The Past, Present, and Future of Complex-Valued FMRI Activation

Daniel B. Rowe
Program in Computational Sciences
Department of Mathematical and Statistical Sciences
Marquette University

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OUTLINE

- **Introduction**

  FMRI Data is Truly Complex-Valued

- **Past**

  Historical Magnitude-Only Analysis

  Complex-Valued R-I or M-P Analysis

- **Present**

  Complex-Valued Time Series Analysis

  Complex-Valued Bayesian Analysis

- **Future**

  Bayesian image reconstruction.

  Activation directly from *k*-space

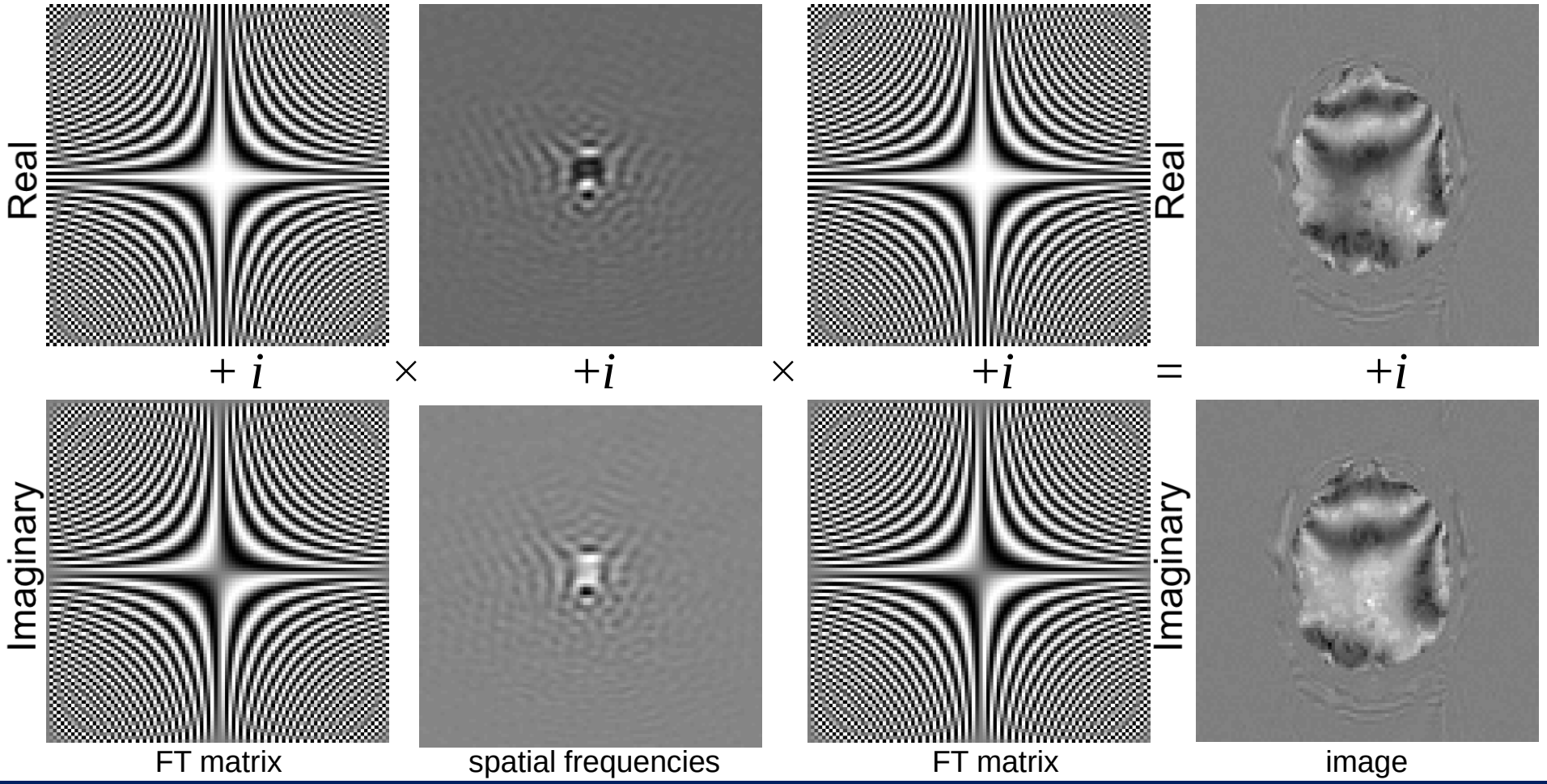
- **Discussion**

  Increased sensitivity/specificity, information, opportunities.

*Similar presented at 2006 JSM

Introduction: Data Complex

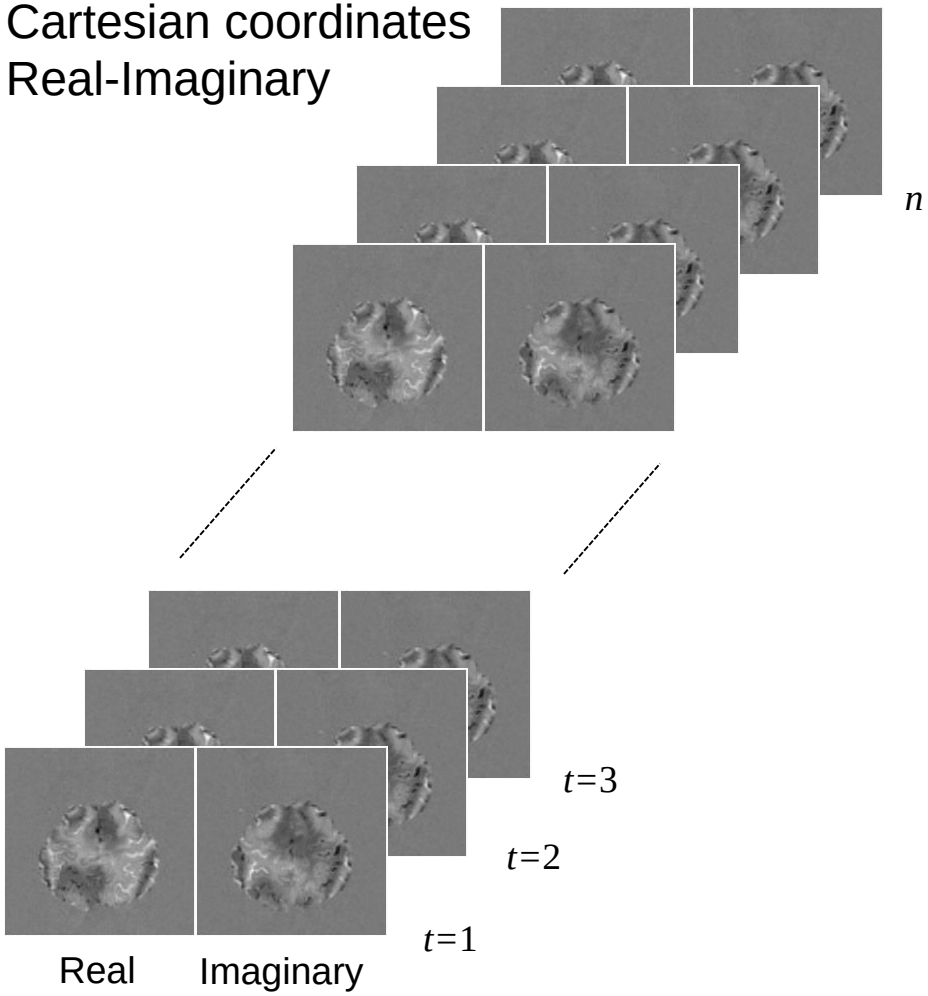
We inverse Fourier transform spatial freqs. to generate image.



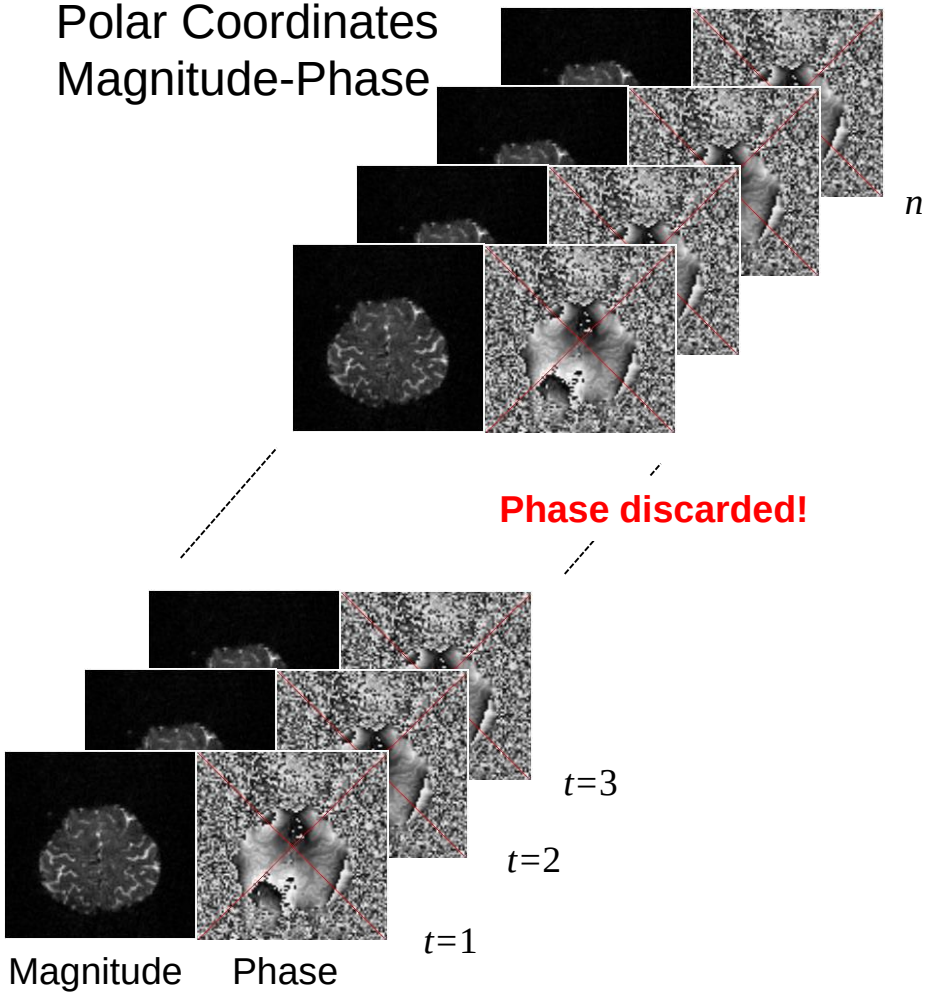
Introduction: Data Complex

***Similar presented at 2004 JSM**

Cartesian coordinates
Real-Imaginary



Polar Coordinates
Magnitude-Phase



Past: Historical Magnitude-Only Analysis

- Thulborn et al. 1982 showed deoxyhemoglobin changed T2*.
- Ogawa et al. 1990 activation in rats with contrast agent.
- Bandettini et al. 1992, Kwong et al. 1992, Ogawa et al. 1992 in close succession demonstrated T2* activation with task
- Bandettini et al. 1993 processing strategies for fMRI
- Cox et al. 1995 real time fMRI correlation activation
- Friston et al. 1995 GLM fMRI activation

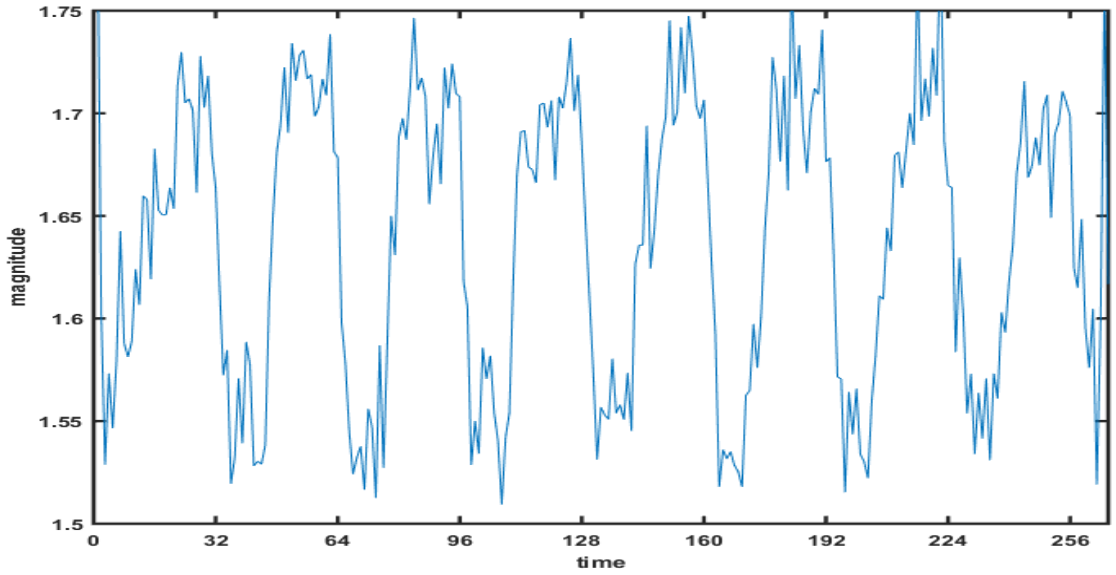
Since early 90's, magnitude-only analysis the norm.

Past: Historical Magnitude-Only Analysis

A linear model is applied to magnitude-only time series

$$y = \beta_0 + \beta_1 x_q + \ldots + \beta_q x_q + \varepsilon \qquad E(\varepsilon) = 0 \qquad \text{cov}(\varepsilon) = \Sigma$$

Statistical significance for task coefficient!



Past: Complex-Valued R-I or M-P Analysis

- Lai and Glover 1997.

$$cc = \frac{\left\{\left[\Sigma(x - \bar{x})\,(r - \bar{r})\right]^2 + \left[\Sigma(y - \bar{y})\,(r - \bar{r})\right]^2\right\}^{1/2}}{(N-1)\,\sigma_r \sqrt{\sigma_x^2 + \sigma_y^2}}$$

- Nan and Nowak 1999.

$$\mathbf{x} = (a\mathbf{1} + b\mathbf{r})(\cos\vartheta + i\sin\vartheta) + \sigma.\mathbf{n}_c.$$

$$L_3(\mathbf{y}) = \frac{2\|P_{\mathbf{S}}^{\perp}\mathbf{y}\|^2}{\|P_{\mathbf{H}}^{\perp}\mathbf{y}\|^2 + \|P_{\mathbf{S}}^{\perp}\mathbf{y}\|^2 - \sqrt{\|P_{\mathbf{H}}\mathbf{y}\|^4 + \|P_{\mathbf{S}}\mathbf{y}\|^4 + 2\|P_{\mathbf{H}}\mathbf{y}\|^2\|P_{\mathbf{S}}\mathbf{y}\|^2\cos 2\varphi}}$$

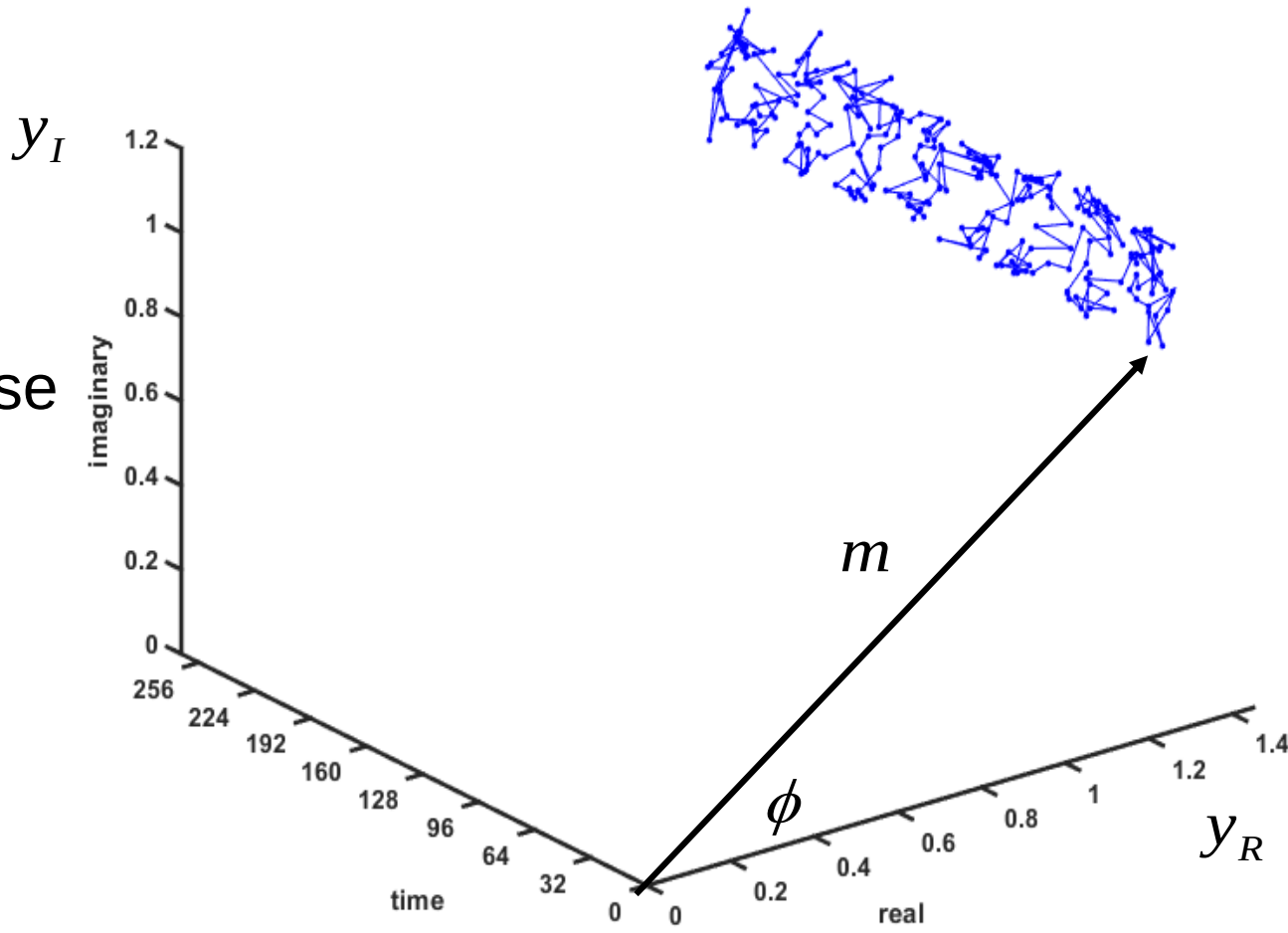
These initial models were limiting.

Past: Complex-Valued R-I or M-P Analysis

***Similar presented at 2004 JSM**

Complex-valued activation and/or Complex-valued correlation?

Magnitude and Phase or equivalently Real and Imaginary

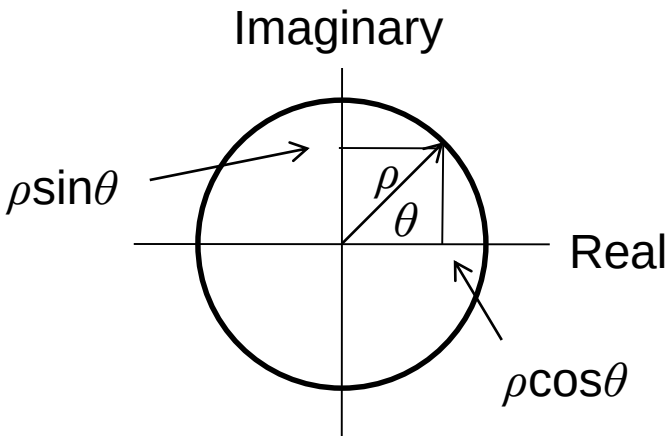


Past: Complex-Valued R-I or M-P Analysis

Voxel measurements are described as
(absorption/dispersion, in-phase/quadrature)

$$y_R = \rho \cos\theta + \eta_R$$

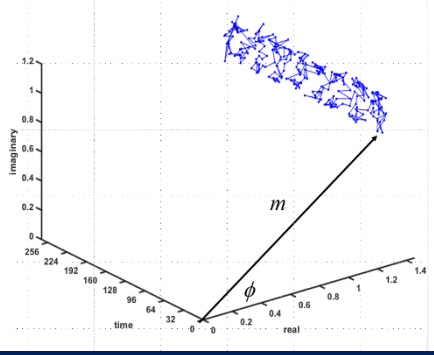
$$y_I = \rho \sin\theta + \eta_I$$



y_R and y_I are measurements for the real and imaginary parts

η_R and η_I are error terms for the real and imaginary parts

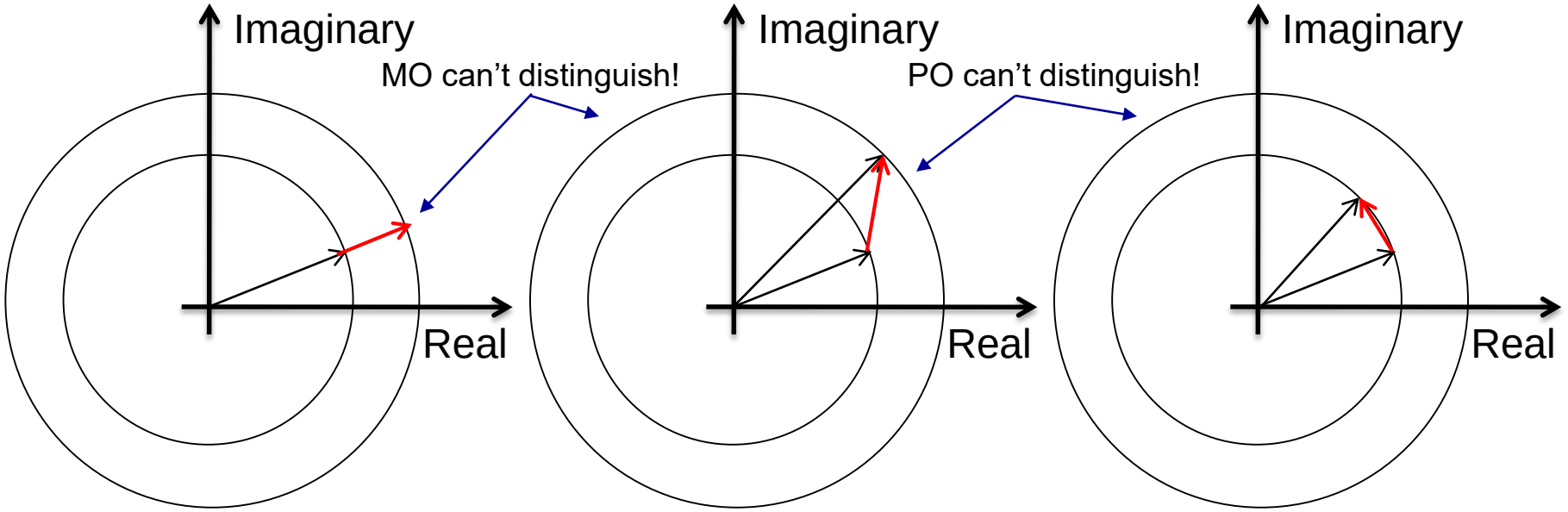
ρ and θ are the population magnitude and phase.



*Similar presented at 2006 JSM

Past: Complex-Valued R-I or M-P Analysis

Three non-zero changes possible



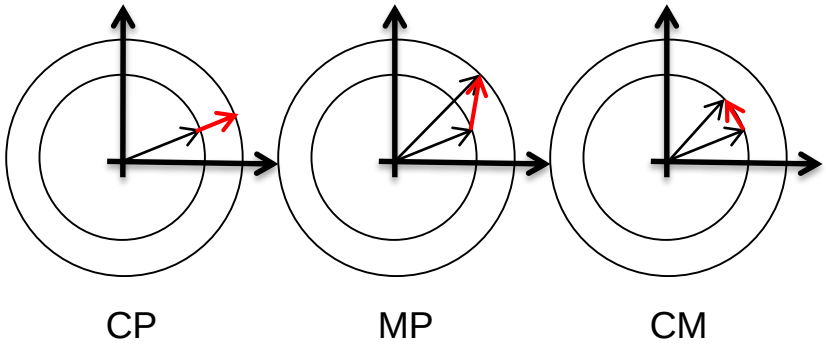
- Complex Magnitude w/ Constant Phase (CP) Activation[1,2]
- Complex Magnitude and/or Phase (MP) Activation[3,7]
- Real Magnitude-Only (MO/UP) Activation (Discard Phase)[4,5]
- Complex Phase Constant Magnitude (CM) Activation[6]

[1]Rowe and Logan: NIMG, 23:1078-1092, 2004.
[2]Rowe: NIMG 25:1124-1132, 2005a.
[3]Rowe: NIMG, 25:1310-1324, 2005b.
[4]Bandettini et al.: MRM, 30:161-173, 1993.
[5]Rowe and Logan: NIMG 24:603-606, 2005.
[6]Rowe, et al.: JNSM, 161:331-341, 2007.
[7]Rowe: MRM, 62:1356-1357, 2009.

Past: Complex-Valued R-I or M-P Analysis

In each voxel at time t:

$$\begin{pmatrix} y_{Rt} \\ y_{It} \end{pmatrix} = \begin{pmatrix} \rho_t \cos\theta_t \\ \rho_t \sin\theta_t \end{pmatrix} + \begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix}$$



CP[1,2]: $\rho_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t}$

$\theta_t = \theta$

MO/UP[4,5]: $\rho_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t}$

$\theta_t \neq \theta_{t'}$

MP[3,7]: $\rho_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t}$

$\theta_t = \gamma_0 + \gamma_1 u_{1t} + \cdots + \gamma_{q_2} u_{q_2 t}$

PO/UM[6]: $\beta_t \neq \beta_{t'}$

$\theta_t = \gamma_0 + \gamma_1 u_{1t} + \cdots + \gamma_{q_2} u_{q_2 t}$

$$E\begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} = 0, \ \text{cov}\begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} = \Sigma_t$$

[1]Rowe and Logan: NIMG, 23:1078-1092, 2004.
[3]Rowe: NIMG, 25:1310-1324, 2005b.
[5]Rowe and Logan: NIMG 24:603-606, 2005.
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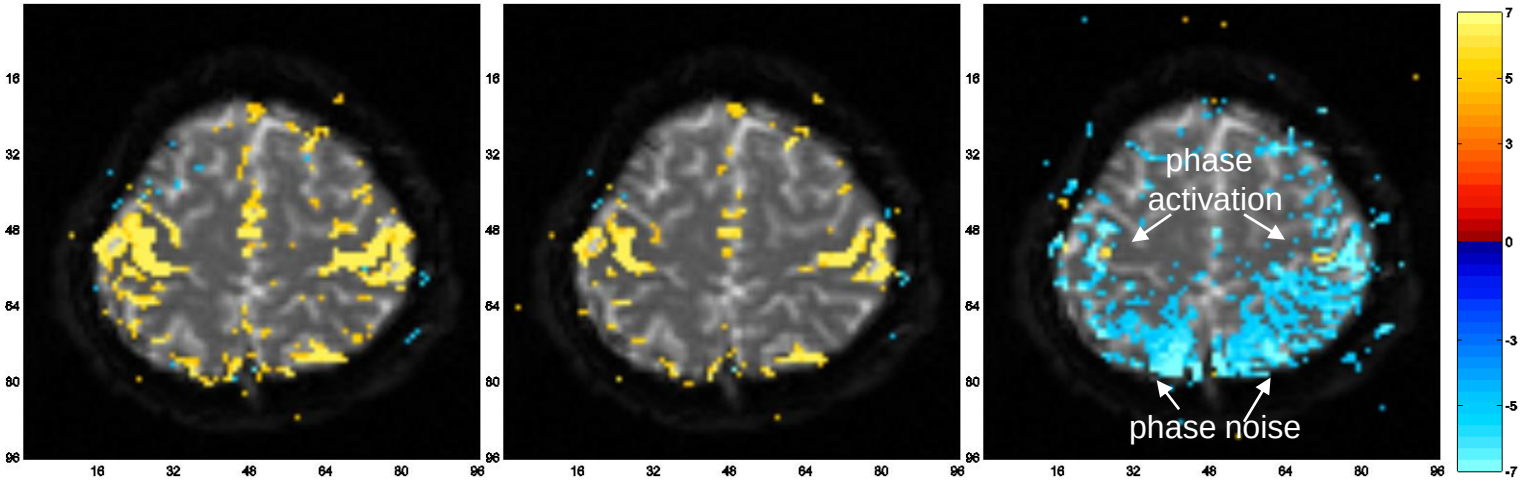
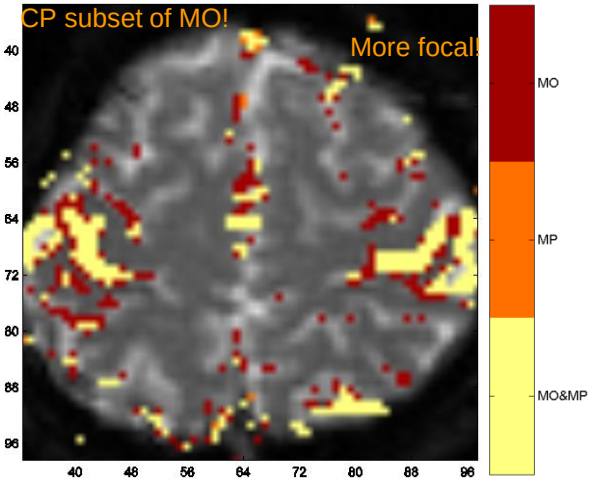
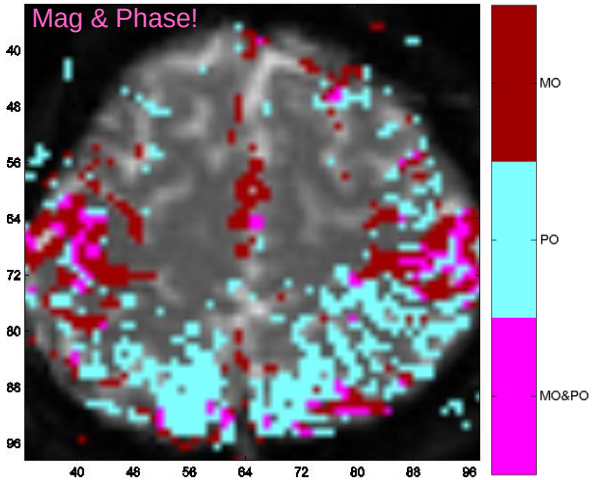
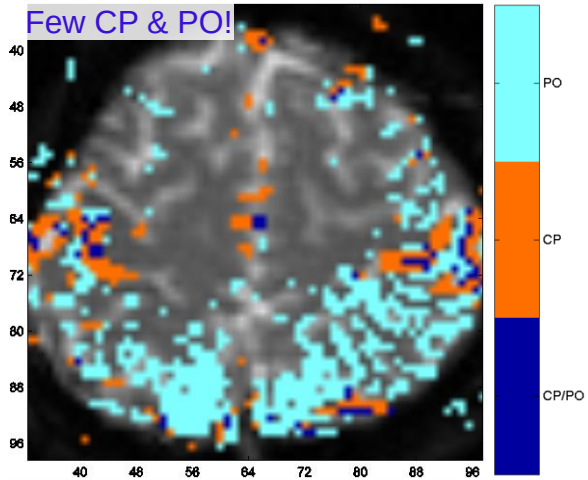
Past: CV R-I or M-P Analysis

***Presented at 2005 JSM**









Rowe and Logan: NIMG, 23:1078-1092, 2004.

Rowe: NIMG, 25:1310-1324, 2005b.

Past: CV R-I or M-P Analysis

3.0T GE LX

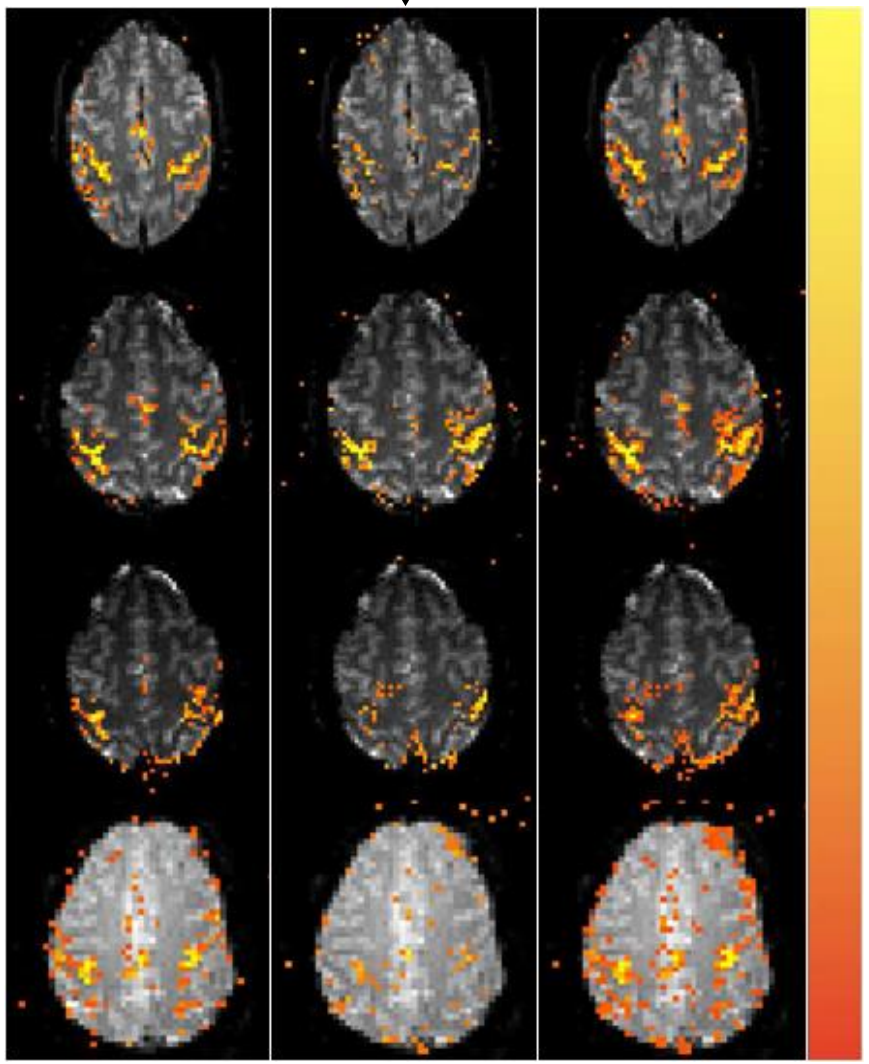
20s off+16×(8 s on 8 s off), 276 TRs
12 axial slices, 96 × 96, FOV = 24 cm
TH = 2.5 mm, TR = 1 s, TE = 34.6 ms
FA = 45◦, BW = 125 kHz, ES = .708 ms

20s off+16×(8 s on 8 s off), 276 TRs
10 axial slices, 96 × 96, FOV = 24 cm
TH = 2.5 mm, TR = 1 s, TE = 42.8 ms
FA = 45◦, BW = 125 kHz, ES = .768 ms

20s off+16×(8 s on 8 s off), 276 TRs
10 axial slices, 96 × 96, FOV = 24 cm,
TH = 2.5 mm, TR = 1 s, TE = 42.8 ms
FA = 45◦, BW = 125 kHz, ES = . 768 ms

20s off+10×(8 s on 8 s off), 180 TRs
9 axial slices, 64 × 64, FOV = 24 cm
TH = 3.8 mm, TR = 1 s, TE = 26.0 ms
FA = 45◦, BW = 125 kHz, ES = .680 ms

Hahn, Nencka, Rowe: HBM, 33:288–306, 2012.



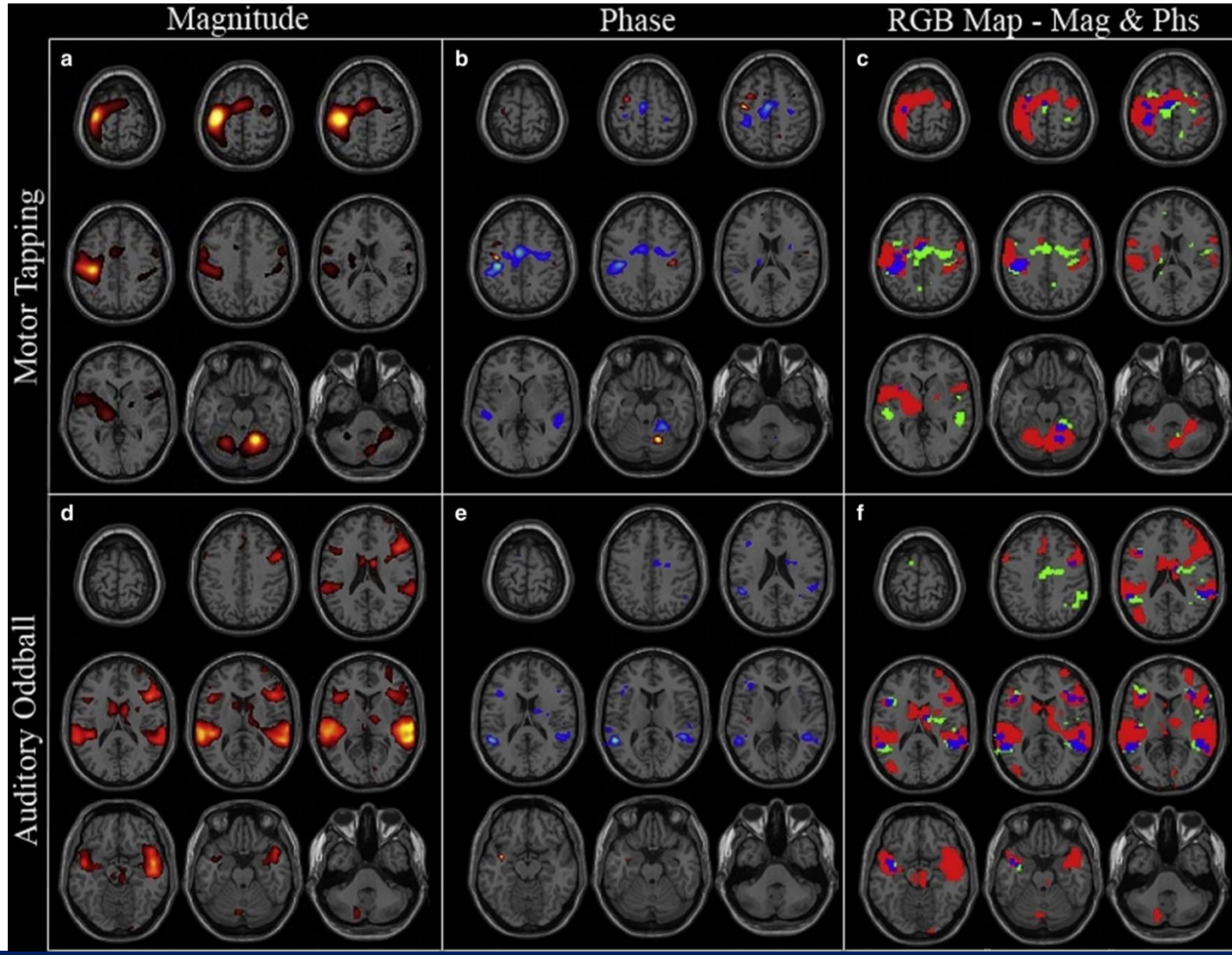
Past: CV R-I or M-P Analysis

Group analysis.
Not just single subject effect!

n=20

n=34

No MP analysis!



Arja, et al.: Changes in fMRI magnitude data and phase data observed in block-design and event-related tasks. NIMG 3149-3160, 2010.

Present: Complex-Valued Time Series Analysis

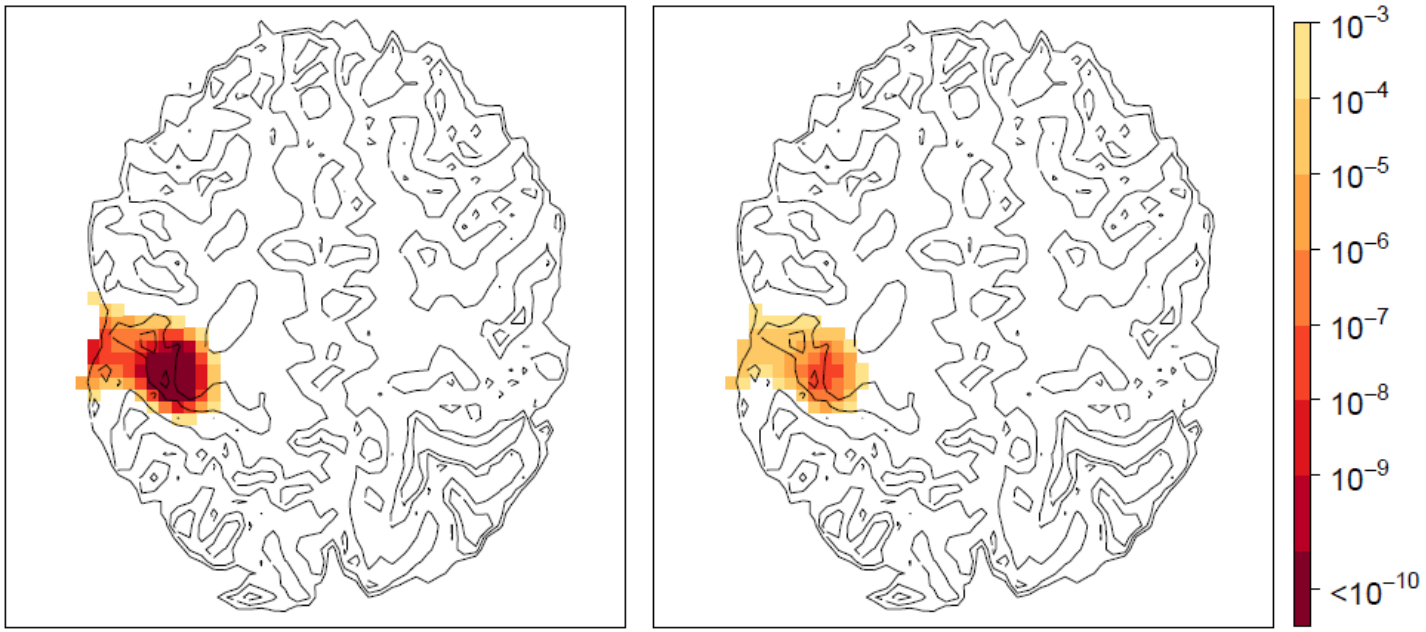
Complex-valued AR model

$$\begin{pmatrix} \boldsymbol{y}_R \\ \boldsymbol{y}_I \end{pmatrix} = \begin{pmatrix} \boldsymbol{X}\boldsymbol{\beta}\cos\theta \\ \boldsymbol{X}\boldsymbol{\beta}\sin\theta \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}_R \\ \boldsymbol{\eta}_I \end{pmatrix}$$

$$(\boldsymbol{\eta}_R', \boldsymbol{\eta}_I')' \sim N(\mathbf{0}, \boldsymbol{\Sigma} \otimes \boldsymbol{\Phi})$$

$$\boldsymbol{\Phi} = \boldsymbol{R}_n$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_R^2 & \rho\sigma_R\sigma_I \\ \rho\sigma_R\sigma_I & \sigma_I^2 \end{pmatrix}$$



(a) Complex-valued AR(1) model (b) Magnitude-only AR(1) model

Adrian, Maitra, Rowe: AOAS, 12(3), 2018.

Present: Complex-Valued Bayesian Analysis

Bayesian Complex-valued model

$$\begin{pmatrix} \mathbf{y}^{v}_{Re} \\ \mathbf{y}^{v}_{Im} \end{pmatrix} = \begin{pmatrix} \mathbf{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{X} \end{pmatrix} \begin{pmatrix} \boldsymbol{\gamma}^{v}_{Re} \\ \boldsymbol{\gamma}^{v}_{Im} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\eta}^{v}_{Re} \\ \boldsymbol{\eta}^{v}_{Im}, \end{pmatrix} \qquad \boldsymbol{\Sigma}_{v} = \begin{pmatrix} \boldsymbol{\Sigma}^{v}_{Re,Re} & \boldsymbol{\Sigma}^{v}_{Re,Im} \\ \boldsymbol{\Sigma}^{v}_{Im,Re} & \boldsymbol{\Sigma}^{v}_{Im,Im} \end{pmatrix}$$

$$\mathbf{y}^{v} = \mathbf{X}\boldsymbol{\gamma}^{v} + \boldsymbol{\eta}^{v}, \quad \boldsymbol{\eta}^{v} \sim CN_T\left(\mathbf{0}, 2\sigma_v^2\mathbf{I}, \mathbf{0}\right),$$

$$\gamma_j^v \mid \psi_j^v \sim \left(1-\psi_j^v\right) CN_1\left(0, 2v_0\sigma_v^2, 0\right) + \psi_j^v CN_1\left(0, 2v_1\sigma_v^2, 0\right), \quad j = 1, \ldots, p,$$

$$\sigma_v^2 \sim IG\left(a_\sigma, b_\sigma\right), \quad \psi_j^v \mid \theta_j \sim Bernoulli\left(\theta_j\right),$$

$$\theta_j \sim Beta\left(a_\theta, b_\theta\right).$$

Yu, Prado, Ombao, Rowe: JASA, 2018.

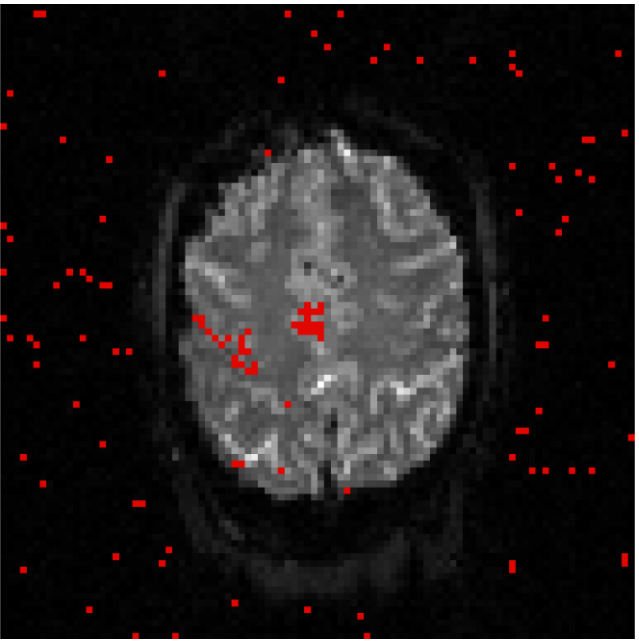
T2* Model $$y_{t_{DT}} = M_t\left(\cos\theta + i\sin\theta\right) + \left(\eta_{R_t} + i\eta_{I_t}\right).$$

$$M_t = \left[M_{t-1}\exp\left(-\frac{TR}{T_1}\right)\cos(\phi) + M_0\left(1-\exp\left(-\frac{TR}{T_1}\right)\right)\right]\sin\left(\phi\right)\exp\left(-\frac{TE_t}{T_2{}^* + \delta z_t}\right) + x'_{1_t}\beta_1$$

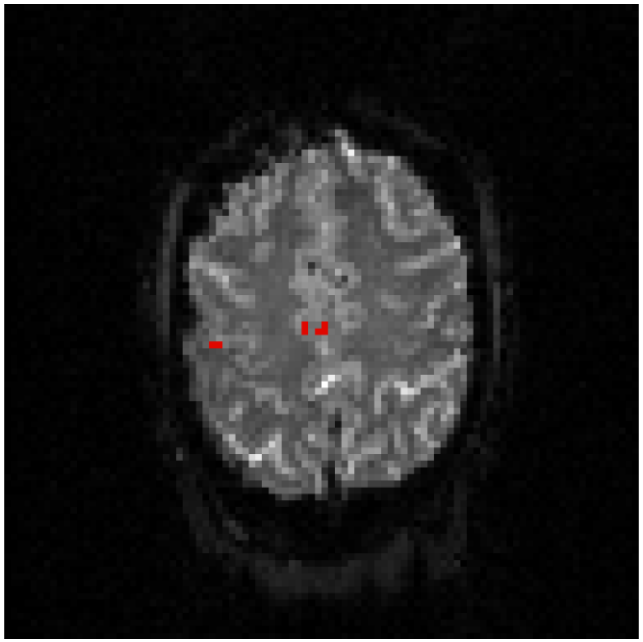
Karaman, Bruce, Rowe: Magn. Reson. Imaging, 32:9-27, 2014.

Present: Complex-Valued Bayesian Analysis

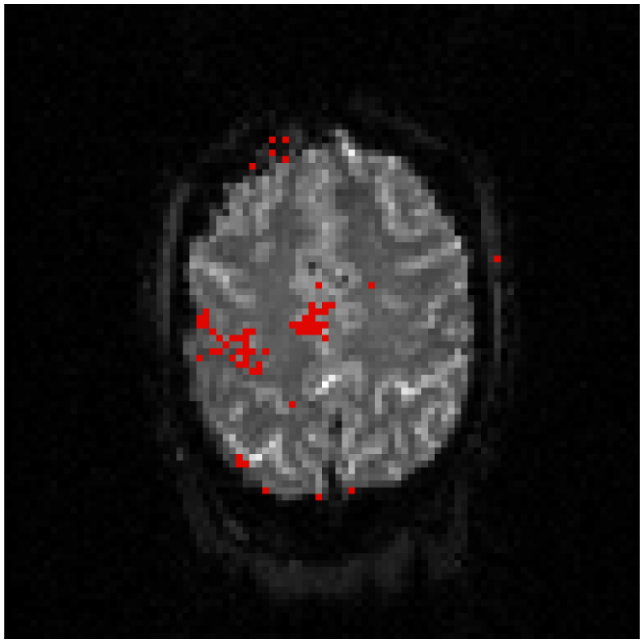
Bayesian Complex-valued model



CV Bayesian Model



MO Bayesian Model



T2* CV Model

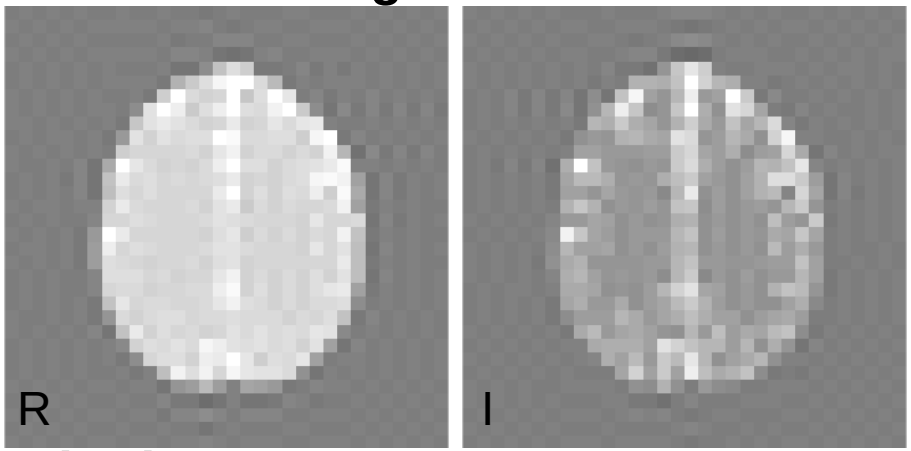
Yu, Prado, Ombao, Rowe: JASA, 2018.

Karaman, Bruce, Rowe: Magn. Reson. Imaging, 32:9-27, 2014.

Future: Bayesian image reconstruction

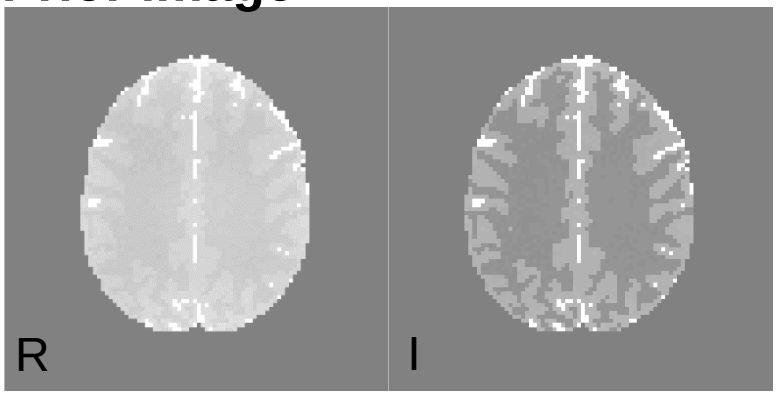
Less time per image by measuring less data.

Observed image at time *t*

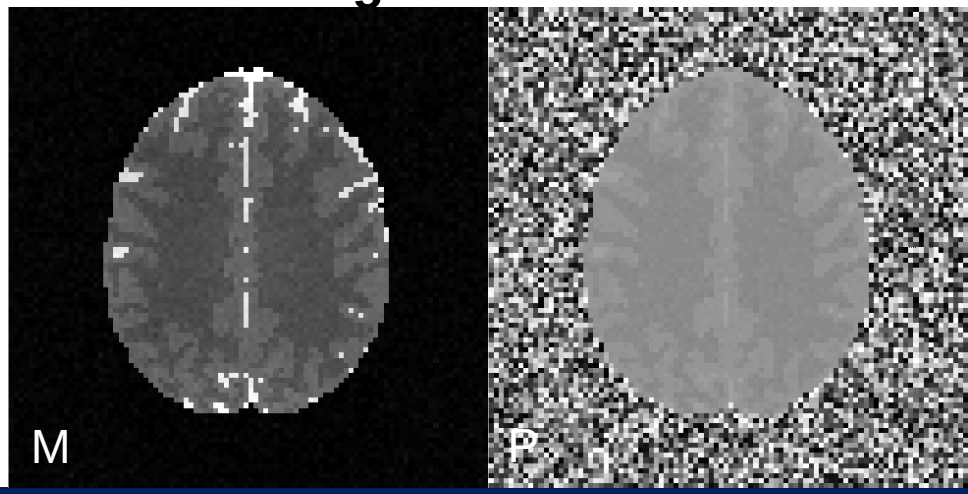


Illustration

Prior image



Posterior image at time t



Future: Bayesian (SENSE) image reconstruction

Less time per image by measuring less data.

N_C=4, A=3

Observed image at time *t*

Coil 4

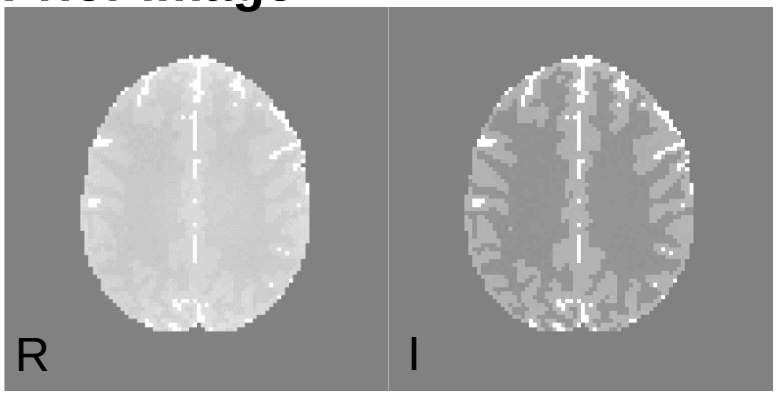
Coil 1

Coil 3

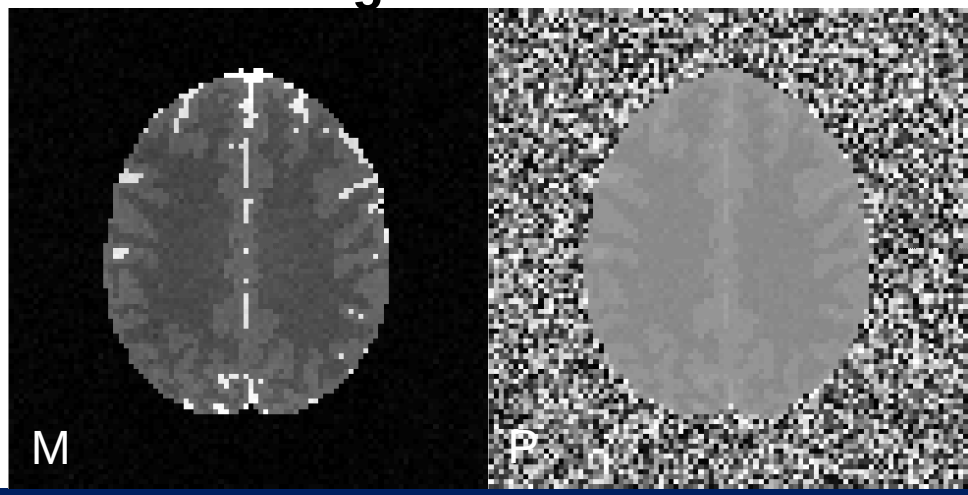
Coil 2

← aliasing

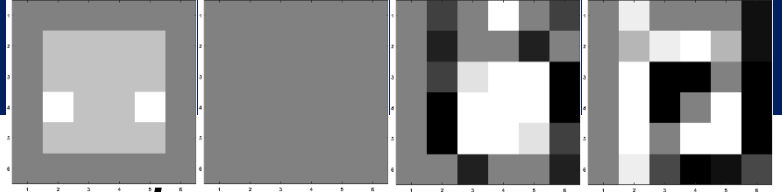
Prior image



Posterior image at time t



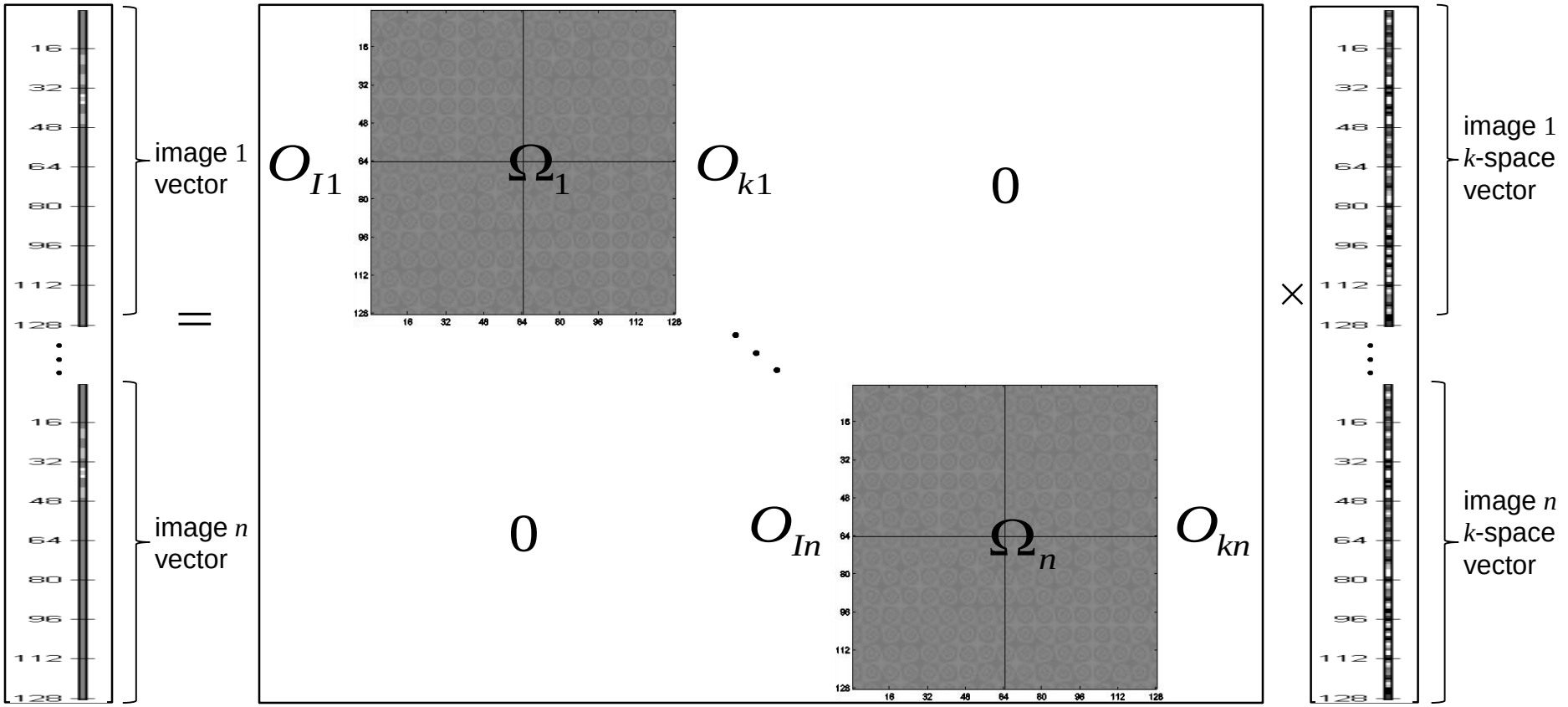
Rowe, Proc JSM, 378-392, 2017

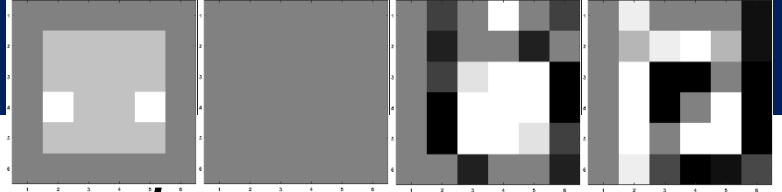


Future: Activation directly from *k*-space

$$x = TIRKs$$

x IRK s





Future: ~~Activation~~ directly from *k*-space

y T P x

ts 1

O_{T1}

0

$\ddots$

0 O_{Tn}

ts *n*

=

0/1 permutation matrix reorder from by image to by voxel.

16 32 48 64 80 96 112 128

iv 1

$\vdots$

16 32 48 64 80 96 112 128

iv *n*

ordered by voxel

Future: Activation directly from *k*-space

Model:

$$y = TIRKs$$

$$y = \Theta X \beta + \eta$$

equate

$$\varepsilon = K^{-1}R^{-1}I^{-1}T^{-1}\eta$$

Invert:

$$\underbrace{s}_{\text{original data}} = \underbrace{K^{-1}R^{-1}I^{-1}T^{-1}}_{\text{processing/reconstruction}}\underbrace{\Theta X}_{\text{fMRI model}}\beta + \underbrace{\varepsilon}_{\text{original error}}$$

known

~~unknown~~ parameters

Now activation in image space in terms of *k*-space.

Rowe, Hahn, Nencka: Magn. Reson. Imaging, 33:1370-1381, 2009.

Discussion: Why Complex-valued?

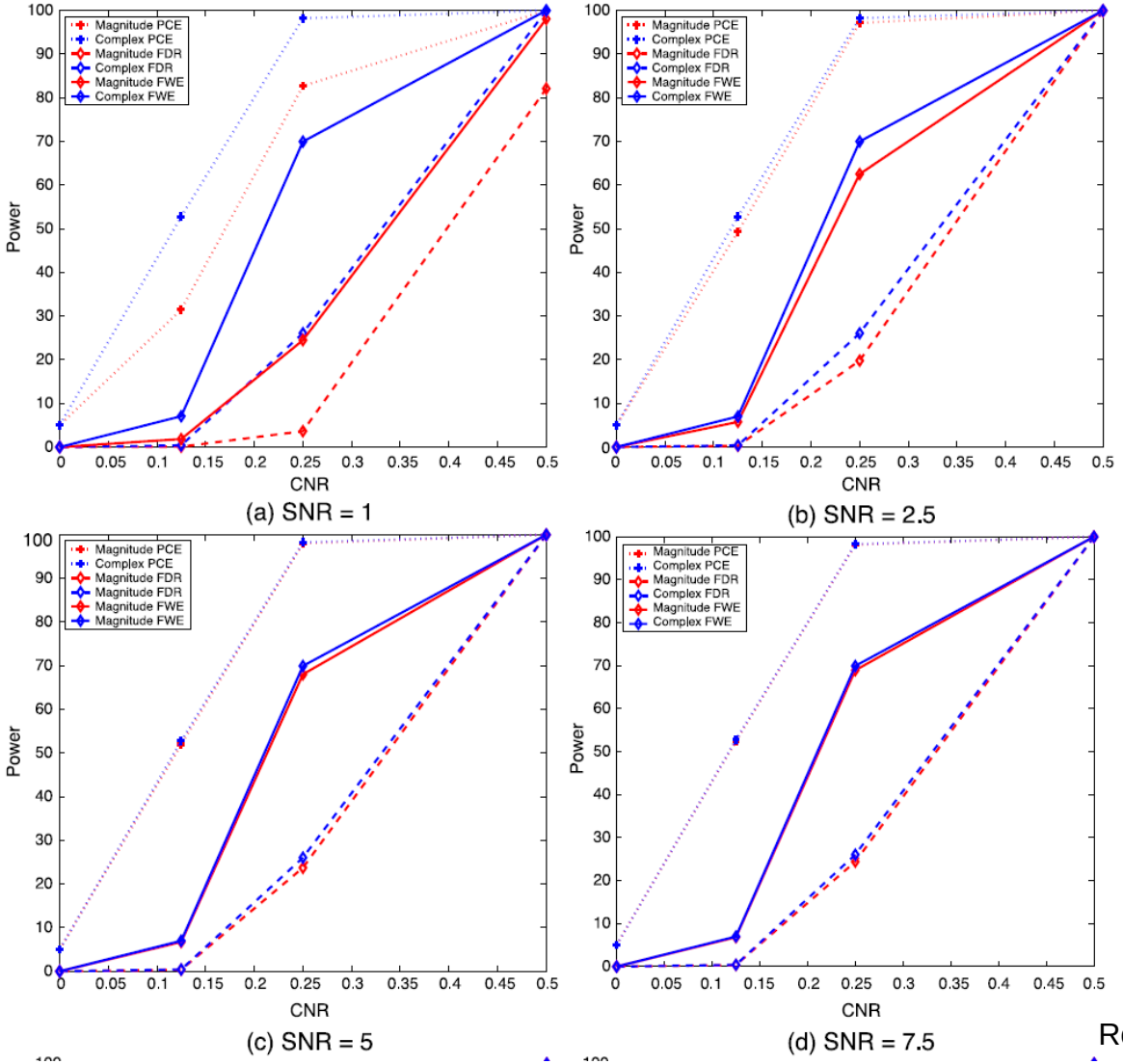
More data quantities. 100% of data

Increased sensitivity/specificity

Increased information content

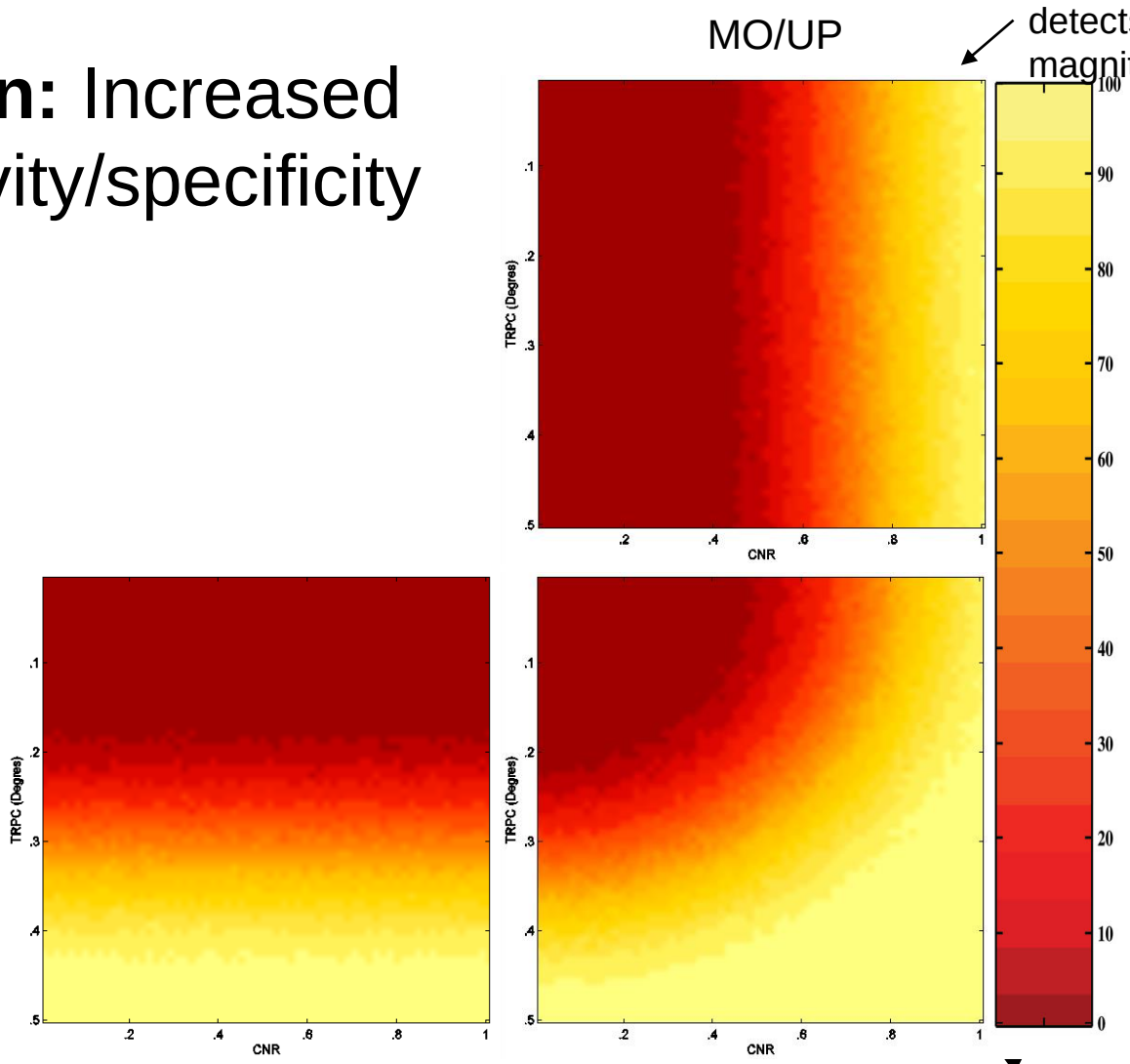
Rowe and Logan: NIMG, 23:1078-1092, 2004.

Discussion: Increased sensitivity/specificity



Rowe and Logan: NIMG, 23:1078-1092, 2004.

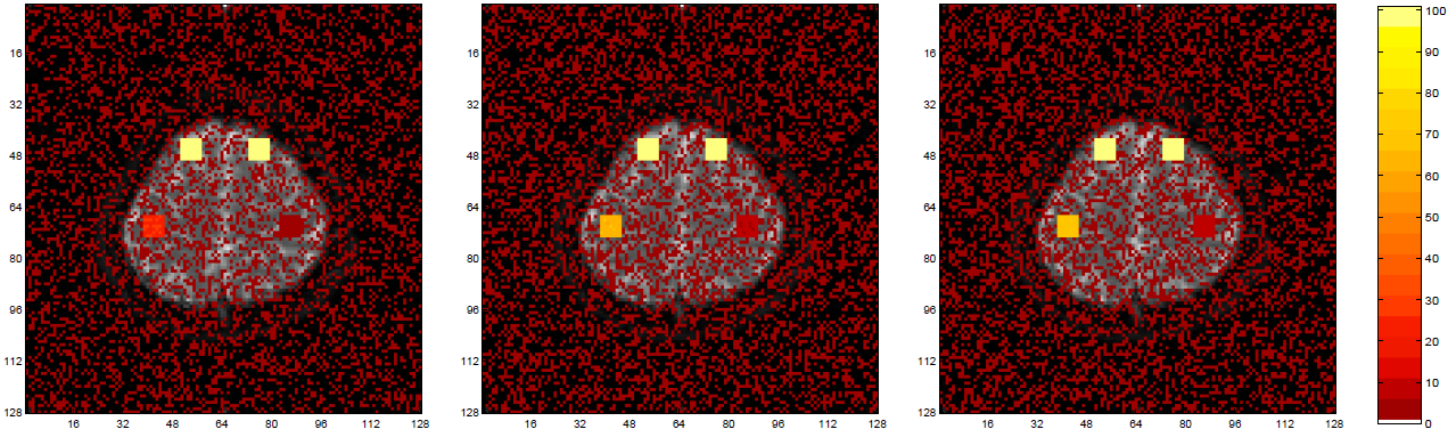
Discussion: Increased sensitivity/specificity



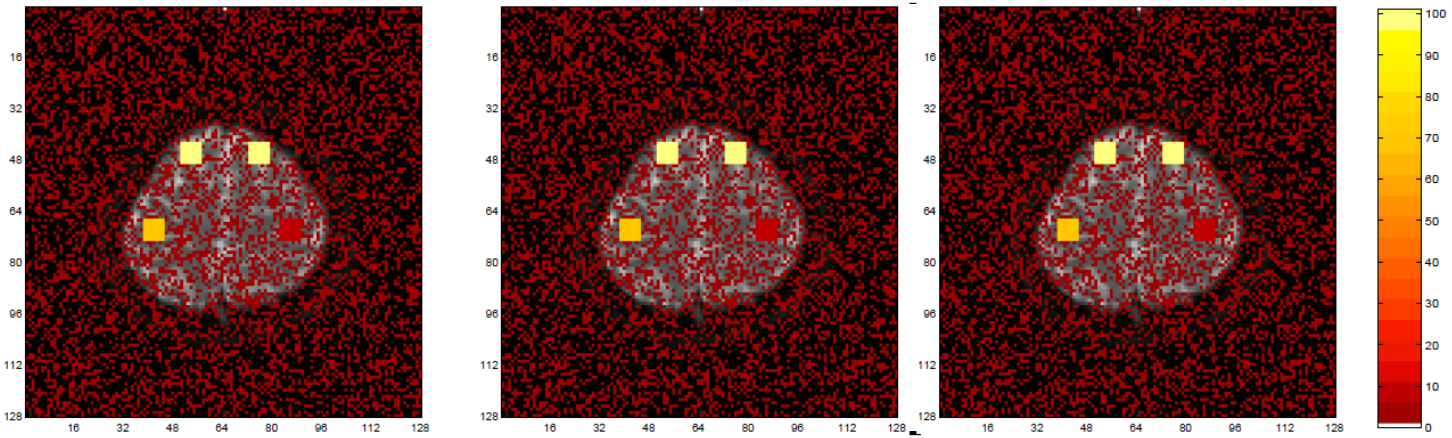
Hernandez-Garcia, Vazquez, Rowe, MRM 62:1597-1608, 2009.

Discussion: Increased sensitivity/specificity

Power: 5% FDR



M: SNR $= 1$ M: SNR $= 2.5$ M: SNR $= 5$



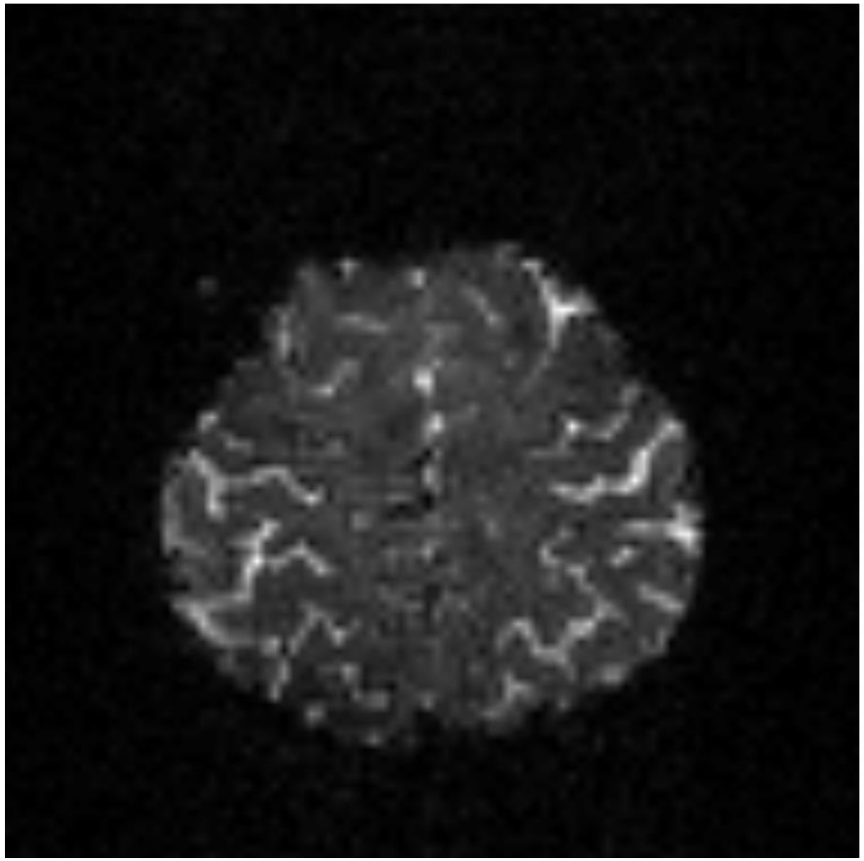
C: SNR $= 1$ C: SNR $= 2.5$ C: SNR $= 5$

Wisc 2003 Talk

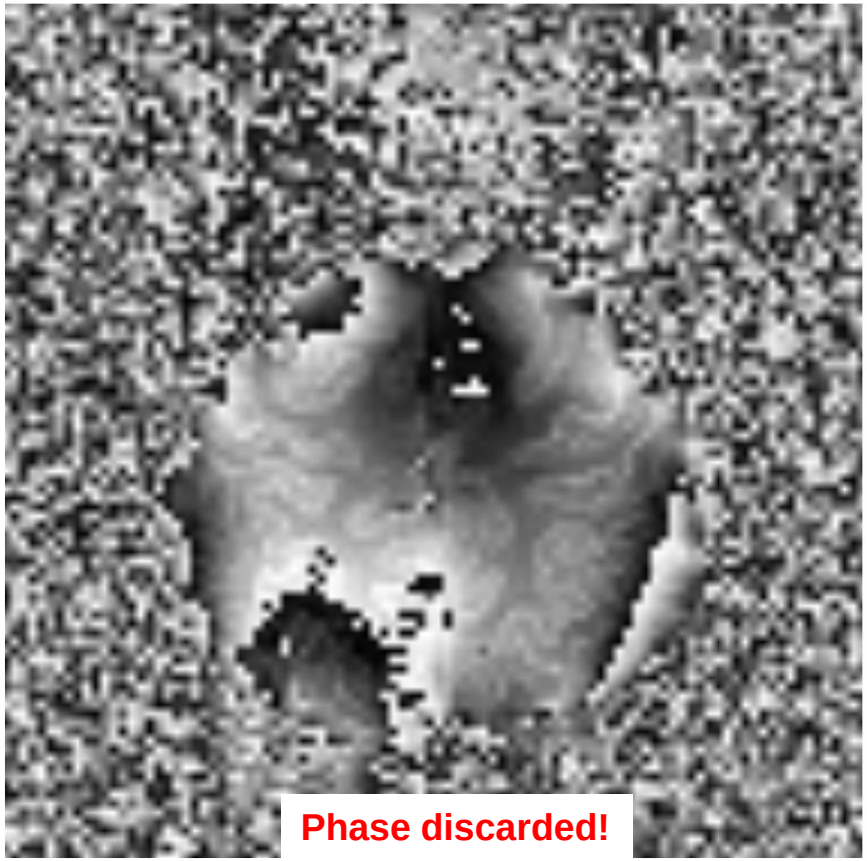
Discussion: Increased Information

There is biological information in the phase!

GRE EPI Image



Magnitude Image

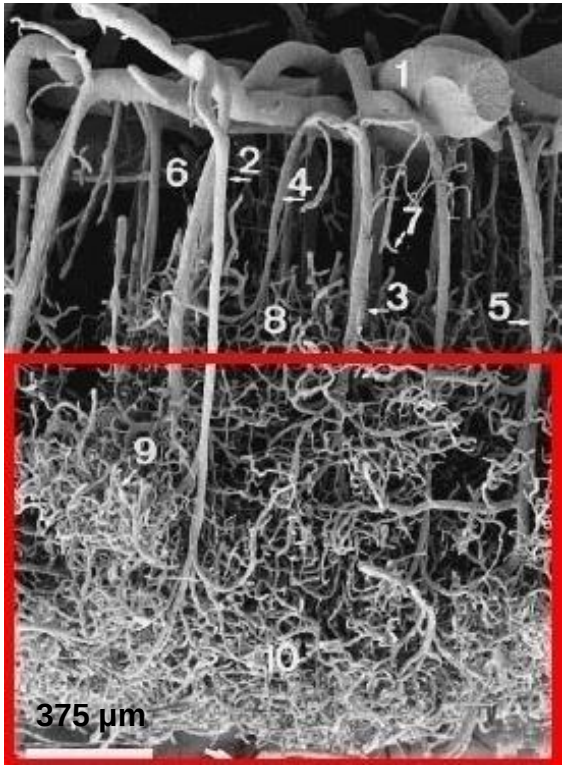


Phase Image

No task related phase changes.

Discussion: Increased Information

There is biological information in the phase!



1. pial artery
2. long cortical artery
3. middle cortical artery
4. short cortical artery
5. cortical vein
6. subpial zone
7. precapillary vessels
8. superficial capillary zone
9. middle capillary zone
10. deep capillary zone

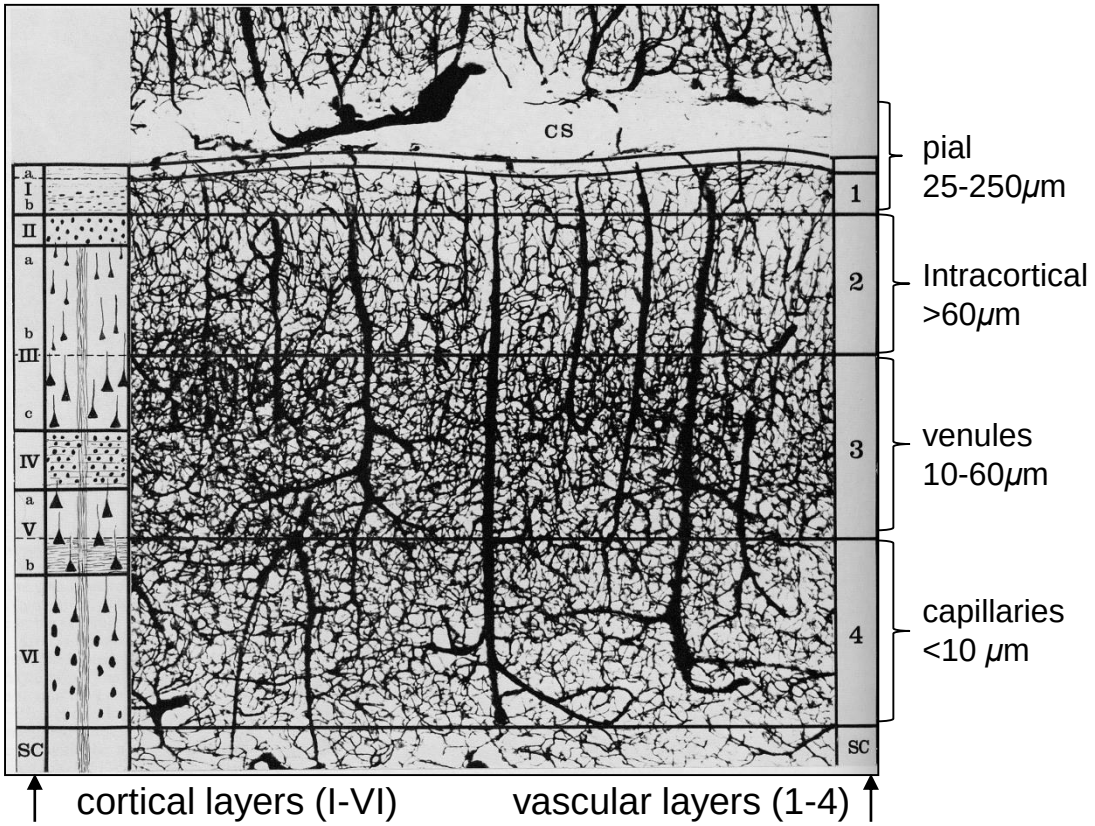


Figure (left) Reina-de la Torre et al.: The Anatomical Record, 1998.
Figure (right) Duvernoy et al. Brain Res Bull 7:519-579, 1981.
Data (right) Yamaguchi et al. Int J Microcirc Clin Exp 1992.

[1]Menon: MRM, 47:1-9, 2002., [2]Nencka, Rowe: NIMG, 177-88, 2007.

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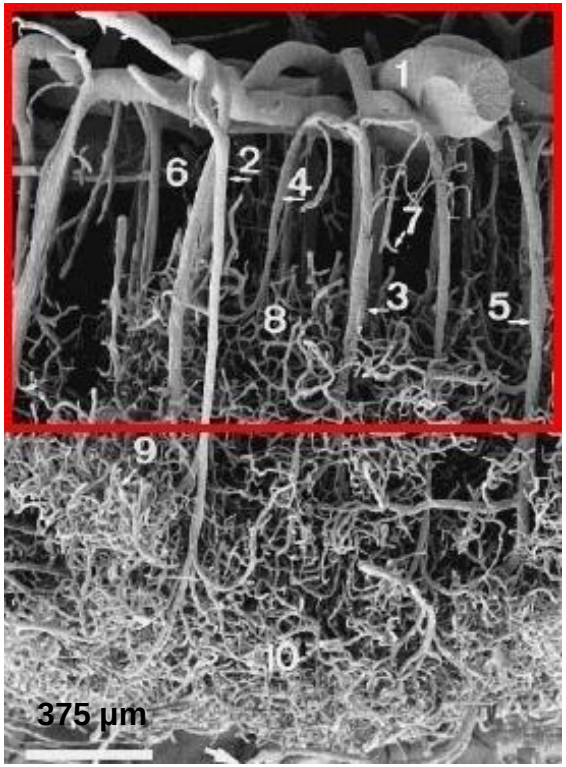

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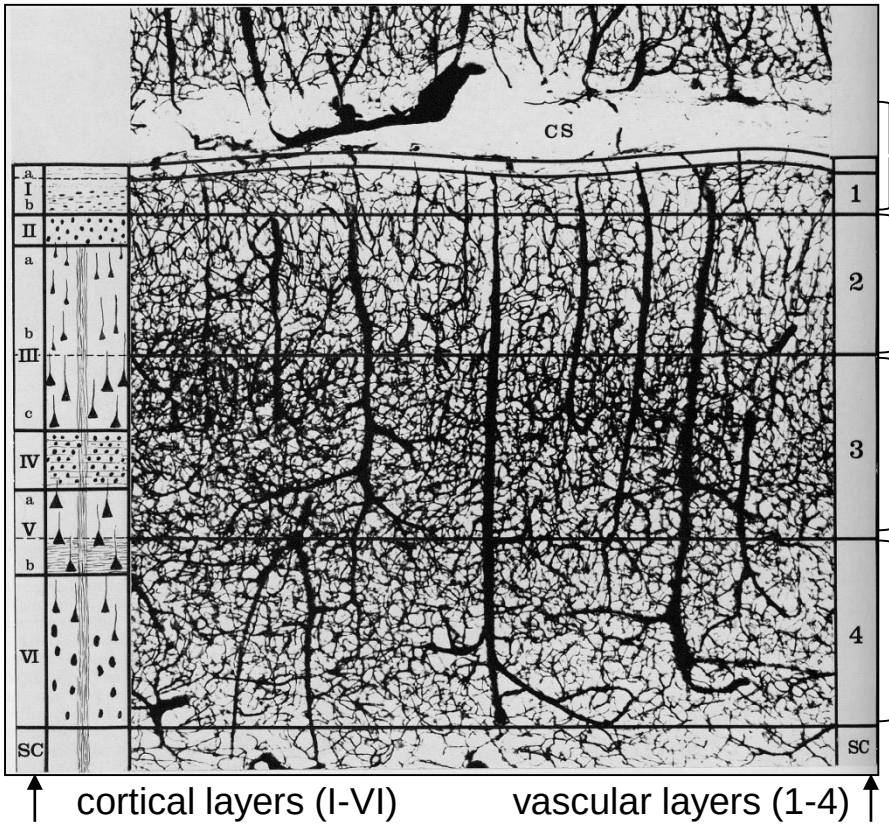

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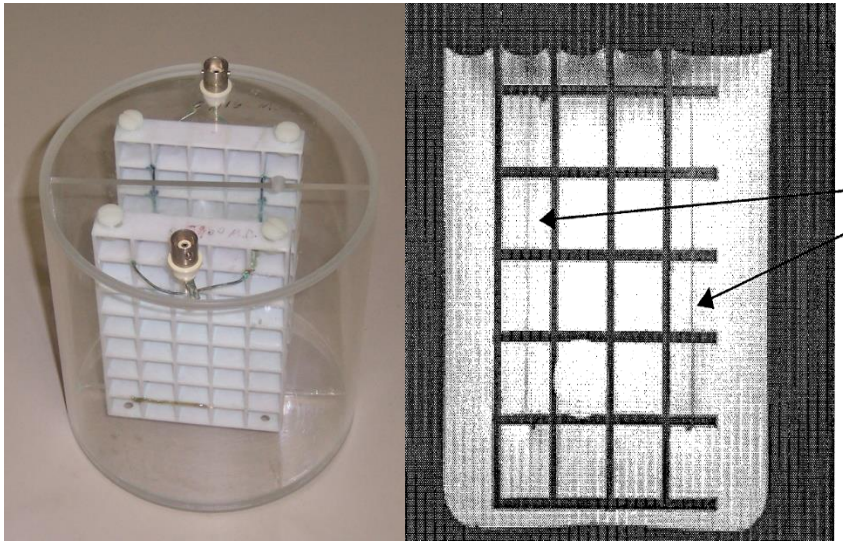
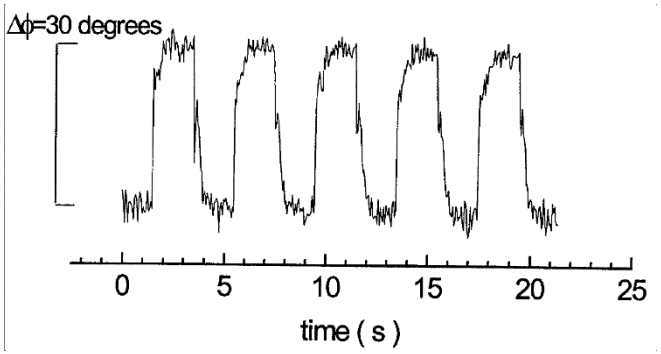


Bodurka et al.: JMR, 1999.

Chow et al.: NIMG, 2006.



phase time series

Phase contains other magnetic field change info: respiration, motion!

Discussion

Time for Statisticians to ask for 100% of data!

More can be extracted from complex-valued data!

Still many opportunities for CV research!

Discussion:

Started work on complex-valued fMRI 15 years ago.

First talk given at U. Wisconsin.

TITLE: **A COMPLEX WAY TO COMPUTE fMRI ACTIVATION**

SPEAKER: Daniel B. Rowe
Department of Biophysics
Medical College of Wisconsin

TIME: 4:00 P.M.

DATE: Wednesday, November 19, 2003

ROOM: 1221 CSSC

ABSTRACT:

In functional magnetic resonance imaging, Fourier "image reconstruction" results in complex valued proton spin densities that make up our voxel time course observations. The complex part of the proton spin density is a result of phase errors due to magnetic field inhomogeneities. Nearly all fMRI studies obtain a statistical measure of functional "activation" based on magnitude image time courses. However, it is the real and imaginary parts of the original signal that are measured with (normally distributed) error, and not the magnitude. The two error specifications are equivalent for "large" signal to noise ratios. The image information is contained in both the real and imaginary parts or in the magnitude and phase. A more accurate model should use the correct distributional specification and all the information contained in the data. A model is presented that uses the original complex form of the data and not the magnitude. By doing this, there are approximately twice as many quantities used to estimate the model parameters which results in improved power.

Coffee and Cookies at 3:30 p.m. in Room 4331 CSSC

Thank You!

Looking for New PhD Students!
Daniel.Rowe@Marquette.Edu