A Bayesian Approach to SENSE Image Reconstruction in FMRI

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July 31, 2017

NIHR21 NS087450

Outline

Introduction

Full Sampled (k-Space) Reconstruction

Data fully sampled and whole images are reconstructed from single channel and multi-channel data.

Sub Sampled (k-Space) Reconstruction

Multi-channel data subsampled and aliased images are reconstructed. Need to be unaliased and combined.

Bayesian Reconstruction

Prior distributions on coil sensitivities and voxel values then estimated *a posteriori*. Similar to Bayesian factor analysis.

Discussion

Estimated latent variable parameters and reconstructed images.

Introduction

In fMRI and MRI, voxel values re NOT measured by the machine, the measurements taken by the machine are an array of complex-valued spatial frequencies.

This array of complex-valued spatial frequencies needs to be reconstructed into an image for us to see, analyze, and interpret.

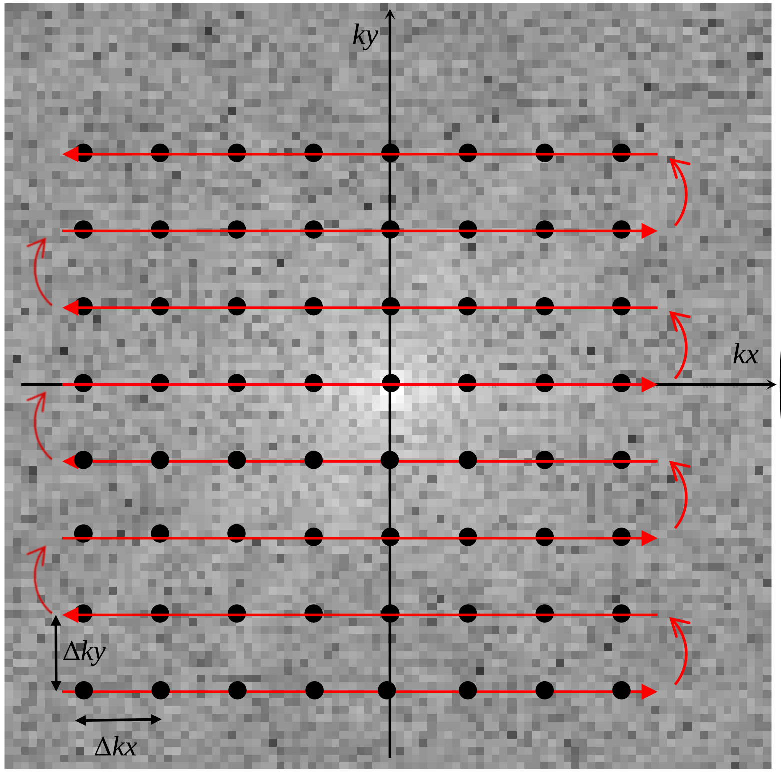
The array of complex-valued spatial frequencies are reconstructed into an image via the inverse Fourier transform.

Lets briefly remind ourselves what the FT and IFT are.

Full Reconstruction

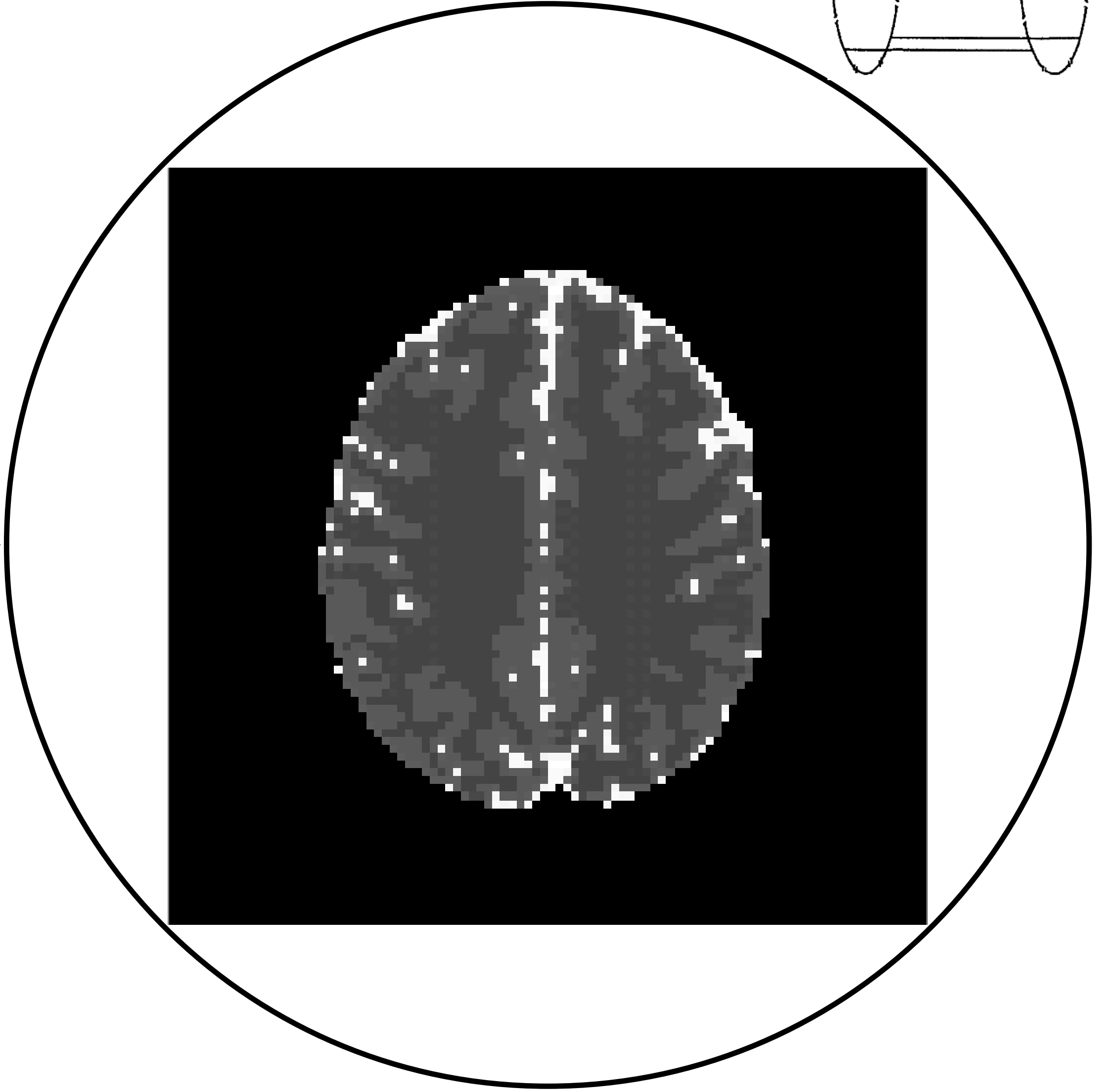
Single Coil Acquisition

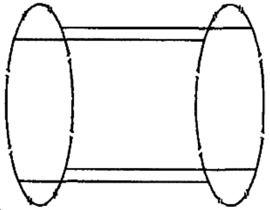
Coil measures k-space.



Unaccelerated Acquisition (A=1).

Coil





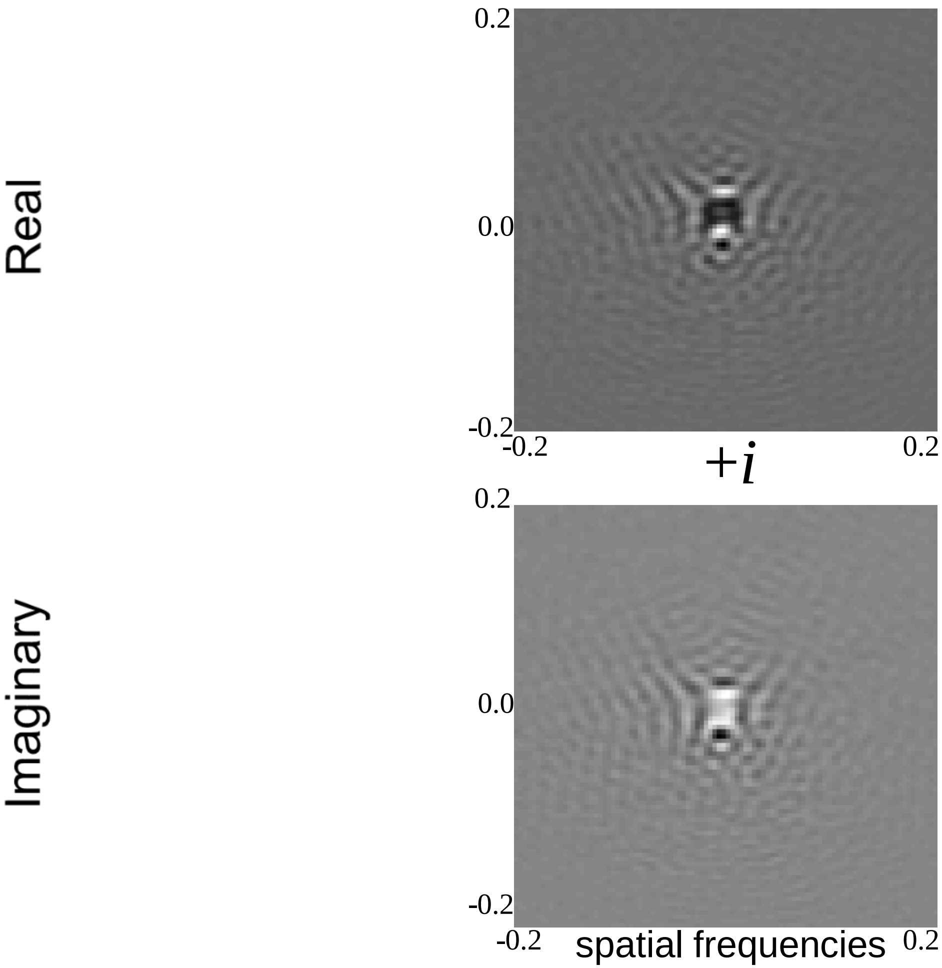
Kumar et al: JMR, 18(1):69-83,1975.

(FOV=240 mm)

(n_x=n_y=96, Δx=Δy=2.5 mm)

Full Reconstruction

We inverse Fourier transform spatial freqs to generate image.

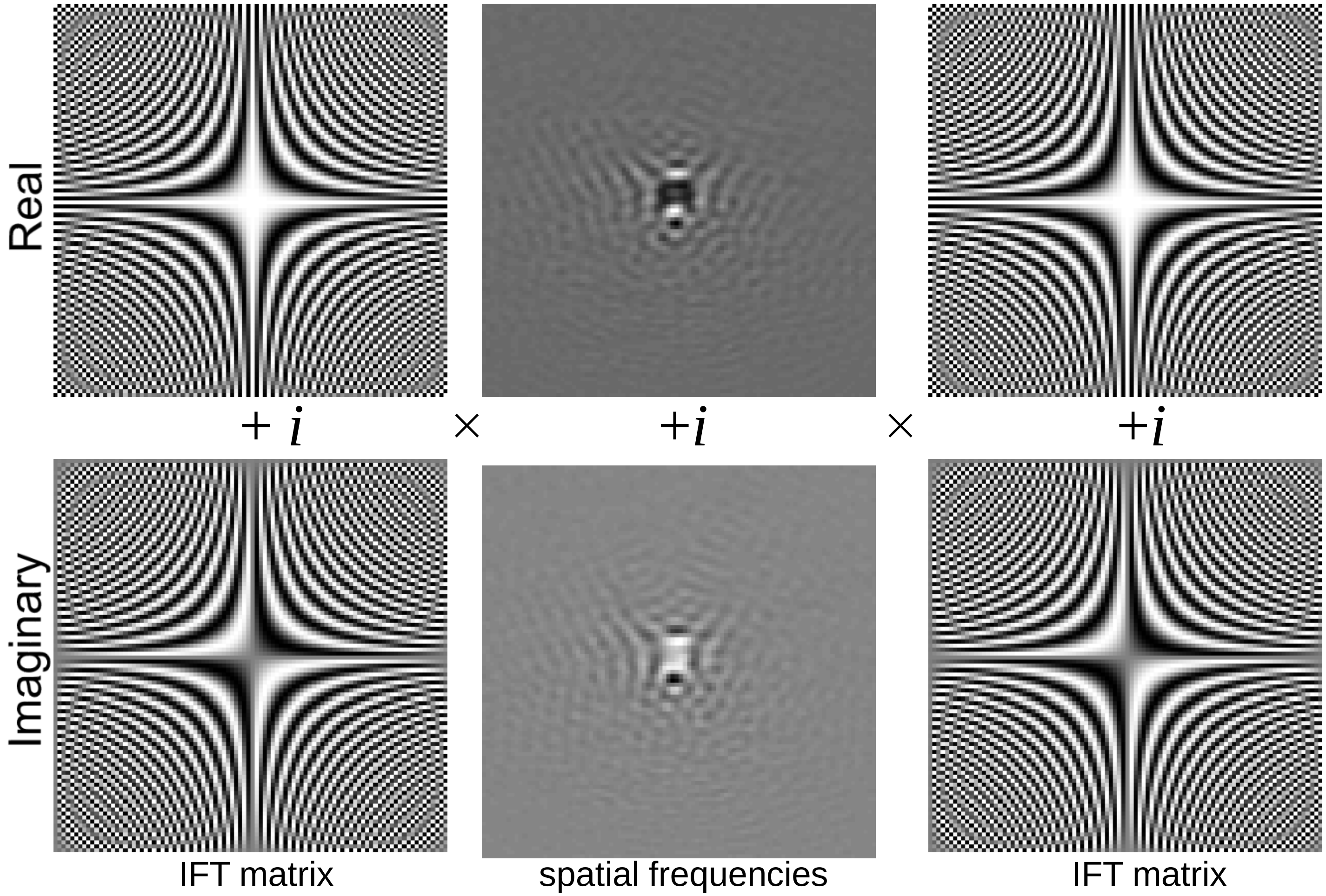


(FOV=240 mm)

Full Reconstruction

($n_x = n_y = 96$, $\Delta x = \Delta y = 2.5$ mm)

We inverse Fourier transform spatial freqs to generate image.



(FOV=240 mm)

(n_x=n_y=96, Δx=Δy=2.5 mm)

Full Reconstruction

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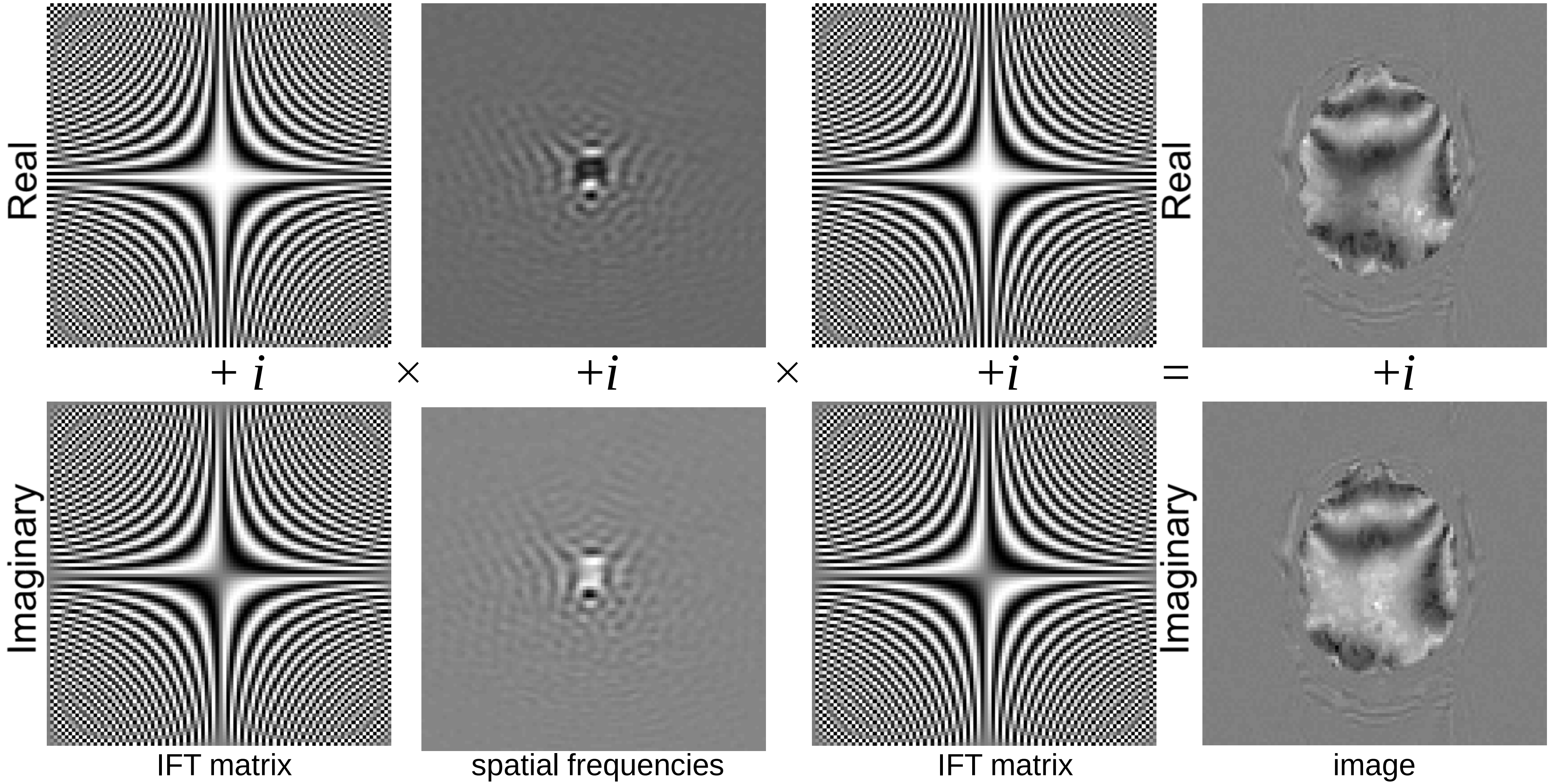
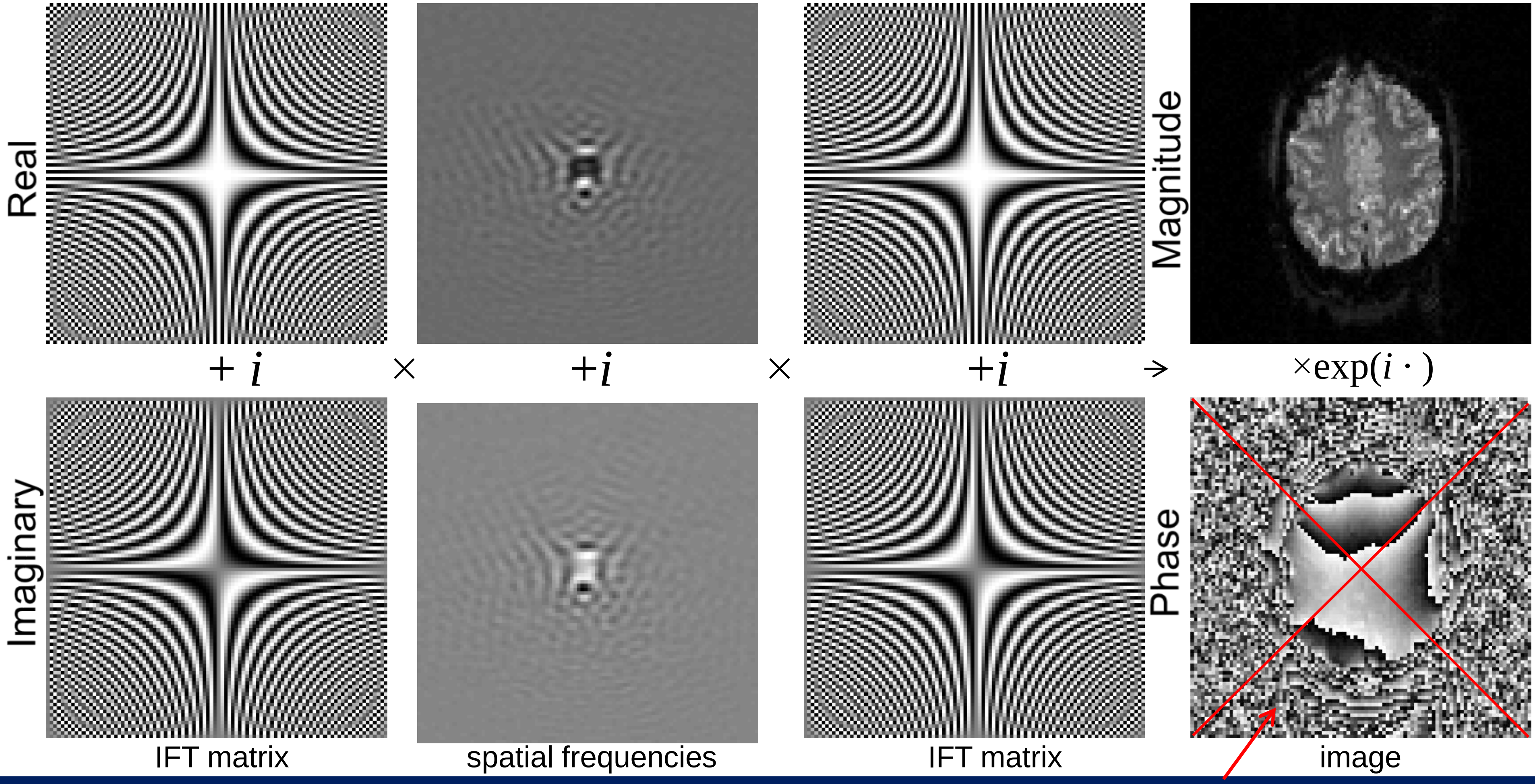


(FOV=240 mm)

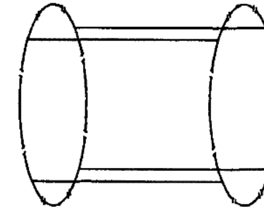
(n_x=n_y=96, Δx=Δy=2.5 mm)

Full Reconstruction

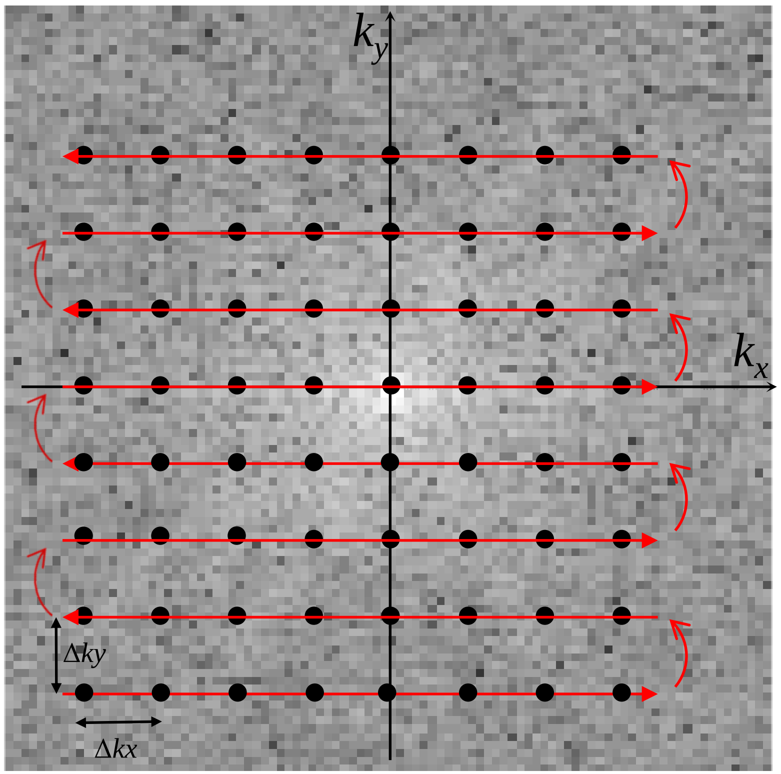
We inverse Fourier transform spatial freqs to generate image.



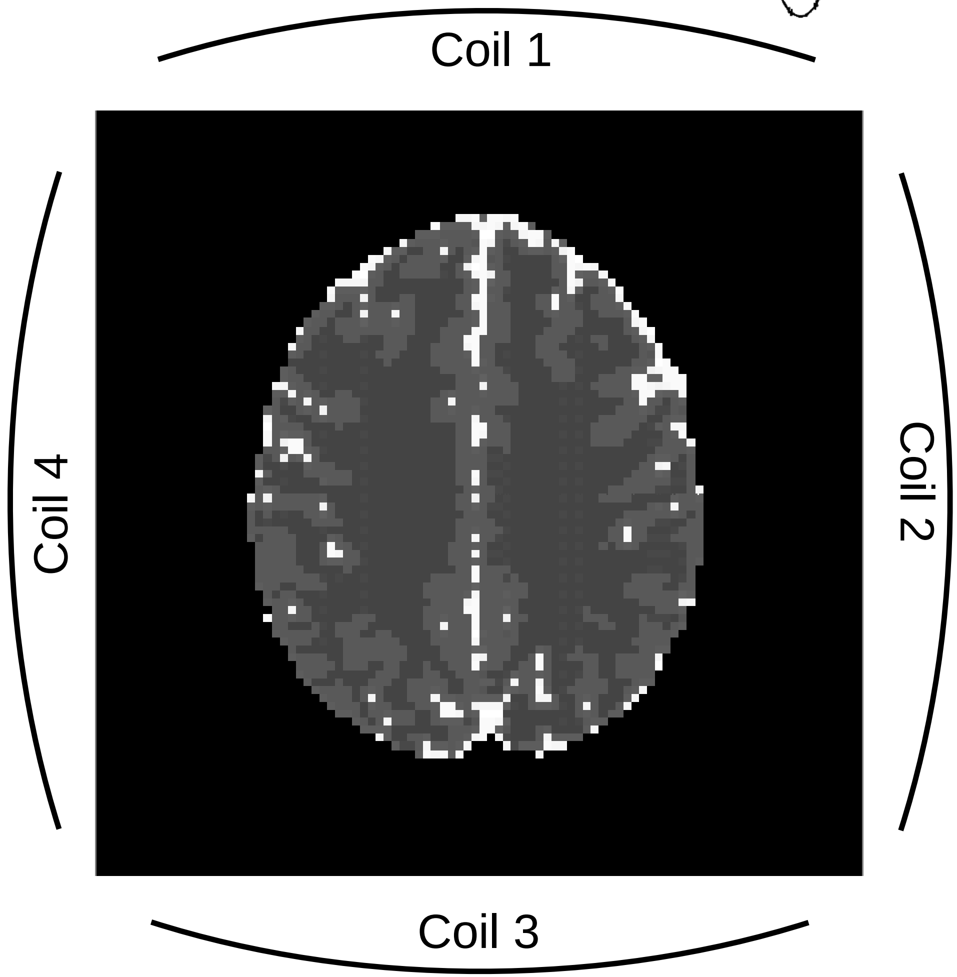
Phase discarded. Ask for the other ½ of YOUR data.



Full Reconstruction
Multi-Coil Acquisition



Each coil measures k-space.



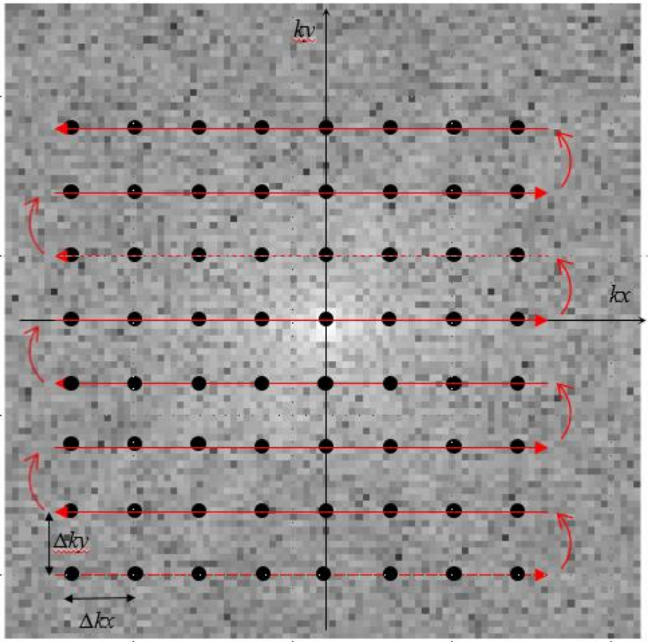
N_C=4, A=1

Hyde et al.: JMR, 70:512–517, 1986. Pruessmann et al.: Proc. ISRM, 579,1998.

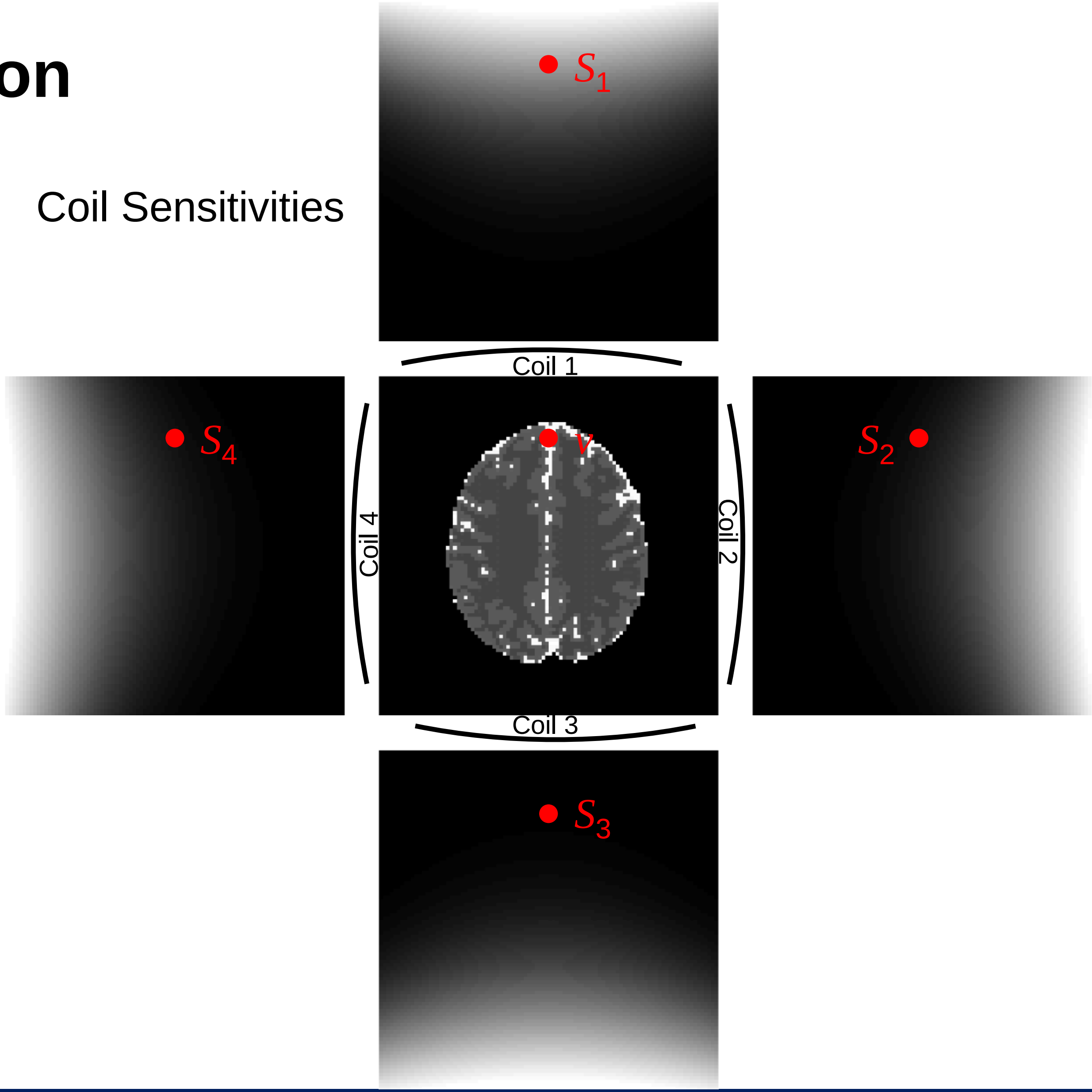
Full Reconstruction

Multi-Coil Acquisition

Coil Sensitivities



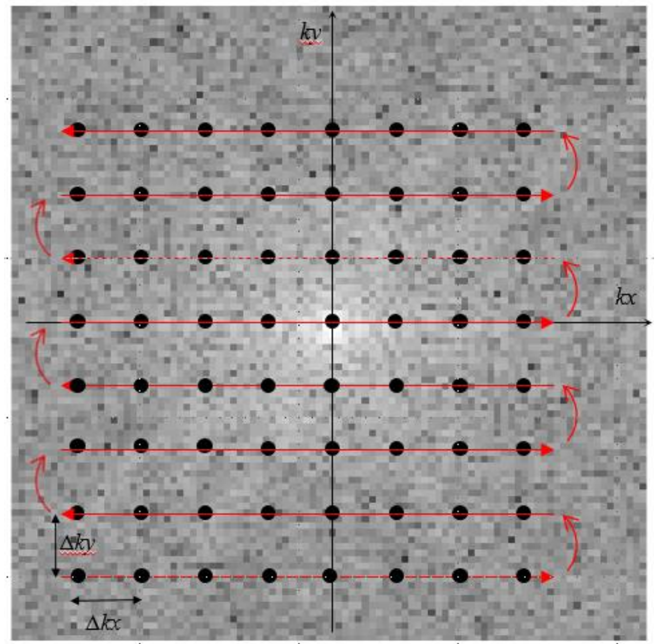
Each coil measures *k*-space.



N_C=4, A=1

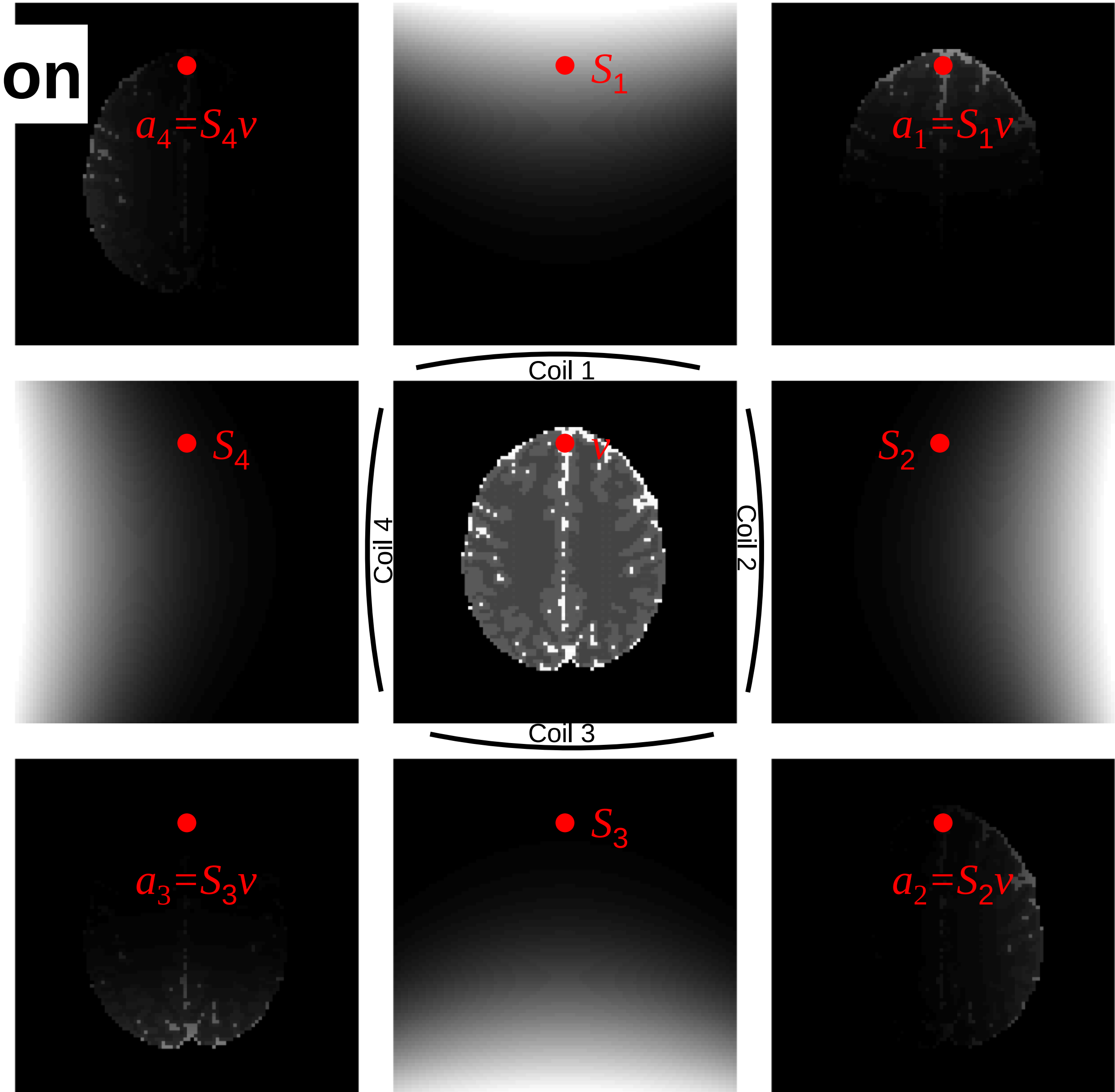
Full Reconstruction

Multi-Coil Acquisition



Each coil measures *k*-space.

N_C=4, A=1



Full Reconstruction

Multi-Coil SENSE

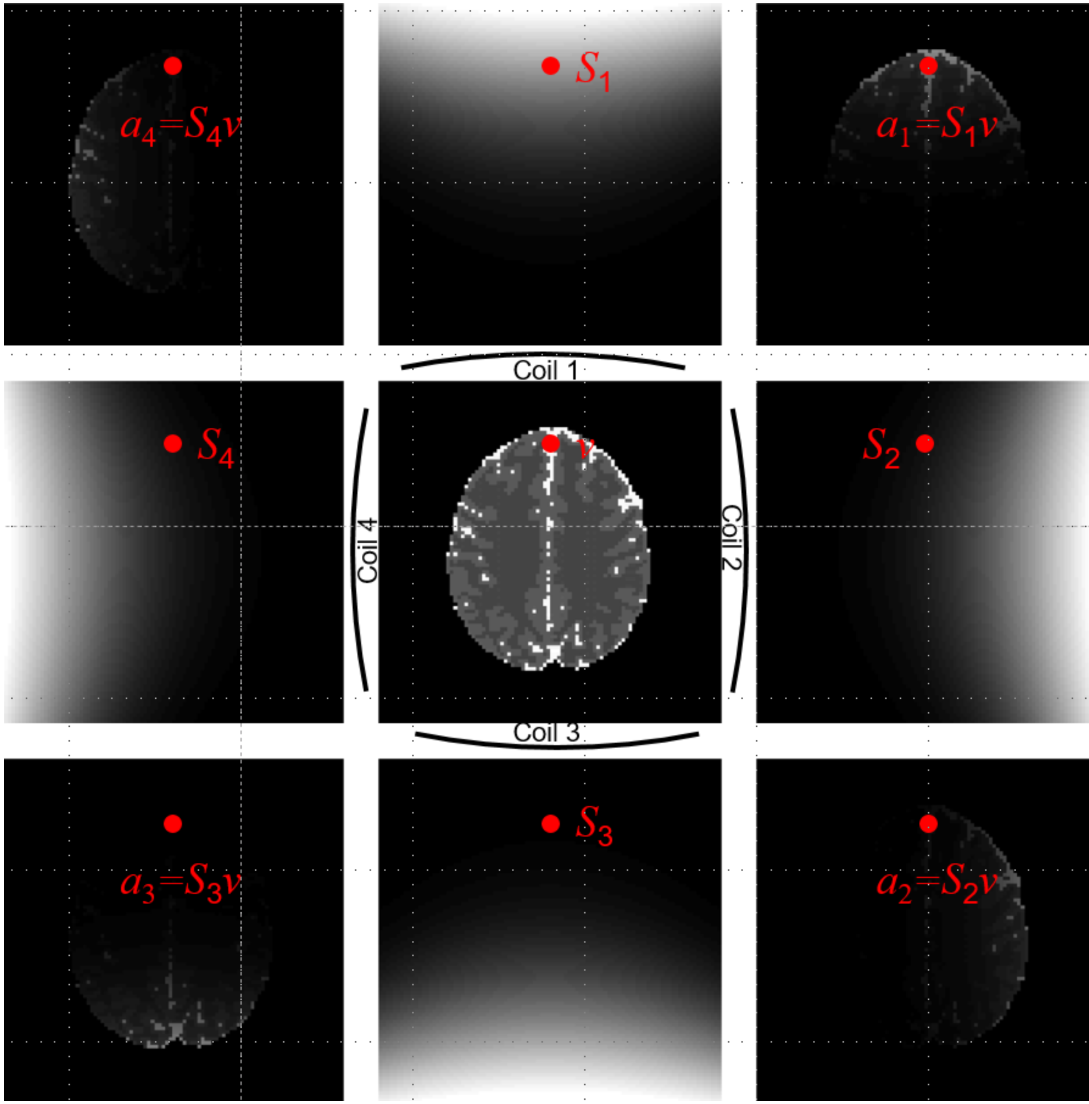
$a_1 = S_1 v + \varepsilon_1$
$a_2 = S_2 v + \varepsilon_2$
$a_3 = S_3 v + \varepsilon_3$
$a_4 = S_4 v + \varepsilon_4$ $N_C=4$

$a = Sv + \varepsilon$

a observed
S and v unobserved

Ad-hoc estimate of S, then

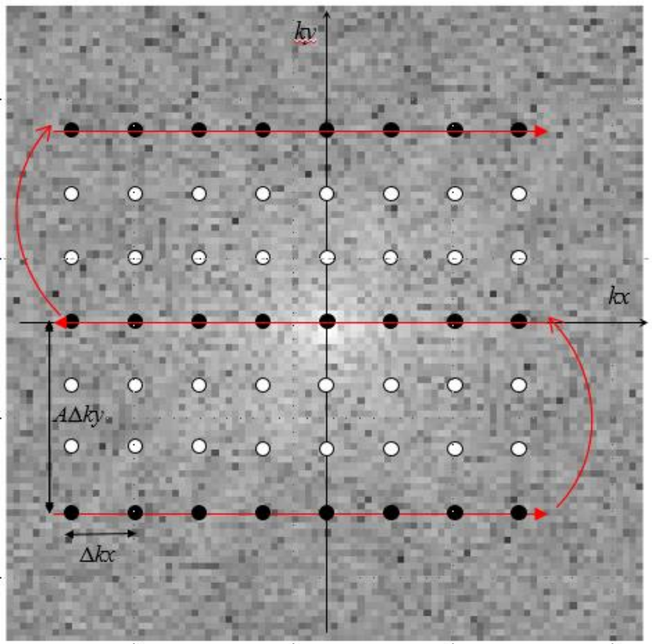
$\hat{v} = (S'S)^{-1} S'a$



But the goal is to get images faster!

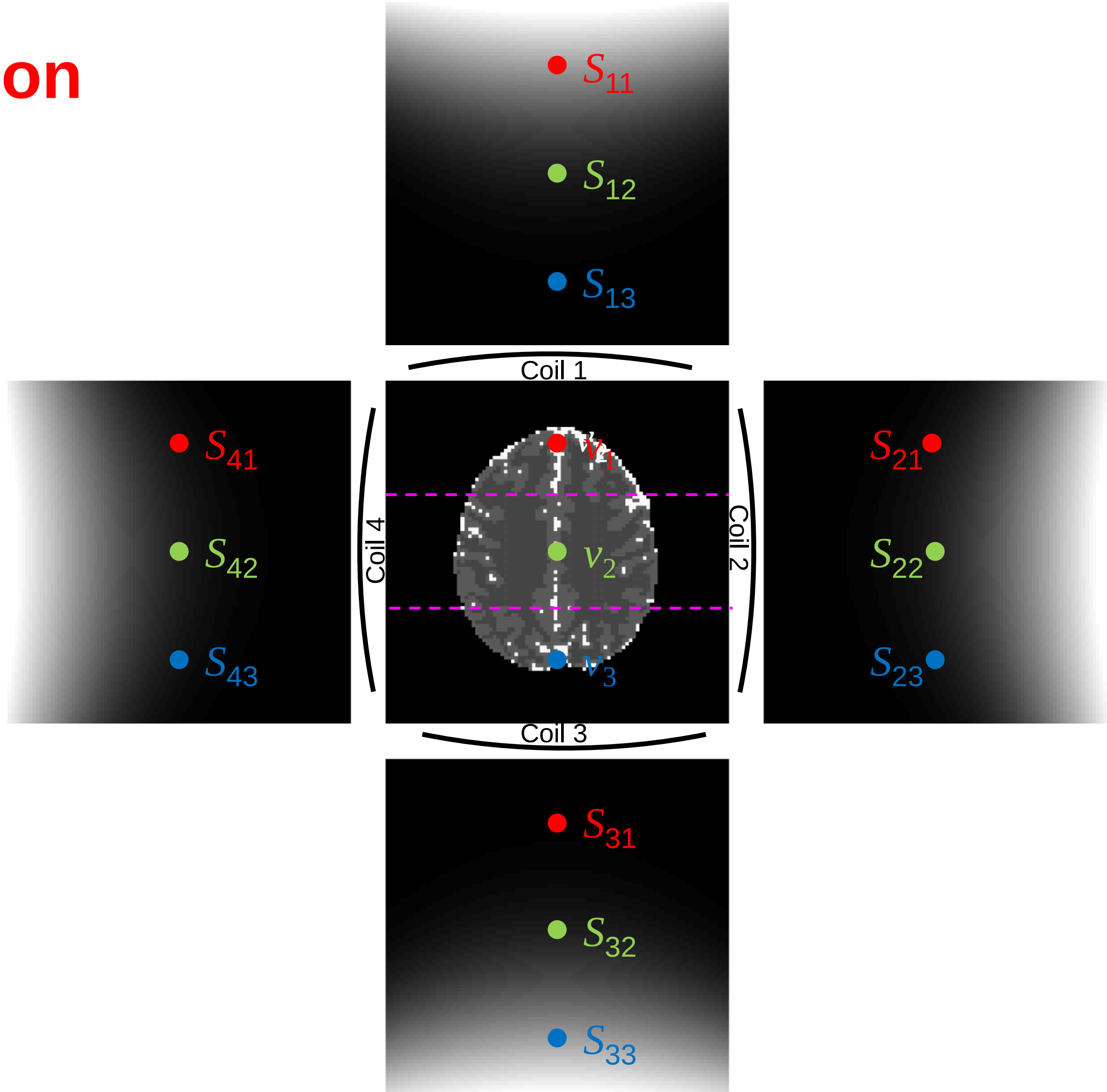
Sub Reconstruction

Multi-Coil Acquisition



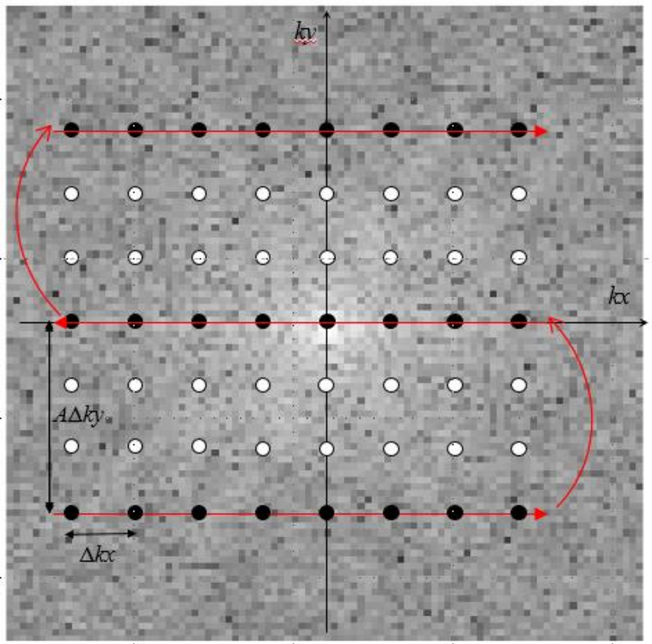
Each coil measures k-space.

N_C=4, A=3



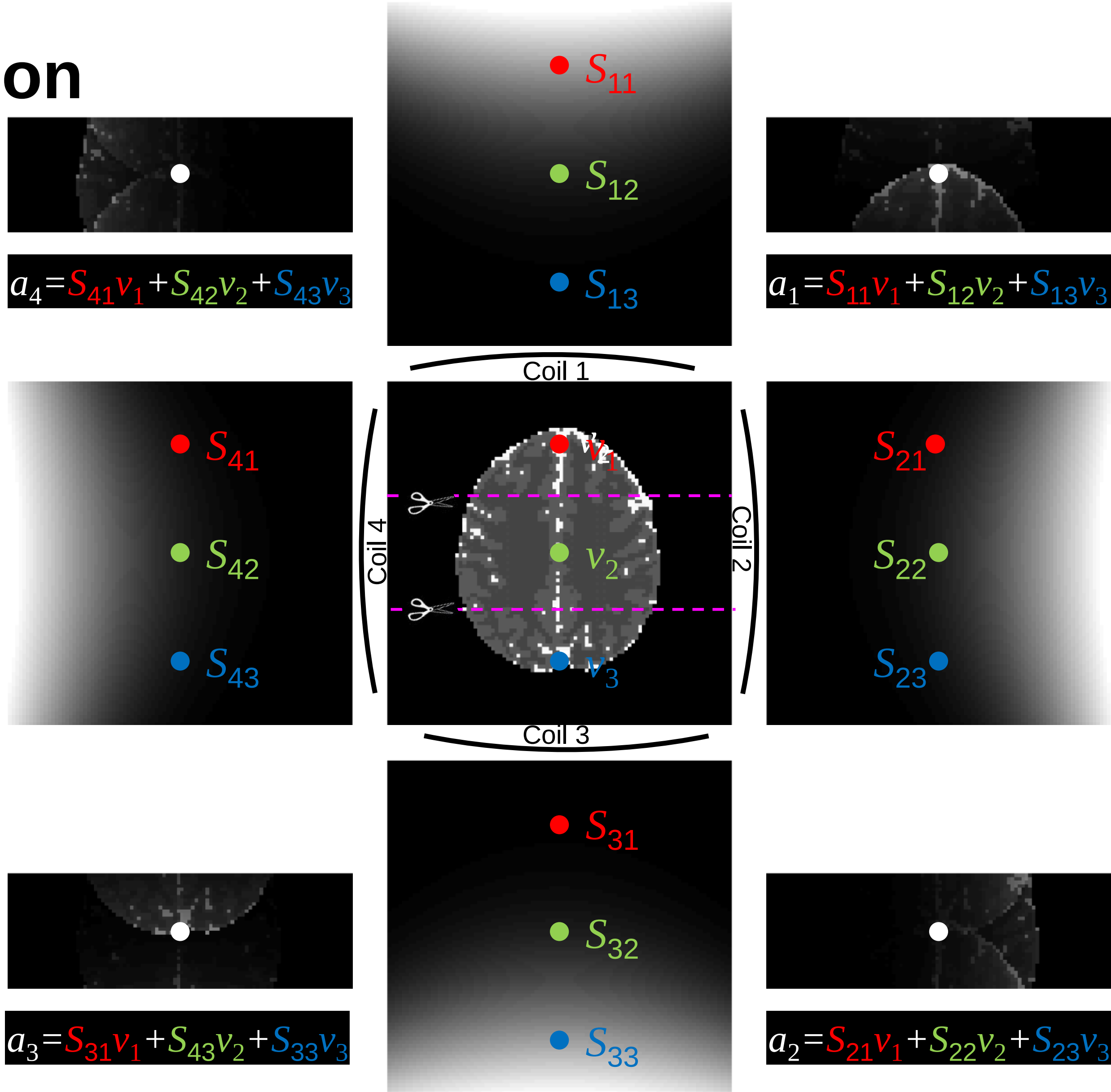
Sub Reconstruction

Multi-Coil Acquisition



Each coil measures k-space.

N_C=4, A=3



Sub Reconstruction

Multi-Coil SENSE

$$a_1 = S_{11}v_1 + S_{12}v_2 + S_{13}v_3 + \varepsilon_1$$
$$a_2 = S_{21}v_1 + S_{22}v_2 + S_{23}v_3 + \varepsilon_2$$
$$a_3 = S_{31}v_1 + S_{43}v_2 + S_{33}v_3 + \varepsilon_3$$
$$a_4 = S_{41}v_1 + S_{42}v_2 + S_{43}v_3 + \varepsilon_4$$

$$a = Sv + \varepsilon$$

a observed aliased

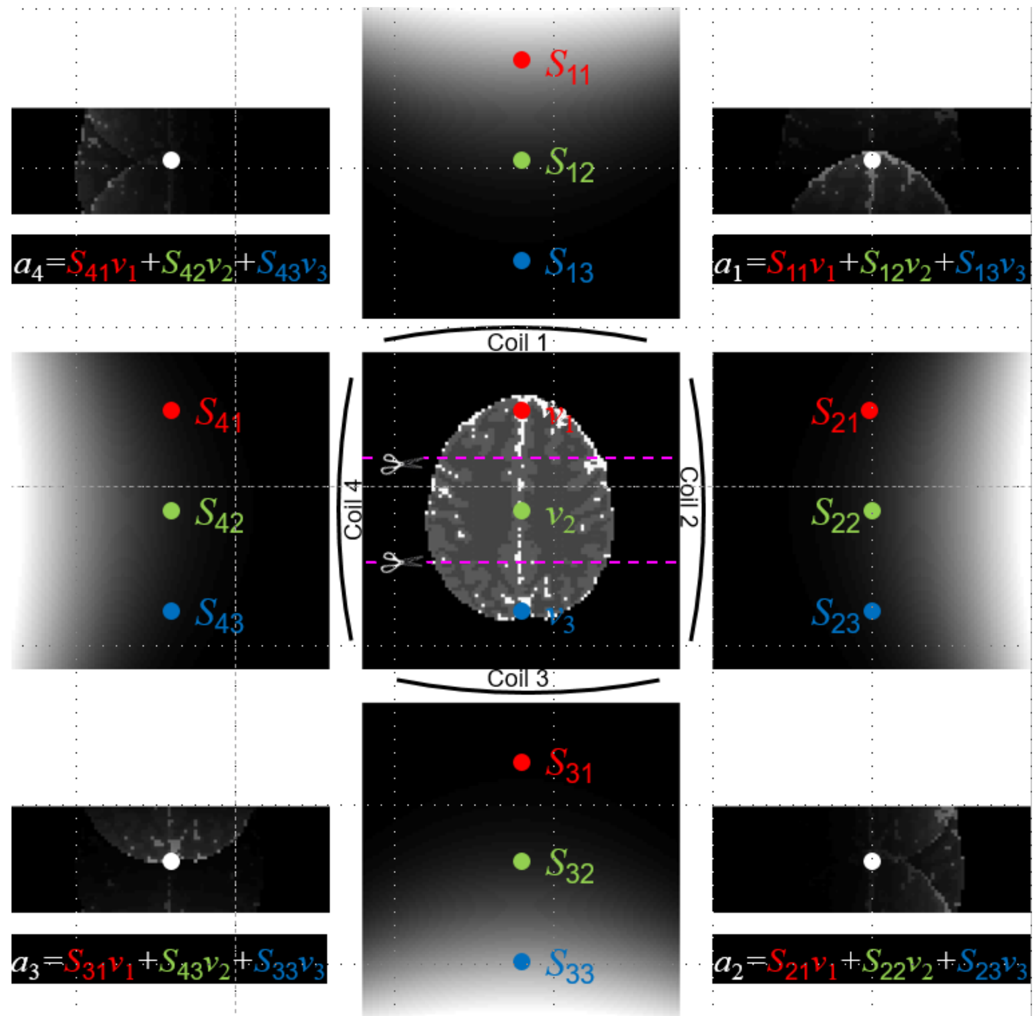
S and v unobserved

Ad-hoc estimate of S, then

$$\hat{v} = (S'S)^{-1} S'a$$

$(S'S)$ not generally pos def

The goal is to get images fast!



Bayesian Reconstruction

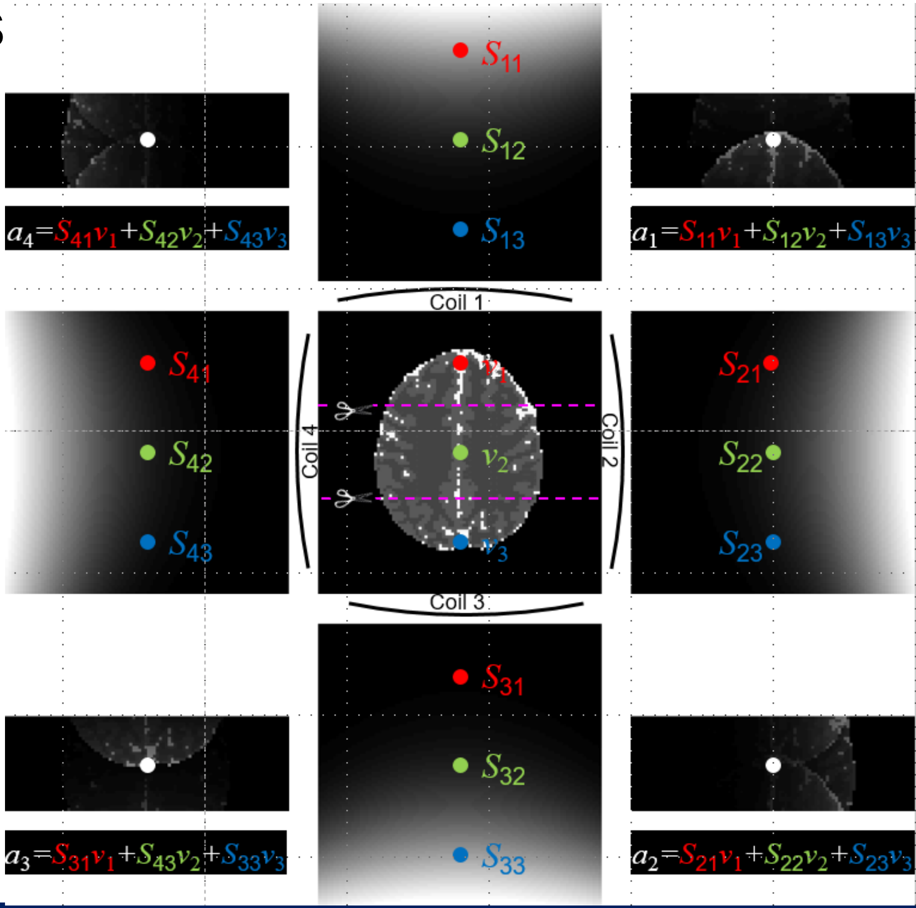
Multi-Coil SENSE

Latent variable model similar to Bayesian factor analysis, but …have complex-valued variables

$$a_C = S_C v_C + \varepsilon_C$$

a_C observed aliased
S_C unobserved sensitivities
v_C unobserved voxel values

$$a_C = a_R + i a_I$$
$$S_C = S_R + i S_I$$
$$v_C = v_R + i v_I$$
$$\varepsilon_C = \varepsilon_R + i \varepsilon_I$$



Rowe. Multivariate Bayesian Statistics. CRC Press. 2003.

Bayesian Reconstruction

Multi-Coil SENSE

Complex-valued representation

$$\underset{Nc\times 1}{a_C} = \underset{Nc\times A}{S_C}\,\underset{A\times 1}{v_C} + \underset{Nc\times 1}{\varepsilon_C}$$

$$a_C = a_R + i a_I$$
$$S_C = S_R + i S_I$$
$$v_C = v_R + i v_I$$
$$\varepsilon_C = \varepsilon_R + i \varepsilon_I$$

Real-valued isomorphism representation

$$\begin{bmatrix} a_R \\ a_I \end{bmatrix} = \begin{bmatrix} S_R & -S_I \\ S_I & S_R \end{bmatrix} \begin{bmatrix} v_R \\ v_I \end{bmatrix} + \begin{bmatrix} \varepsilon_R \\ \varepsilon_I \end{bmatrix}$$

$$\underset{2Nc\times 1}{a} = \underset{2Nc\times 2A}{S}\;\underset{2A\times 1}{v} + \underset{2Nc\times 1}{\varepsilon}$$

a observed aliased
S unobserved sensitivities
v unobserved voxel values

Bayesian Reconstruction

Multi-Coil SENSE

$$\underset{2Nc\times 1}{a} = \begin{bmatrix} a_R \\ a_I \end{bmatrix}$$

$$\underset{2Nc\times 2A}{S} = \begin{bmatrix} S_R & -S_I \\ S_I & S_R \end{bmatrix}$$

$$\underset{2A\times 1}{v} = \begin{bmatrix} v_R \\ v_I \end{bmatrix}$$

$$\underset{2Nc\times 1}{\varepsilon} = \begin{bmatrix} \varepsilon_R \\ \varepsilon_I \end{bmatrix}$$

Isomorphism Model

$$\underset{2Nc\times 1}{a} = \underset{2Nc\times 2A}{S} \; \underset{2A\times 1}{v} + \underset{2Nc\times 1}{\varepsilon}$$

Likelihood

$$\varepsilon \sim N(0, \sigma^2 I)$$

$$p(a \mid S, v, \sigma^2) \propto (\sigma^2)^{-\frac{2N_C}{2}} \exp\left[-\frac{1}{2\sigma^2}(a - Sv)'(a - Sv) \right]$$

a observed aliased

S unobserved sensitivities

v unobserved voxel values

Bayesian Reconstruction

Multi-Coil SENSE

Isomorphism Model

$$\underset{2Nc\times 1}{a} = \underset{2Nc\times 2A}{S}\ \underset{2A\times 1}{v} + \underset{2Nc\times 1}{\varepsilon}$$

$$\underset{2Nc\times 1}{a} = \begin{bmatrix} a_R \\ a_I \end{bmatrix}$$

$$\underset{2Nc\times 2A}{S} = \begin{bmatrix} S_R & -S_I \\ S_I & S_R \end{bmatrix}$$

$$\underset{2A\times 1}{v} = \begin{bmatrix} v_R \\ v_I \end{bmatrix}$$

$$\underset{2Nc\times 1}{\varepsilon} = \begin{bmatrix} \varepsilon_R \\ \varepsilon_I \end{bmatrix}$$

Priors

$$p(S\,|\,\sigma^2) \propto (\sigma^2)^{-\frac{2N_C A}{2}} \exp\left[-\frac{g_S}{2\sigma^2} tr(S-S_0)'(S-S_0)\right]$$

$$p(v\,|\,\sigma^2) \propto (\sigma^2)^{-\frac{2N_C A}{2}} \exp\left[-\frac{g_v}{2\sigma^2} (v-v_0)'(v-v_0)\right]$$

$$p(\sigma^2) \propto (\sigma^2)^{-\frac{d}{2}-1} \exp\left[-\frac{q}{2\sigma^2}\right]$$

a observed aliased

S unobserved sensitivities

v unobserved voxel values

Bayesian Reconstruction

Multi-Coil SENSE

Isomorphism Model

$$\underset{2Nc\times 1}{a} = \underset{2Nc\times 2A}{S}\ \underset{2A\times 1}{v} + \underset{2Nc\times 1}{\varepsilon}$$

Posterior

$$p(S,v,\sigma^2 \mid a) \propto p(S \mid \sigma^2)\, p(v \mid \sigma^2)\, p(\sigma^2)\, p(a \mid S,v,\sigma^2)$$

Modes for ICM

$$\hat{S} = (g_S S_0 + av')(g_S I + v'v)^{-1}$$

$$\hat{v} = (g_v I + S'S)^{-1}(g_v v_0 + S'a)$$

$$\hat{\sigma}^2 = [(a - Sv)'(a - Sv) + g_v (v - v_0)'(v - v_0) + dq + g_S tr(S - S_0)(S - S_0)] / [2(2N_C + 2A + d + 2N_C A + 1)]$$

Bayesian Reconstruction

Multi-Coil SENSE

Hyperparameter Assessment

$$p(S\,|\,\sigma^2):\;\; g_S, S_0$$

$$p(v\,|\,\sigma^2):\;\; g_v, v_0$$

$$p(\sigma^2): d, q$$

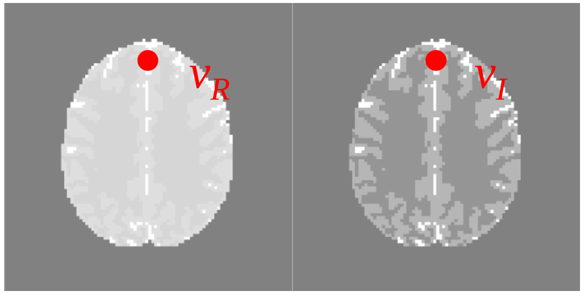
Measure m full data multi coil sensitivity weighted images.

Before we perform fMRI experiment. Calibration images.

m full data calibration images	n full data experimental images
Non-Task	Task
Hyperparameter Assessment	For Analysis

Bayesian Reconstruction

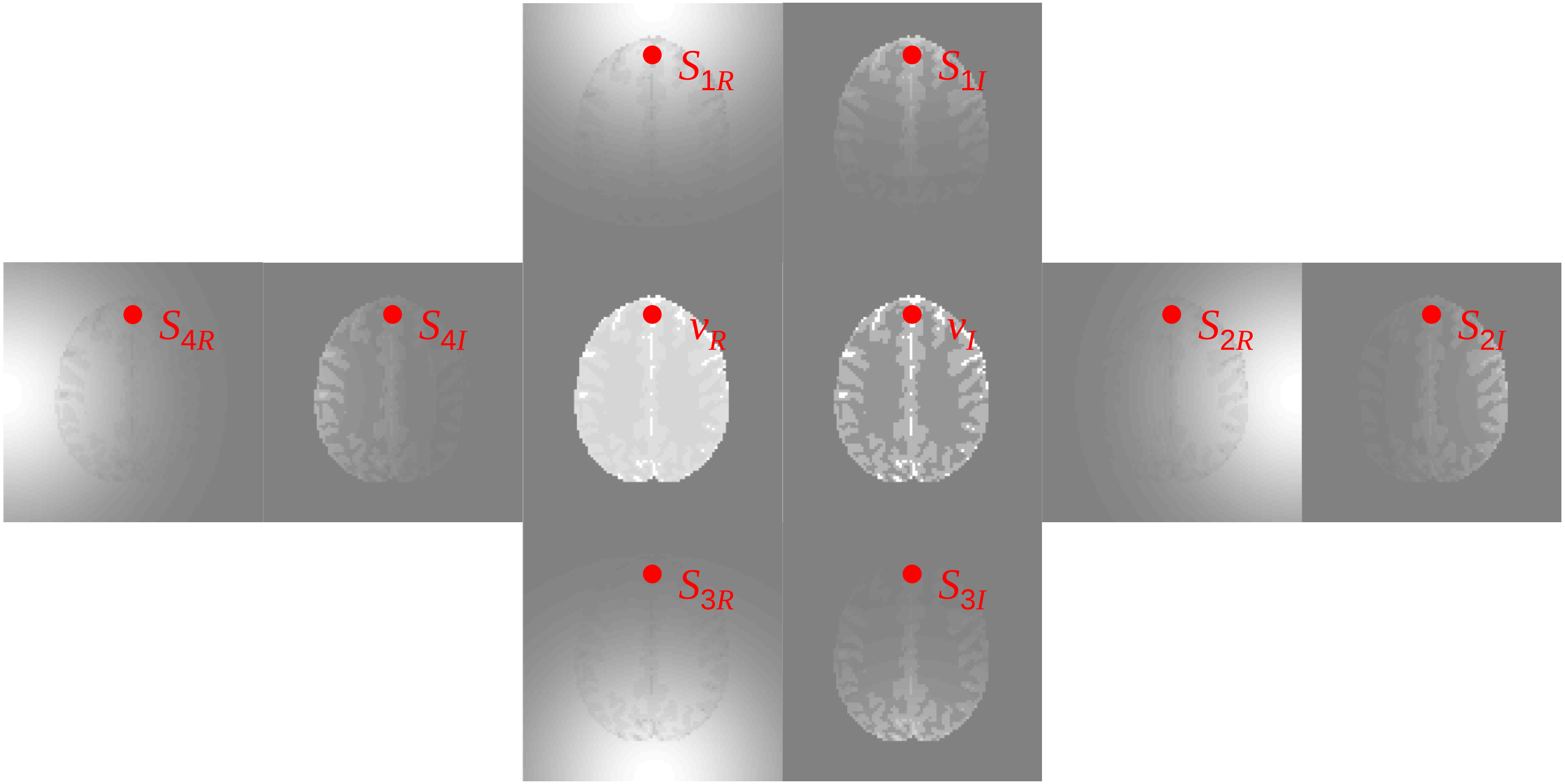
m full data calibration images



True Slice Image

Bayesian Reconstruction

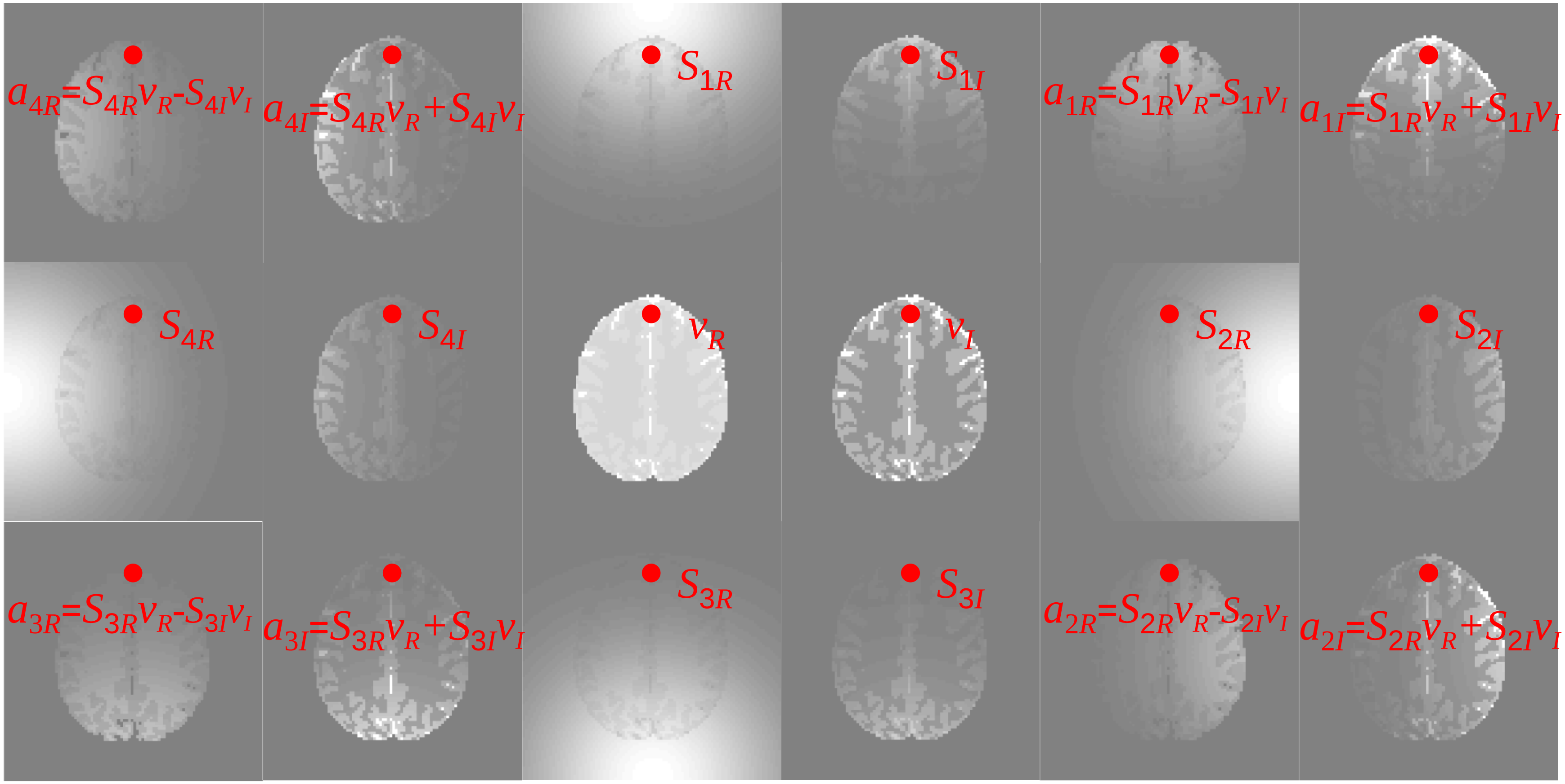
m full data calibration images



True Slice Coil Sensitivities

Bayesian Reconstruction

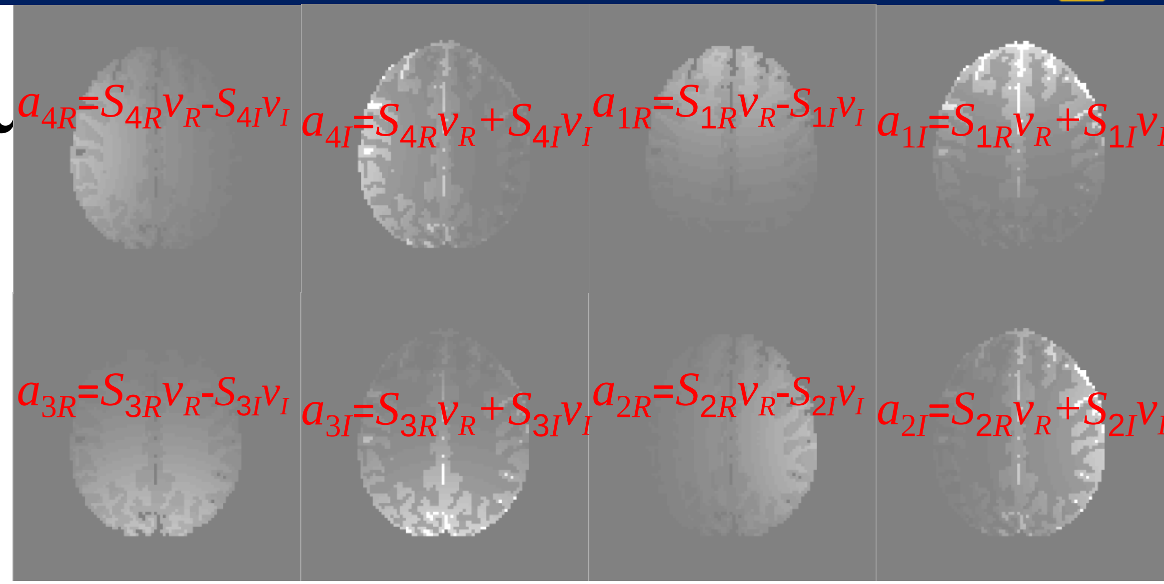
m full data calibration images



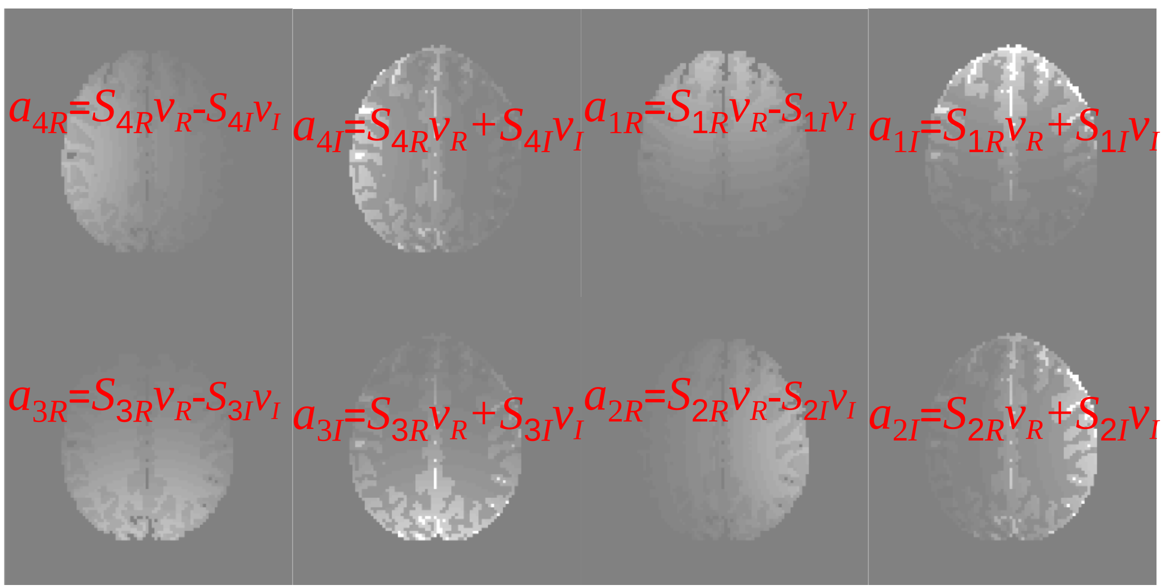
True Slice Coil Images

Bayesian Reconstru

m **full data calibration images**



m



1

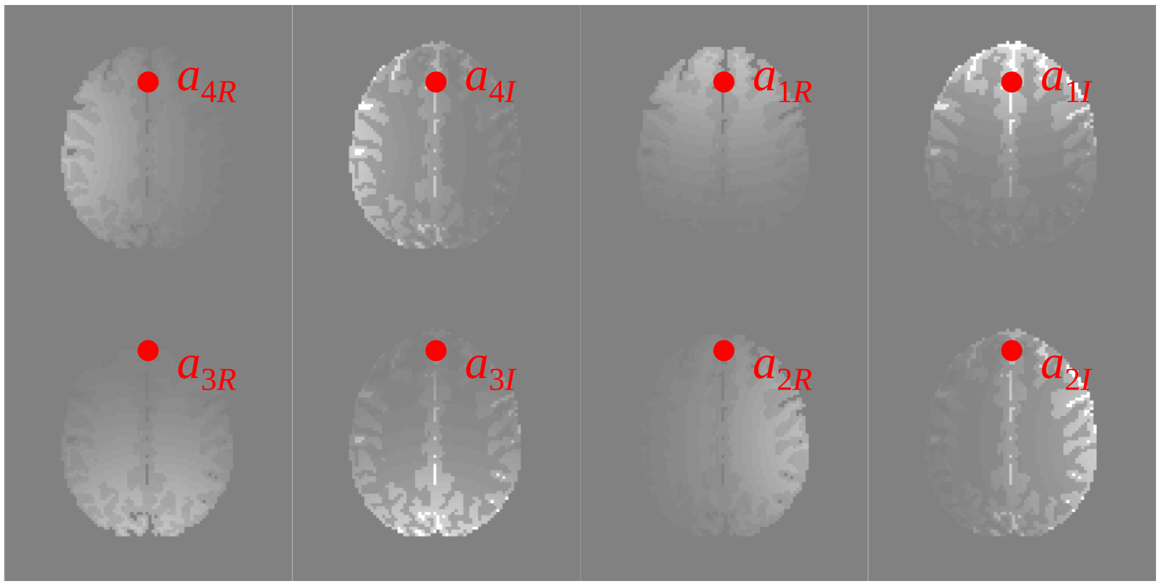
Add $N(0,1)$ noise to R and I and average

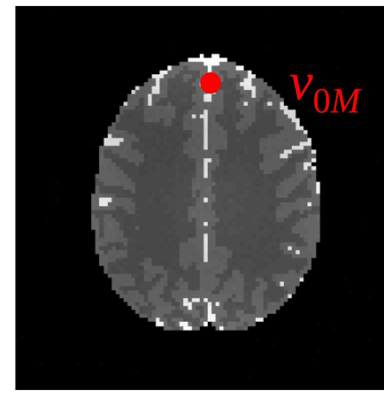
Bayesian Reconstruction

Multi-Coil SENSE

$$p(S \mid \sigma^2):\ \ g_S, S_0$$

Hyperparameter Assessment





RSOS for magnitude

average

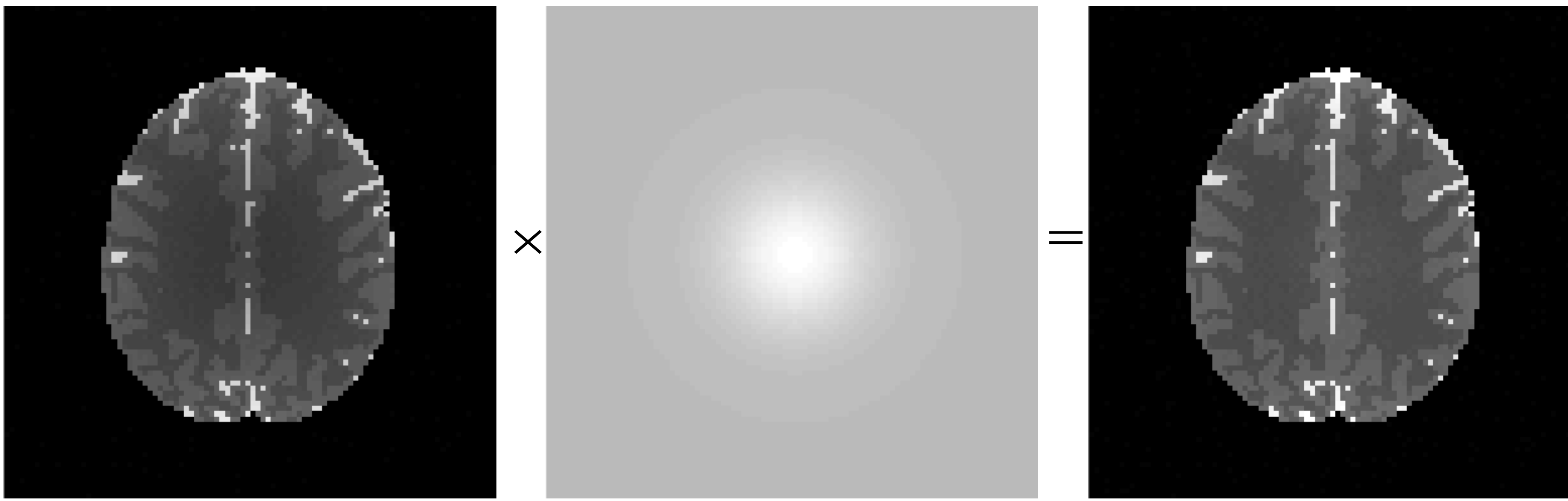
$$v_{0M} = \left[a_{R1}^2 + a_{1I}^2 + a_{2R}^2 + a_{2I}^2 + a_{3R}^2 + a_{3I}^2 + a_{4R}^2 + a_{4I}^2 \right]^{1/2}$$

Bayesian Reconstruction

Hyperparameter Assessment

$$p(S\,|\,\sigma^2):\;\; g_S, S_0$$

Actually weighted RSOS with scaled normal plus constant.



$$h(x,y) = 1.1 + .4 \times \exp\left\{ -\frac{1}{2n_{xy}} \left[(x - n_{xy}/2 - 1)^2 + (-n_{xy}/2 - 1)^2 \right] \right\}$$

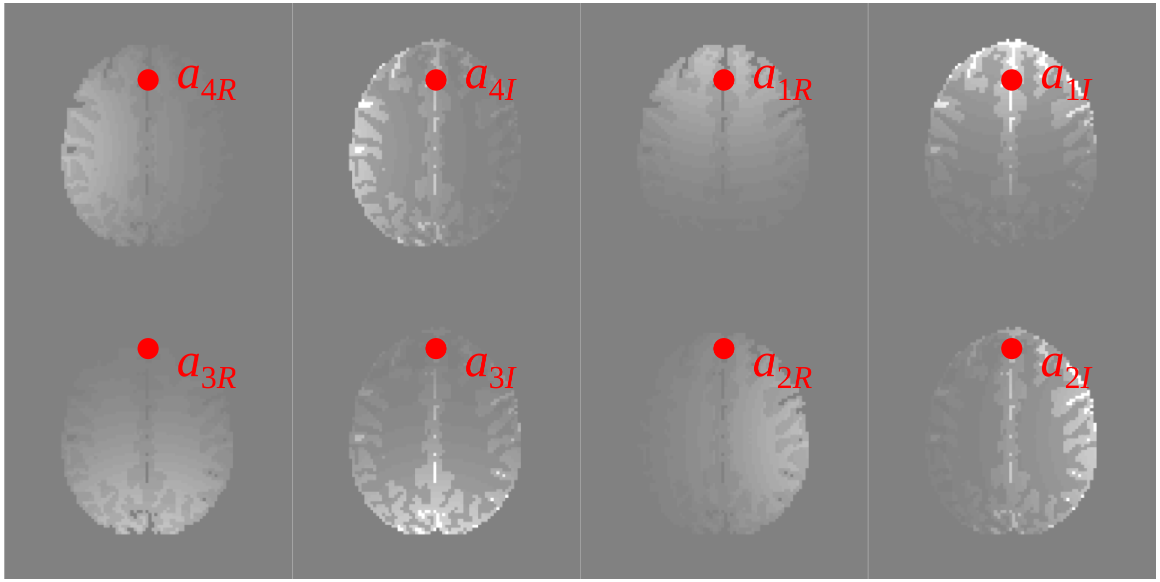
$$n_{xy} = 96$$

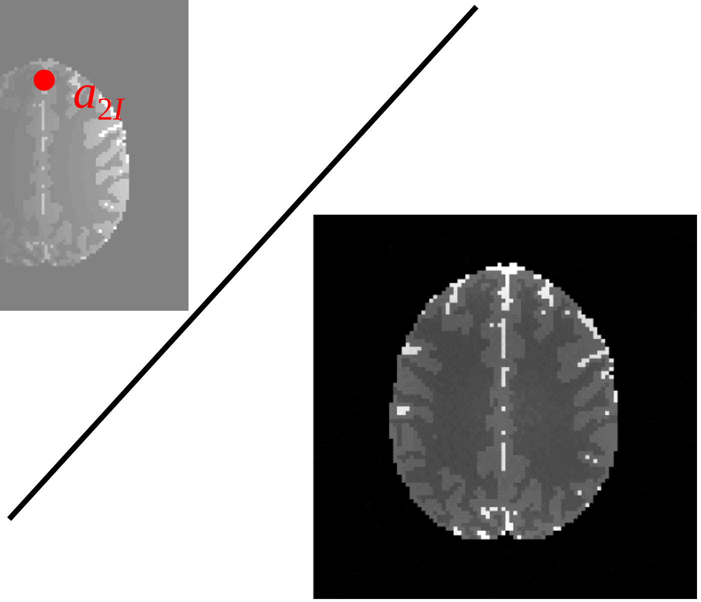
Bayesian Reconstruction

Hyperparameter Assessment

$$p(S \mid \sigma^2): \; g_S, S_0$$

Divide each averaged image by RSOS for Sensitivities



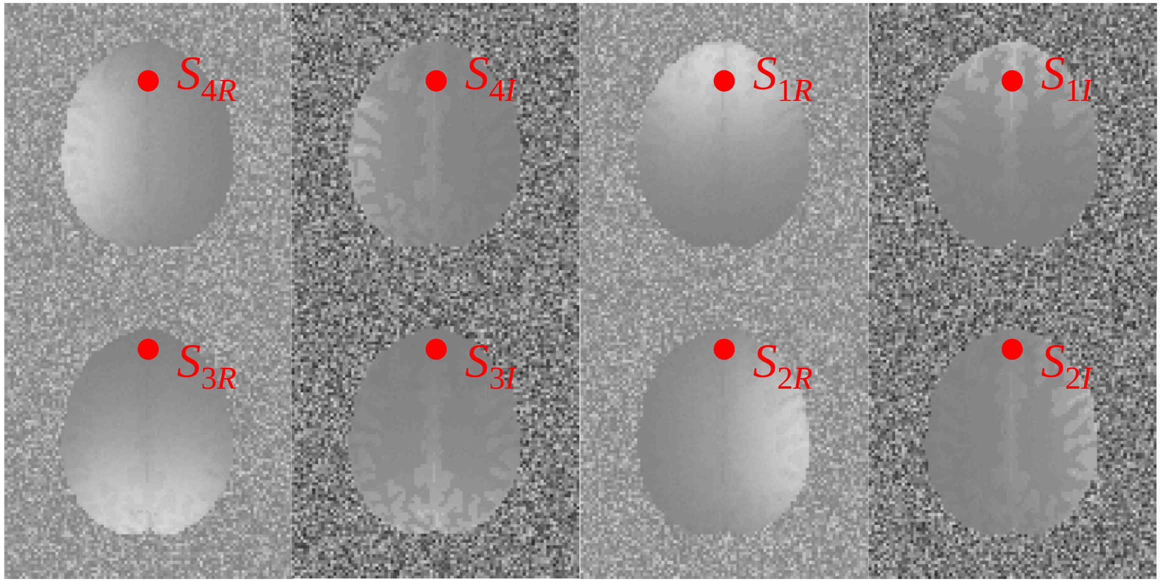


Bayesian Reconstruction

Hyperparameter Assessment

$$p(S \mid \sigma^2): \; g_S, S_0$$

Divide each averaged image by RSOS for Sensitivities



S_0

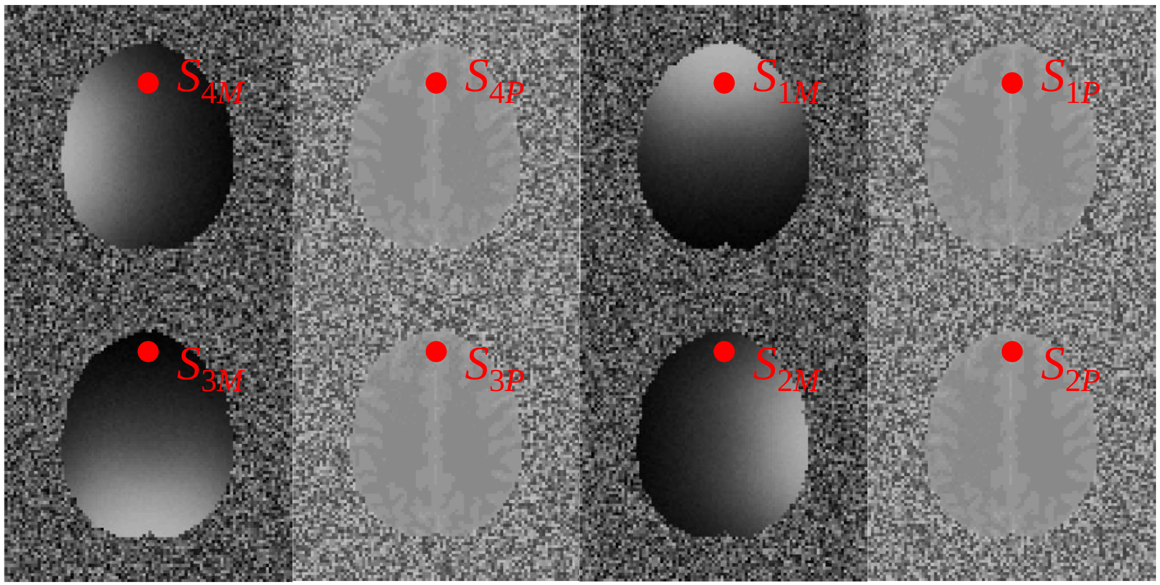
$g_S = 0.1$

Bayesian Reconstruction

Hyperparameter Assessment

$$p(S \mid \sigma^2): \;\; g_S, S_0$$

Divide each averaged image by RSOS for Sensitivities



S_0

$g_S = 0.1$

Bayesian Reconstruction

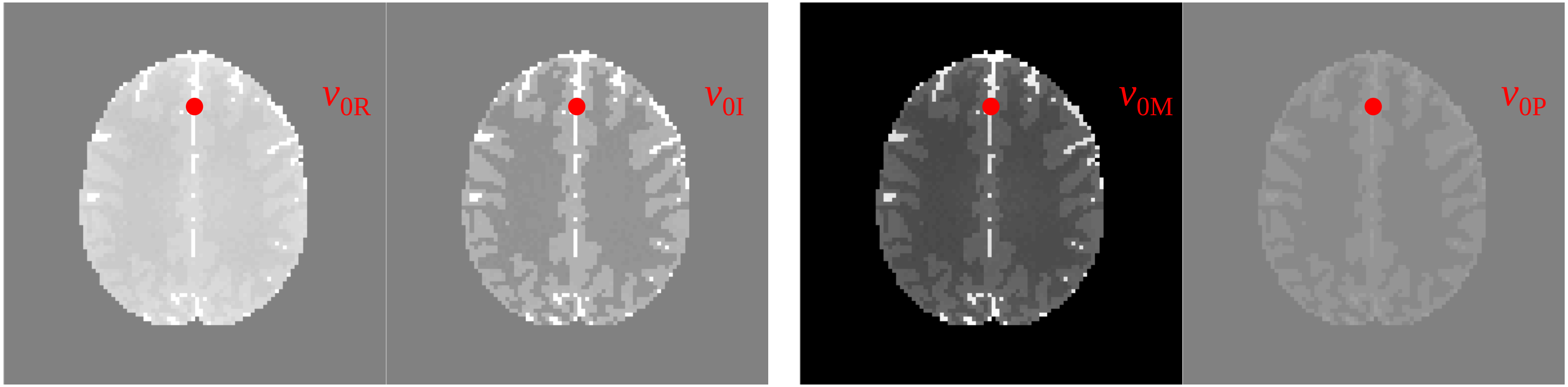
Hyperparameter Assessment

$p(v \mid \sigma^2): \; g_v, v_0$

Have S_0, assess v_0 from

$$\underset{2Nc\times 1}{a} = \underset{2Nc\times 2}{S_0} \; \underset{2\times 1}{v} + \underset{2Nc\times 1}{\varepsilon}$$

$$\underset{2\times 1}{v_0} = (S_0' S_0)^{-1} S' a \qquad g_v = 0.1$$



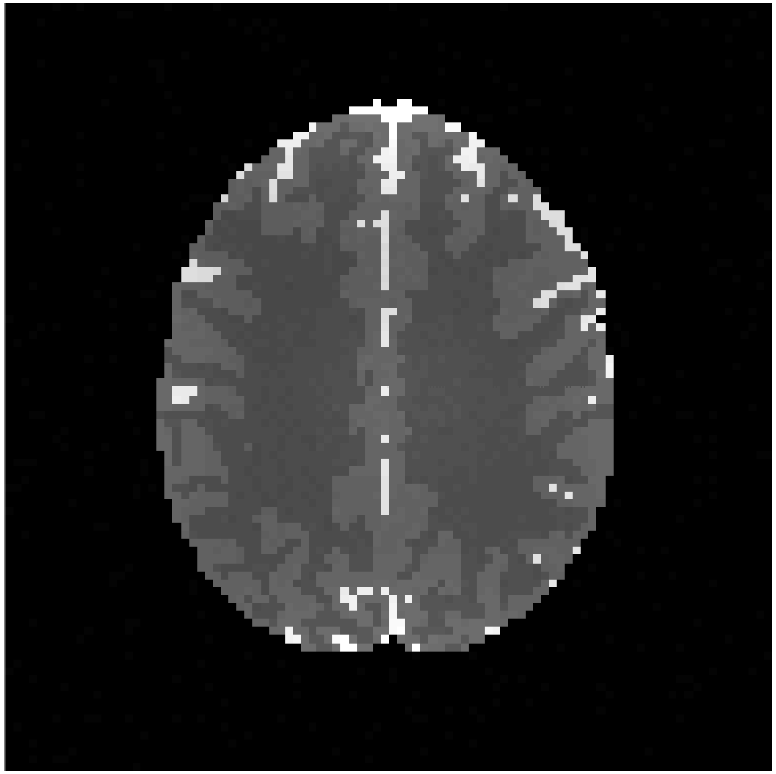
v_0

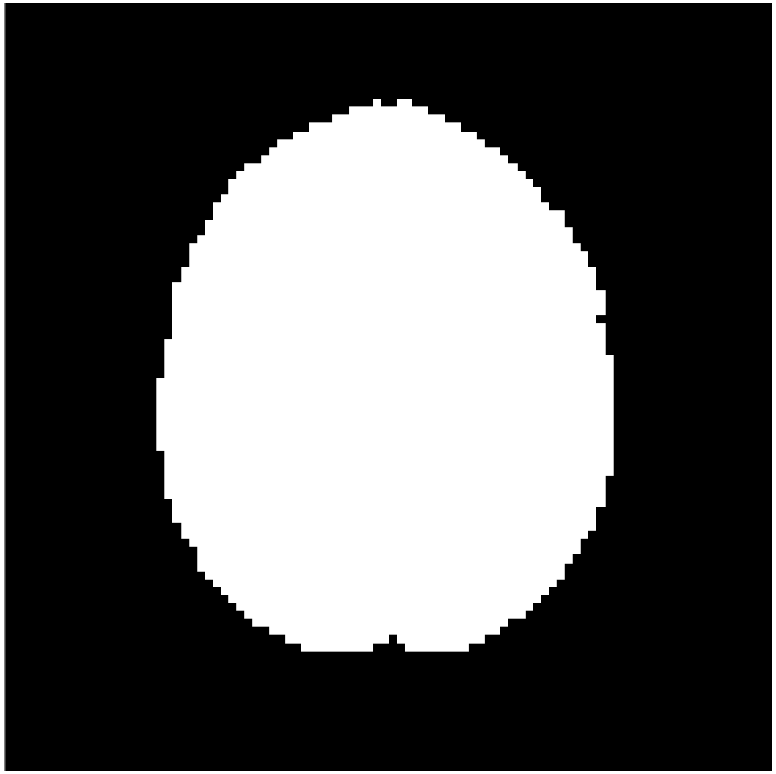
Bayesian Reconstruction

Hyperparameter Assessment

$p(v\,|\,\sigma^2):\;\; g_v, v_0$

Also generated a mask from voxels >.1 max RSOS.
And zeroed out voxel prior mean v_0 outside brain.



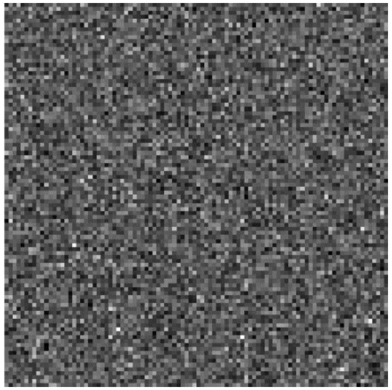


Bayesian Reconstruction

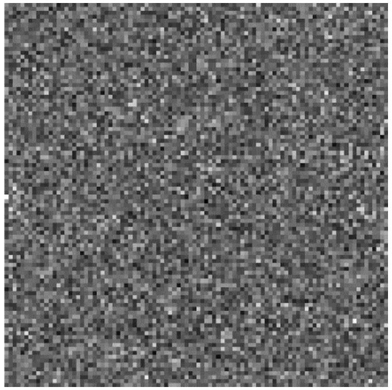
Hyperparameter Assessment

$$p(\sigma^2) : d, q$$

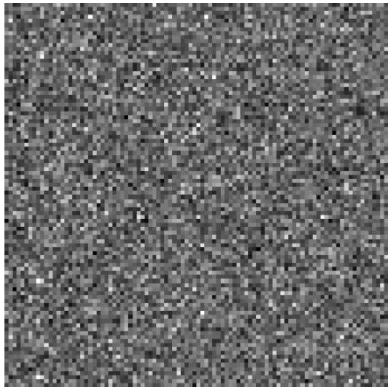
Calculated the average real and imaginary voxel sample variances from the calibration images



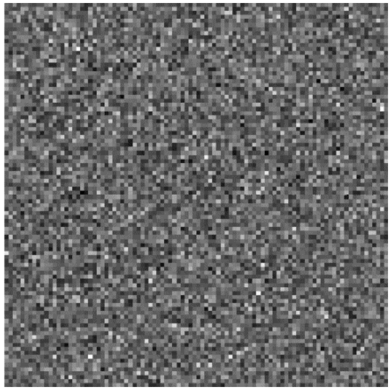
Coil 1



Coil 2



Coil 3

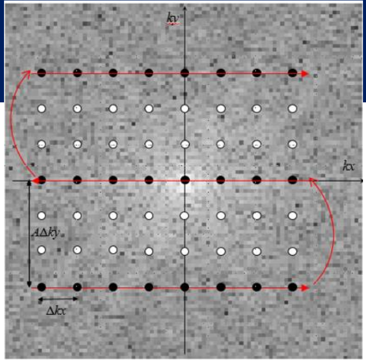


Coil 4

and averaged to obtain single s_v^2.

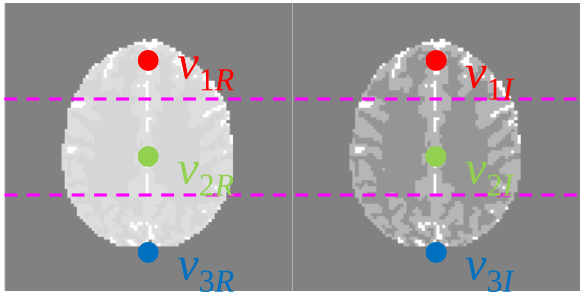
Assessed a prior df d=10, then $q = 2ds_v^2$.

$$q = 20.01$$



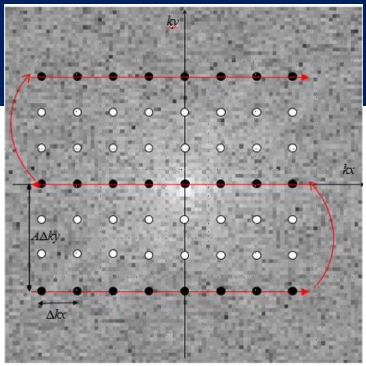
Bayesian Reconstruction

n **sub sampled data fMRI images**



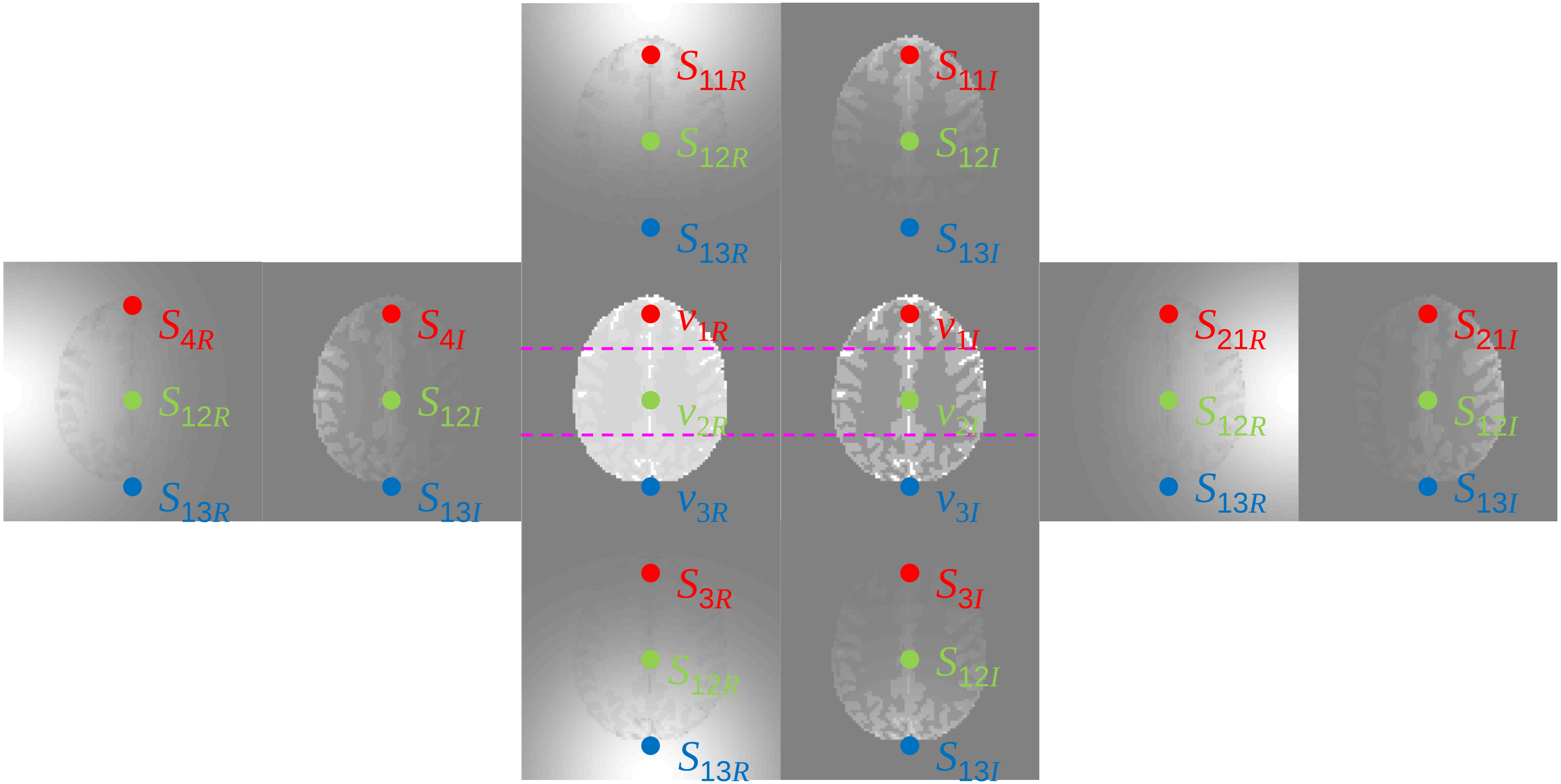
True Slice Coil Images

N_C=4, A=3



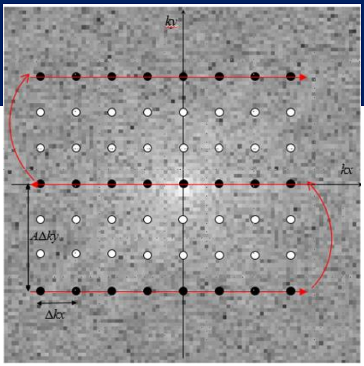
Bayesian Reconstruction

n sub sampled data fMRI images



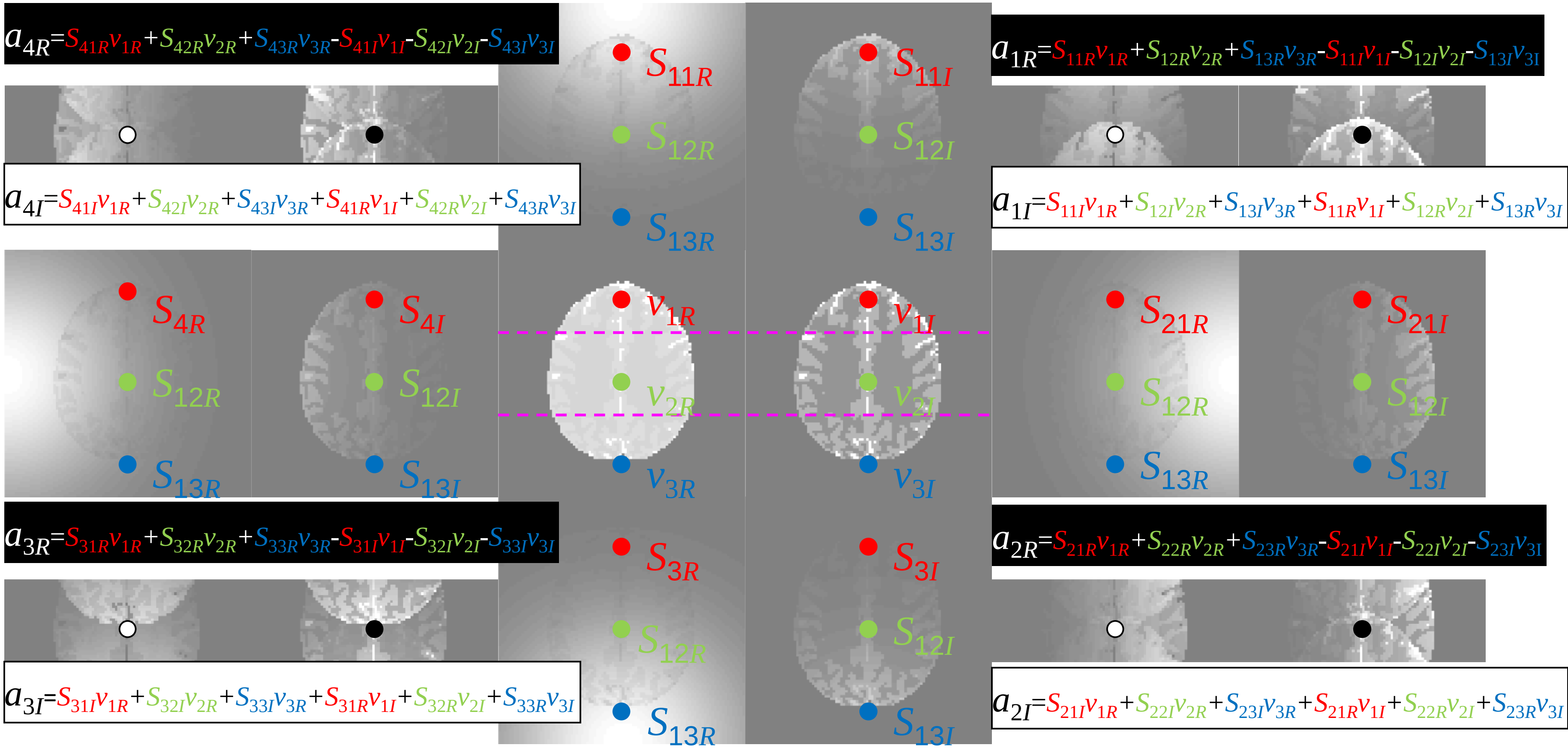
N_C=4, A=3

True Slice Coil Images



Bayesian Reconstruction

n sub sampled data fMRI images



N_C=4, A=3

True Slice Coil Images

Bayesian Reconstruction

n sub sampled data fMRI images

Coil 4

Coil 1

Coil 3

Coil 2

n

$\cdot^{\cdot^{\cdot}}$

Coil 4

Coil 1

Coil 3

Coil 2

1

Add $N(0,1)$ noise to R and I.

N_C=4, A=3

Bayesian Reconstruction

n sub sampled data fMRI images

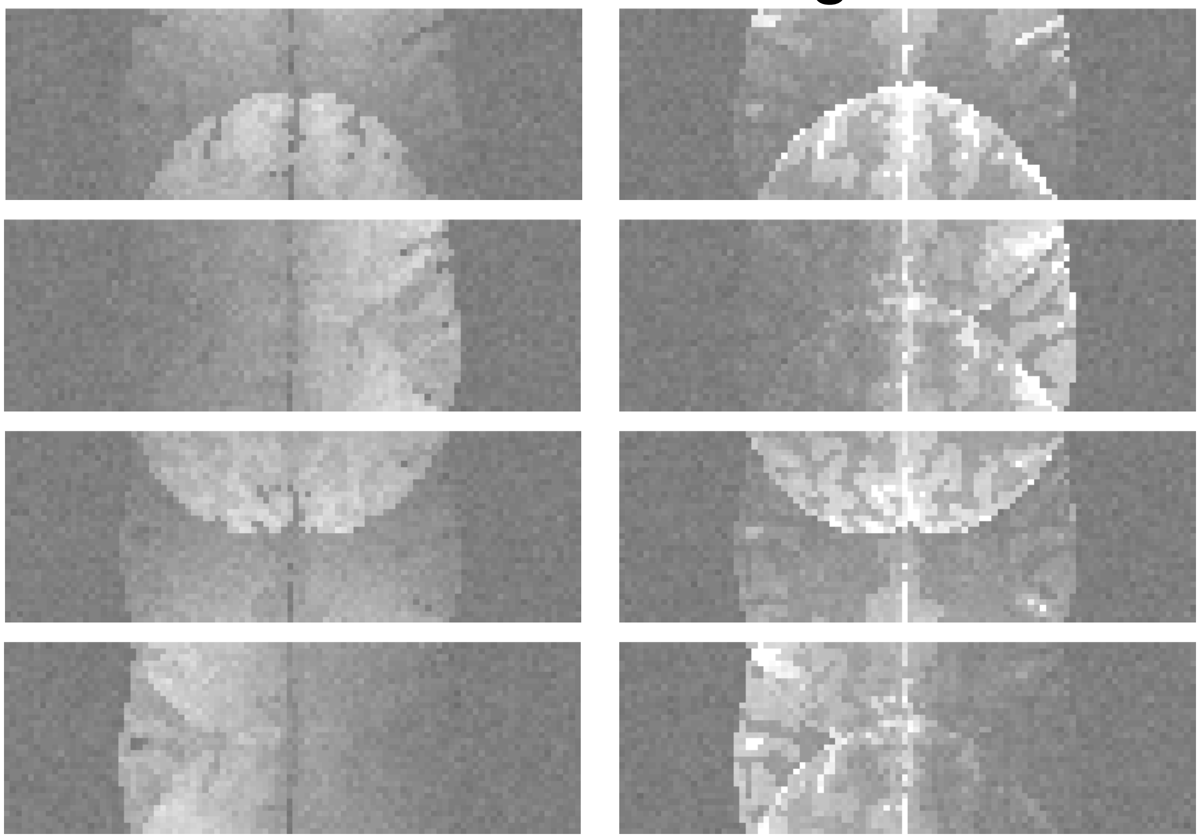
Observed aliased slice coil images

Coil 1

Coil 2

Coil 3

Coil 4



Real Imaginary

N_C=4, A=3

Bayesian Reconstruction

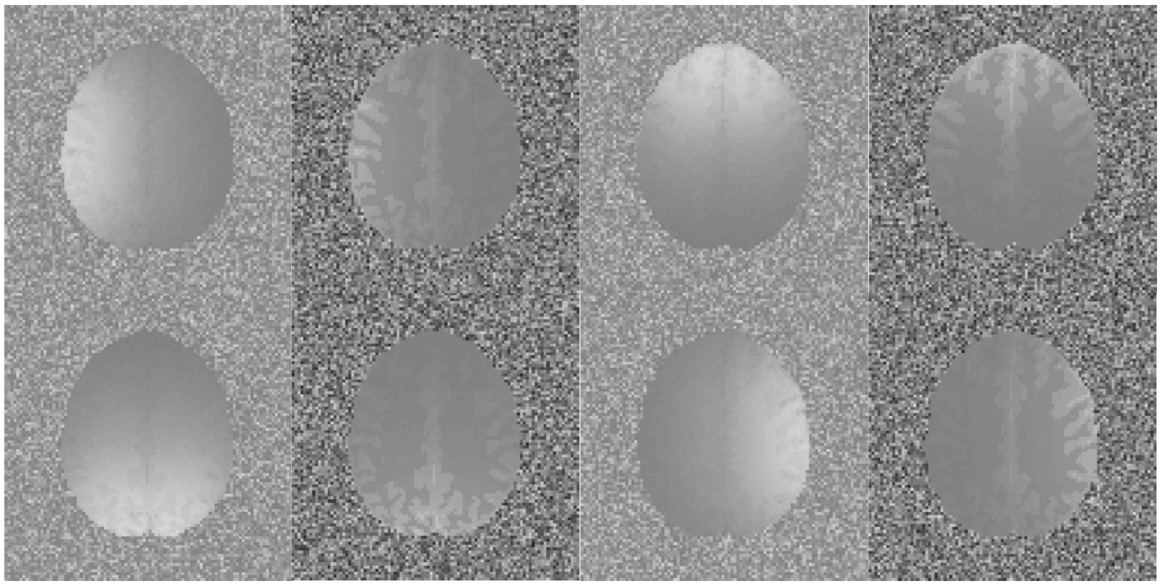
n sub sampled data fMRI images

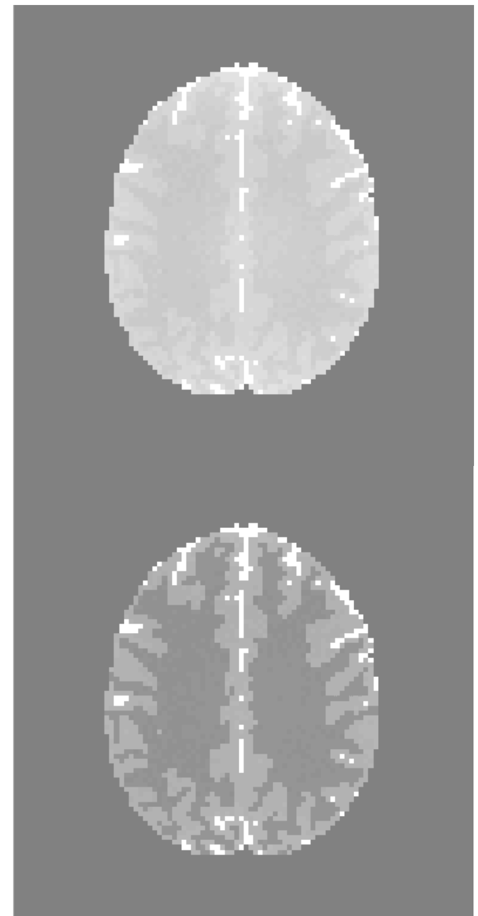
$$p(S\,|\,\sigma^2):\ g_S, S_0$$

$$p(v\,|\,\sigma^2):\ g_v, v_0$$

$$p(\sigma^2): d, q$$

Prior Hyperparameters





d=10

$q = 2ds_v^2$

S_0

$g_S = 0.1$

v_0

$g_v = 0.1$

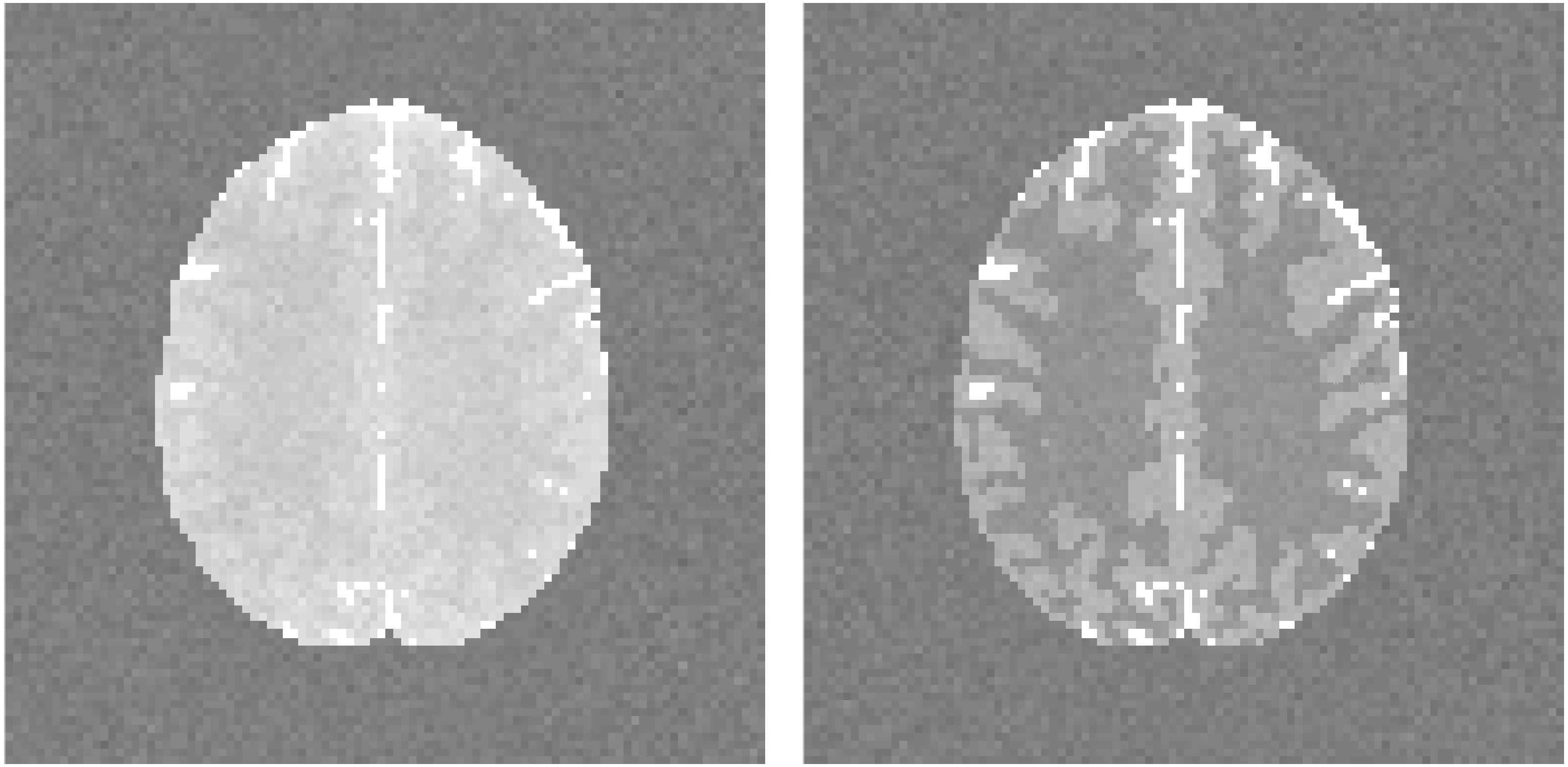
N_C=4, A=3

Bayesian Reconstruction

n sub sampled data fMRI images

Unaliased (Estimated) MAP Image (Parameters).

$\hat{v}$



Real Imaginary

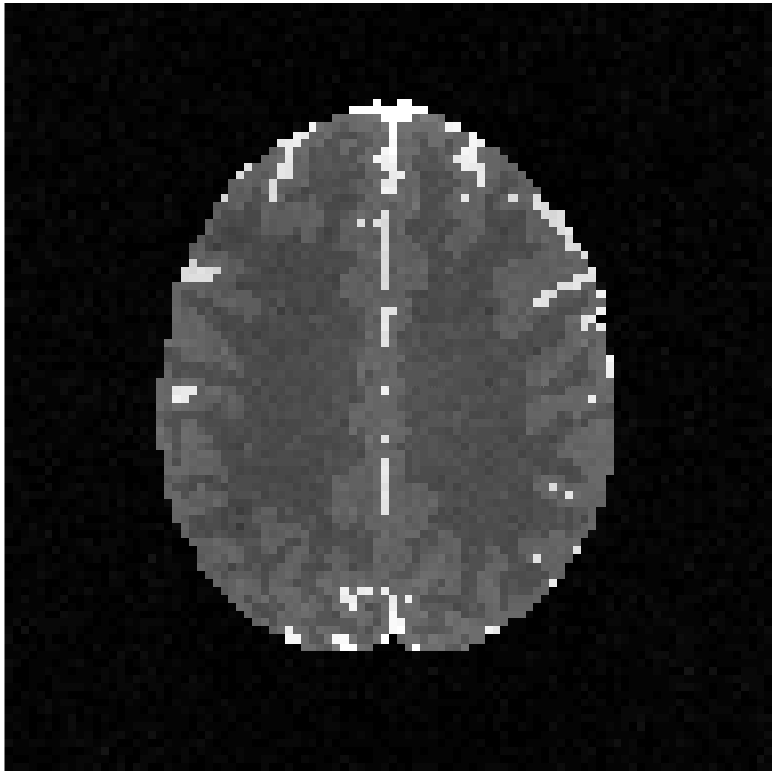
N_C=4, A=3

Bayesian Reconstruction

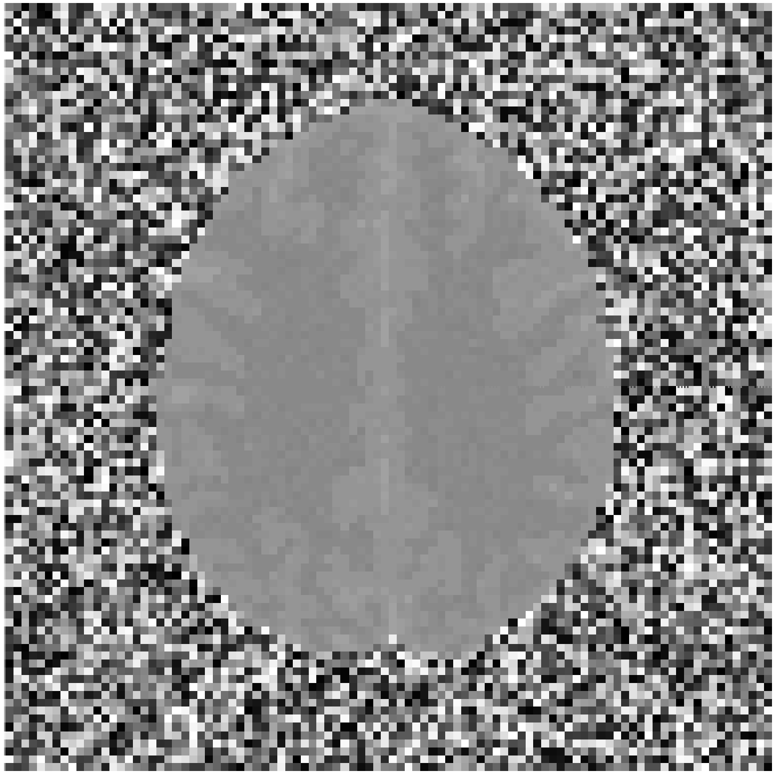
n sub sampled data fMRI images

Unaliased (Estimated) MAP Image (Parameters).

$\hat{v}$



Magnitude



Phase

N_C=4, A=3

Can now discarded phase ½ of YOUR data.

Discussion

Voxel values are not measured by the machine.

It is important to obtain data in raw measured form.

Subsampled data reconstruction a statistical problem.

This area is still the wild west with many opportunities.

Lots of competition in statistical modeling after reconstructed into images. Freely downloadable data.

Thank You!

Looking for New PhD Students!

Daniel.Rowe@Marquette.Edu