Statistical Image Separation of Multiple Simultaneously Excited FMRI Slices Using a Single Coil

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Introduction

FMRI takes a nontrivial amount of time to measure an image.

Within slice techniques such as SENSE[1] and GRAPPA[2] have been developed to decrease image measurement time.

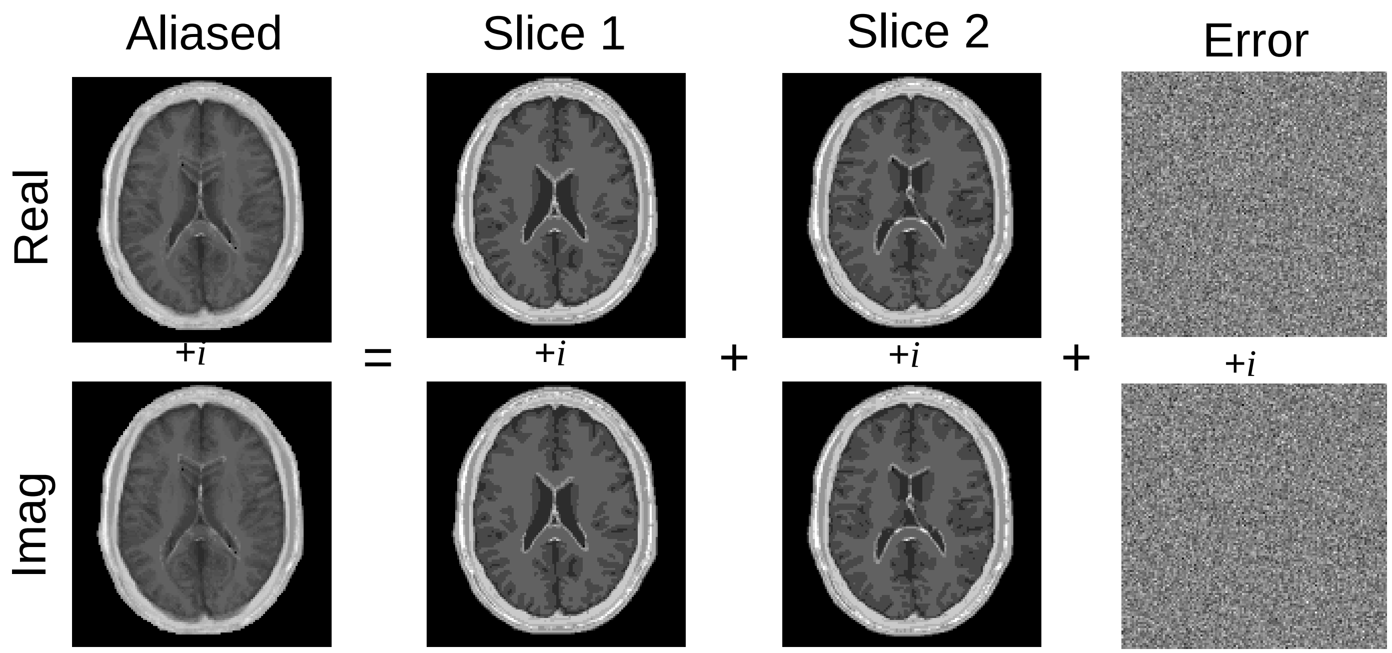
Recent work[3,4] has demonstrated that SENSE and GRAPPA unfolded aliased images yield long range spatial correlation.

Multiband image aliasing has recently been developed but preliminary results[5] indicate high correlation between slices.

Here the SPECS model is presented to separate aliased multiband images without induced between slice correlation.

Image Aliasing

A single complex-valued summed image is measured



In each voxel

$$a_R + i a_I = (\beta_{R1} + i\beta_{I1}) + (\beta_{R2} + i\beta_{I2}) + (\varepsilon_R + i\varepsilon_I)$$

One measurement error added to the sum

Image Aliasing

A single complex-valued summed image is measured

$$a_R + i a_I = (\beta_{1R} + \beta_{2R}) + i(\beta_{1I} + \beta_{2I}) + (\varepsilon_R + i\varepsilon_I)$$

$$\begin{pmatrix} a_R \\ a_I \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \beta_{1R} \\ \beta_{2R} \\ \beta_{1I} \\ \beta_{2I} \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix} \longrightarrow a = X_A \beta + \varepsilon$$

2×1 Aliased image; 2×4 Aliasing matrix; 4×1 True un-aliased images; 2×1 Error

Want $\hat{\beta} = (X_A ' X_A)^{-1} X_A ' a$, but X has 2 equations & 4 unknowns.

The matrix $X_A ' X_A$ is not square, invertible or of full rank

Image Un-Aliasing

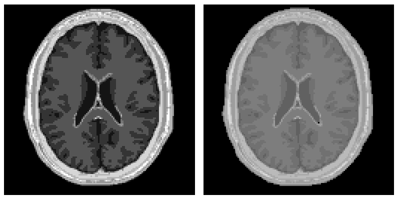
$$\nu_{Rj} = \mu_{R1} + \eta_{Rj}$$
$$\nu_{Ij} = \mu_{I1} + \eta_{Ij}$$

Full Calibration Reference Images

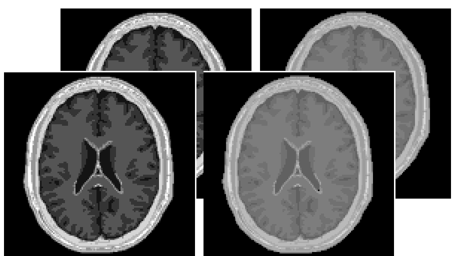
Slice 1

μ_{1R}, μ_{1I}

True



m

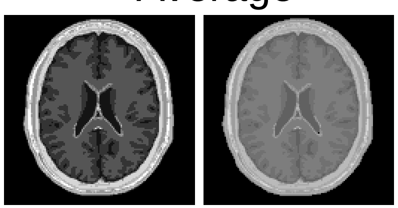


Real Imaginary

1 2 $\cdots$

Average

$\bar{\nu}_{1R}$

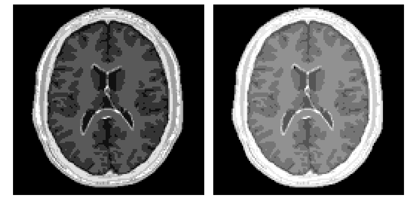


$\bar{\nu}_{1I}$

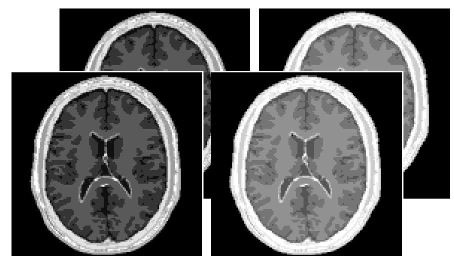
Slice 2

μ_{2R}, μ_{2I}

True



m

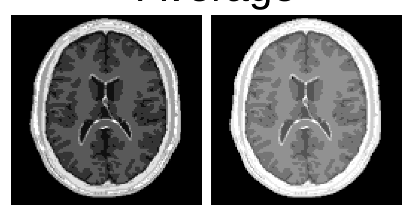


Real Imaginary

1 2 $\cdots$

Average

$\bar{\nu}_{2R}$



$\bar{\nu}_{2I}$

$$\bar{\nu}_R = \begin{pmatrix} \bar{\nu}_{1R} \\ \bar{\nu}_{2R} \end{pmatrix}$$

$$\bar{\nu}_I = \begin{pmatrix} \bar{\nu}_{1I} \\ \bar{\nu}_{2I} \end{pmatrix}$$

Image Un-Aliasing

Incorporate artificially aliased mean calibration values by:

Acquired aliased voxel

Artificially aliased voxel

$$\begin{pmatrix} a_R \\ C\bar{v}_R \\ a_I \\ C\bar{v}_I \end{pmatrix} = \begin{pmatrix} X_A & 0 \\ C & 0 \\ 0 & X_A \\ 0 & C \end{pmatrix} \begin{pmatrix} \beta_{1R} \\ \beta_{2R} \\ \beta_{1I} \\ \beta_{1I} \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \eta_R \\ \varepsilon_I \\ \eta_I \end{pmatrix}$$

$$\underset{4\times 1}{a} = \underset{4\times 4}{X} \quad \underset{4\times 1}{\beta} + \underset{4\times 1}{\varepsilon}$$

$$\begin{bmatrix} X_A \\ \hdashline C \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \hdashline 1 & -1 \end{bmatrix}$$

$$E(\varepsilon) = 0$$

$$\text{cov}(\varepsilon) = I_2 \otimes \begin{bmatrix} \sigma^2 & 0 \\ 0 & \tau^2 \end{bmatrix}$$

With the invertible matrix X we can un-alias via: $\hat{\beta} = X^{-1}a$.

Using mean of all m calibration images.

Image Un-Aliasing Statistics

We can obtain the statistical properties of the unaliased images.

Using the average of all the calibration images:

$$E\begin{bmatrix}\hat{\beta}_{1R}\\ \hat{\beta}_{2R}\\ \hat{\beta}_{1I}\\ \hat{\beta}_{1I}\end{bmatrix}=X^{-1}E\begin{bmatrix}a_R\\ C\bar{v}_R\\ a_I\\ C\bar{v}_I\end{bmatrix}=\begin{bmatrix}\frac{1}{2}(\beta_{1R}+\mu_{1R})+\frac{1}{2}(\beta_{2R}-\mu_{2R})\\ \frac{1}{2}(\beta_{2R}+\mu_{2R})+\frac{1}{2}(\beta_{1R}-\mu_{1R})\\ \frac{1}{2}(\beta_{1I}+\mu_{1I})+\frac{1}{2}(\beta_{2I}-\mu_{2I})\\ \frac{1}{2}(\beta_{2I}+\mu_{2I})+\frac{1}{2}(\beta_{1I}-\mu_{1I})\end{bmatrix}$$

$$\text{cov}(\hat{\beta})=X^{-1}\text{cov}(a)(X^{-1})'=\frac{\sigma^2}{4}I_2\otimes\begin{pmatrix}1&1\\1&1\end{pmatrix}$$

perfectly correlated reals
perfectly correlated imaginaries

Image Un-Aliasing Statistics

We can obtain the statistical properties of the unaliased images.

If we average 2 calibration images for each separated image:

$$E\begin{bmatrix}\hat{\beta}_{1R}\\ \hat{\beta}_{2R}\\ \hat{\beta}_{1I}\\ \hat{\beta}_{1I}\end{bmatrix} = X^{-1}E\begin{bmatrix}a_R\\ C\bar{v}_R\\ a_I\\ C\bar{v}_I\end{bmatrix} = \begin{bmatrix}\frac{1}{2}(\beta_{1R}+\mu_{1R})+\frac{1}{2}(\beta_{2R}-\mu_{2R})\\ \frac{1}{2}(\beta_{2R}+\mu_{2R})+\frac{1}{2}(\beta_{1R}-\mu_{1R})\\ \frac{1}{2}(\beta_{1I}+\mu_{1I})+\frac{1}{2}(\beta_{2I}-\mu_{2I})\\ \frac{1}{2}(\beta_{2I}+\mu_{2I})+\frac{1}{2}(\beta_{1I}-\mu_{1I})\end{bmatrix}$$

$$\mathrm{cov}(\hat{\beta}) = X^{-1}\,\mathrm{cov}(a)(X^{-1})' = \frac{\sigma^2}{4} I_4$$

perfectly uncorrelated reals
perfectly uncorrelated imaginaries

Using mean if two calibration images.

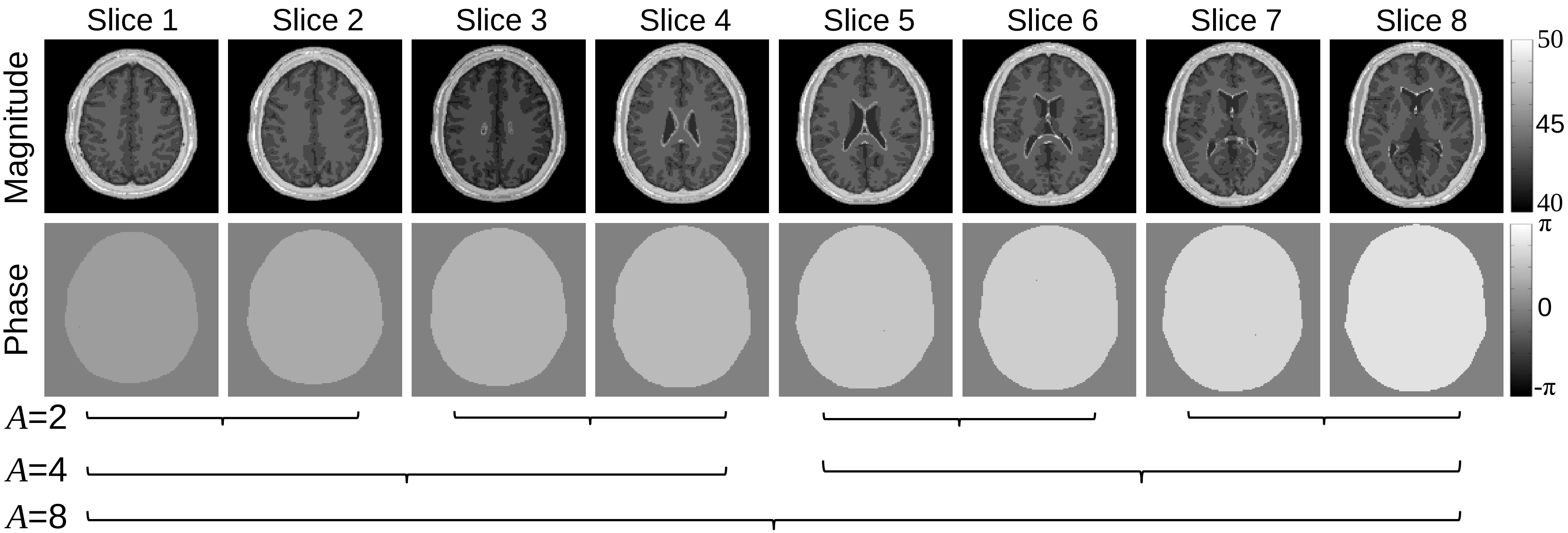
Image Un-Aliasing Simulation

128×128 3D brain phantom with 8 slices

Packets of adjacent slices aliased in k-space with A=2,4,8

$N(0, \sqrt{128 \times 128})$ noise added to 500 k-space arrays in each slice

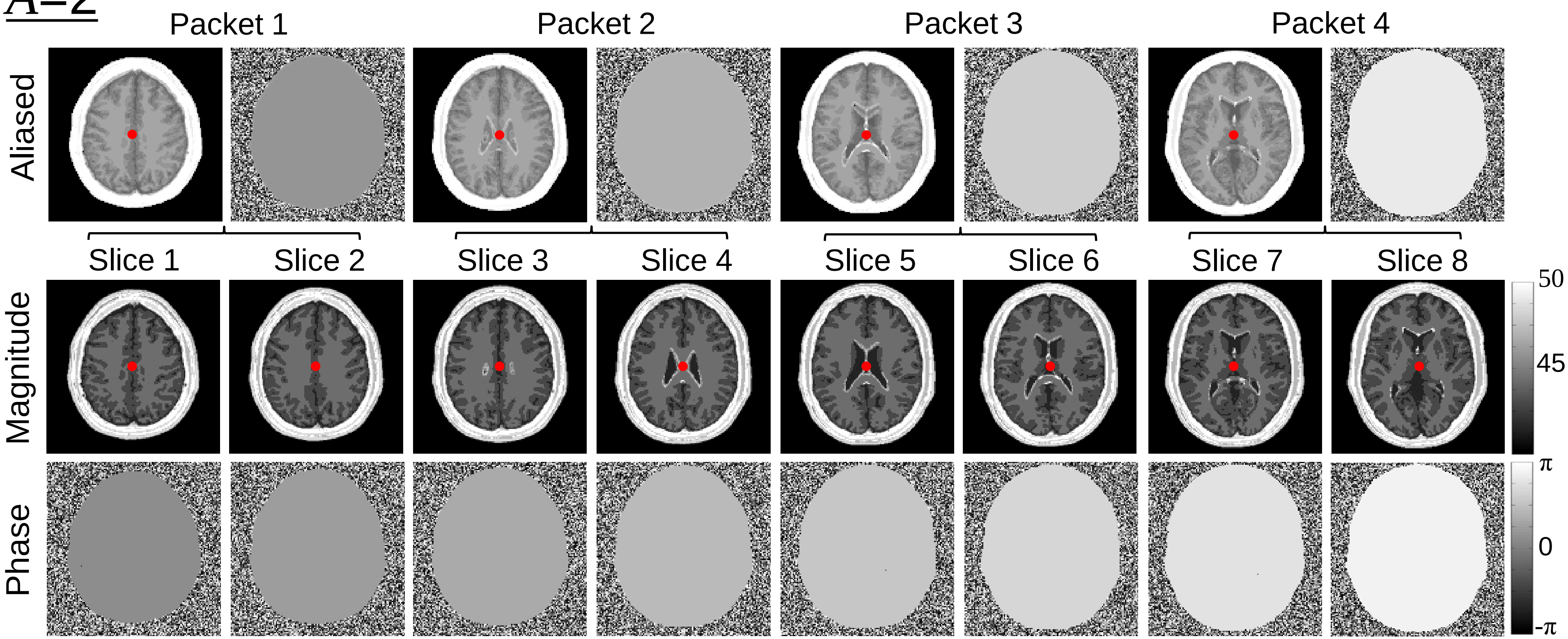
Hadamard coefficients, C, used in SPECS reconstruction



True images. Noise added so SNR=50.

Un-Aliased Results: Using All Calibration Images

$A=2$



Magnitude^2 correlations about center voxel

1	0.9388
0.9388	1

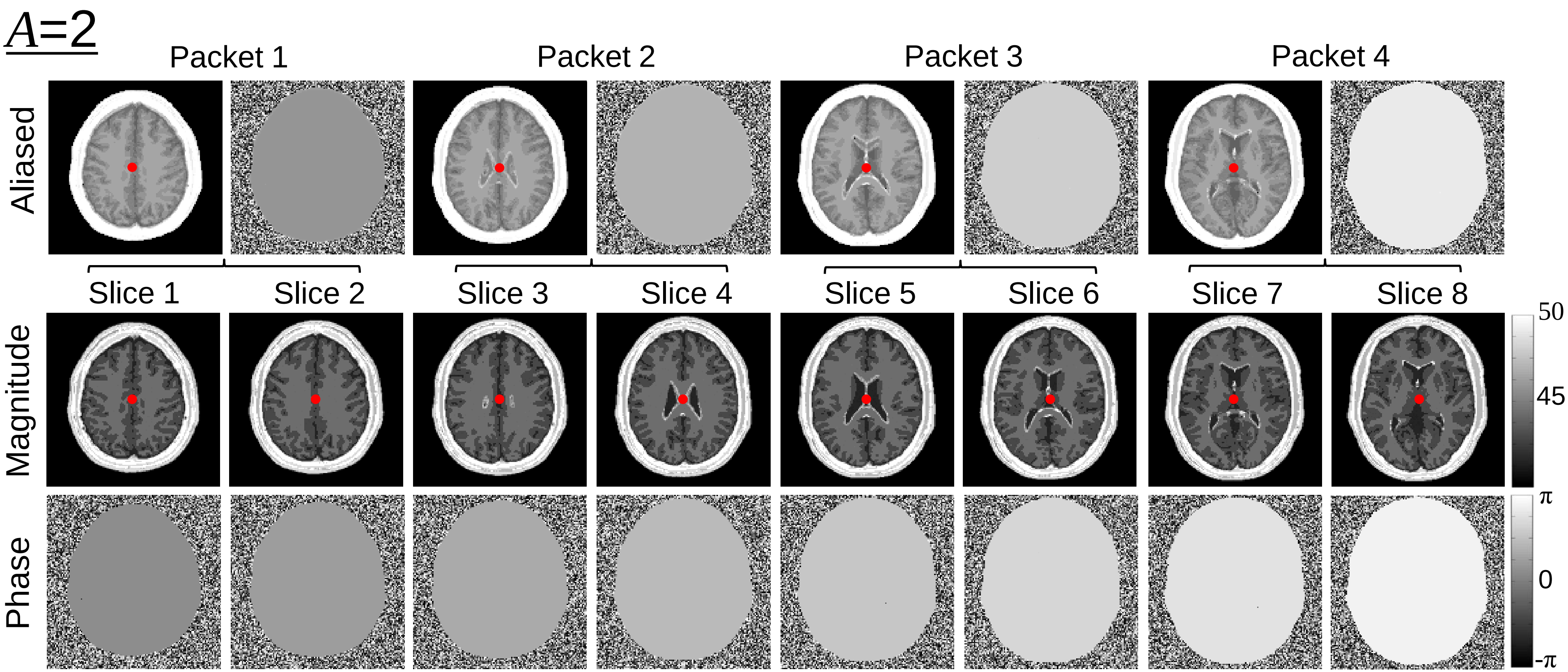
1	0.9338
0.9338	1

1	0.9382
0.9382	1

1	0.9333
0.9333	1

Note correlations close to 1 …. Bad

Un-Aliased Results: Bootstrapping Approach



Magnitude^2 correlations about center voxel

1	0.0160
0.0160	1

1	-0.0369
-0.0369	1

1	-0.0393
-0.0393	1

1	-0.0736
-0.0736	1

Note correlations close to 0 … Good.

Summary

Introduced 1 coil 2 slice image acquisition.

Images were acquired A=2 times as fast.

No correlation was induced between voxels in the A=2 slices.

This technique can be applied to A>2 slices.

Higher acceleration factors A have been achieved.

Additional results at eposter 14 in CC-Exhibit Hall B2.

Thank You!

This work is joint with:
Dr. Iain P. Bruce, Duke
Dr. Andrew S. Nencka, MCW
Dr. James S. Hyde, MCW
Dr. Andrez Jesmanowicz, MCW

References

1) Pruessman et al.: MRM 42:952-962, 1999.
2) Griswold et al.: MRM 47:1202-1210, 2002.
3) Bruce et al.: MRI 29(9):1267-1287, 2011.
4) Bruce & Rowe: IEEE-TMI 33(2):495-503, 2014.
5) Rowe et al.: Proc ISMRM 20:123, 2013.