Separation of Several Aliased Images to Increase Volume Speed

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Outline:

1. Coil Arrays

2. Simultaneous Multi-Slice

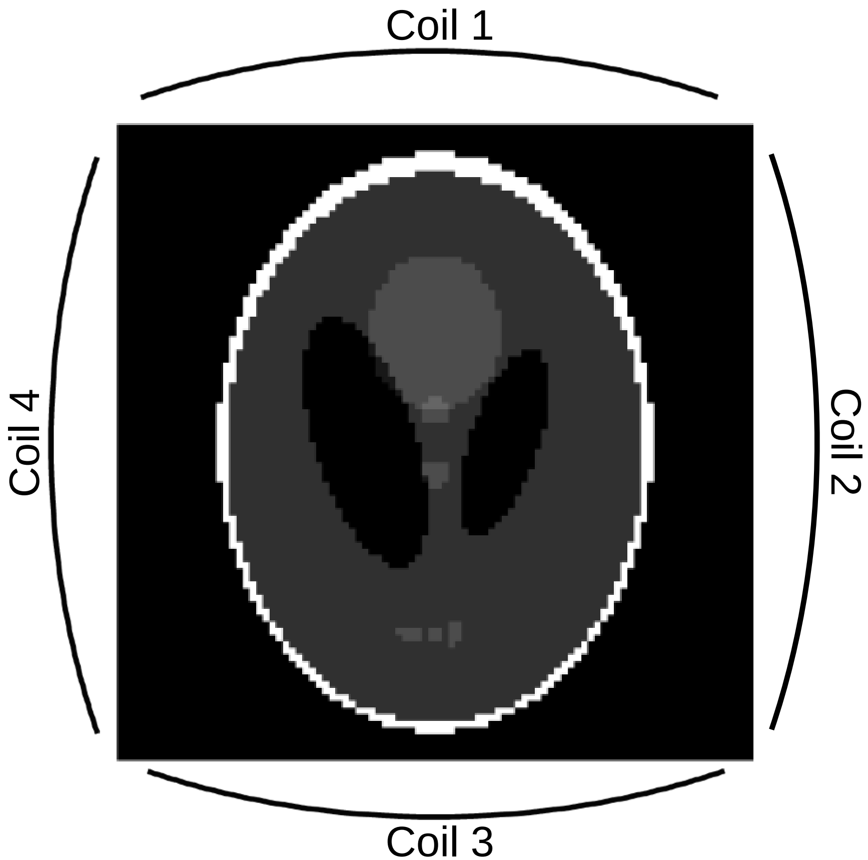
3. Image Separation

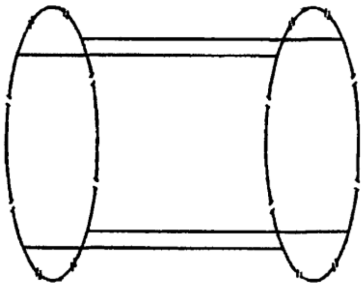
4. Simulation Results

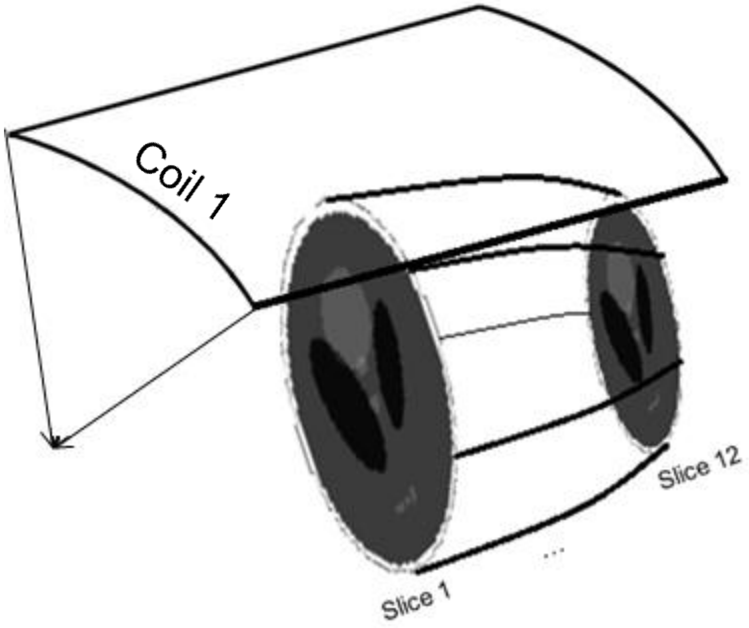
5. Discussion

1. Coil Arrays

A 4 Channel Coil Array.



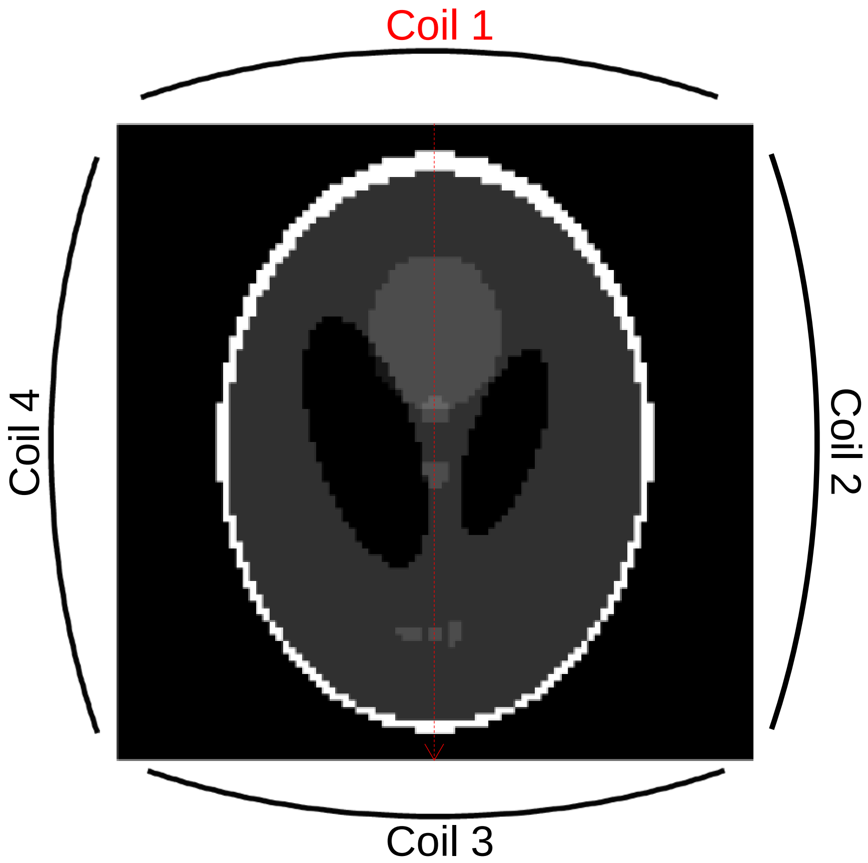


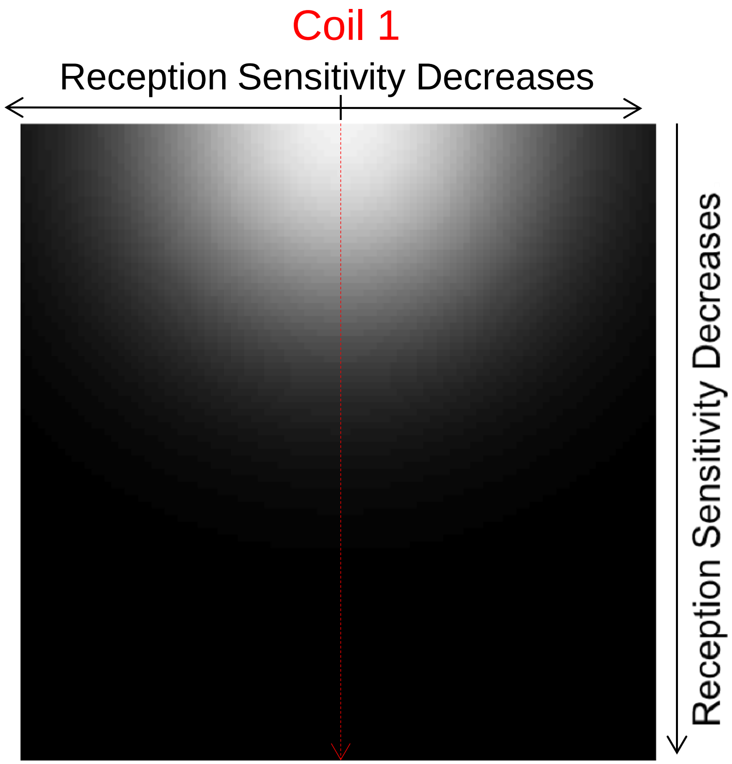


Each slice has a different width and depth profile.

1. Coil Arrays

A 4 Channel Coil Array. Coils Sensitivity Decreases.





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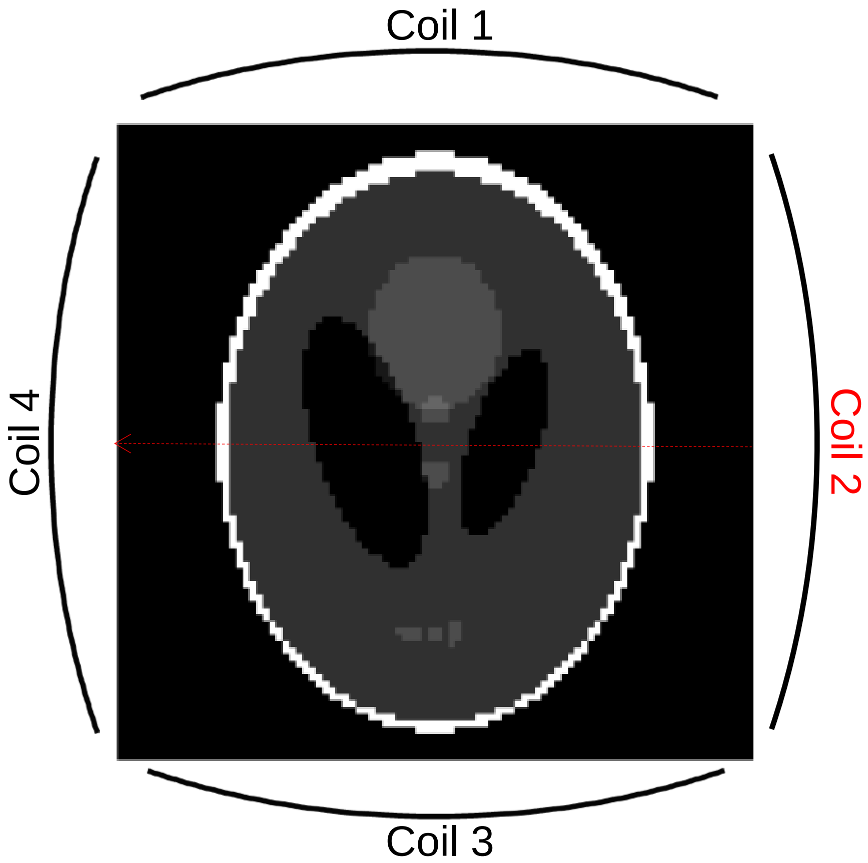
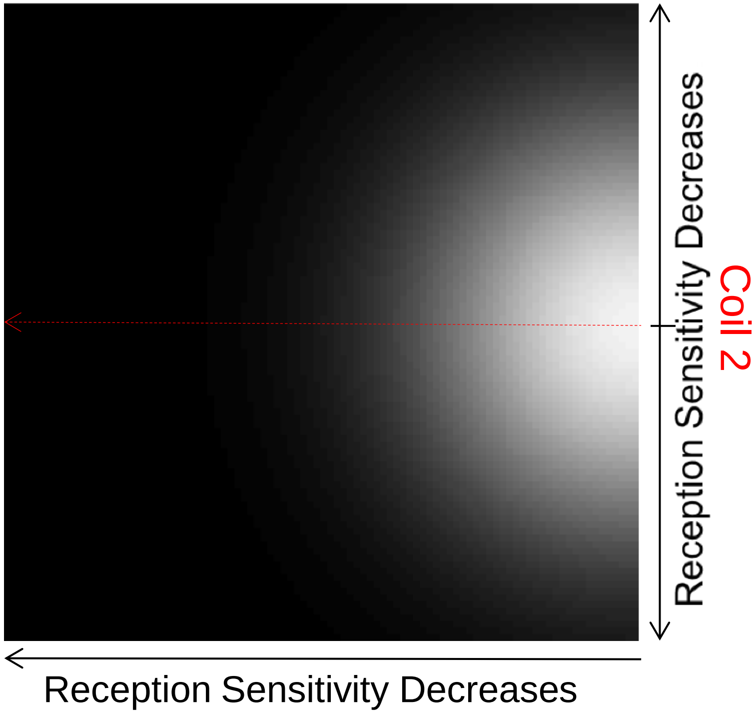


Coils Sensitivity Decreases.



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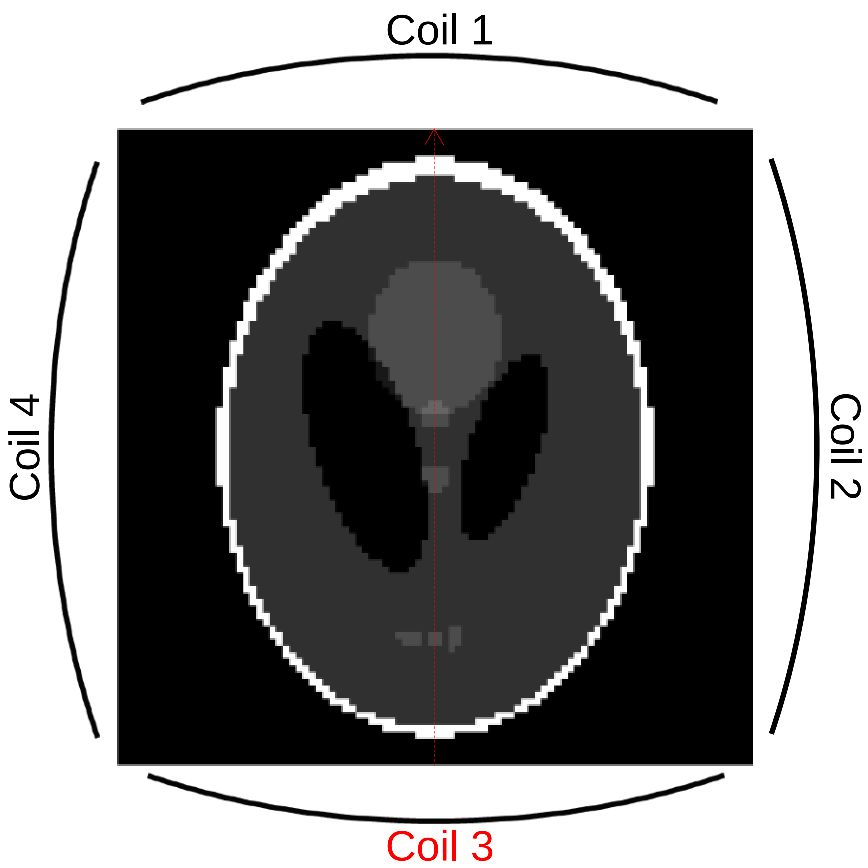

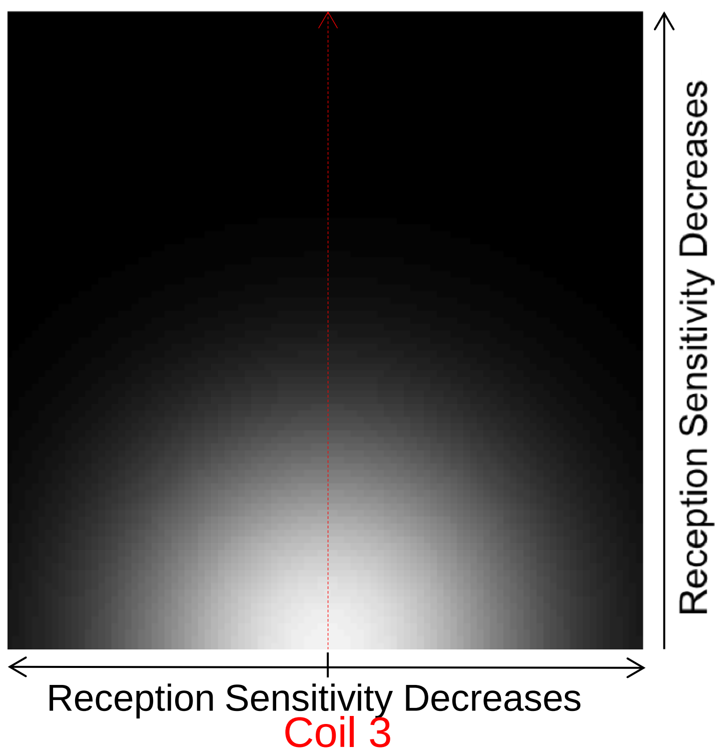

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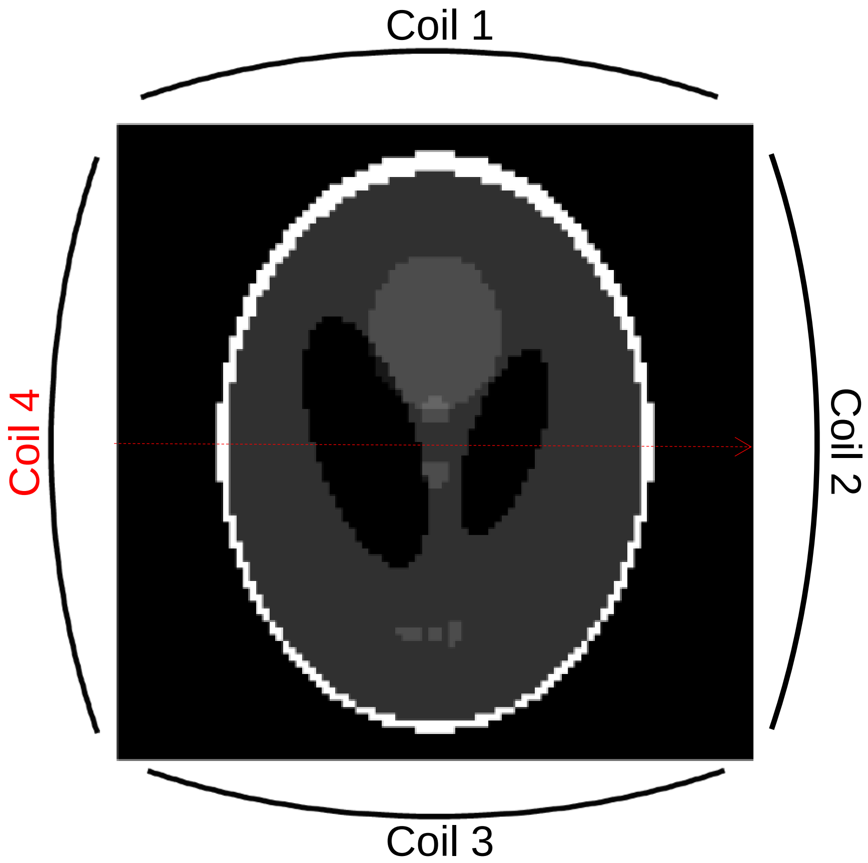

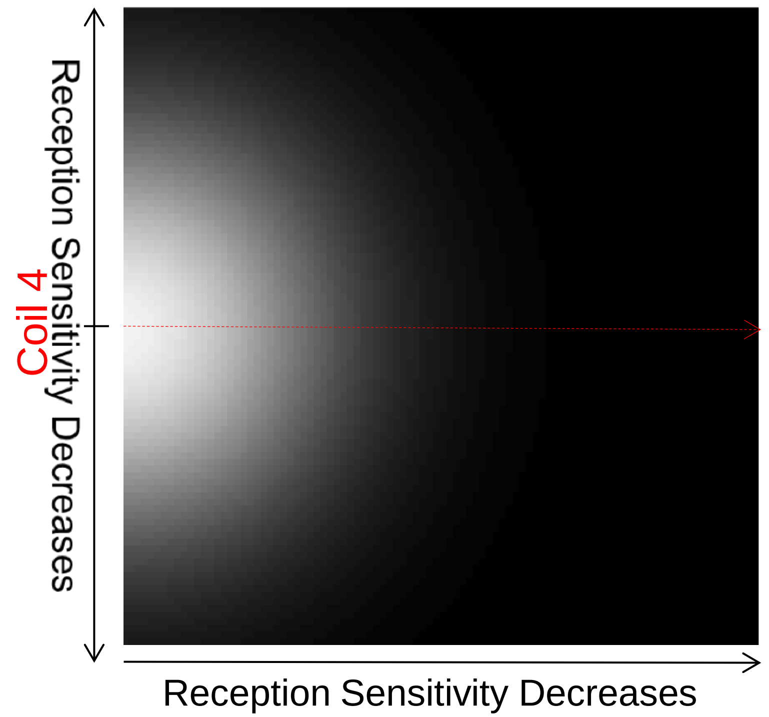

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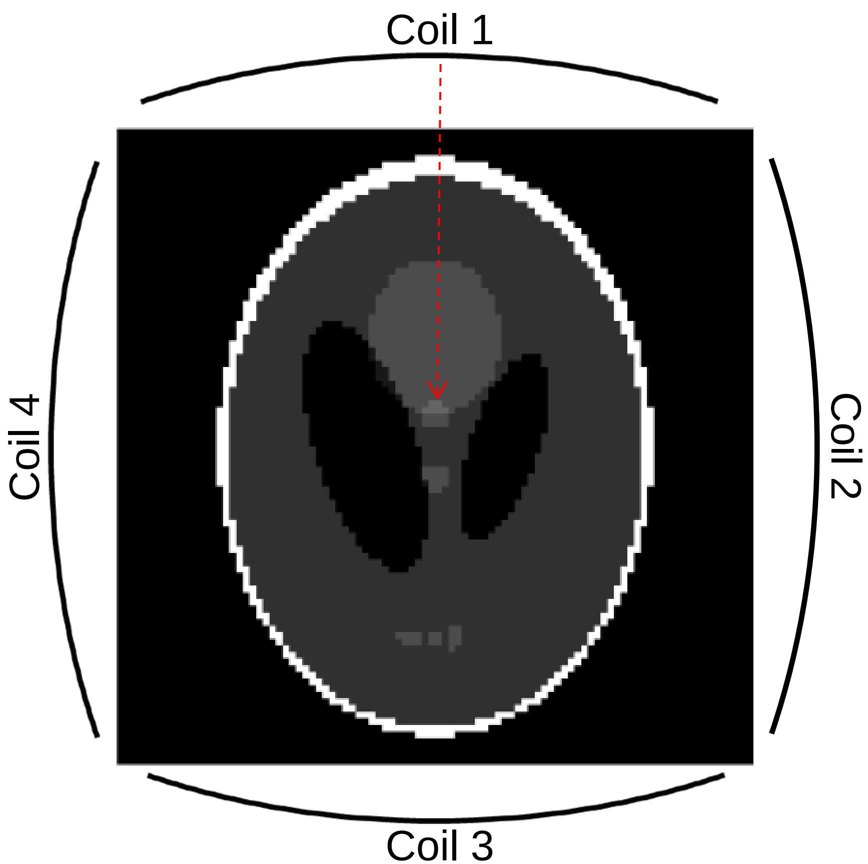
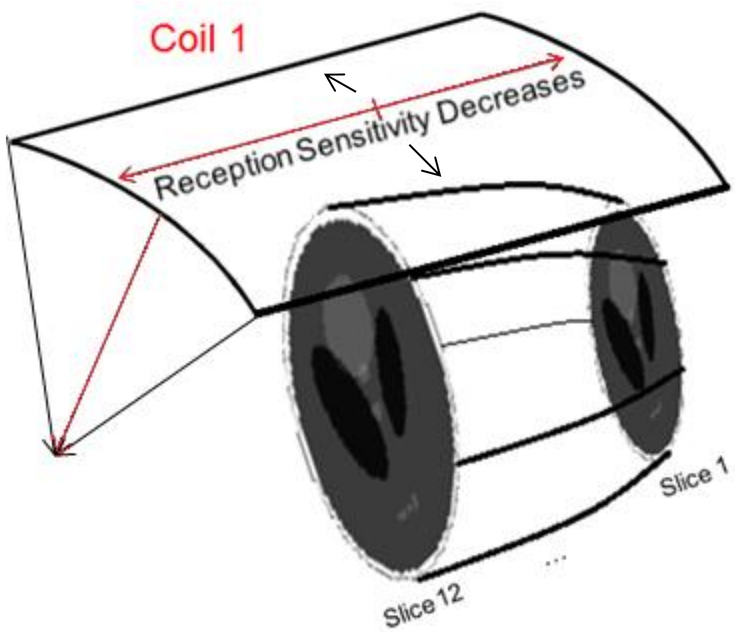
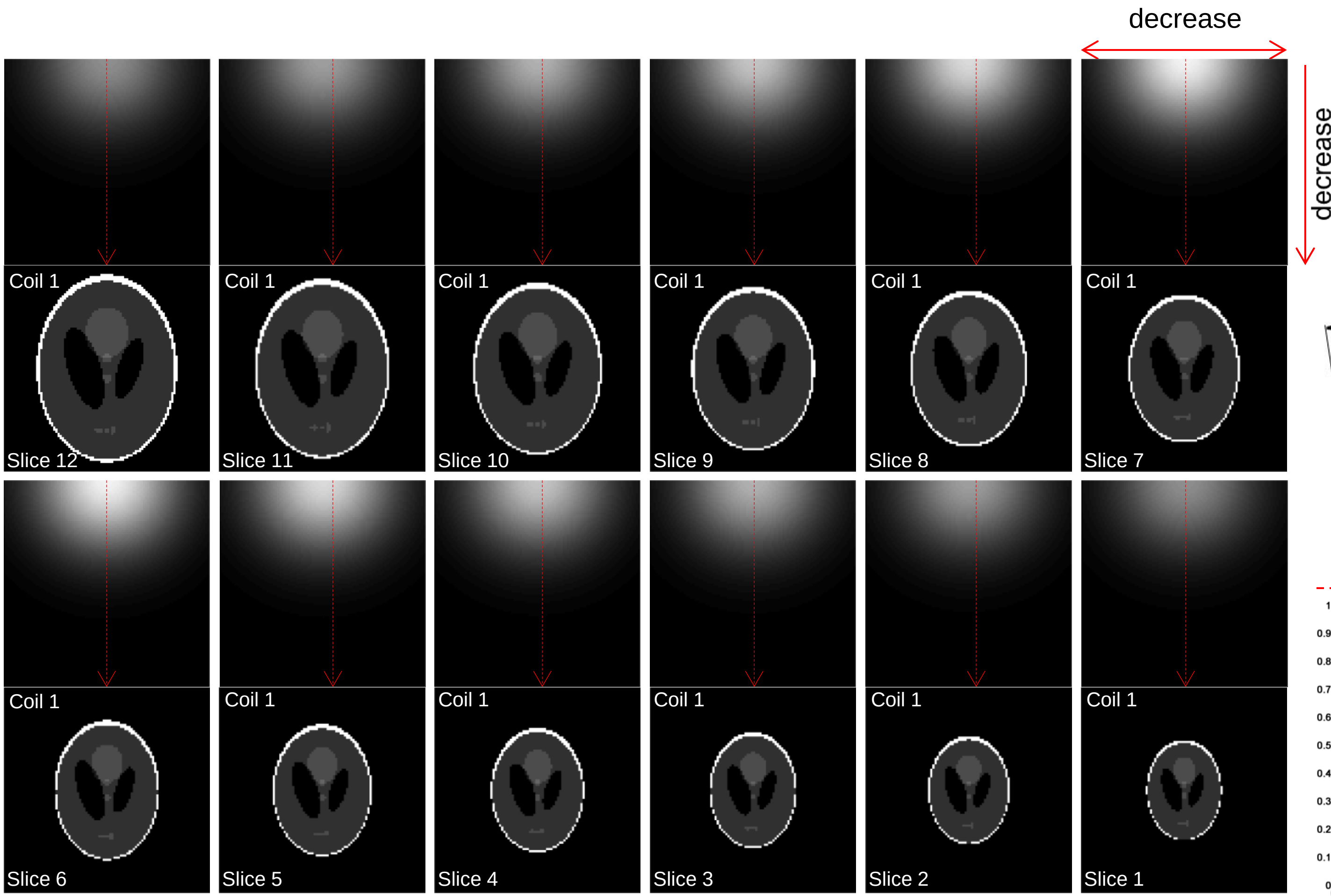
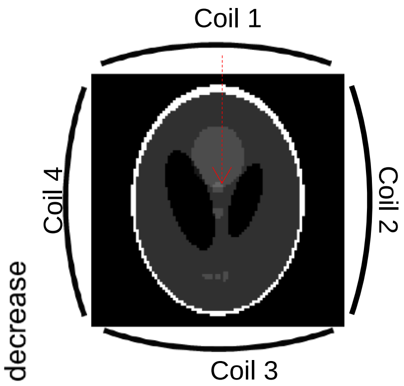
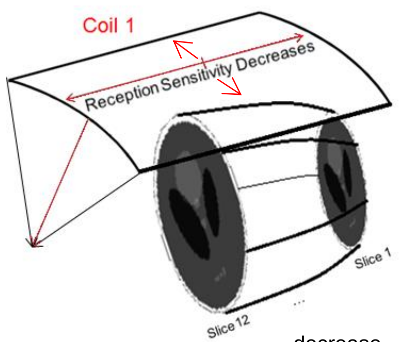


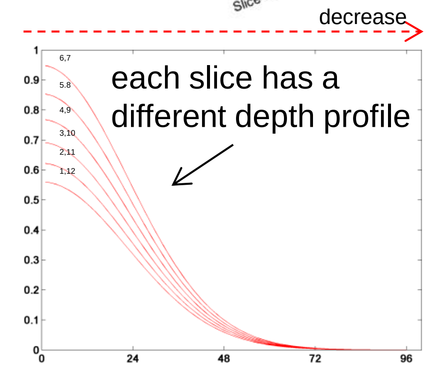


1. Coil Arrays







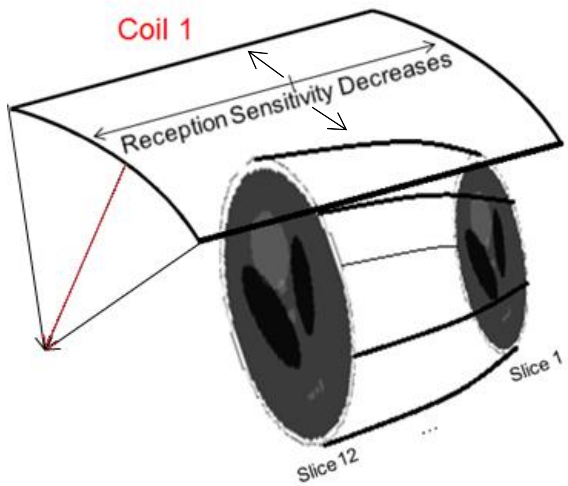


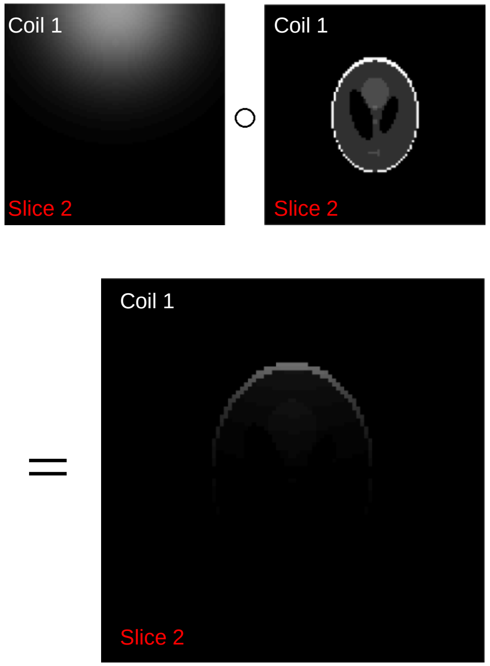
Each slice has a different width and depth profile.

1. Coil Arrays

A 4 Channel Coil Array and 12 Slices.

Coil 1 sees.





Fully Sampled

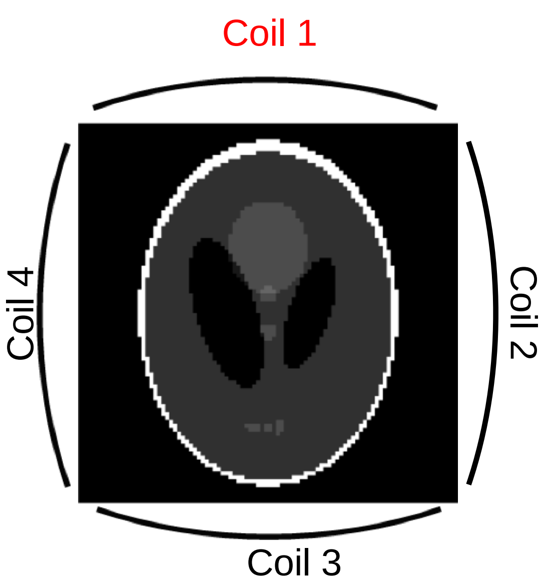
Hadamard Product: $\circ = .*$

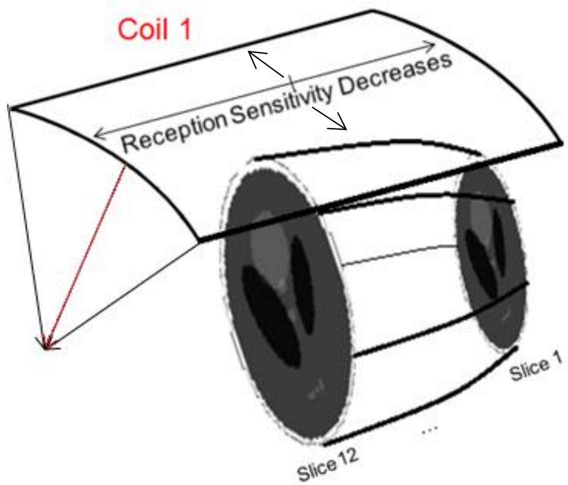
Each slice has a different width and depth profile.

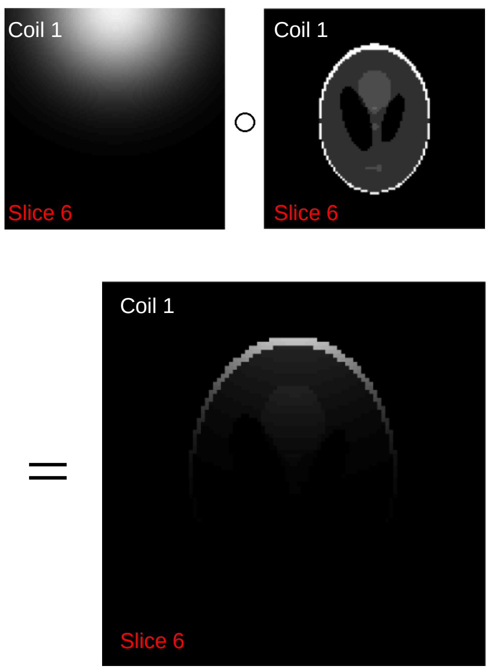
1. Coil Arrays

A 4 Channel Coil Array and 12 Slices.

Coil 1 sees.







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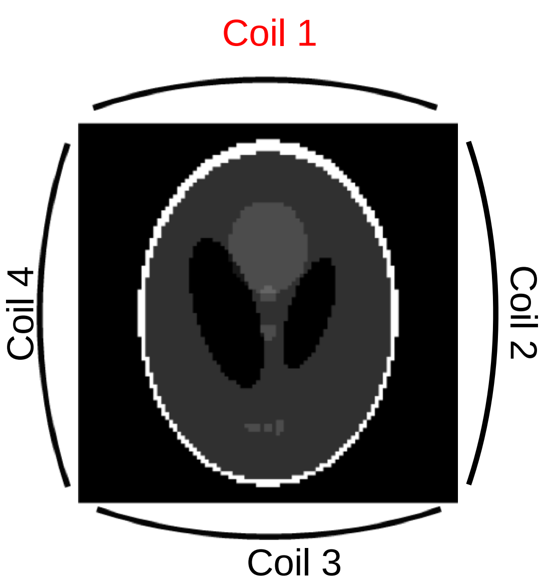
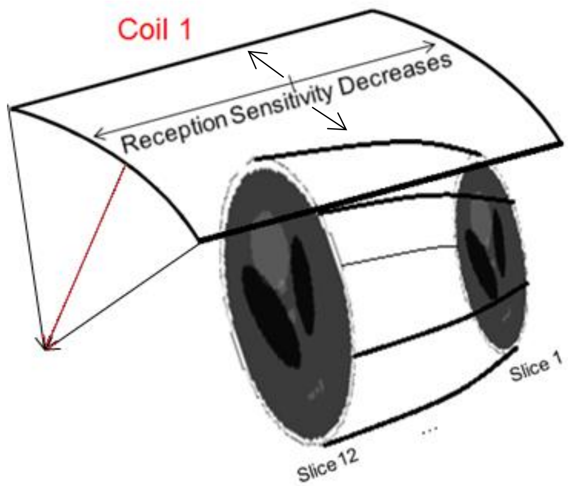
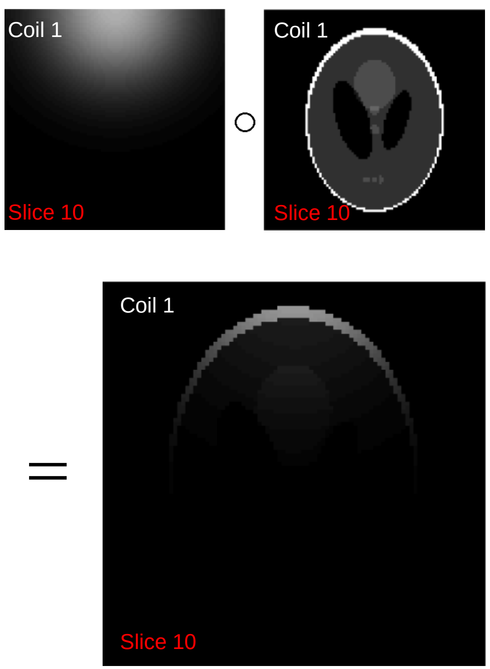




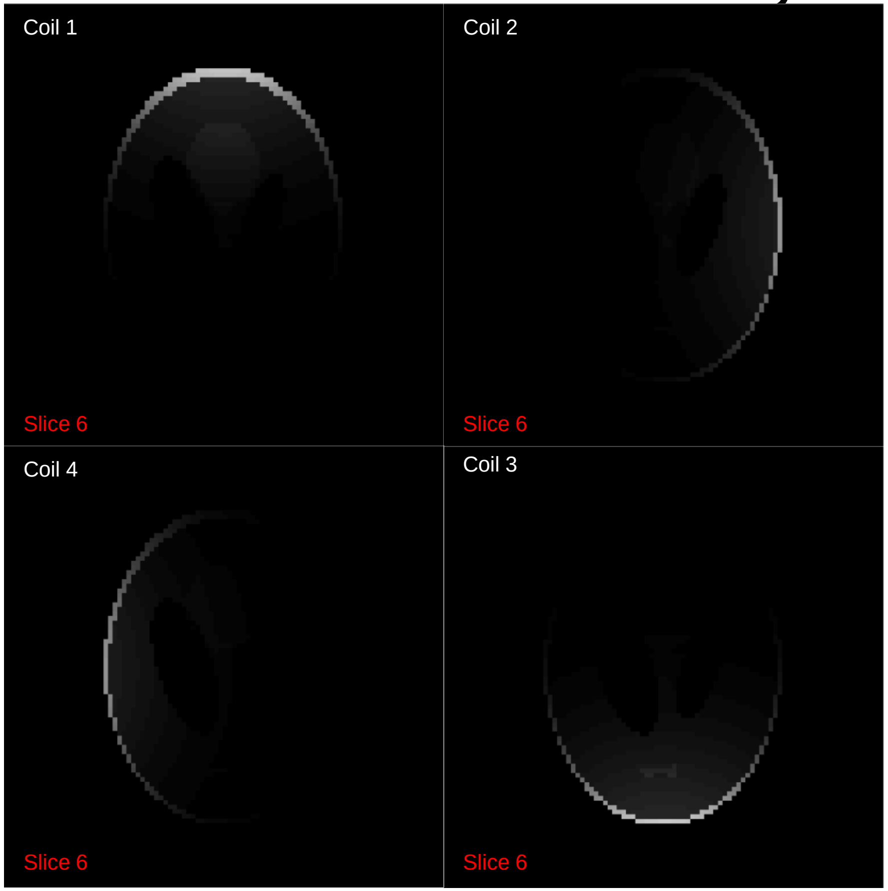


Fully Sampled

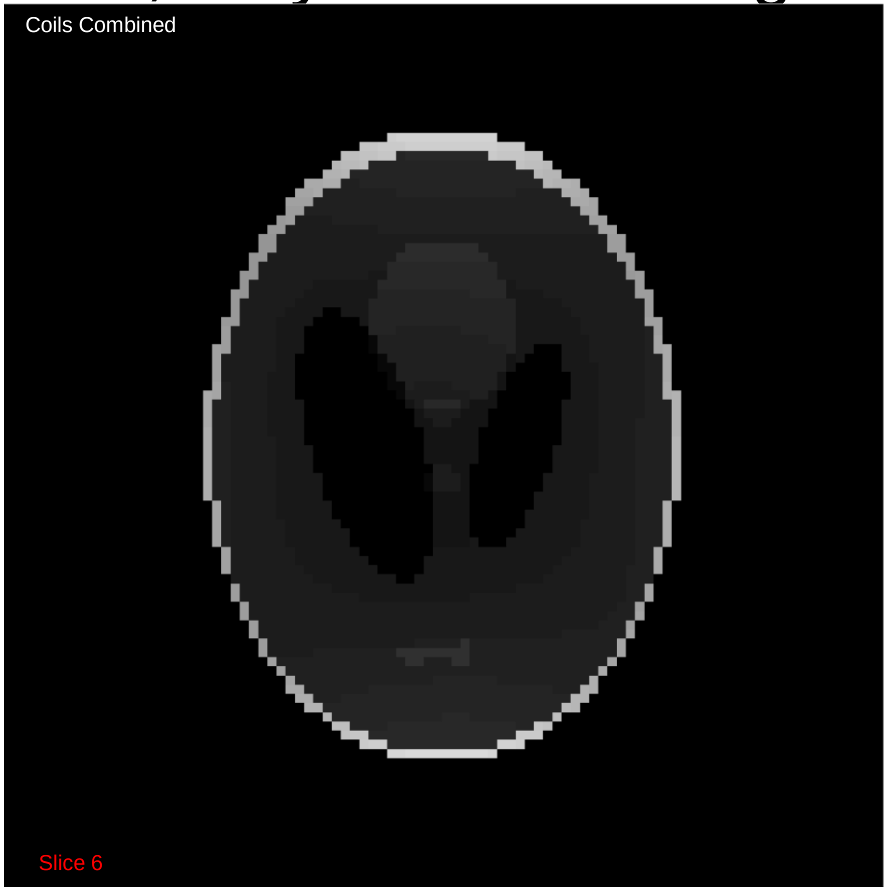
Hadamard Product: $\circ = .*$

1. Coil Arrays

A 4 channel Coil Array. In Slice 6, they Combine to get.

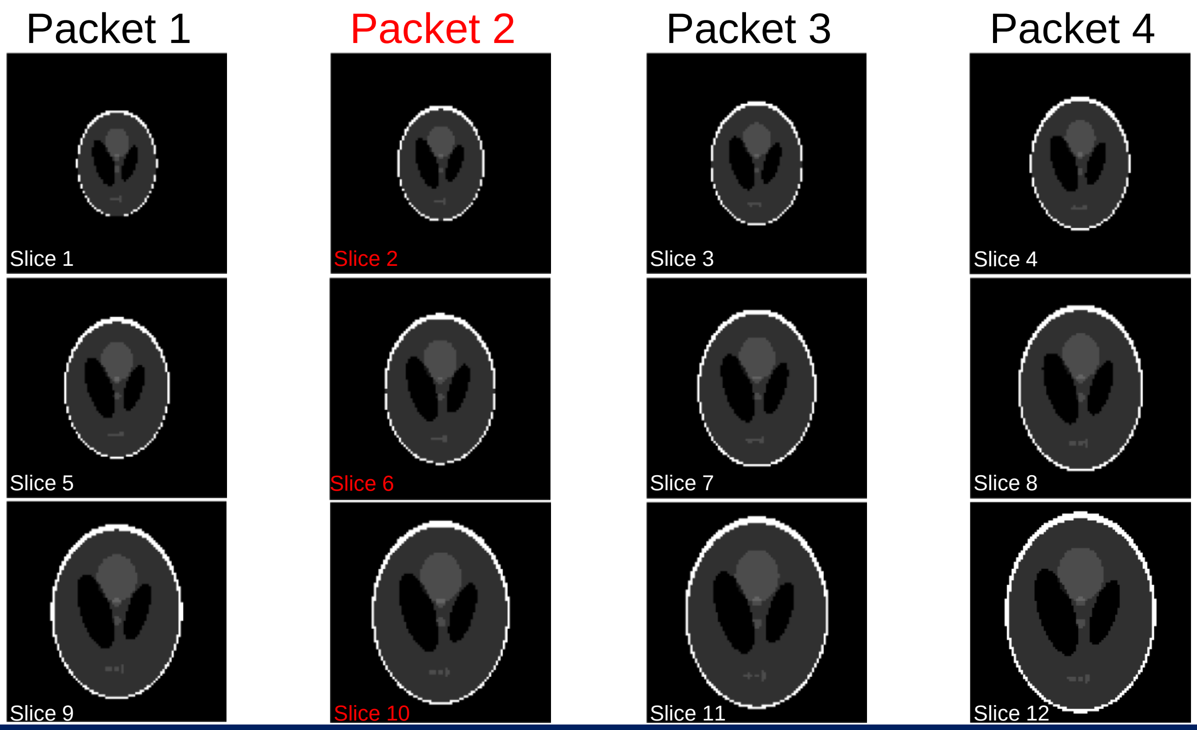


Coil Sensitivities
Coil Covariance



2. Simultaneous Multi-Slice

New techniques have been/are being developed to simultaneously encode, measure, and reconstruct packets of multiple slices. Three-Slices Encoded.

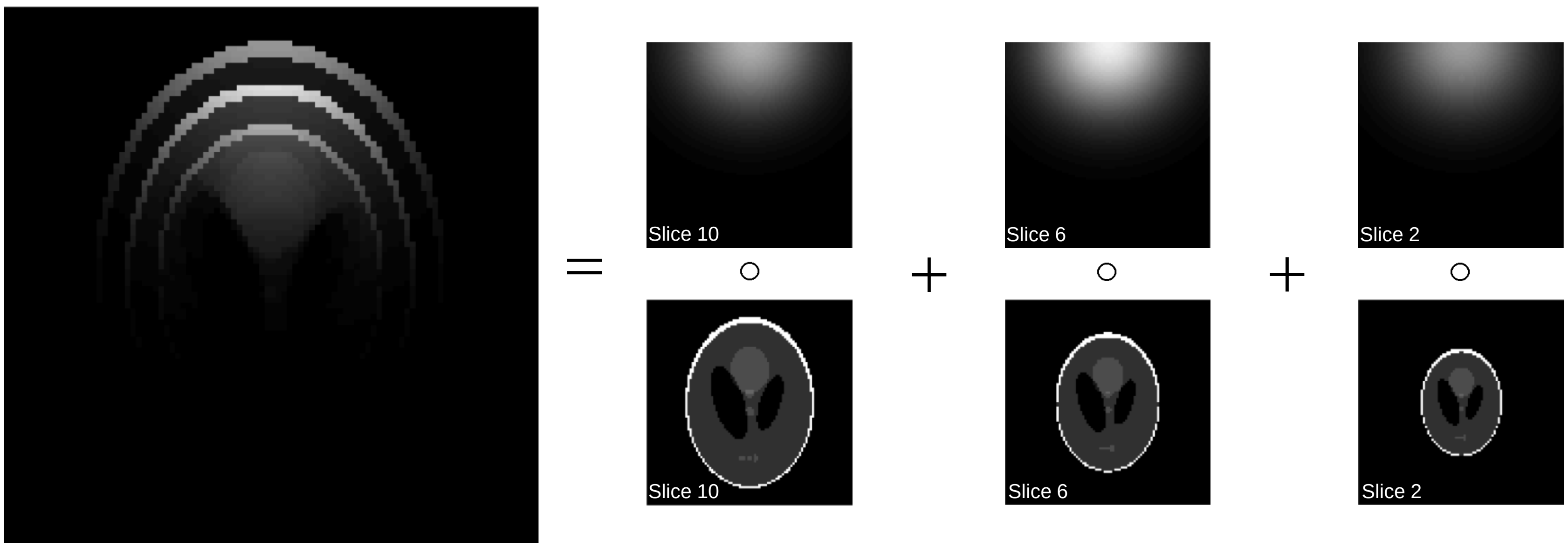


1	2	3	4
5	6	7	8
9	10	11	12

2. Simultaneous Multi-Slice

A 4 Channel Coil Array. 3 Slices Encoded of 12.

Coil 1 Receives in Packet 2

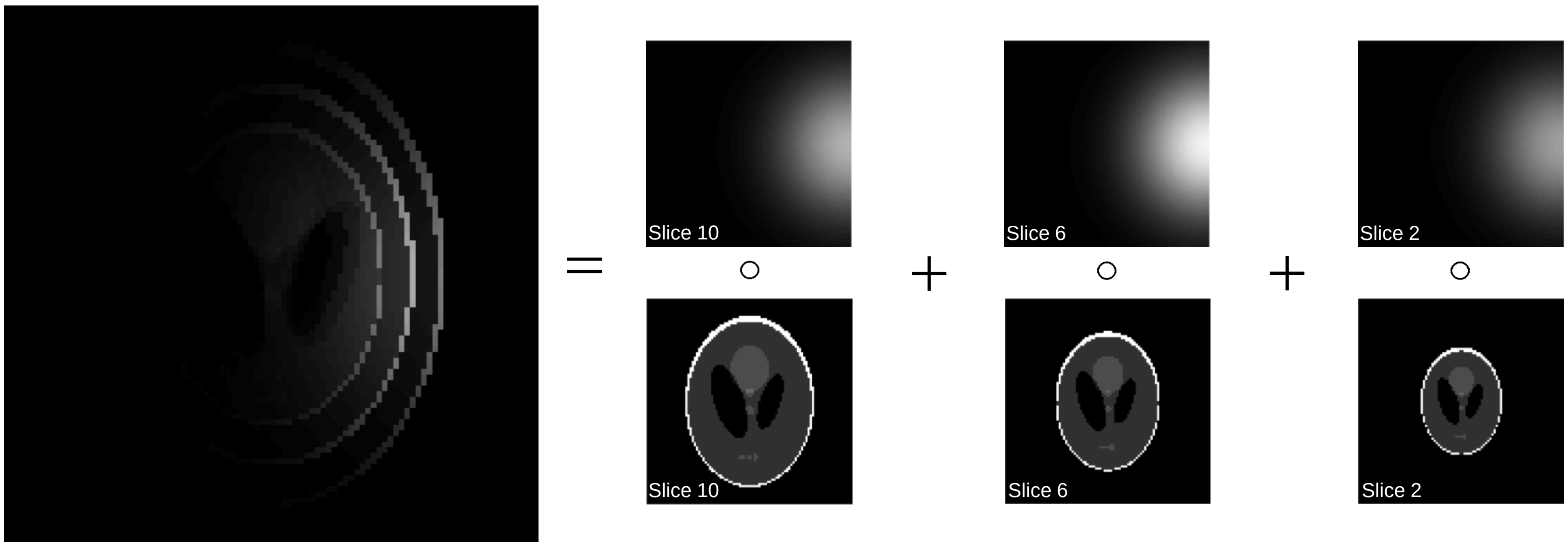


Hadamard Product: $\circ = .*$

2. Simultaneous Multi-Slice

A 4 Channel Coil Array. 3 Slices Encoded of 12.

Coil 2 Receives in Packet 2

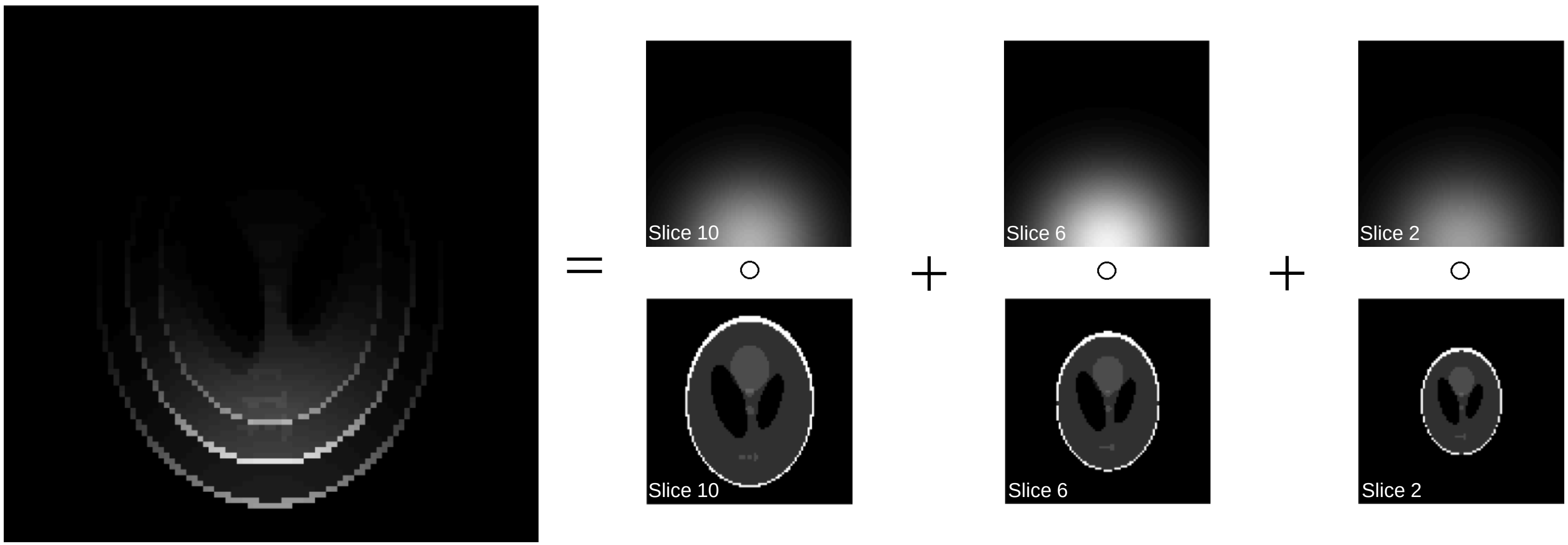


Hadamard Product: $\circ = .*$

2. Simultaneous Multi-Slice

A 4 Channel Coil Array. 3 Slices Encoded of 12.

Coil 3 Receives in Packet 2

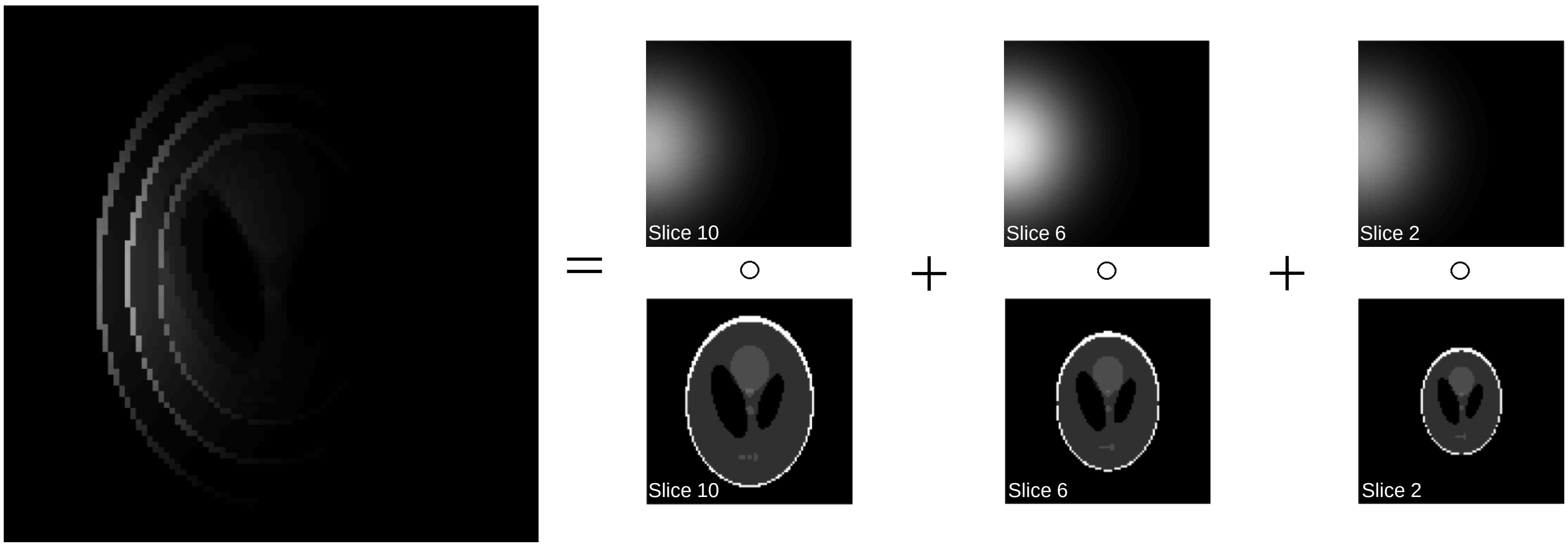


Hadamard Product: $\circ = .*$

2. Simultaneous Multi-Slice

A 4 Channel Coil Array. 3 Slices Encoded of 12.

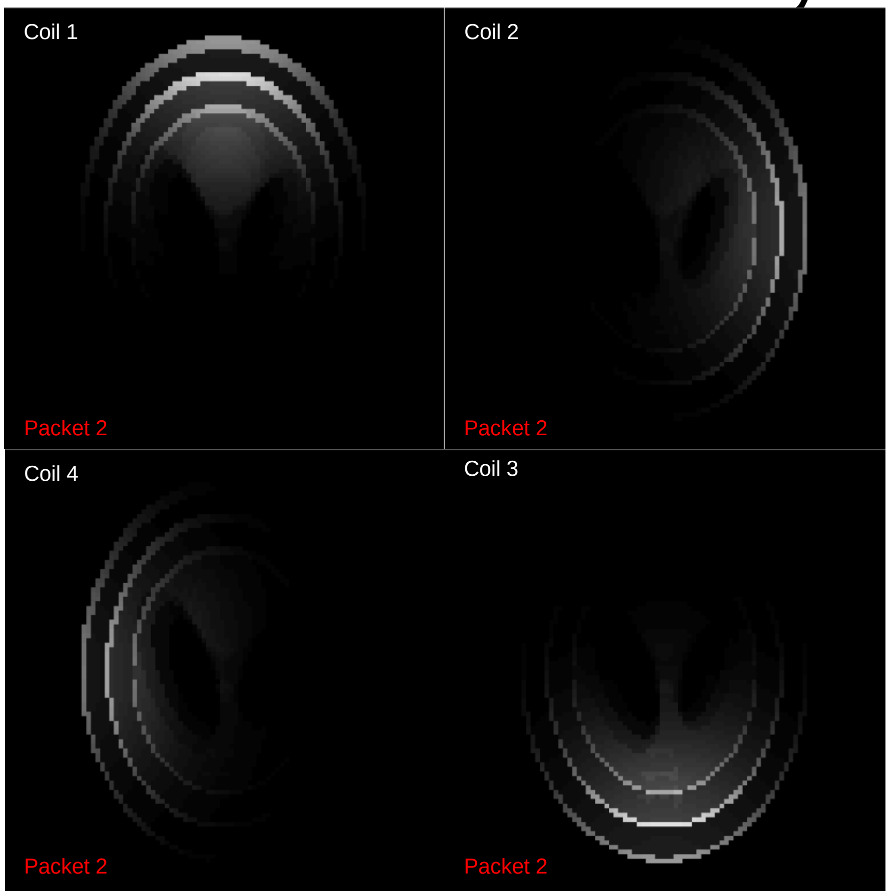
Coil 4 Receives in Packet 2

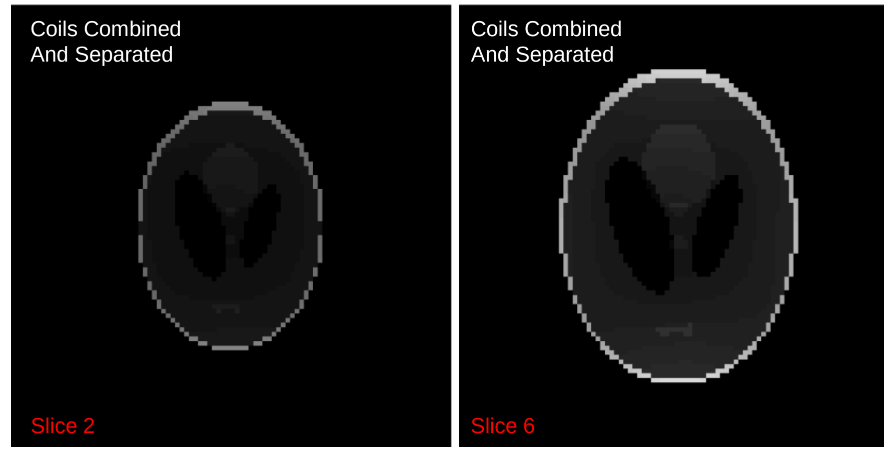


Hadamard Product: $\circ = .*$

2. Simultaneous Multi-Slice

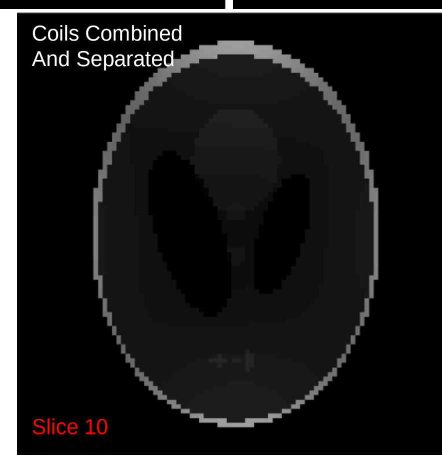
A 4 Channel Coil Array. 3 Slices Encoded of 12.





Separate

Coil Sensitivities
Coil Covariance
Calibration Images

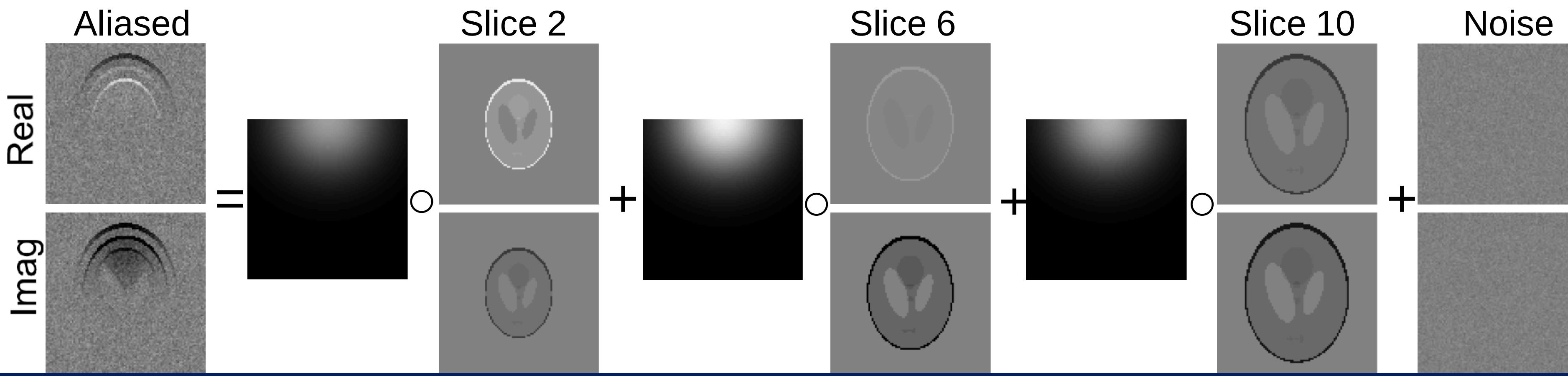


2. Simultaneous Multi-Slice

In each voxel of coil 1 for packet 2 (2,6,10):

$$\begin{aligned}
(y_{1R} + i y_{1I}) &= S_{1,2}(\rho_2 \cos\theta_2 + i\rho_2 \sin\theta_2) \longleftarrow \text{True Slice 2} \\
&+ S_{1,6}(\rho_6 \cos\theta_6 + i\rho_6 \sin\theta_6) \longleftarrow \text{True Slice 6} \\
&+ S_{1,10}(\rho_{10} \cos\theta_{10} + i\rho_{10} \sin\theta_{10}) \longleftarrow \text{True Slice 10} \\
&+ (\varepsilon_{1R} + i\varepsilon_{1I}) \longleftarrow \text{Additive Noise}
\end{aligned}$$

Observed Image

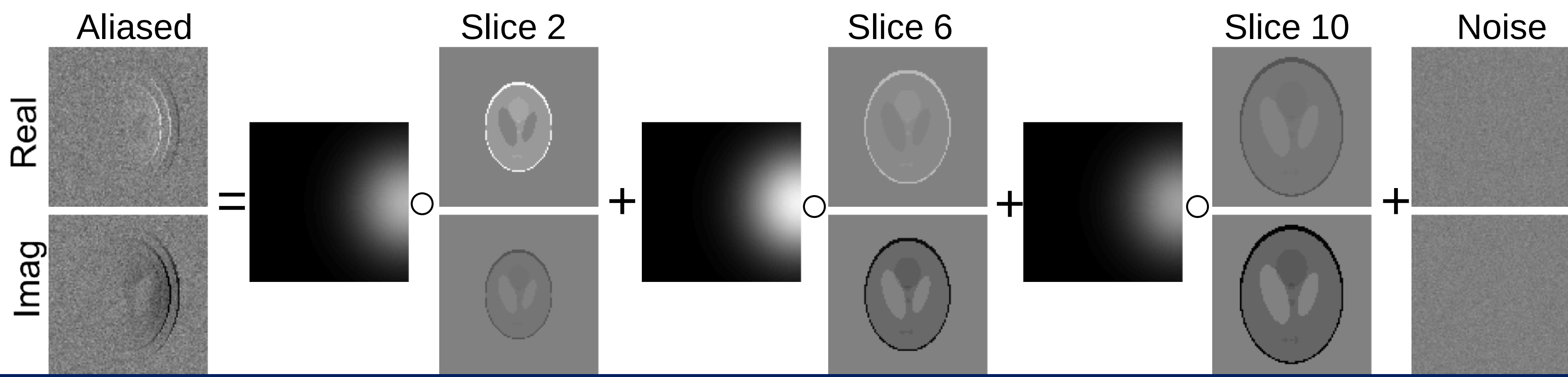


2. Simultaneous Multi-Slice

In each voxel of coil 2 for packet 2 (2,6,10):

$$
\begin{aligned}
(y_{2R}+iy_{2I}) &= S_{2,2}(\rho_2\cos\theta_2+i\rho_2\sin\theta_2) &\longleftarrow\ &\text{True Slice 2}\\
&+ S_{2,6}(\rho_6\cos\theta_6+i\rho_6\sin\theta_6) &\longleftarrow\ &\text{True Slice 6}\\
&+ S_{2,10}(\rho_{10}\cos\theta_{10}+i\rho_{10}\sin\theta_{10}) &\longleftarrow\ &\text{True Slice 10}\\
&+ (\varepsilon_{2R}+i\varepsilon_{2I}) &\longleftarrow\ &\text{Additive Noise}
\end{aligned}
$$

Observed Image

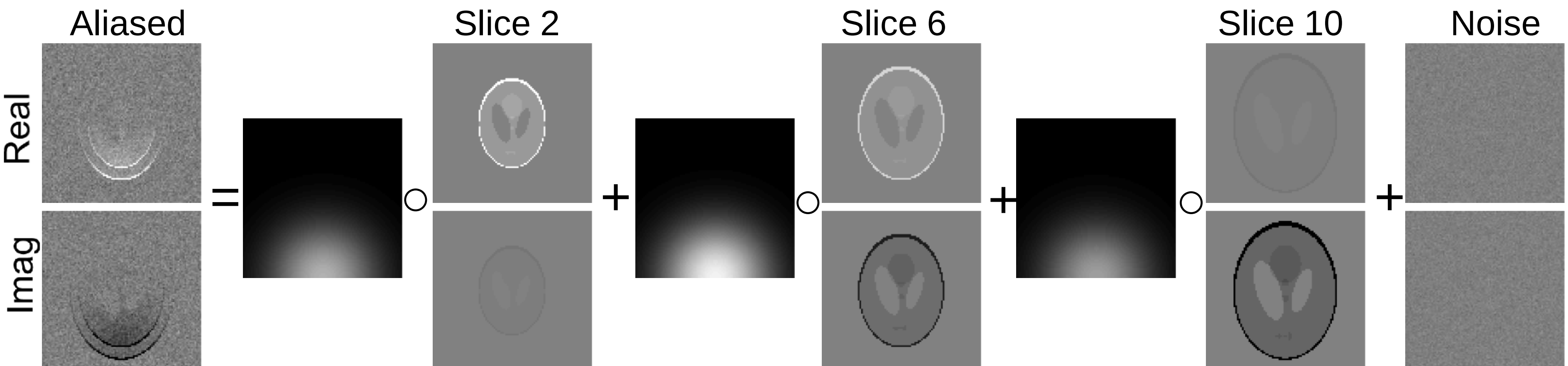


2. Simultaneous Multi-Slice

In each voxel of coil 3 for packet 2 (2,6,10):

$$
\begin{aligned}
(y_{3R} + i y_{3I}) &= S_{3,2}(\rho_2 \cos\theta_2 + i\rho_2 \sin\theta_2) && \longleftarrow \text{True Slice 2} \\
&+ S_{3,6}(\rho_6 \cos\theta_6 + i\rho_6 \sin\theta_6) && \longleftarrow \text{True Slice 6} \\
&+ S_{3,10}(\rho_{10} \cos\theta_{10} + i\rho_{10} \sin\theta_{10}) && \longleftarrow \text{True Slice 10} \\
&+ (\varepsilon_{3R} + i\varepsilon_{3I}) && \longleftarrow \text{Additive Noise}
\end{aligned}
$$

Observed Image

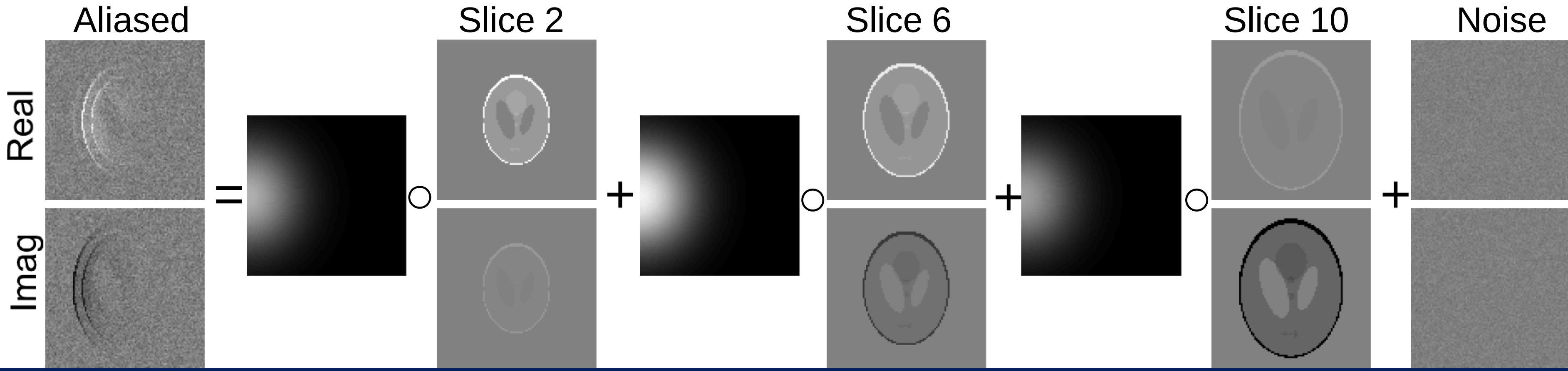


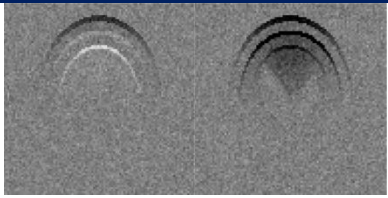
2. Simultaneous Multi-Slice

In each voxel of coil 4 for packet 2 (2,6,10):

$$
\begin{array}{rcll}
(y_{4R} + iy_{4I}) & = & S_{4,2}(\rho_2 \cos\theta_2 + i\rho_2 \sin\theta_2) & \longleftarrow \text{True Slice 2} \\
 & + & S_{4,6}(\rho_6 \cos\theta_6 + i\rho_6 \sin\theta_6) & \longleftarrow \text{True Slice 6} \\
 & + & S_{4,10}(\rho_{10} \cos\theta_{10} + i\rho_{10} \sin\theta_{10}) & \longleftarrow \text{True Slice 10} \\
 & + & (\varepsilon_{4R} + i\varepsilon_{4I}) & \longleftarrow \text{Additive Noise}
\end{array}
$$

Observed Image





2. Simultaneous Multi-Slice

In each voxel of coil 1 for packet 2 (2,6,10):

$$\begin{pmatrix} y_{1R} \\ y_{1I} \end{pmatrix} = \begin{pmatrix} S_{1,2} & S_{1,6} & S_{1,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{1,2} & S_{1,6} & S_{1,10} \end{pmatrix} \begin{pmatrix} \rho_2 \cos\theta_2 \\ \rho_6 \cos\theta_6 \\ \rho_{10} \cos\theta_{10} \\ \rho_2 \sin\theta_2 \\ \rho_6 \sin\theta_6 \\ \rho_{10} \sin\theta_{10} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1R} \\ \varepsilon_{1I} \end{pmatrix}$$

↑ Aliased Image

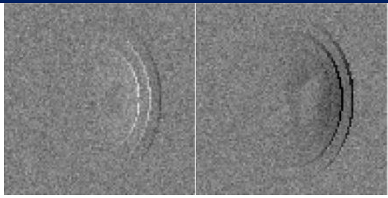
↑ Aliasing Matrix

↑ True Unaliased Images

↑ Measurement Error

$$y_1 \quad = \quad X_1 \quad \beta \quad + \quad \varepsilon_1$$

(2 linear equations and 6 unknowns)



2. Simultaneous Multi-Slice

In each voxel of coil 2 for packet 2 (2,6,10):

$$\begin{pmatrix} y_{2R} \\ y_{2I} \end{pmatrix} = \begin{pmatrix} S_{2,2} & S_{2,6} & S_{2,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{2,2} & S_{2,6} & S_{2,10} \end{pmatrix} \begin{pmatrix} \rho_2 \cos\theta_2 \\ \rho_6 \cos\theta_6 \\ \rho_{10} \cos\theta_{10} \\ \rho_2 \sin\theta_2 \\ \rho_6 \sin\theta_6 \\ \rho_{10} \sin\theta_{10} \end{pmatrix} + \begin{pmatrix} \varepsilon_{2R} \\ \varepsilon_{2I} \end{pmatrix}$$

Aliased Image

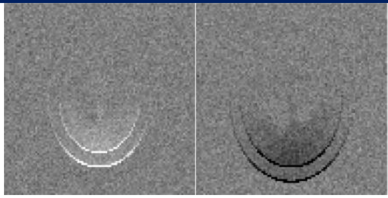
Aliasing Matrix

True Unaliased Images

Measurement Error

$$y_2 = X_2 \beta + \varepsilon_2$$

(2 linear equations and 6 unknowns)



2. Simultaneous Multi-Slice

In each voxel of coil 3 for packet 2 (2,6,10):

$$\begin{pmatrix} y_{3R} \\ y_{3I} \end{pmatrix} = \begin{pmatrix} S_{3,2} & S_{3,6} & S_{3,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{3,2} & S_{3,6} & S_{3,10} \end{pmatrix} \begin{pmatrix} \rho_2 \cos\theta_2 \\ \rho_6 \cos\theta_6 \\ \rho_{10} \cos\theta_{10} \\ \rho_2 \sin\theta_2 \\ \rho_6 \sin\theta_6 \\ \rho_{10} \sin\theta_{10} \end{pmatrix} + \begin{pmatrix} \varepsilon_{3R} \\ \varepsilon_{3I} \end{pmatrix}$$

Aliased Image

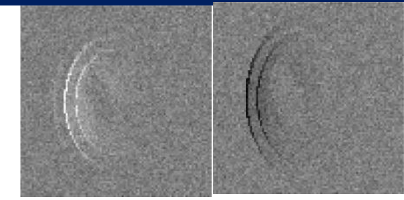
Aliasing Matrix

True Unaliased Images

Measurement Error

$$y_3 = X_3 \qquad \beta + \varepsilon_3$$

(2 linear equations and 6 unknowns)



2. Simultaneous Multi-Slice

In each voxel of coil 4 for packet 2 (2,6,10):

$$\begin{pmatrix} y_{4R} \\ y_{4I} \end{pmatrix} = \begin{pmatrix} S_{4,2} & S_{4,6} & S_{4,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{4,2} & S_{4,6} & S_{4,10} \end{pmatrix} \begin{pmatrix} \rho_2 \cos\theta_2 \\ \rho_6 \cos\theta_6 \\ \rho_{10} \cos\theta_{10} \\ \rho_2 \sin\theta_2 \\ \rho_6 \sin\theta_6 \\ \rho_{10} \sin\theta_{10} \end{pmatrix} + \begin{pmatrix} \varepsilon_{4R} \\ \varepsilon_{4I} \end{pmatrix}$$

↑ Aliased Image

↑ Aliasing Matrix

↑ True Unaliased Images

↑ Measurement Error

$$y_4 \quad = \quad X_4 \quad \beta \quad + \quad \varepsilon_4$$

(2 linear equations and 6 unknowns)

2. Simultaneous Multi-Slice

In the same voxel of the 4 coils for packet 2 (2,6,10):

$$\begin{pmatrix} y_{1R} \\ y_{2R} \\ y_{3R} \\ y_{4R} \\ y_{1I} \\ y_{2I} \\ y_{3I} \\ y_{4I} \end{pmatrix} = \left(\begin{array}{ccc|ccc} S_{1,2} & S_{1,6} & S_{1,10} & 0 & 0 & 0 \\ S_{2,2} & S_{2,6} & S_{2,10} & 0 & 0 & 0 \\ S_{3,2} & S_{3,6} & S_{3,10} & 0 & 0 & 0 \\ S_{4,2} & S_{4,6} & S_{4,10} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & S_{1,2} & S_{1,6} & S_{1,10} \\ 0 & 0 & 0 & S_{2,2} & S_{2,6} & S_{2,10} \\ 0 & 0 & 0 & S_{3,2} & S_{3,6} & S_{3,10} \\ 0 & 0 & 0 & S_{4,2} & S_{4,6} & S_{4,10} \end{array}\right) \begin{pmatrix} \rho_2 \cos\theta_2 \\ \rho_6 \cos\theta_6 \\ \rho_{10} \cos\theta_{10} \\ \rho_2 \sin\theta_2 \\ \rho_6 \sin\theta_6 \\ \rho_{10} \sin\theta_{10} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1R} \\ \varepsilon_{2R} \\ \varepsilon_{3R} \\ \varepsilon_{4R} \\ \varepsilon_{1I} \\ \varepsilon_{2I} \\ \varepsilon_{3I} \\ \varepsilon_{4I} \end{pmatrix}$$

$$y_A = X_A \beta + \varepsilon_A$$

Aliased Image | Aliasing Matrix | True Unaliased Images | Measurement Error

2. Simultaneous Multi-Slice

In the same voxel of the 4 coils for packet 2 (2,6,10):

$$y_A \quad = \quad X_A \quad \beta \quad + \quad \varepsilon_A$$

Aliased Image — Aliasing Matrix — True Unaliased Images — Measurement Error

$$E(\varepsilon_A) = 0$$

$$\text{cov}(\varepsilon_A) = \Psi$$

X_A is severely rank deficient. rank(X_A)≥2

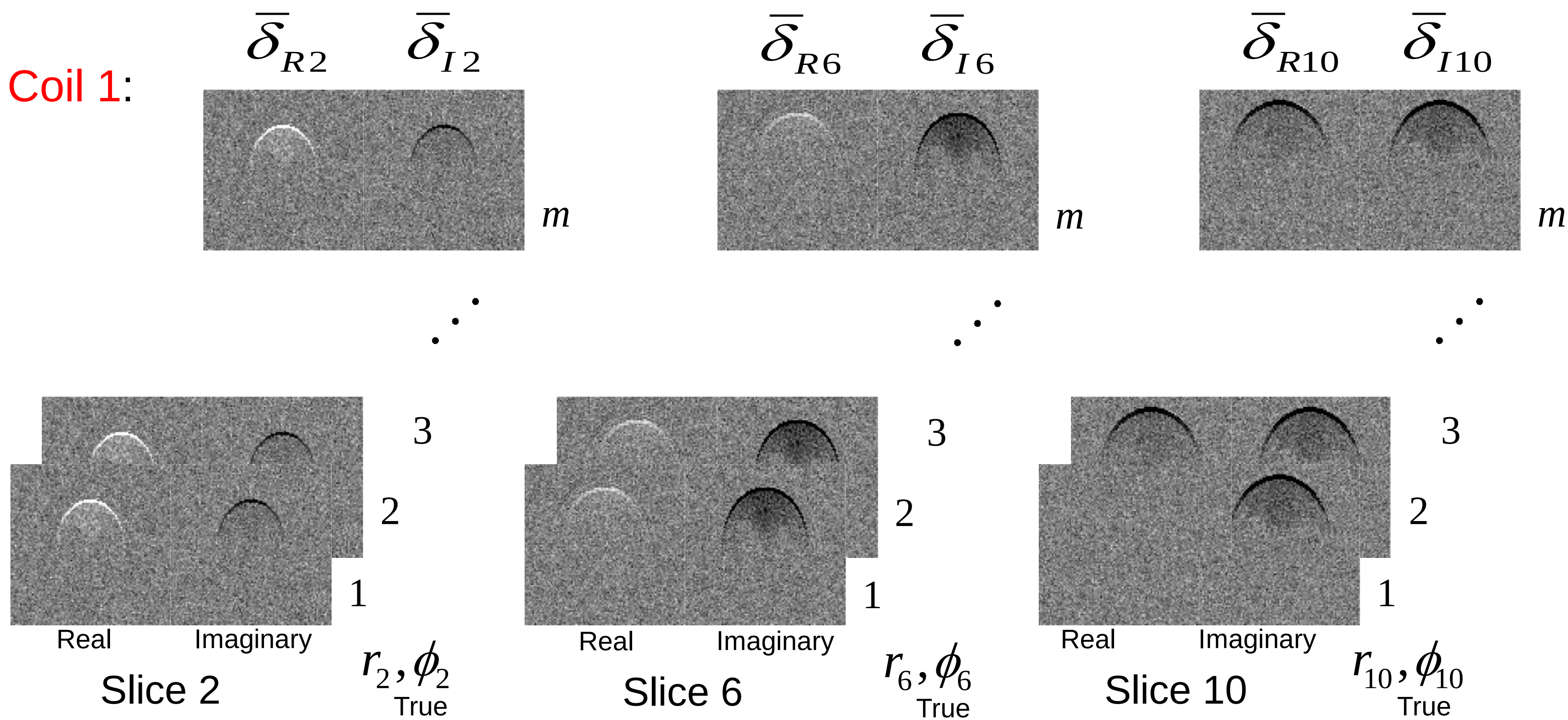
So can't invert X_A. or $X_A{}'X_A$.

$$X_A = \begin{pmatrix} S_{1,2} & S_{1,6} & S_{1,10} & 0 & 0 & 0 \\ S_{2,2} & S_{2,6} & S_{2,10} & 0 & 0 & 0 \\ S_{3,2} & S_{3,6} & S_{3,10} & 0 & 0 & 0 \\ S_{4,2} & S_{4,6} & S_{4,10} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{1,2} & S_{1,6} & S_{1,10} \\ 0 & 0 & 0 & S_{2,2} & S_{2,6} & S_{2,10} \\ 0 & 0 & 0 & S_{3,2} & S_{3,6} & S_{3,10} \\ 0 & 0 & 0 & S_{4,2} & S_{4,6} & S_{4,10} \end{pmatrix}$$

3. Image Separation

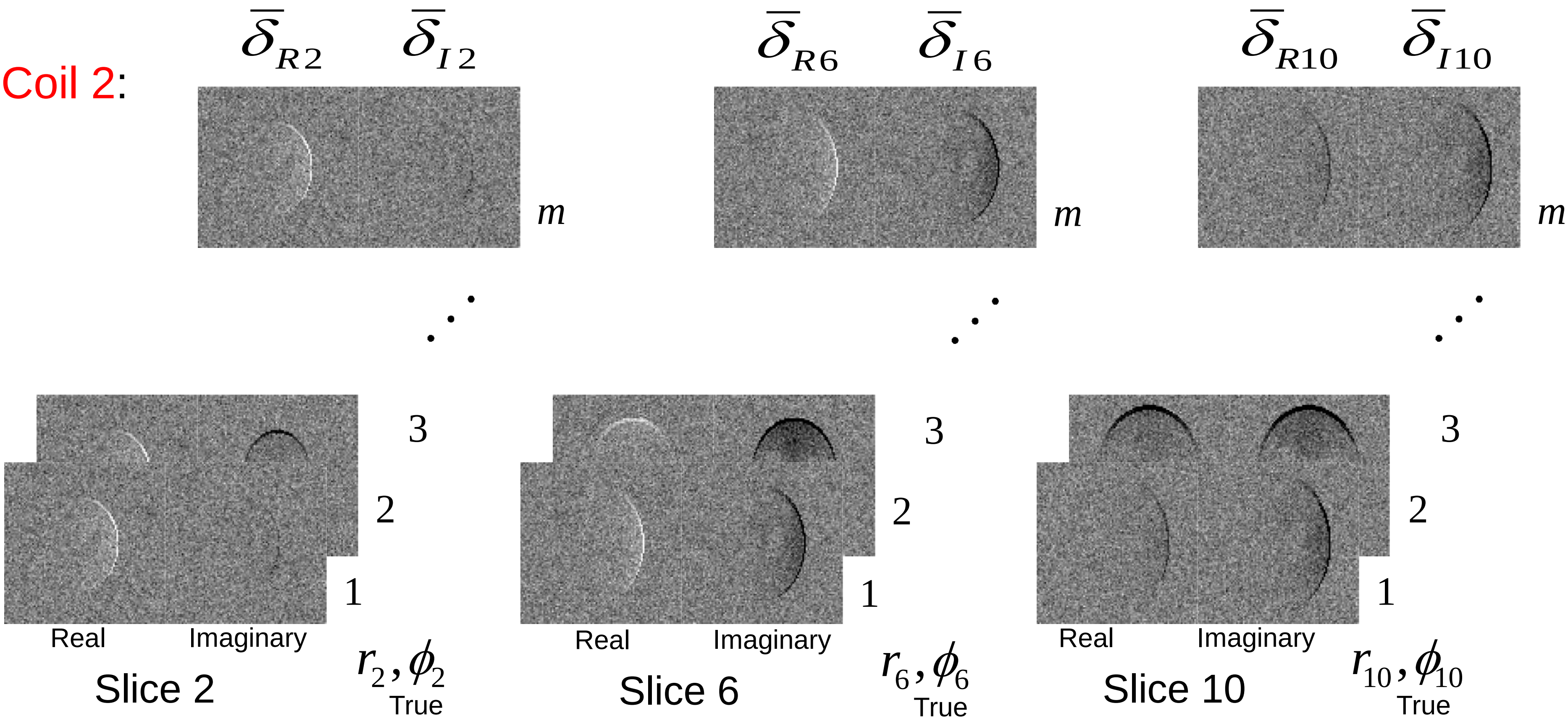
Full Calibration Images Acquired

Average



3. Image Separation

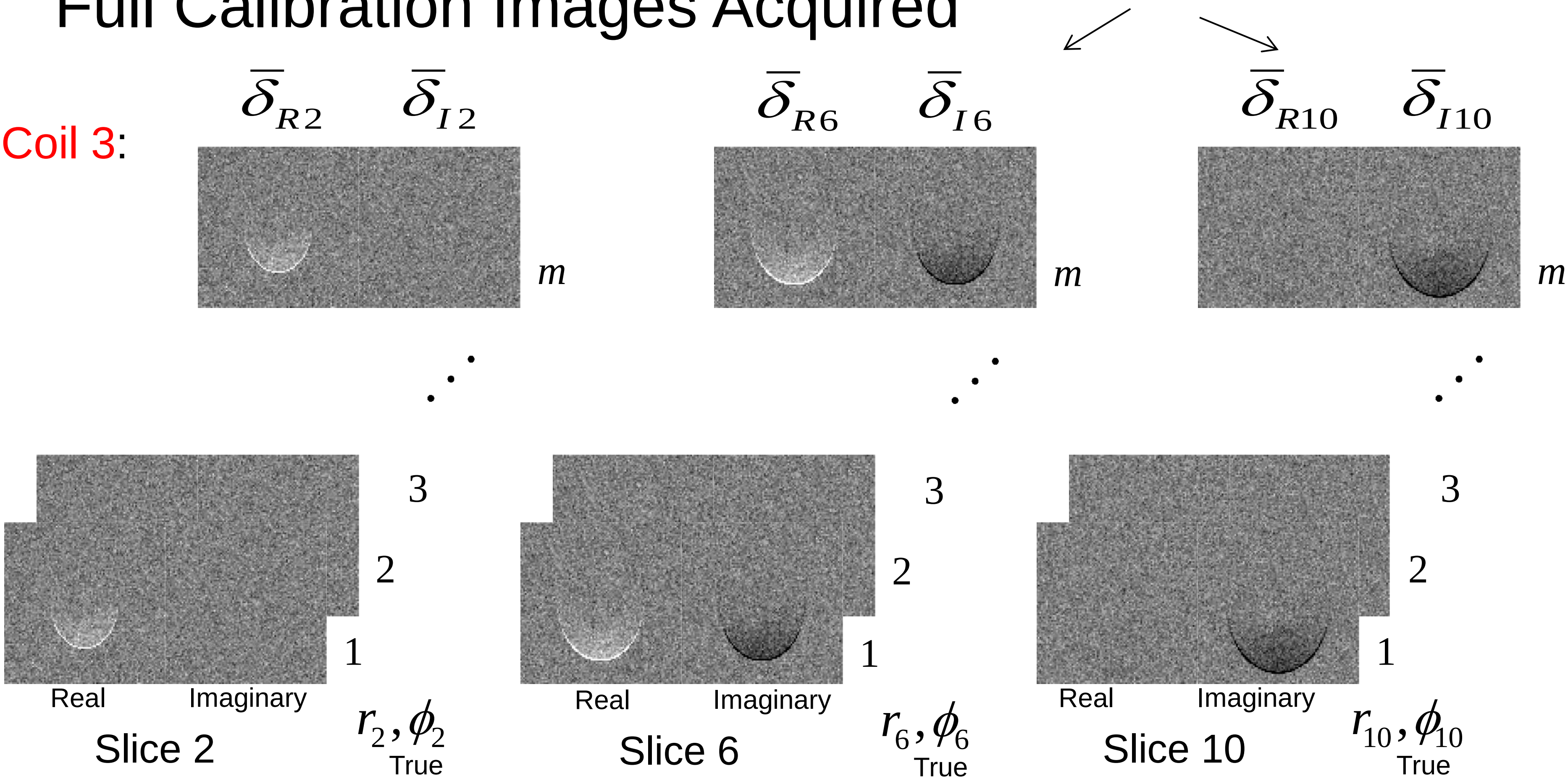
Full Calibration Images Acquired



3. Image Separation

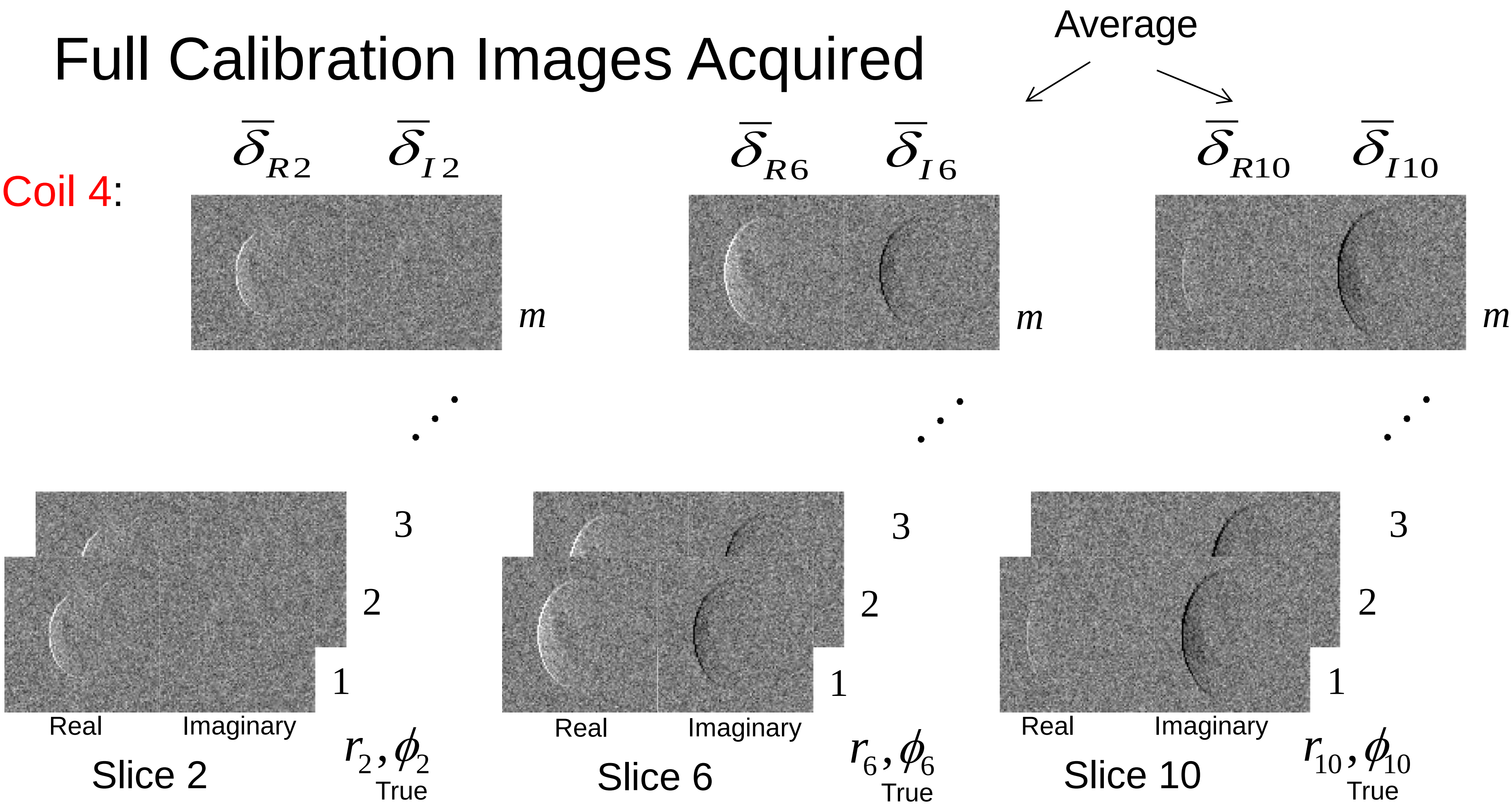
Full Calibration Images Acquired

Average



3. Image Separation

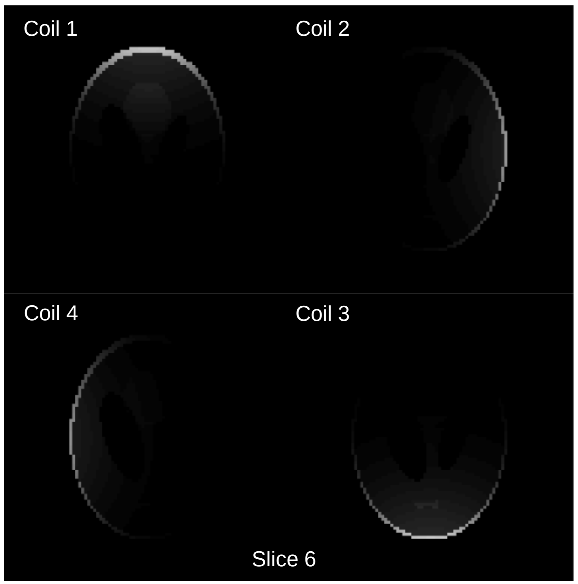
Full Calibration Images Acquired



3. Image Separation

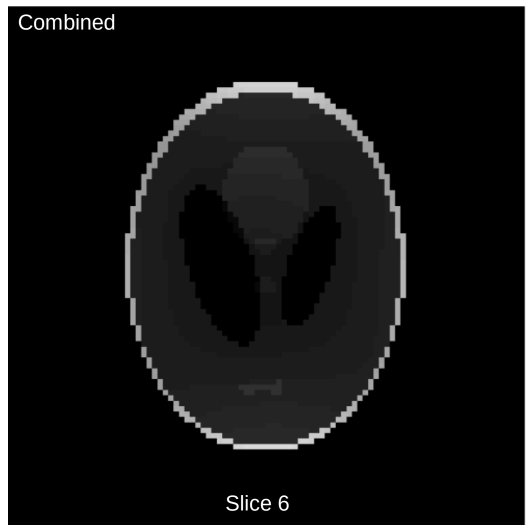
magnitude of

The coil sensitivities are obtained from $\overset{\downarrow}{m}$ full images.



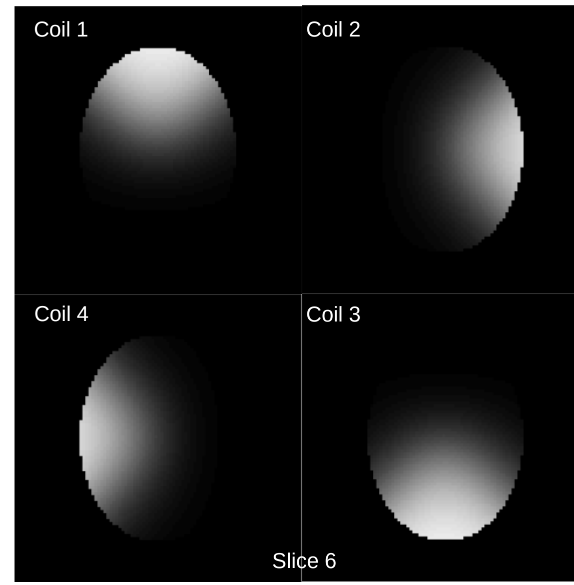
Individual Coil Images
(from average of m)

./



Combined Coil Image
(from average of m)

=



Individual Coil Sensitivities
(from average of m)

Coil covariances are also obtained from the full scans.

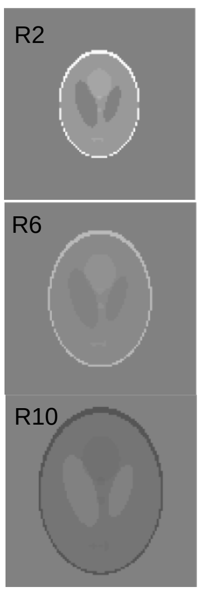
3. Image Separation

Due to rank deficiency, add rows to X_A , y_A, and ε_A.

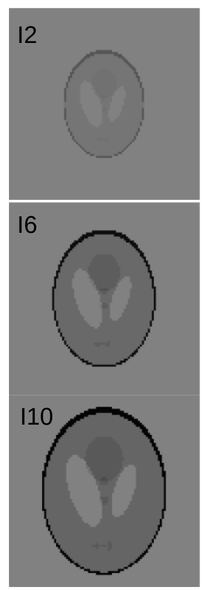
$$y_A = X_A \beta + \varepsilon_A$$

complex-valued

Time average same $\downarrow$ combined calibration images



$$\longrightarrow \quad \bar{\delta}_R = \begin{pmatrix} \bar{\delta}_{R2} \\ \bar{\delta}_{R6} \\ \bar{\delta}_{R10} \end{pmatrix}$$



$$\longrightarrow \quad \bar{\delta}_I = \begin{pmatrix} \bar{\delta}_{I2} \\ \bar{\delta}_{I6} \\ \bar{\delta}_{I10} \end{pmatrix} \quad \longrightarrow \quad \bar{\delta} = \begin{pmatrix} \bar{\delta}_R \\ \bar{\delta}_I \end{pmatrix}$$

Similar to
Rowe et al.: Proc. JSM 2012.
Rowe et al. Proc. ISMRM, 2013

3. Image Separation

Add constraints & artificially aliased data.

$$S = \begin{pmatrix} S_{1,2} & S_{1,6} & S_{1,10} \\ S_{2,2} & S_{2,6} & S_{2,10} \\ S_{3,2} & S_{3,6} & S_{3,10} \\ S_{4,2} & S_{4,6} & S_{4,10} \end{pmatrix}$$

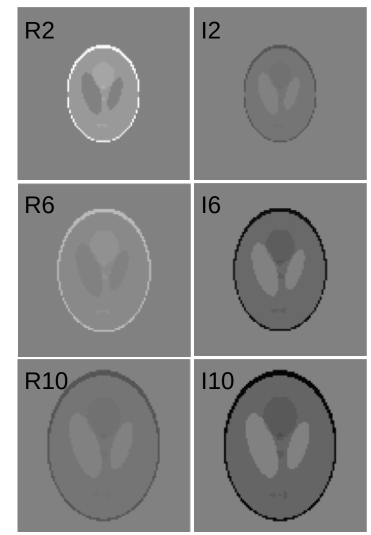
$$\begin{pmatrix} y_{AR} \\ y_{AI} \\ \nu_{AR} \\ \nu_{AI} \end{pmatrix} = \begin{pmatrix} S & 0 \\ 0 & S \\ C & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} \beta_R \\ \beta_I \end{pmatrix} + \begin{pmatrix} \varepsilon_{AR} \\ \varepsilon_{AI} \\ \eta_{AR} \\ \eta_{AR} \end{pmatrix}$$

$$y \quad = \quad X \qquad \beta \; + \; \varepsilon$$

$$\begin{pmatrix} \nu_{AR} \\ \nu_{AI} \end{pmatrix} = \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} \bar{\delta}_R \\ \bar{\delta}_I \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}$$

$\bar{\delta}_R$ $\bar{\delta}_I$



Least squares image separation

$$\hat{\beta} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}y \longleftarrow$$ separated images

Extension of
Rowe et al.: Proc. JSM 2012.
Rowe et al. Proc. ISMRM, 2013

3. Image Separation

Mean and Covariance of separated image

$$\hat{\beta} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}y \longleftarrow$$ separated images

$$y = X\beta + \varepsilon$$

can be computed.

$$E(\varepsilon_A) = 0$$
$$\mathrm{cov}(\varepsilon_A) = \Psi$$

If $E(\bar{\delta}) = \delta$ and $\mathrm{cov}(\bar{\delta}) = \Gamma$, then

$$E(\hat{\beta}) = \begin{pmatrix} (S'\Psi^{-1}S + C'C)^{-1}(S'\Psi^{-1}S\beta_R + C'C\delta_R) \\ (S'\Psi^{-1}S + C'C)^{-1}(S'\Psi^{-1}S\beta_I + C'C\delta_I) \end{pmatrix}$$

$$\mathrm{cov}(\hat{\beta}) = I_2 \otimes (S'\Psi^{-1}S + C'C)^{-1}(S'\Psi^{-1}S)(S'\Psi^{-1}S + C'C)^{-1}$$

4. Results

Generated 50 simulated calibration images.
First 5 deleted and remainder averaged
for sensitivities and artificially aliased data.

Generated n=256 simulated aliased slice images.
First 5 deleted, remaining separated.

Show calibration image used for separating.

Show coil sensitivities used for separation.

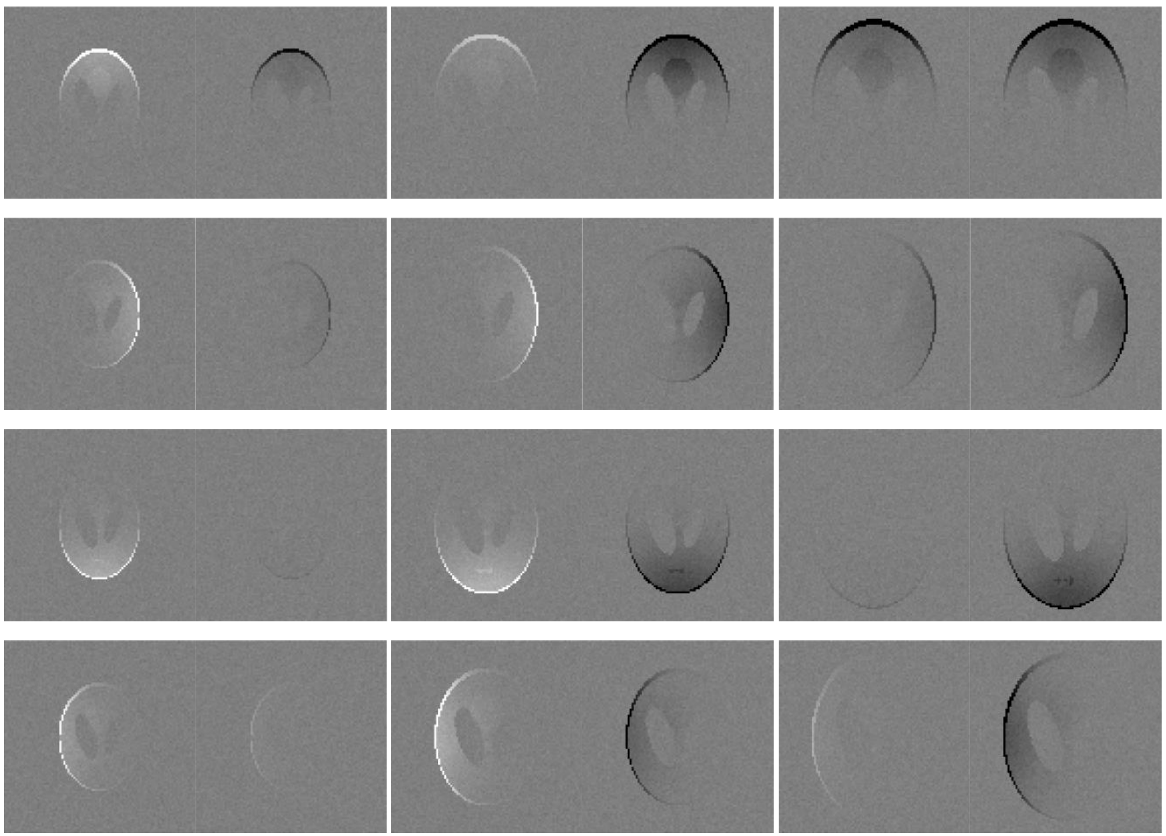
Show first separated and average separated.

4. Results

$$C = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \end{pmatrix}$$

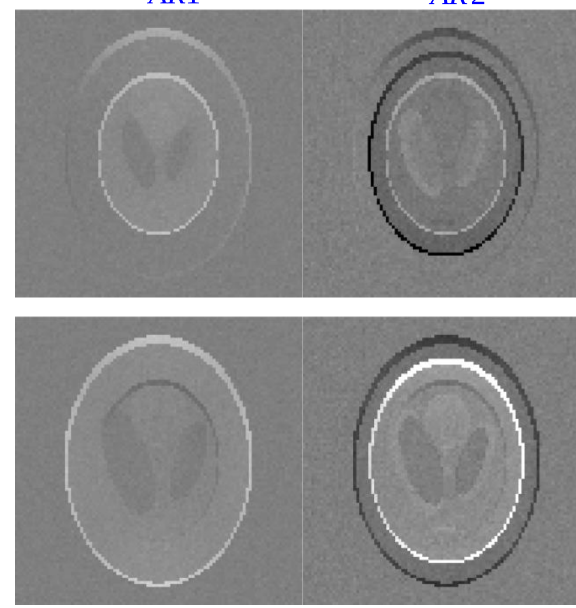
Slice 2 | Slice 6 | Slice 10

Coil 1: | Coil 2: | Coil 3: | Coil 4:



Real | Imaginary | Real | Imaginary | Real | Imaginary

ν_{AR1} | ν_{AR2}



ν_{AI1} | ν_{AI2}

$$\begin{pmatrix} y_{AR} \\ y_{AI} \\ \nu_{AR} \\ \nu_{AI} \end{pmatrix} = \begin{pmatrix} S & 0 \\ 0 & S \\ C & 0 \\ 0 & C \end{pmatrix} \begin{pmatrix} \beta_R \\ \beta_I \end{pmatrix} + \begin{pmatrix} \varepsilon_{AR} \\ \varepsilon_{AI} \\ \eta_{AR} \\ \eta_{AR} \end{pmatrix}$$

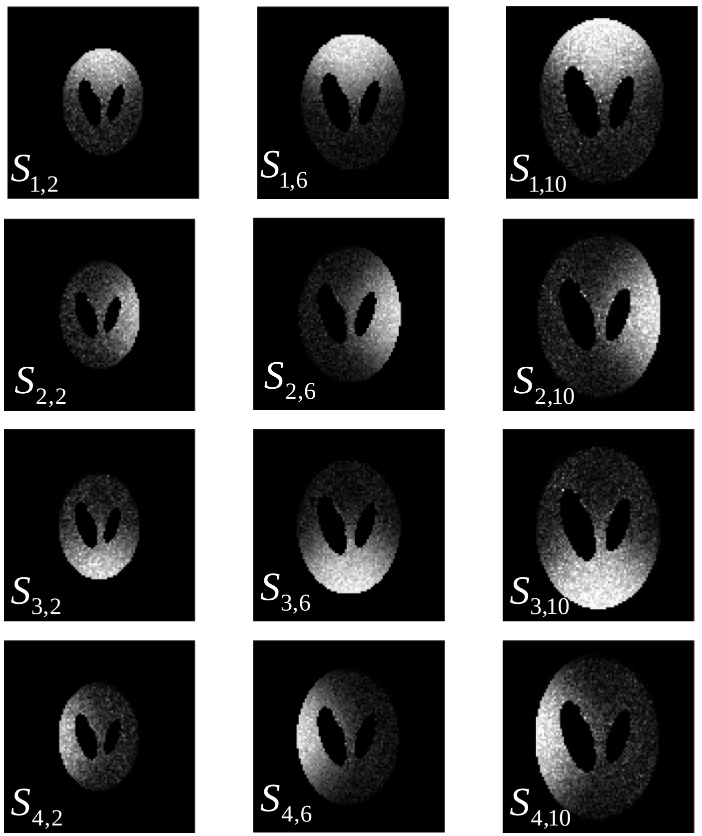
4. Results

Sensitivities.

$$C=\begin{pmatrix}-1 & 0 & 1\\ 1 & -2 & 1\end{pmatrix}$$

Slice 2 Slice 6 Slice 10

Coil 1:
Coil 2:
Coil 3:
Coil 4:

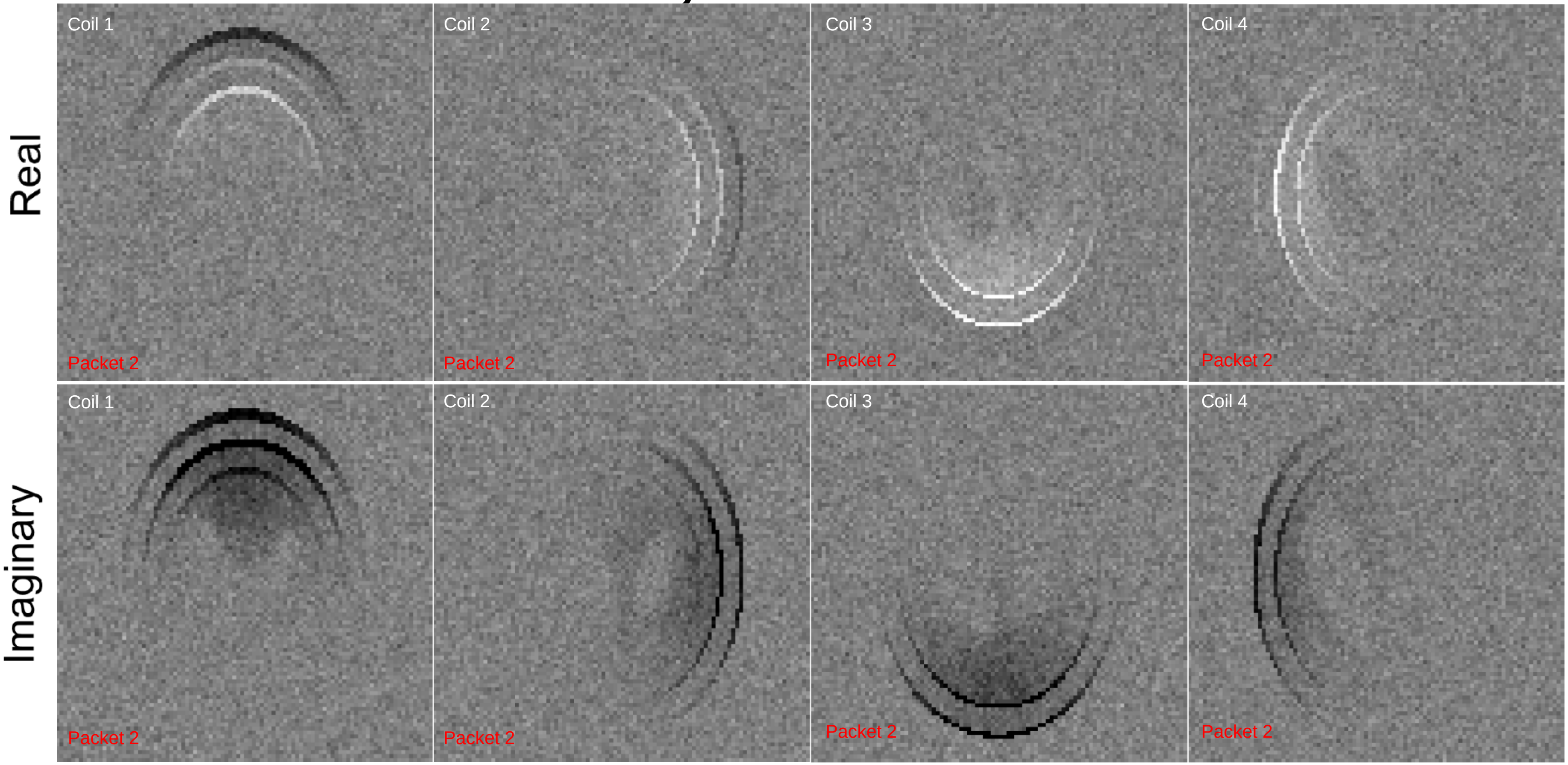


$$S=\begin{pmatrix}S_{1,2} & S_{1,6} & S_{1,10}\\ S_{2,2} & S_{2,6} & S_{2,10}\\ S_{3,2} & S_{3,6} & S_{3,10}\\ S_{4,2} & S_{4,6} & S_{4,10}\end{pmatrix}$$

$$\begin{pmatrix}y_{AR}\\ y_{AI}\\ \nu_{AR}\\ \nu_{AI}\end{pmatrix}=\begin{pmatrix}S & 0\\ 0 & S\\ C & 0\\ 0 & C\end{pmatrix}\begin{pmatrix}\beta_R\\ \beta_I\end{pmatrix}+\begin{pmatrix}\varepsilon_{AR}\\ \varepsilon_{AI}\\ \eta_{AR}\\ \eta_{AR}\end{pmatrix}$$

4. Results

A 4 Channel Coil Array. 3 Slices Encoded of 12.



2. Simultaneous Multi-Slice

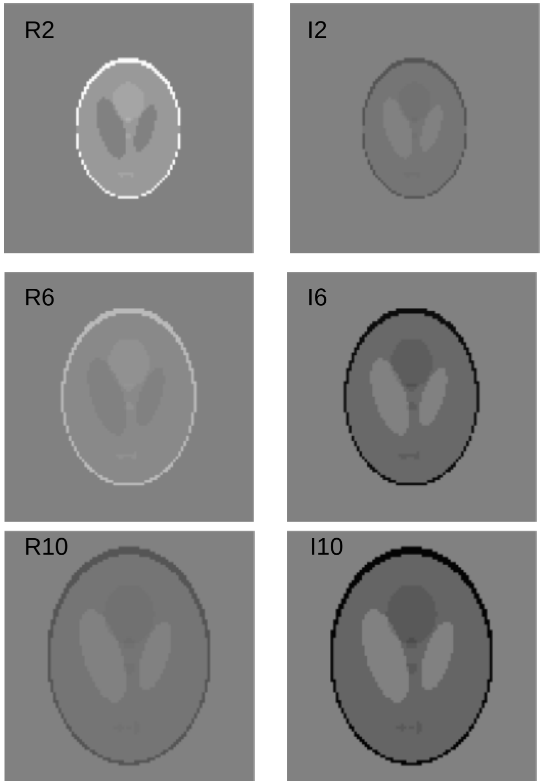
A 4 Channel Coil Array. 3 Slices Encoded of 12.

$$\hat{\beta} = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}y$$

Separate

Coil Sensitivities
Coil Covariance
Calibration Images

Depiction



5. Discussion

Description of the N_A slice N_C coil aliasing process.

New complex-valued multislice multicoil separation.

Statistical properties of the CV multislice multicoil separation.

Simulated data results described.

Did not calculate from the simulated data, but as the theory implies, any subsampling yields correlated voxels.

Thank You!