Single Coil Multi-Slice Aliasing and Separation for FMRI

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Outline

1. Introduction
SMS as an incomplete CAIPI or Hadamard block design.

2. Theory
The SPECS model utilizing both CAIPI and Hadamard aliasing.

3. Materials and Methods
Describe simulation to compare CAIPI and Hadamard.

4. Results
Present SPECS SMS results comparing CAIPI and Hadamard.

5. Discussion
Review and relate what theory and experiment demonstrate.

1. Introduction

FMRI takes a nontrivial amount of time to measure an image.

Within slice techniques such as SENSE[1] and GRAPPA[2] have been developed to decrease image measurement time.

Some work[3,4] has demonstrated that SENSE and GRAPPA unfolded aliased images yield long range spatial correlation.

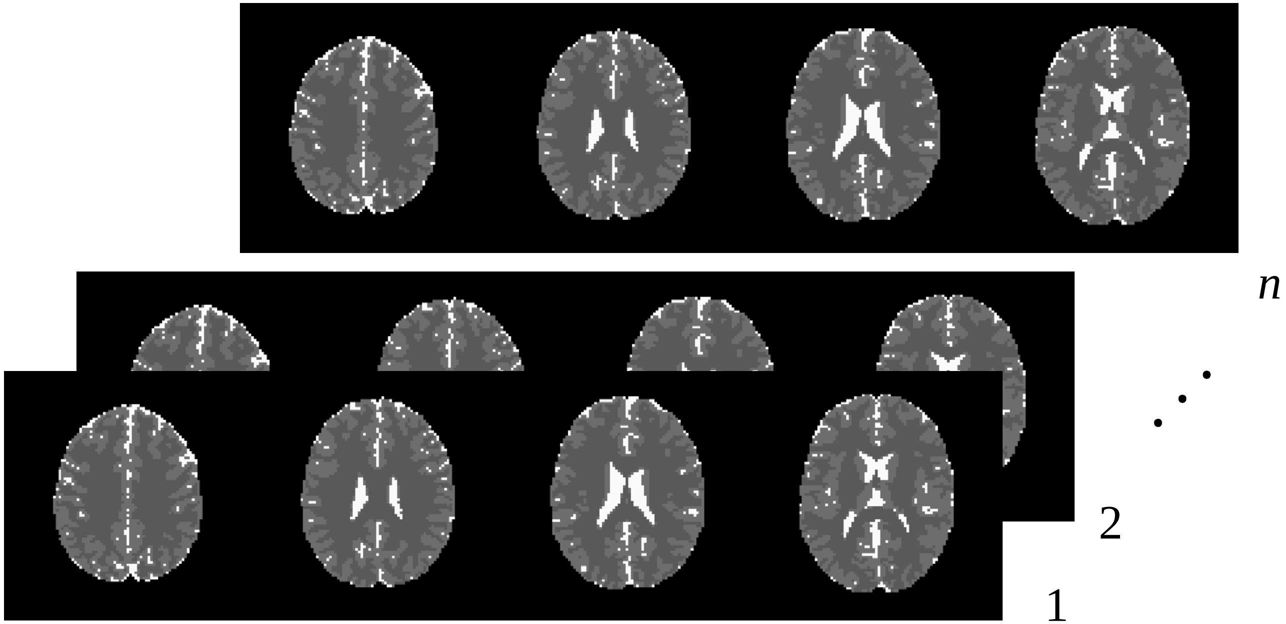
Multiband image aliasing has recently been developed but results[5,6] indicate there can be high correlation between slices.

The SPECS separation model[7] does not induce between slice correlation is utilized on CAIPI and Hadamard aliased images.

1. Introduction

Traditionally in fMRI the images would be acquired individually as quickly as possible while the subject is performing the task.

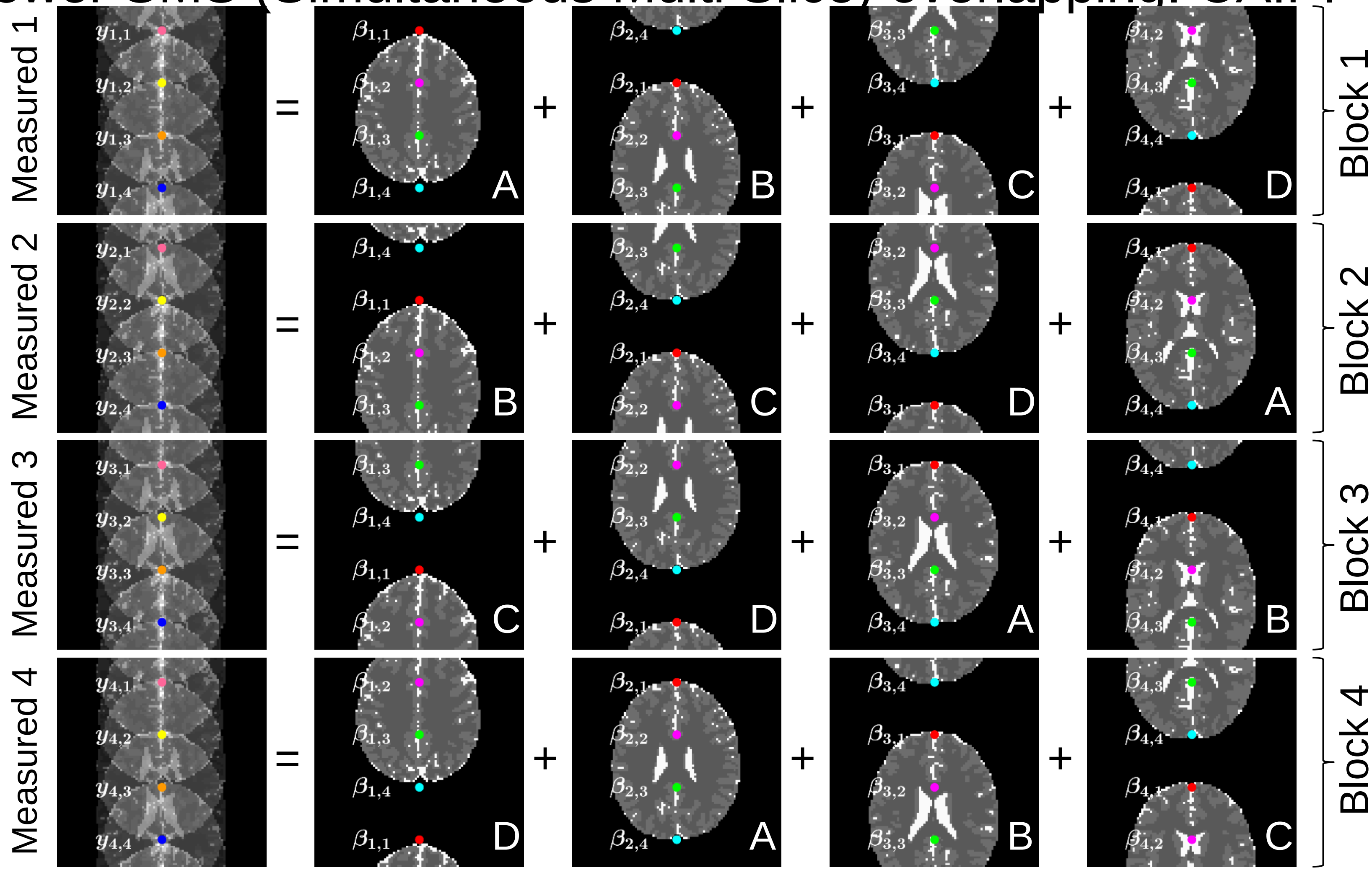
TR=1,2,3



Latin Square

1. Introduction

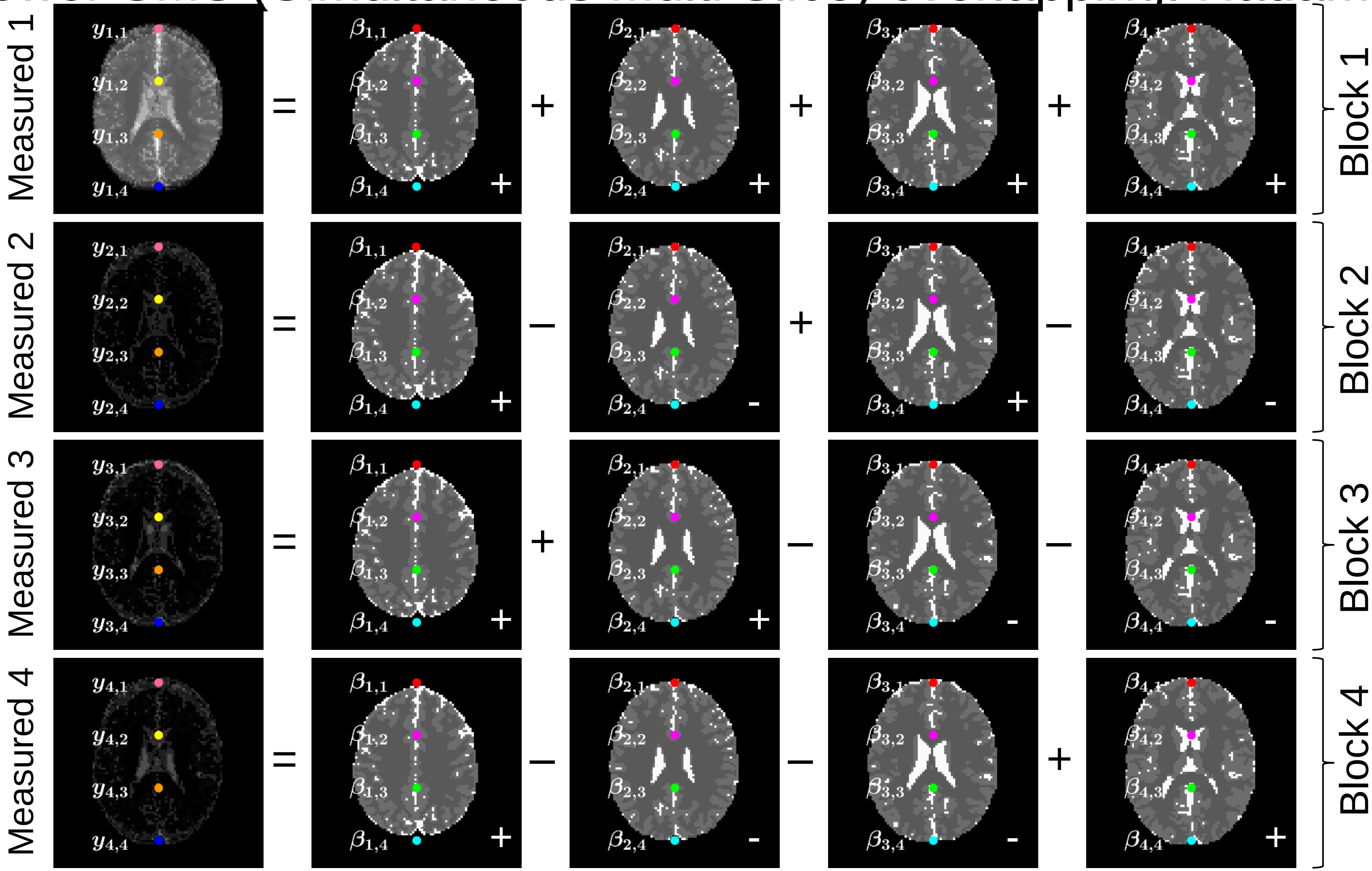
Newer SMS (Simultaneous Multi Slice) overlapping. CAIPI



1. Introduction

Orthogonal

Newer SMS (Simultaneous Multi Slice) overlapping. Hadamard



1. Introduction

But if we have to measure 4 aliased slices to get 4 non-aliased slices, then we have no time gain.

So what we want to do is measure a fraction of the blocks to save time.

But there are estimation challenges when doing so.

We need additional information.

How should we optimally alias? CAIPI* or Hadamard

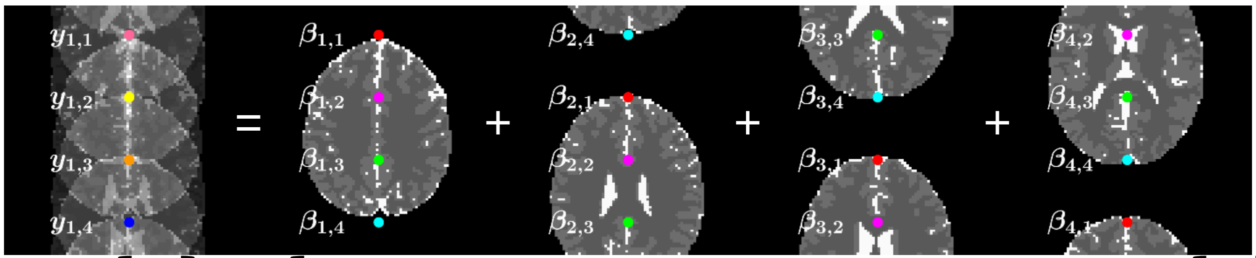
*Reconstruction challenges with overlapping CAIPI shifted images.

2. Theory

A single complex-valued “summed” image is measured

Aliased Slice 1 Slice 2 Slice 3 Slice 4

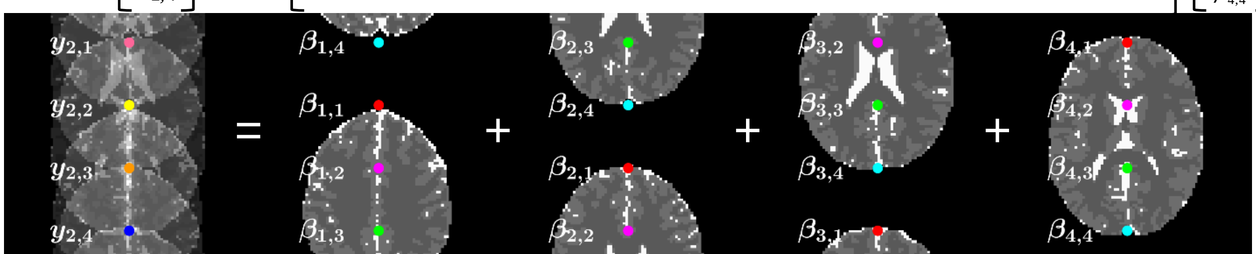
Measured 1



1st CAPI Encoding

$$\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0 \\ 0&1&0&0&1&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&1&0&0&1&0&0&1&0&0&0 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

Measured 2



2nd CAPI Encoding

$$\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix} = \begin{bmatrix} 0&0&0&1&0&0&1&0&0&1&0&0&1&0&0&0 \\ 1&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0 \\ 0&1&0&0&1&0&0&0&0&0&0&1&0&0&1&0 \\ 0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

$$\begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \\ \beta_{1,4} \\ \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \\ \beta_{2,4} \\ \beta_{3,1} \\ \beta_{3,2} \\ \beta_{3,3} \\ \beta_{3,4} \\ \beta_{4,1} \\ \beta_{4,2} \\ \beta_{4,3} \\ \beta_{4,4} \end{bmatrix}$$

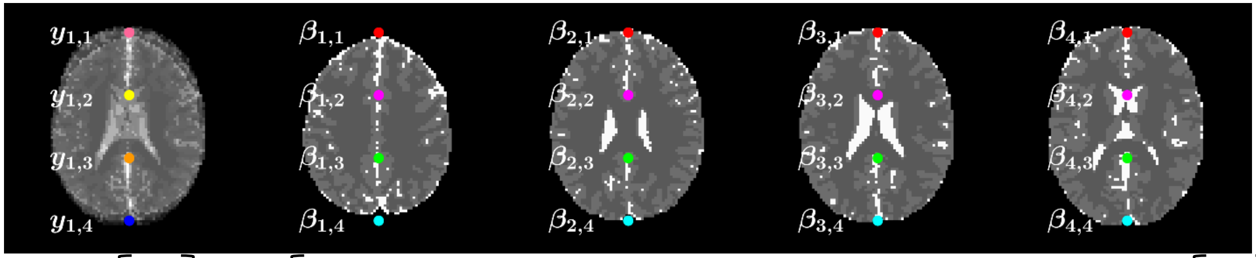
Breuer et al. 2005.

2. Theory

A single complex-valued "summed" image is measured

Aliased | Slice 1 | Slice 2 | Slice 3 | Slice 4

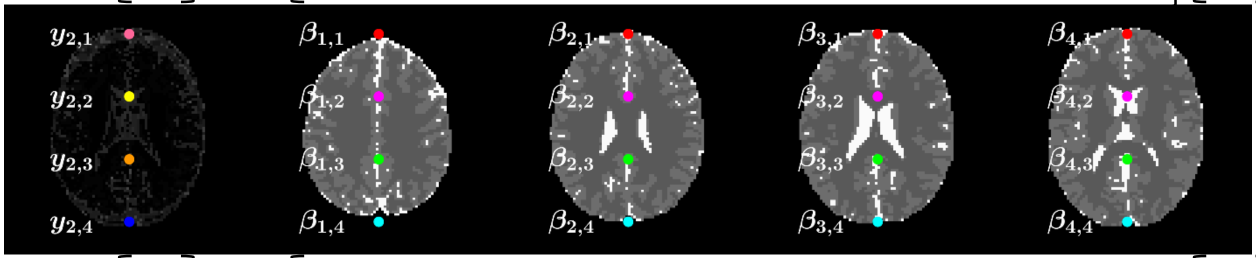
Measured 1



$$\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix} = \begin{bmatrix} 1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

1st Hadamard Encoding

Measured 2



$$\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix} = \begin{bmatrix} 1&0&0&0&-1&0&0&0&1&0&0&0&-1&0&0&0 \\ 0&1&0&0&0&-1&0&0&0&1&0&0&0&-1&0&0 \\ 0&0&1&0&0&0&-1&0&0&0&1&0&0&0&-1&0 \\ 0&0&0&1&0&0&0&-1&0&0&0&1&0&0&0&-1 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

2nd Hadamard Encoding

$$\begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \\ \beta_{1,4} \\ \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \\ \beta_{2,4} \\ \beta_{3,1} \\ \beta_{3,2} \\ \beta_{3,3} \\ \beta_{3,4} \\ \beta_{4,1} \\ \beta_{4,2} \\ \beta_{4,3} \\ \beta_{4,4} \end{bmatrix}$$

Mueller et al. 1988., Souza et al. 1988.

2. Theory

A single complex-valued "summed" image is measured

Slice 1 Slice 2 Slice 3 Slice 4

$$
\begin{array}{c}\text{Measured 1}\\ \\ \\ \text{Measured 2}\end{array}
\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \\ \hdashline y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hdashline
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \\ \beta_{1,4} \\ \hdashline \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \\ \beta_{2,4} \\ \hdashline \beta_{3,1} \\ \beta_{3,2} \\ \beta_{3,3} \\ \beta_{3,4} \\ \hdashline \beta_{4,1} \\ \beta_{4,2} \\ \beta_{4,3} \\ \beta_{4,4} \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \varepsilon_{1,3} \\ \varepsilon_{1,4} \\ \hdashline \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \varepsilon_{1,3} \\ \varepsilon_{1,4} \end{bmatrix}
$$

Aliased Aliasing Matrix True Error

CAIPI Aliased

8 equations and 16 unknowns

$$y_A = X_A \beta + \varepsilon_A$$

2. Theory

A single complex-valued “summed” image is measured

Slice 1 | Slice 2 | Slice 3 | Slice 4

$$
\begin{array}{c}\text{Measured 1}\\ \\ \text{Measured 2}\end{array}
\begin{bmatrix} y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \\ \hdashline y_{1,1} \\ y_{1,2} \\ y_{1,3} \\ y_{1,4} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
\hdashline
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \\ \beta_{1,4} \\ \hdashline \beta_{2,1} \\ \beta_{2,2} \\ \beta_{2,3} \\ \beta_{2,4} \\ \hdashline \beta_{3,1} \\ \beta_{3,2} \\ \beta_{3,3} \\ \beta_{3,4} \\ \hdashline \beta_{4,1} \\ \beta_{4,2} \\ \beta_{4,3} \\ \beta_{4,4} \end{bmatrix}
+
\begin{bmatrix} \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \varepsilon_{1,3} \\ \varepsilon_{1,4} \\ \hdashline \varepsilon_{1,1} \\ \varepsilon_{1,2} \\ \varepsilon_{1,3} \\ \varepsilon_{1,4} \end{bmatrix}
$$

Aliased | Aliasing Matrix | True | Error

Hadamard Aliased

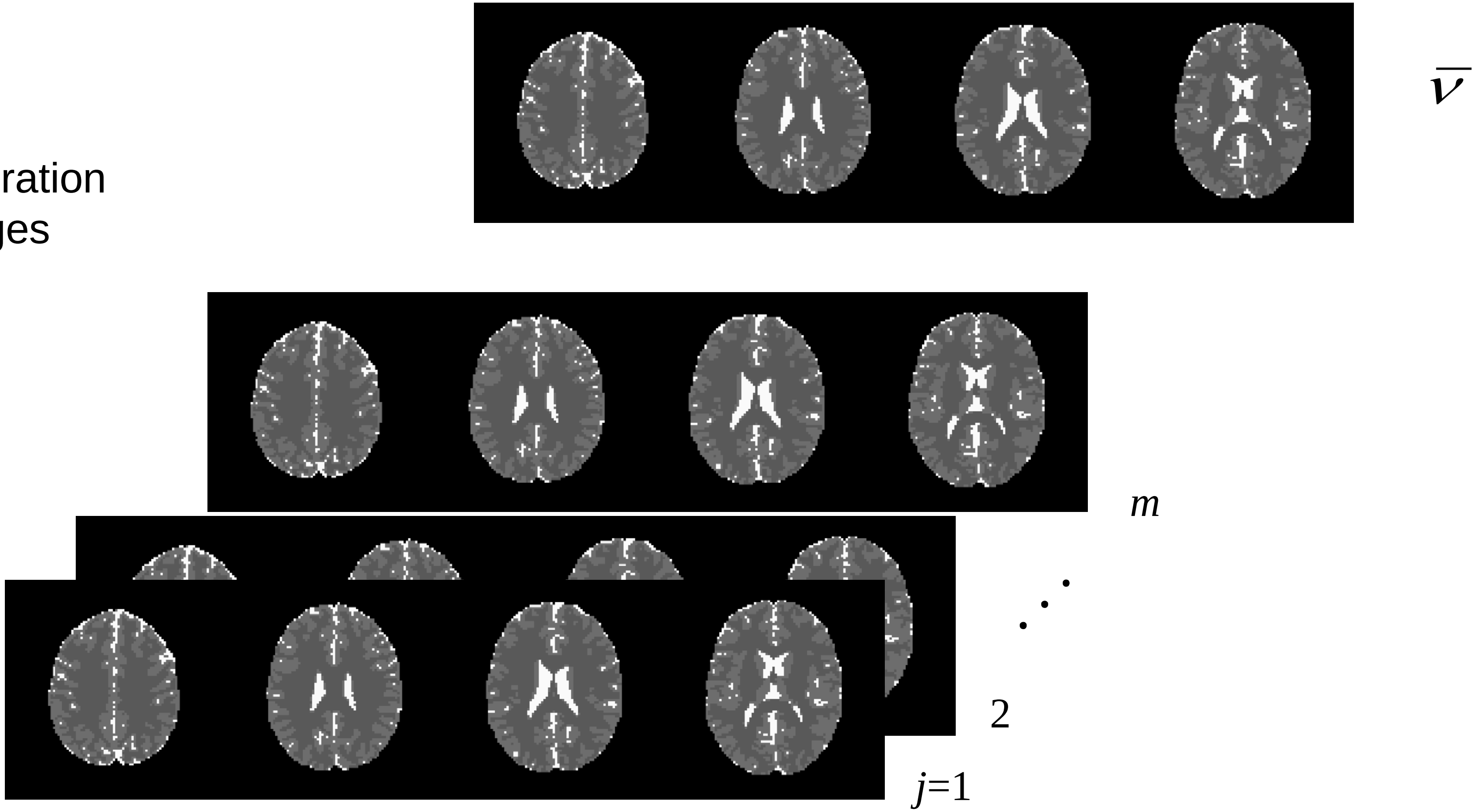
8 equations and 16 unknowns.

$$y_A = X_A \beta + \varepsilon_A$$

2. Theory

The SPECS model utilizes previous full measured images.

calibration images



Separation of Parallel Encoded Complex-Valued Slices

Rowe et al.: ISMRM 2013. Rowe et al. In submission, 2015.

2. Theory

SPECS model incorporates previously measured images to increase the rank of the design matrix.

Partition aliasing design matrix.

$$\underset{8\times 16}{X_A} = [X_{A1}, X_{A2}, X_{A3}, X_{A4}]$$

Partition Hadmard orthogonal matrix.

$$\underset{3\times 16}{C} = [C_1, C_2, C_3, C_4]$$

Generate aliasing contrast matrix.

$$\underset{24\times 16}{C_A} = [X_{A1} \otimes C_1, X_{A2} \otimes C_2, X_{A3} \otimes C_3, X_{A4} \otimes C_4]$$

Measured aliased voxel →

Artificially aliased voxel - - - →

$$\underset{32\times 1}{\begin{bmatrix} y_A \\ \hdashline C_A \bar{v} \end{bmatrix}} = \underset{32\times 16}{\begin{bmatrix} X_A \\ \hdashline C_A \end{bmatrix}} \underset{16\times 1}{\beta} + \underset{32\times 1}{\begin{bmatrix} \varepsilon_A \\ \hdashline \eta \end{bmatrix}}$$

2. Theory

SPECS model incorporates previously measured images to increase the rank of the design matrix.

$$\underset{32\times1}{\begin{bmatrix} y_A \\ \hdashline C_A\bar{v} \end{bmatrix}} = \underset{32\times16}{\begin{bmatrix} X_A \\ \hdashline C_A \end{bmatrix}} \underset{16\times1}{\beta} + \underset{32\times1}{\begin{bmatrix} \varepsilon_A \\ \hdashline \eta \end{bmatrix}}$$

$$\underset{32\times1}{y} = \underset{32\times16}{X} \underset{16\times1}{\beta} + \underset{32\times1}{\varepsilon}$$

$$E(\varepsilon) = 0$$

$$\text{cov}(\varepsilon) = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \tau^2 \end{bmatrix}$$

$$\hat{\beta} = (X'X)^{-1}X'y$$

$$E(\hat{\beta}) = ? \qquad \text{cov}(\hat{\beta}) = ?$$

2. Theory

Taking a closer look at the estimated images

$$y = \begin{bmatrix} y_A \\ C_A \bar{v} \end{bmatrix}$$

$$\hat{\beta} = (X'X)^{-1} X' y$$

$$X = \begin{bmatrix} X_A \\ C_A \end{bmatrix}$$

$$\hat{\beta} = \left[\begin{pmatrix} X_A \\ C_A \end{pmatrix} \begin{pmatrix} X_A' & C' \end{pmatrix} \right]^{-1} \left[X_A' y_A + C_A' C_A \bar{v} \right]$$

$$\varepsilon = \begin{bmatrix} \varepsilon_A \\ \eta \end{bmatrix}$$

$$\hat{\beta} = \left[X_A' X_A + C_A' C_A \right]^{-1} \left[X_A' y_A + C_A' C_A \bar{v} \right]$$

$$\hat{\beta} = \frac{1}{N_z N_{acq}} \left[X_A' y_A + C_A' C_A \bar{v} \right]$$

But $y_A, \bar{v}$, X_A, and C_A are different for the two methods so we need to look at their means, variances, and correlations.

Note that $X_A' X_A + C_A' C_A$ is the same for both methods

2. Theory

In the SPECS model, the calibration image mean is assumed to be μ which is not necessarily the same as the true mean of the images β.

$$\hat{\beta} = \frac{1}{N_z N_{acq}} \left[X_A' y_A + C_A' C_A \bar{v} \right]$$

$$E(\hat{\beta}) = \frac{1}{N_z N_{acq}} \left[X_A' E(y_A) + C_A' C_A E(\bar{v}) \right]$$

$$E(\hat{\beta}) = \frac{1}{N_z N_{acq}} \left[X_A' X_A \beta + C_A' C_A \mu \right]$$

$E(\hat{\beta}) = \beta$ if $\mu = \beta$

2. Theory

In the SPECS model, a bootstrap sample of calibration images is taken so that var($\bar{v}$)=$\sigma^2 I$.

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$\text{cov}(\hat{\beta}) = (X'X)^{-1} X' \text{cov}(y) X (X'X)^{-1}$$

$$\text{cov}(\hat{\beta}) = (X'X)^{-1} X' (\sigma^2 I) X (X'X)^{-1}$$

$$\text{cov}(\hat{\beta}) = \sigma^2 (X'X)^{-1}$$

$$\text{cov}(\hat{\beta}) = \frac{\sigma^2}{N_z N_{acq}}$$

No correlation between the separated slices.

3. Materials and Methods

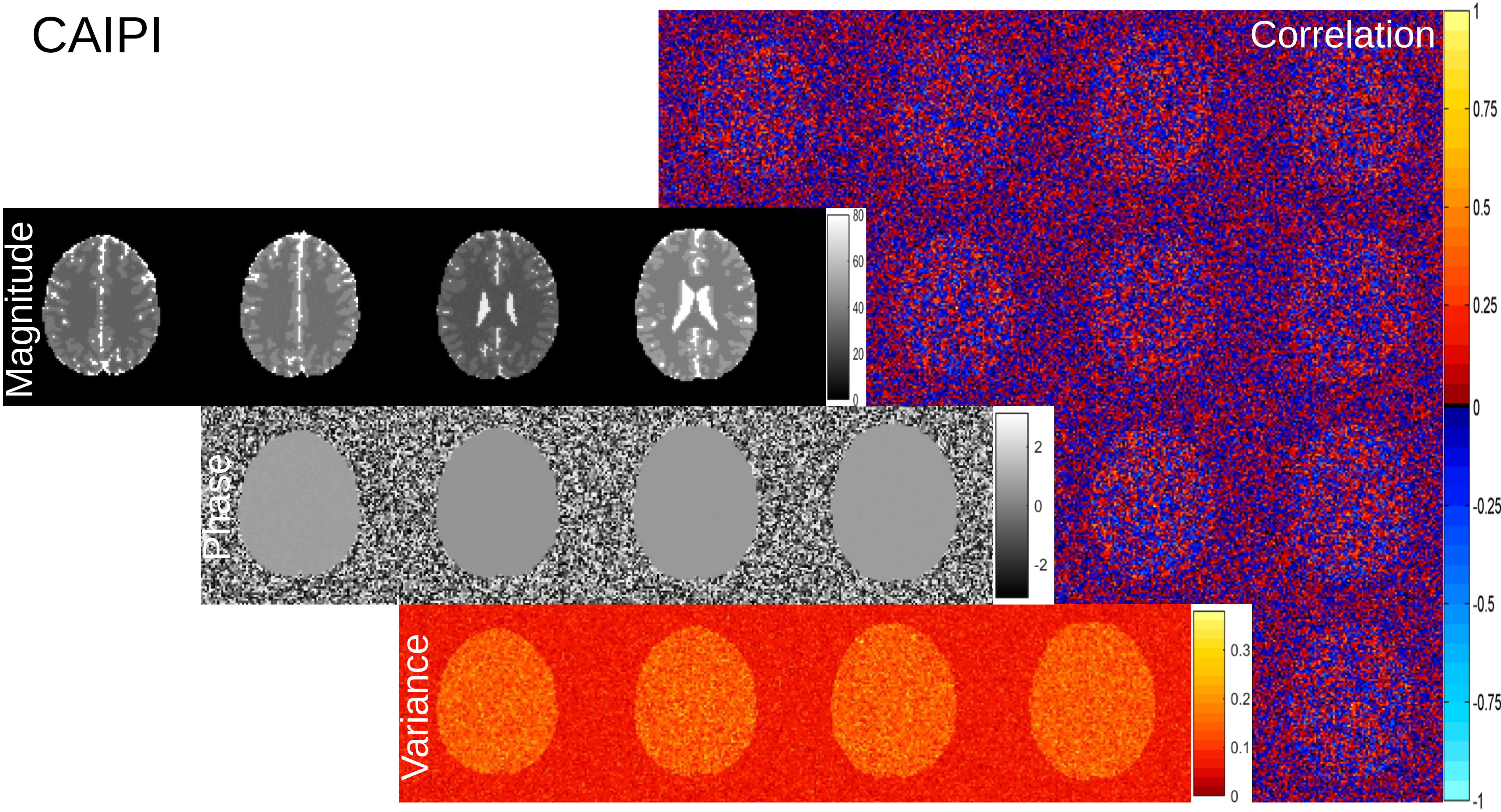
In order to demonstrate the SPECS and CAIPI models with the aliasing, a T_2* weighted 96×96 digital phantom is generated with 720 TRs for 4 slices. For the optimal separation, a unique magnitude and phase is added to each slice, with an average SNR = 50.

One voxel region in each slice, with the ocations rotating clockwise, has a block design task simulated of sixteen 22-second periods, added to its magnitude with a CNR = ½.

In both models the initial off-task portion of the time-series is used for the calibration images in the slice separation.

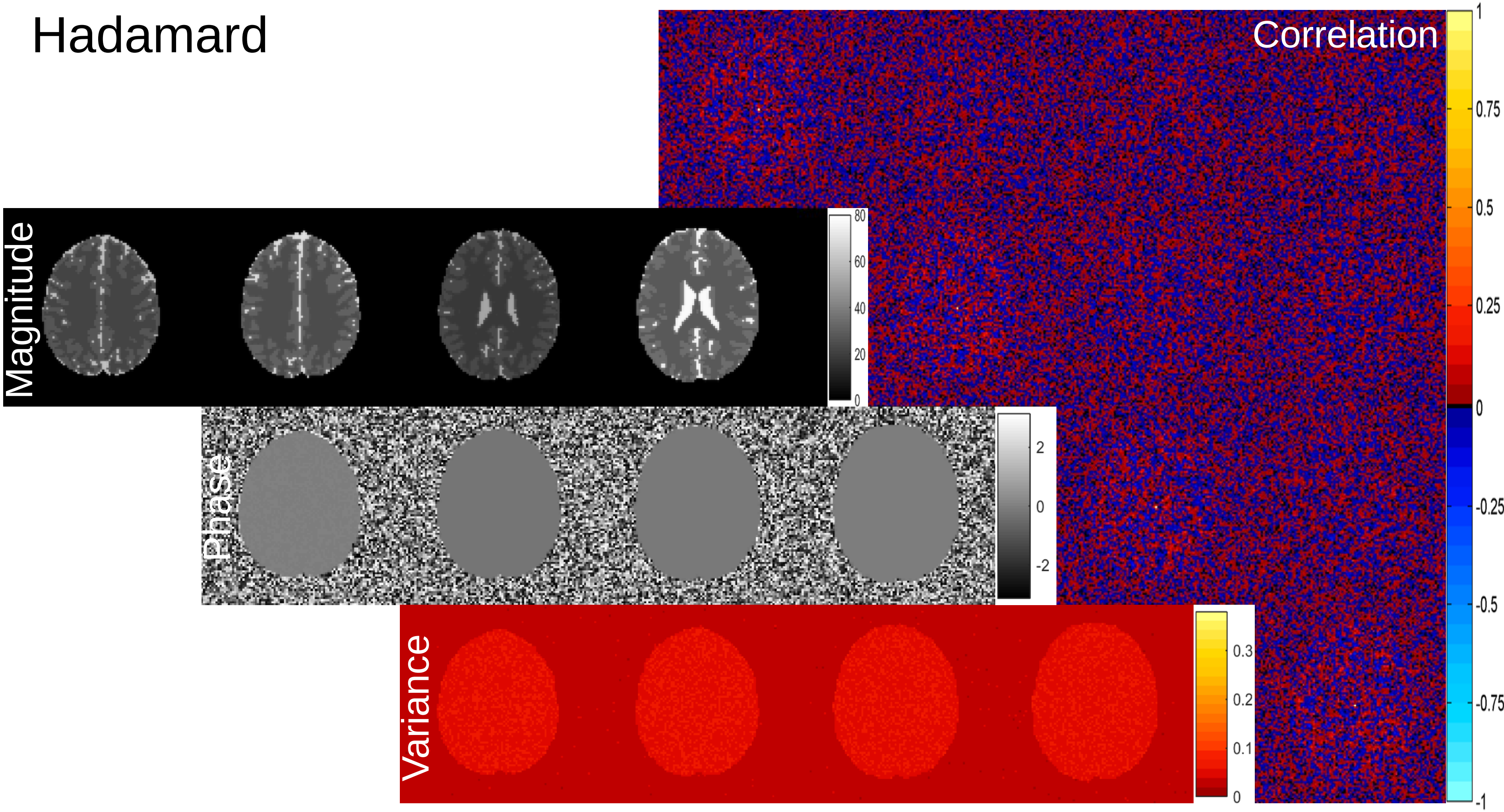
4. Results

CAIPI



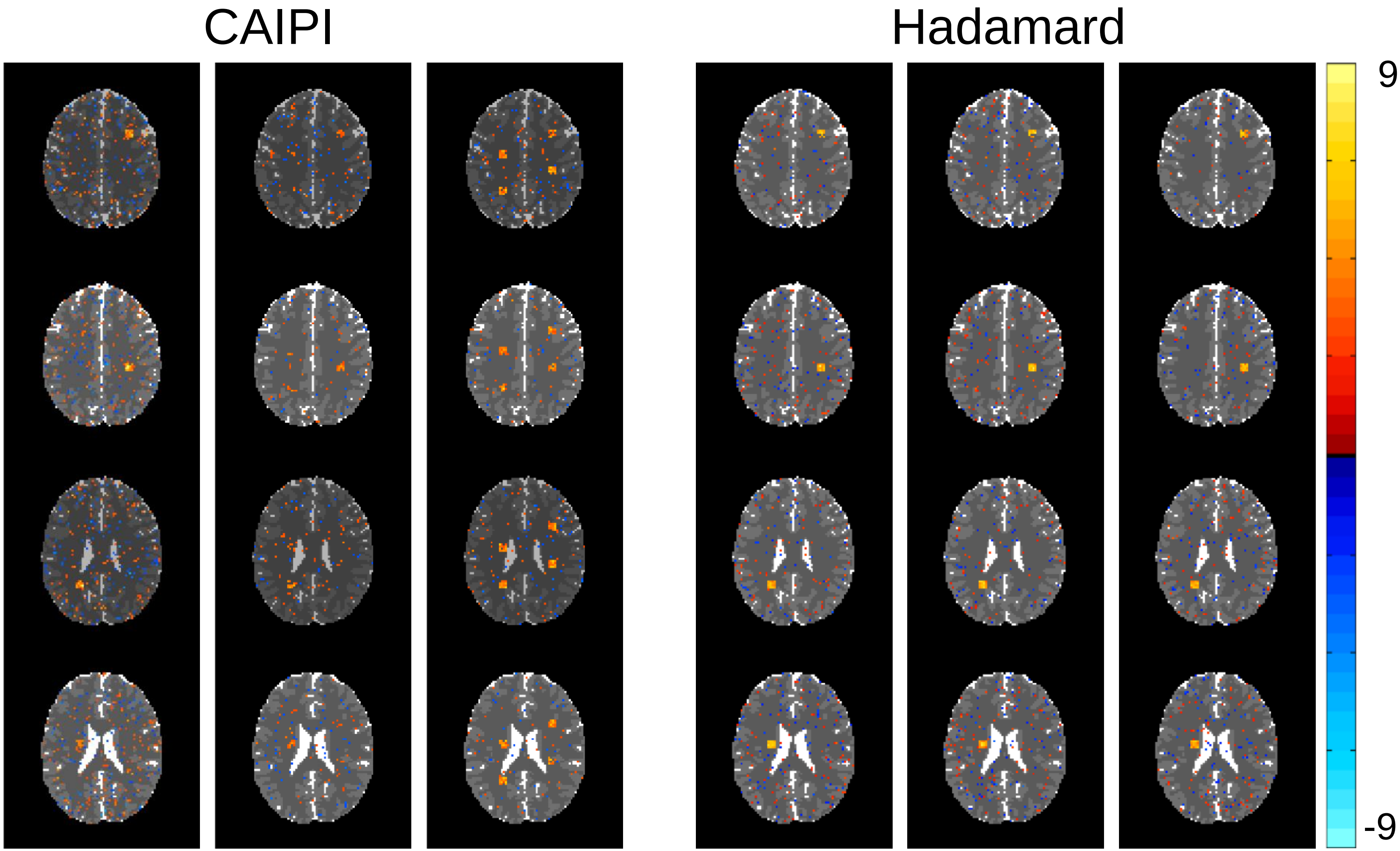
4. Results

Hadamard



4. Results

TH=2



5. Discussion

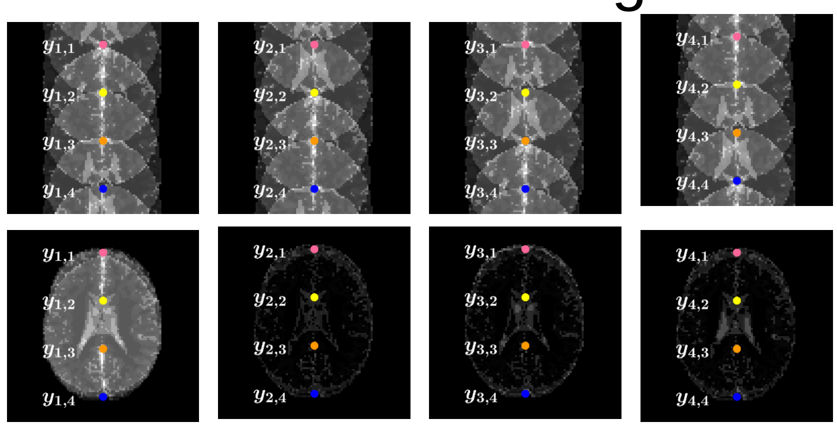
It was theoretically demonstrated that the SPECS model separates images with the same mean and covariance.

It was in simulation demonstrated that the SPECS model separates images with the same mean and covariance.

Since experimentally it is difficult to reconstruct a CAIPI aliased image and not a Hadamard aliased image, it is recommended that Hadamard aliasing be utilized.

CAIPI

Hadamard



Thank You!

References

1) Pruessman et al.: MRM 42:952-962, 1999.
2) Griswold et al.: MRM 47:1202-1210, 2002.
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10) Muller: MRM 6:364-371, 1988.