\ w_{21} & w_{22} & \cdots & w_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_C 1} & w_{n_C 2} & \cdots & w_{n_C p} \end{bmatrix}$$

$$n_C = 4$$
$$k_{rows} = 2$$
$$k_{cols} = 1$$
$$p = n_C k_{rows} k_{cols}$$

$f_l = acquired$
$f_{calib} = calibration$

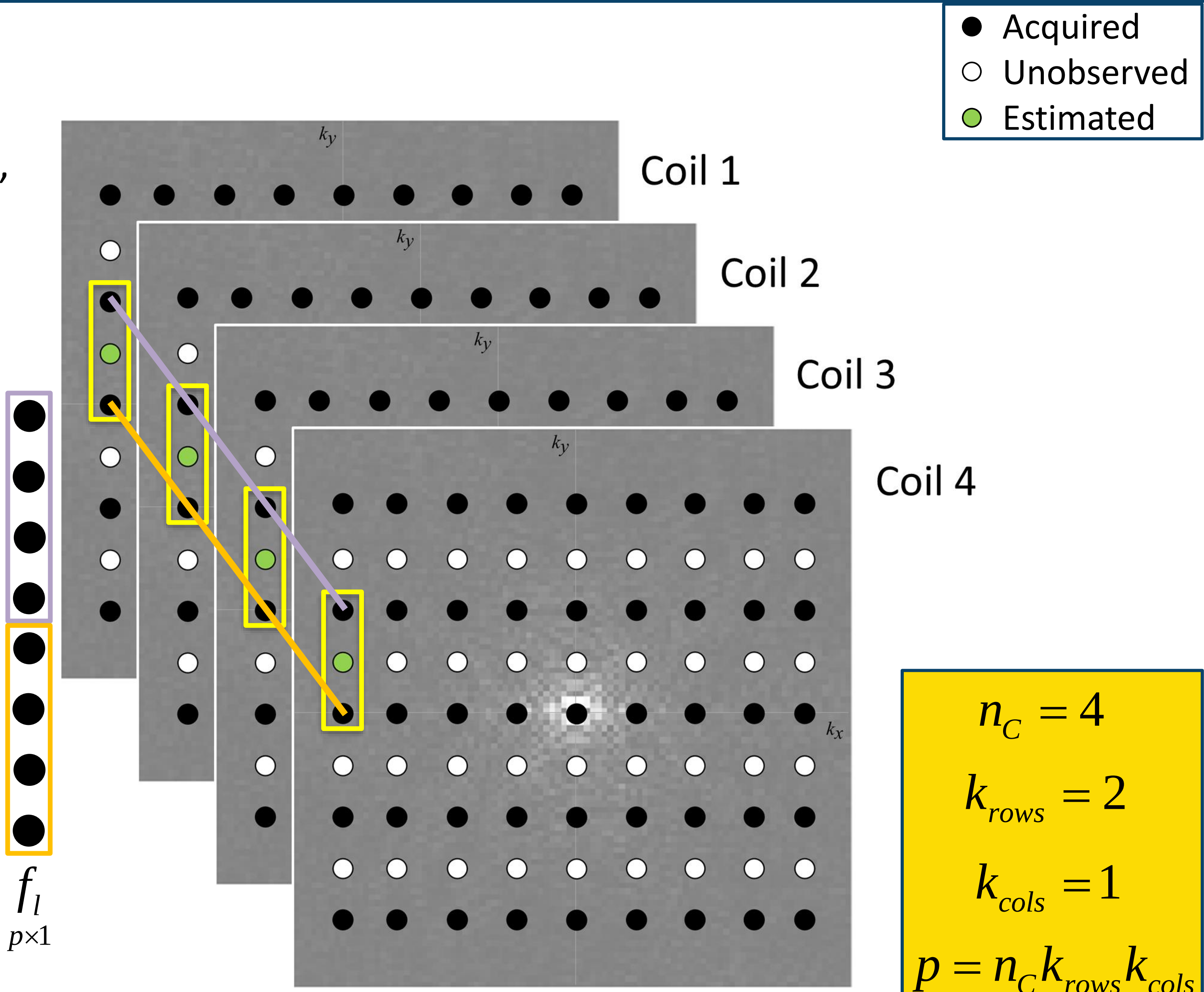
2. GRAPPA

Estimating Missing Spatial Frequencies

- Once when the weights are calculated, they are used to interpolate the missing spatial frequencies
- Weights are used for each time point in the fMRI series

$$\underset{n_C \times 1}{f_k} = \underset{n_C \times p}{w} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1p} \\ w_{21} & w_{22} & \cdots & w_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n_C 1} & w_{n_C 2} & \cdots & w_{n_C p} \end{bmatrix}$$

$\underset{p \times 1}{f_l}$



$f_l = acquired$
$f_k = estimated$

$$f_k = wf_l$$

$n_C = 4$

$k_{rows} = 2$

$k_{cols} = 1$

$p = n_C k_{rows} k_{cols}$

3. Bayesian GRAPPA (BGRAPPA)

Model Parameters

- ● Acquired
- ○ Omitted
- ● Calibration

Subsampled k-space array

GRAPPA

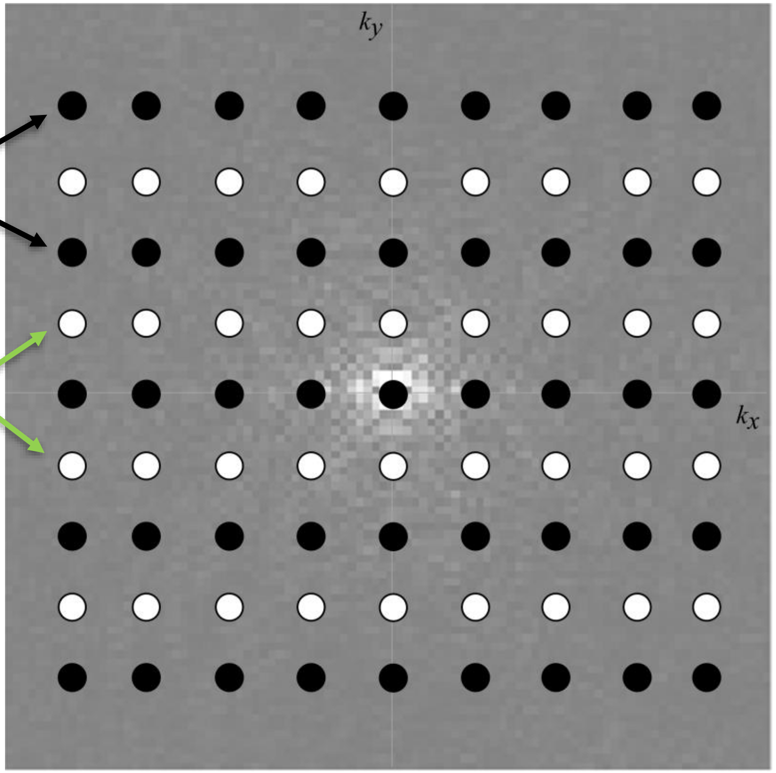
$$f_k = wf_l + \eta$$

f_l : *observed*

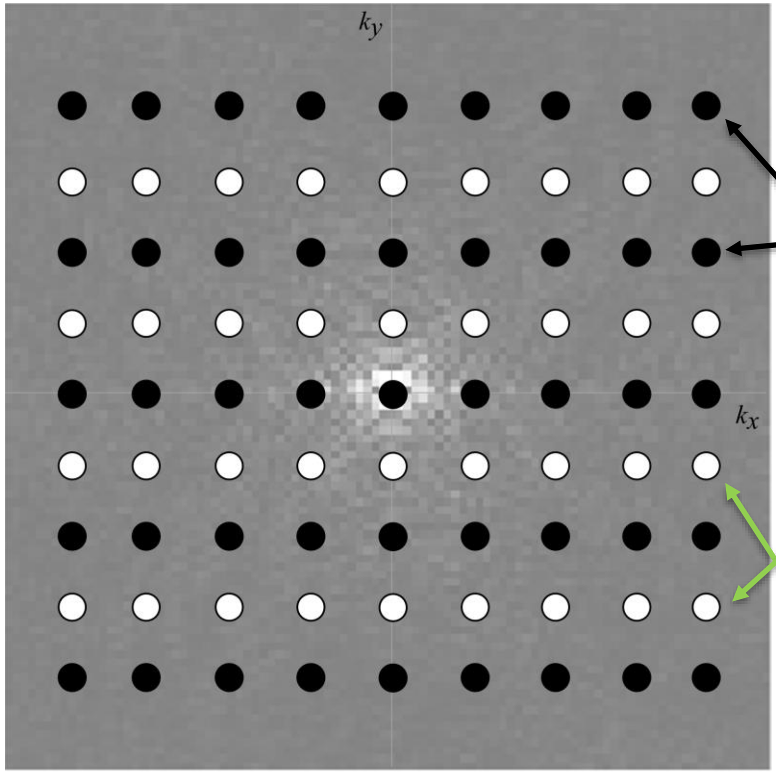
w : *estimated*

f_k : *unobserved*

**What we are interested in



BGRAPPA



$$f_k = wf_l + \eta$$

$$\eta \sim N(0, \tau^2 I_{2n_C})$$

f_k : *observed*

w : *unknown(prior)*

τ^2 : *unknown(prior)*

f_l : *unknown(prior)*

**What we are interested in

3. Bayesian GRAPPA (BGRAPPA)

Model, Likelihood, and Prior Distribution

$n_C = number\ of\ coils$
$n_A = acceleration\ factor$

- Subsampled k-space measurements are observed with random error
  - $f_k = wf_l + \eta$, where $\eta \sim N(0, \tau^2 I_{2n_C})$
- Data Likelihood
  - $P(f_k | w, f_l, \tau^2) \propto (\tau^2)^{-\frac{2n_C}{2}} \exp\left[-\frac{1}{2\sigma^2}(f_k - wf_l)'(f_k - wf_l)\right]$ ⟵ Normal
- Priors
  - $P(D | n_w, D_0, \tau^2) \propto (\tau^2)^{-\frac{2n_C n_A}{2}} \exp\left[-\frac{n_w}{2\tau^2} tr[(D - D_0)'(D - D_0)]\right]$ ⟵ Normal
  - $P(f_l | n_l, f_{l0}, \tau^2) \propto (\tau^2)^{-\frac{2n_A}{2}} \exp\left[-\frac{n_l}{2\tau^2}(f_l - f_{l0})'(f_l - f_{l0})\right]$ ⟵ Normal
  - $P(\tau^2 | \alpha_k, \delta) \propto (\tau^2)^{-(\alpha_k+1)} \exp\left[-\frac{\delta}{\tau^2}\right]$ ⟵ Inverse Gamma
- Assessed Hyperparameters: n_w, D_0, n_l, f_{l0}, α_k, and δ
- Posterior
  - $P(D, f_l, \tau^2 | f_k) \propto P(f_k | w, f_l, \tau^2) P(D | n_w, D_0, \tau^2) P(f_l | n_l, f_{l0}, \tau^2) P(\tau^2 | \alpha_k, \delta)$

$$\underset{2n_C \times 1}{f_k} = \begin{bmatrix} f_{lR} \\ f_{lI} \end{bmatrix}$$

$$\underset{2n_c \times 2n_A}{w} = \begin{bmatrix} w_R & -w_I \\ w_I & w_R \end{bmatrix}$$

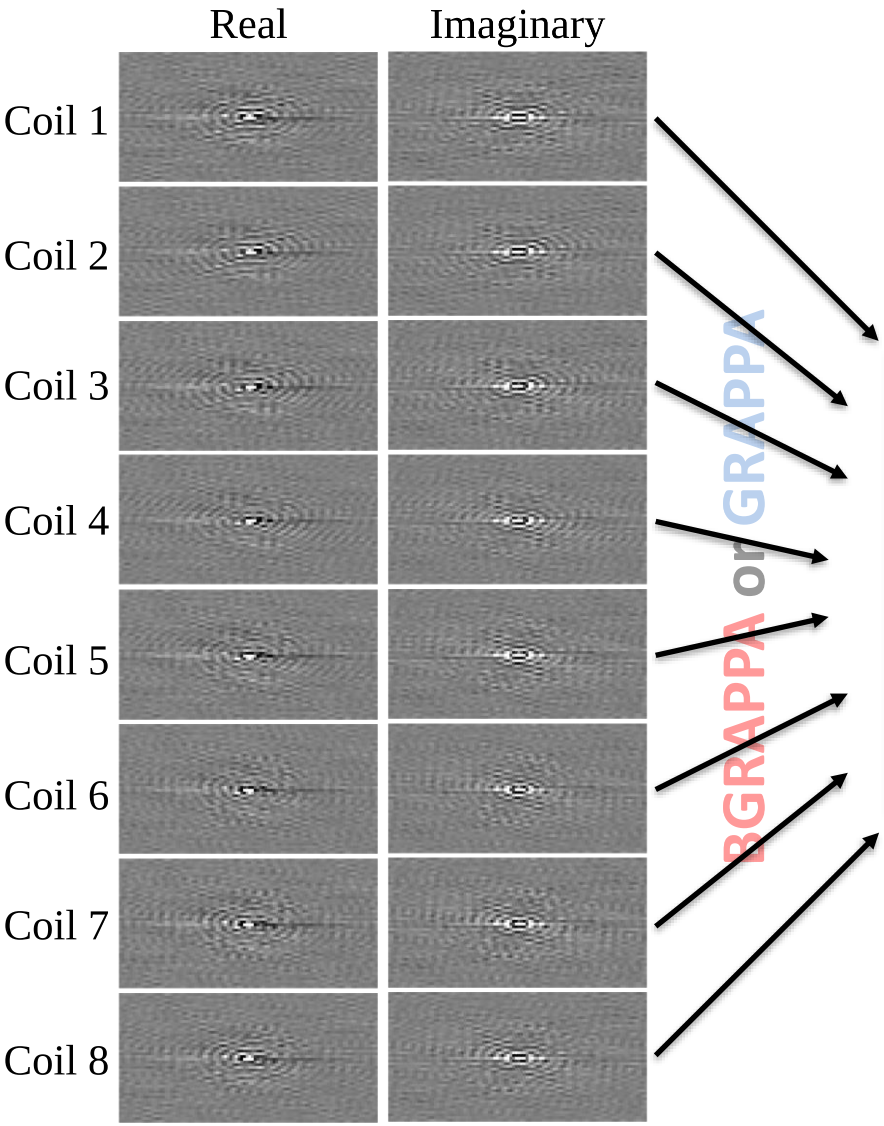
$$\underset{n_c \times 2n_A}{D} = \begin{bmatrix} w_R & w_I \end{bmatrix}$$

$$\underset{2n_A \times 1}{f_l} = \begin{bmatrix} f_{lR} \\ f_{lI} \end{bmatrix}$$

4. Simulation Study

Simulated Data

- 490 time points in the simulated fMRI time series
  - Started with 510 time points discarding first 20 to mimic experimental fMRI
- 30 calibration time points utilized for hyperparameter assessment
  - Calibration time points from a separate simulated series
- Number of coils used is 8 with an acceleration factor of 3
- 2x1 kernel size used for the hyperparameter assessment and parameter estimation
- Reconstruction Method: MAP estimate via ICM for BGRAPPA



Full coil *k*-spaces are averaged together to form one full *k*-space then reconstructed to a full image using the IFT

*Averaging the coil arrays does not properly scale the image since it does not utilize coil sensitivities in the reconstruction

4. Simulation Study

$n_C = number\ of\ coils (8)$

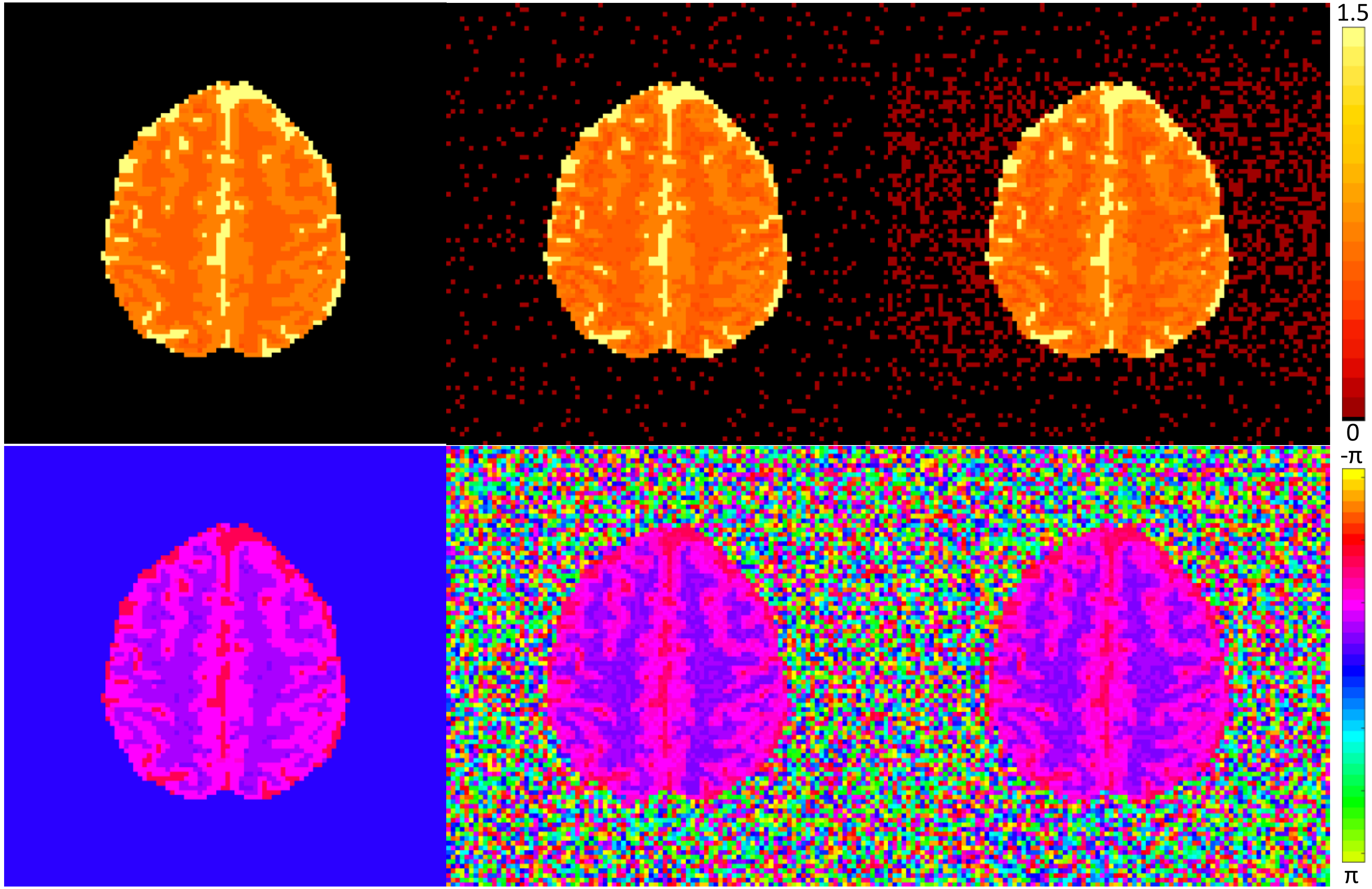
$n_A = acceleration\ factor (3)$

Reconstructed Images for One Time Point

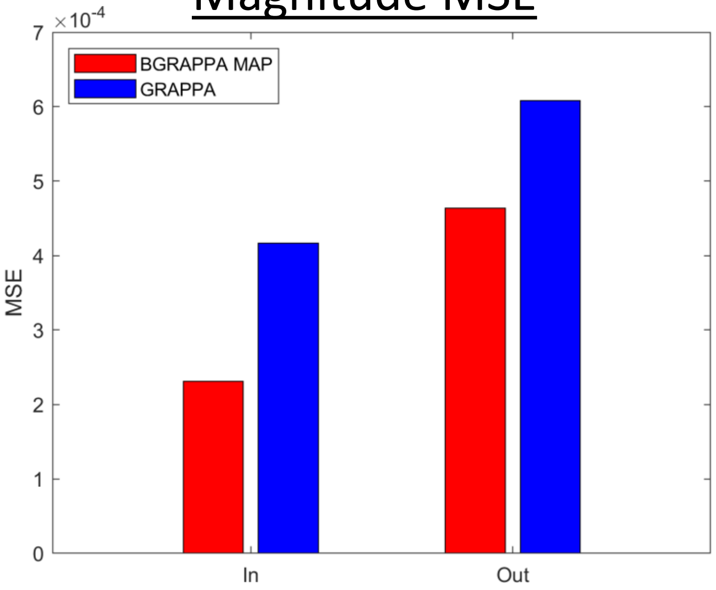
TRUE **BGRAPPA MAP** **GRAPPA MLE**

Magnitude

Phase



Magnitude MSE



- Decreased noise inside and outside the brain for BGRAPPA
- This leads to more accurate reconstruction when compared to the true simulated images
- The MSE plot above shows the larger MSE for the GRAPPA magnitude image (which is statistically significant)

4. Simulation Study

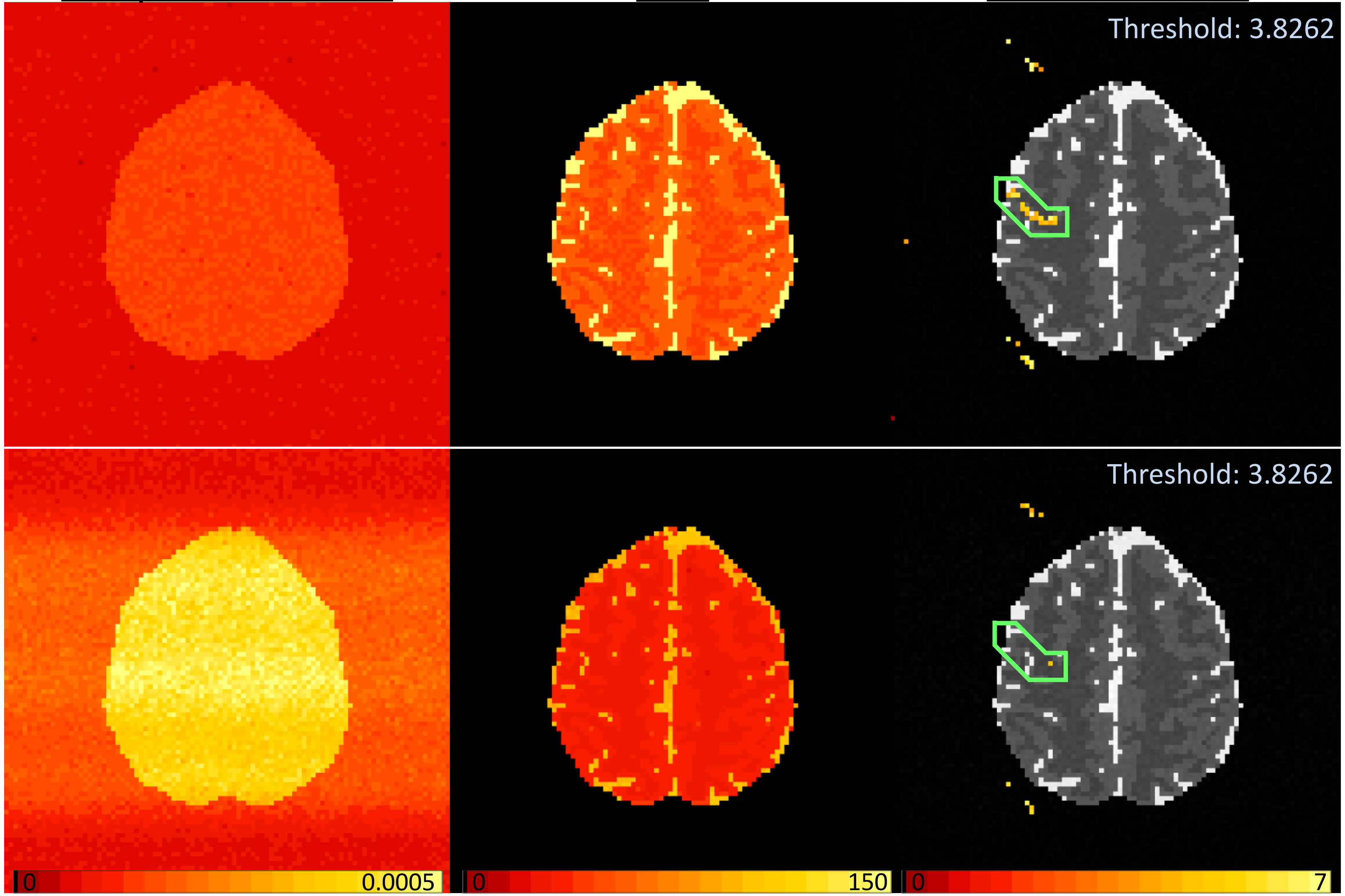
Analysis for Time Series

$n_C = number\ of\ coils (8)$
$n_A = acceleration\ factor (3)$

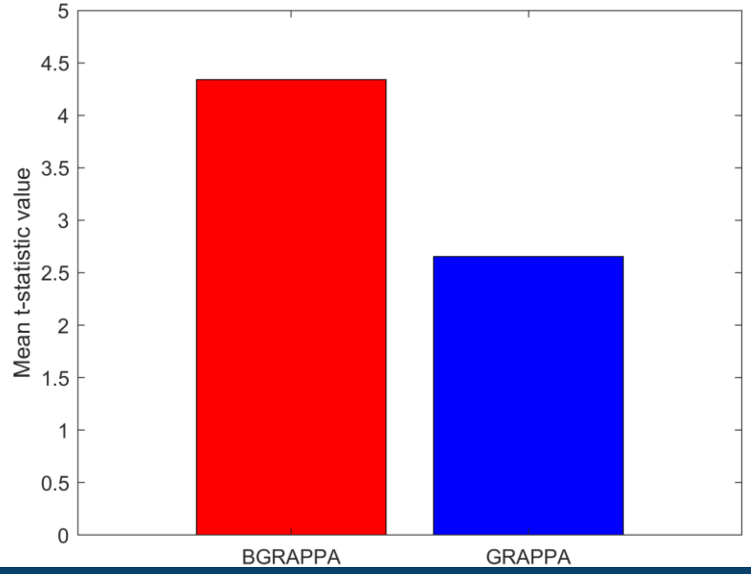
Temporal Variance | SNR | Task Detection

BGRAPPA MAP

GRAPPA MLE



- Lower temporal variance for the BGRAPPA reconstructed time series which leads to higher SNR
- With an FDR threshold of 3.8262, BGRAPPA identified 18/28 task voxels in the ROI while GRAPPA identified 1
- The plot below shows BGRAPPA having a higher mean *t*-stat values in the ROI, increasing the power of task detection



5. Discussion

Conclusion and Future Work

- BGRAPPA is a Bayesian approach to GRAPPA which incorporates more valuable prior information in estimating the missing spatial frequency values
- BGRAPPA reconstructed images more accurately than GRAPPA while decreasing temporal variation which increased SNR and task detection power
- Future work:
  - Analyze correlation between previously aliased voxels and all other voxels in the image
  - Potential bootstrapping of the calibration images
  - Apply BGRAPPA to experimental fMRI data and compare to GRAPPA reconstruction along the way

Thank You

Questions?