Statistical Image Reconstruction of Two Simultaneously Excited FMRI Slices

(With A Single Coil)

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Outline:

1. Two Slice Encoding with a Single-Channel Coil
2. The Jesmanowicz Magnitude-Only Approach
3. The Rowe Complex-Valued Approach
4. Statistical Properties
5. Results
6. Discussion

1. Two Slice Encoding with a Single-Channel Coil

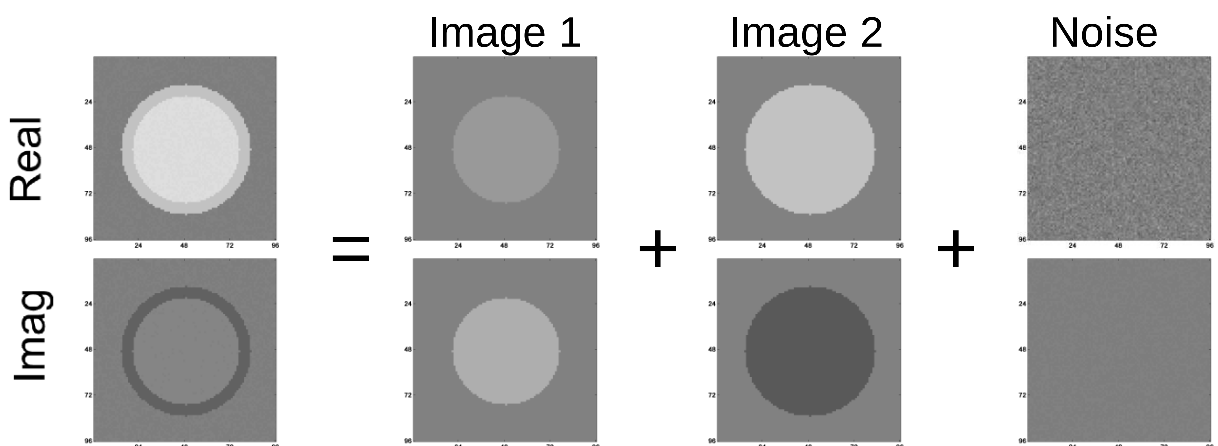
In each voxel:

Observed Aliased Image

$$(y_R + i y_I) = (\rho_1 \cos\theta_1 + i \rho_1 \sin\theta_1) \longleftarrow \text{True Image 1}$$

$$+ (\rho_2 \cos\theta_2 + i \rho_2 \sin\theta_2) \longleftarrow \text{True Image 2}$$

$$+ (\varepsilon_R + i \varepsilon_I) \longleftarrow \text{Additive Noise}$$



Muller: MRM 1988.

1. Two Slice Encoding with a Single-Channel Coil

In each voxel:

$$\begin{aligned}(y_R + i y_I) &= (\rho_1 \cos\theta_1 + i\rho_1 \sin\theta_1)\\ &+ (\rho_2 \cos\theta_2 + i\rho_2 \sin\theta_2)\\ &+ (\varepsilon_R + i\varepsilon_I)\end{aligned}$$

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

↑ Aliased Image

↑ Aliasing Matrix

↑ True Unaliased Images

↑ Measurement Error

$$y = X \beta + \varepsilon$$

(2 linear equations and 4 unknowns)

1. Two Slice Encoding with a Single-Channel Coil

The goal is to estimate (unalias) the two images

$$\hat{\beta} = (X'X)^{-1}X'y \quad .$$

However, we have 2 equations and 4 unknowns and $X'X$ is not square or invertible or of full rank.

Two Approaches:

Previous: Jesmanowicz magnitude-only reconstruction

Current: Rowe complex-valued reconstruction

2. The Jesmanowicz Magnitude-Only Approach

Aliased Image

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

$$\begin{pmatrix} y_R \\ y_I \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \cos\theta_2 \\ \sin\theta_1 & \sin\theta_2 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

equivalent

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = \frac{1}{\sin(\hat{\theta}_1 - \hat{\theta}_2)} \begin{pmatrix} -\sin\hat{\theta}_2 & +\cos\hat{\theta}_2 \\ +\sin\hat{\theta}_1 & -\cos\hat{\theta}_1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

where $\hat{\theta}_1, \hat{\theta}_2$ are phase estimates

$$\hat{\theta}_1 - \hat{\theta}_2 \neq k\pi, \;\; k = 0, \pm 1, \ldots$$

Need phase estimates!

Jesmanowicz, Li, Hyde: ISMRM,2009.
Islam, Glover: ISMRM, 2012.

2. The Jesmanowicz Magnitude-Only Approach

Utilizing complete images called “Reference Images.”

$$\begin{pmatrix} y_{R1t} \\ y_{I1t} \\ y_{R2t} \\ y_{I2t} \end{pmatrix} = \begin{pmatrix} S_1 \cos\phi_1 \\ S_1 \sin\phi_1 \\ S_2 \cos\phi_2 \\ S_2 \sin\phi_2 \end{pmatrix} + \begin{pmatrix} \eta_{R1t} \\ \eta_{I1t} \\ \eta_{R2t} \\ \eta_{I2t} \end{pmatrix}, \qquad \begin{pmatrix} \eta_{R1t} \\ \eta_{I1t} \\ \eta_{R2t} \\ \eta_{I2t} \end{pmatrix} \sim N(0, \sigma^2 I_4) \quad t = 1, \ldots, m$$

$$\begin{pmatrix} \bar{y}_{R1} \\ \bar{y}_{I1} \\ \bar{y}_{R2} \\ \bar{y}_{I2} \end{pmatrix} \sim N\left(\begin{pmatrix} S_1 \cos\phi_1 \\ S_1 \sin\phi_1 \\ S_2 \cos\phi_2 \\ S_2 \sin\phi_2 \end{pmatrix}, \frac{\sigma^2}{m} I_4 \right)$$

acquire m full unaliased images in a prescan

2. The Jesmanowicz Magnitude-Only Approach

Utilizing complete images called “Reference Images.”

$$\begin{pmatrix} \bar{y}_{R1} \\ \bar{y}_{I1} \\ \bar{y}_{R2} \\ \bar{y}_{I2} \end{pmatrix} \longrightarrow \begin{pmatrix} \bar{r}_1 \\ \bar{\phi}_1 \\ \bar{r}_2 \\ \bar{\phi}_2 \end{pmatrix} \longrightarrow \left(\hat{\theta}_1 = \bar{\phi}_1, \hat{\theta}_2 = \bar{\phi}_2\right)$$

phase estimates

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = \frac{1}{\sin(\bar{\phi}_1 - \bar{\phi}_2)} \begin{pmatrix} -\sin\bar{\phi}_2 & +\cos\bar{\phi}_2 \\ +\sin\bar{\phi}_1 & -\cos\bar{\phi}_1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

$$j = 1, 2$$
$$\bar{r}_j = (\bar{y}_{Rj}^2 + \bar{y}_{Ij}^2)^{1/2}$$
$$\bar{\phi}_j = \operatorname{atan}(\bar{y}_{Ij} / \bar{y}_{Rj})$$

3. The Rowe Complex-Valued Approach

$$\begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \\ 0 \\ 0 \end{pmatrix}$$

$$y = X\beta + \varepsilon$$

added two
linear constraints

$$\begin{pmatrix} \nu_R \\ \nu_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{y}_{R1} \\ \bar{y}_{I1} \\ \bar{y}_{R2} \\ \bar{y}_{I2} \end{pmatrix}$$

$$\begin{pmatrix} \nu_R \\ \nu_I \end{pmatrix} \sim N\left(\begin{pmatrix} S_1\cos\phi_1 - S_2\cos\phi_2 \\ S_1\sin\phi_1 - S_2\sin\phi_2 \end{pmatrix}, \frac{2\sigma^2}{m} I_2 \right)$$

3. The Rowe Complex-Valued Approach

$$\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}}_{\text{rank=4}}^{-1} \begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix}$$

$$\hat{\beta} = X^{-1} y$$

$$\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix}$$

4. Statistical Properties of the Approaches

Jesmanowicz Magnitude-Only

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = \frac{1}{\sin(\bar{\phi}_1 - \bar{\phi}_2)} \begin{pmatrix} -\sin\bar{\phi}_2 & +\cos\bar{\phi}_2 \\ +\sin\bar{\phi}_1 & -\cos\bar{\phi}_1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

$$\left(y_R, y_I\right) \sim N\left((\rho_1\cos\theta_1 + \rho_2\cos\theta_2, \rho_1\sin\theta_1 + \rho_2\sin\theta_2)', \sigma^2 I_2\right)$$

$$\left(\bar{y}_{R1}, \bar{y}_{I1}, \bar{y}_{R2}, \bar{y}_{I2}\right) \sim N\left((S_1\cos\phi_1, S_1\sin\phi_1, S_2\cos\phi_2, S_2\sin\phi_2)', \frac{\sigma^2}{m} I_4\right)$$

$$f_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\bar{\phi}_1, \bar{\phi}_2 \mid \bar{r}_1 = 1, \bar{r}_2 = 1\right) = f\left(\bar{r}_1, \bar{\phi}_1, \bar{r}_2, \bar{\phi}_2\right) / f\left(\bar{r}_1 = 1_1, \bar{r}_2 = 1\right)$$

$$(\bar{\phi}_1, \bar{\phi}_2 \mid \bar{r}_1 = 1, \bar{r}_2 = 1) \sim VM(\kappa_1 = mS_1/\sigma^2) \cdot VM(\kappa_2 = mS_2/\sigma^2)$$

4. Statistical Properties of the Approaches

Jesmanowicz Magnitude-Only

$$(\bar{\phi}_1,\bar{\phi}_2 \mid \bar{r}_1=1,\bar{r}_2=1) \sim VM(\kappa_1 = mS_1/\sigma^2)\cdot VM(\kappa_2 = mS_2/\sigma^2)$$

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} \sim N\left(\begin{pmatrix} \rho_1\cos\theta_1+\rho_2\cos\theta_2 \\ \rho_1\sin\theta_1+\rho_2\sin\theta_2 \end{pmatrix}, \sigma^2 I_2\right)$$

$$\hat{\rho}_1 = \frac{-\sin\bar{\phi}_2}{\sin(\bar{\phi}_1-\bar{\phi}_2)}y_R + \frac{\cos\bar{\phi}_2}{\sin(\bar{\phi}_1-\bar{\phi}_2)}y_I\ ,\ \ \hat{\rho}_2 = \frac{\sin\bar{\phi}_1}{\sin(\bar{\phi}_1-\bar{\phi}_2)}y_R - \frac{\cos\bar{\phi}_1}{\sin(\bar{\phi}_1-\bar{\phi}_2)}y_I$$

$$E(XY) = E(X)E(Y) + \cancelto{0}{\mathrm{cov}(XY)}$$

$$E\left(\hat{\rho}_1\right) = \rho_1 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\theta_1-\bar{\phi}_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right) + \rho_2 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\theta_2-\bar{\phi}_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)$$

$$E\left(\hat{\rho}_2\right) = \rho_2 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\bar{\phi}_1-\theta_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right) + \rho_1 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\bar{\phi}_1-\theta_1)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)$$

4. Statistical Properties of the Approaches

Jesmanowicz Magnitude-Only

$$E\left(\hat{\rho}_1\right)=\rho_1 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\theta_1-\bar{\phi}_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)+\rho_2 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\theta_2-\bar{\phi}_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)$$

$$E\left(\hat{\rho}_2\right)=\rho_2 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\bar{\phi}_1-\theta_2)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)+\rho_1 E_{\bar{\Phi}_1,\bar{\Phi}_1}\left(\frac{\sin(\bar{\phi}_1-\theta_1)}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\right)$$

working on integrals

$$E\left(\hat{\rho}_1\right)\approx\rho_1\frac{\sin(\theta_1-\phi_2)}{\sin(\phi_1-\phi_2)}+\rho_2\frac{\sin(\theta_2-\phi_2)}{\sin(\phi_1-\phi_2)}$$

$$E\left(\hat{\rho}_2\right)\approx\rho_2\frac{\sin(\phi_1-\theta_2)}{\sin(\phi_1-\phi_2)}+\rho_1\frac{\sin(\phi_1-\theta_1)}{\sin(\phi_1-\phi_2)}$$

If $\phi_1\approx\theta_1$ and $\phi_2\approx\theta_2$, then $E\left(\hat{\rho}_1\right)\approx\rho_1$ and $E\left(\hat{\rho}_2\right)\approx\rho_2$.

4. Statistical Properties of the Approaches

Rowe Complex-Valued

$$\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix}$$

$$\nu_R = \bar{y}_{R1} - \bar{y}_{R2}$$
$$\nu_I = \bar{y}_{I1} - \bar{y}_{I2}$$

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} \sim N\left(\begin{pmatrix} \rho_1\cos\theta_1 + \rho_2\cos\theta_2 \\ \rho_1\sin\theta_1 + \rho_2\sin\theta_2 \end{pmatrix}, \sigma^2 I_2\right)$$

$$\begin{pmatrix} \bar{y}_{R1} \\ \bar{y}_{I1} \\ \bar{y}_{R2} \\ \bar{y}_{I2} \end{pmatrix} \sim N\left(\begin{pmatrix} S_1\cos\phi_1 \\ S_1\sin\phi_1 \\ S_2\cos\phi_2 \\ S_2\sin\phi_2 \end{pmatrix}, \frac{\sigma^2}{m} I_4\right)$$

$$\begin{pmatrix} \nu_R \\ \nu_I \end{pmatrix} \sim N\left(\begin{pmatrix} S_1\cos\phi_1 - S_2\cos\phi_2 \\ S_1\sin\phi_1 - S_2\sin\phi_2 \end{pmatrix}, \frac{2\sigma^2}{m} I_2\right)$$

4. Statistical Properties of the Approaches

Rowe Complex-Valued

$$E\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} E\begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix}$$

$$E\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \begin{bmatrix} \frac{1}{2}(\rho_1 \cos\theta_1 + S_1 \cos\phi_1) + \frac{1}{2}(\rho_2 \cos\theta_2 - S_2 \cos\phi_2) \\ \frac{1}{2}(\rho_1 \sin\theta_1 + S_1 \sin\phi_1) + \frac{1}{2}(\rho_2 \sin\theta_2 - S_2 \sin\phi_2) \\ \frac{1}{2}(\rho_2 \cos\theta_2 + S_2 \cos\phi_2) + \frac{1}{2}(\rho_1 \cos\theta_1 - S_1 \cos\phi_1) \\ \frac{1}{2}(\rho_2 \sin\theta_2 + S_2 \sin\phi_2) + \frac{1}{2}(\rho_1 \sin\theta_1 - S_1 \sin\phi_1) \end{bmatrix}$$

4. Statistical Properties of the Approaches

Rowe Complex-Valued

$$\operatorname{cov}\begin{pmatrix}\hat\rho_1\cos\hat\theta_1\\ \hat\rho_1\sin\hat\theta_1\\ \hat\rho_2\cos\hat\theta_2\\ \hat\rho_2\sin\hat\theta_2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{pmatrix}\operatorname{cov}\begin{pmatrix}y_R\\y_I\\v_R\\v_I\end{pmatrix}\frac{1}{2}\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&-1&0\\0&1&0&-1\end{pmatrix}$$

$$\operatorname{cov}\begin{pmatrix}\hat\rho_1\cos\hat\theta_1\\ \hat\rho_1\sin\hat\theta_1\\ \hat\rho_2\cos\hat\theta_2\\ \hat\rho_2\sin\hat\theta_2\end{pmatrix}=\frac{\sigma^2}{4}\begin{pmatrix}1+2/m&0&1-2/m&0\\0&1+2/m&0&1-2/m\\1-2/m&0&1+2/m&0\\0&1-2/m&0&1+2/m\end{pmatrix}$$

$\longleftarrow$ use m=2

5. Results from the Approaches

Data 1: Sim Constant Magnitude and Constant Phase

$\rho_1=S_1=1$, $\rho_2=S_2=1.5$, $\theta_1=\phi_1=\pi/2-\pi/6$, $\theta_2=\phi_2=-\pi/6$, $m=2$, $n=720$, $\sigma=.01$

Data 2: Sim Average Experimental Images (not shown)

Acquired 720 full volumes and averaged slices for $\rho_1=S_1$, $\rho_2=S_2$, $\theta_1=\phi_1$, $\theta_2=\phi_2$, $m=2$, $n=720$, $\sigma=.01$

Data 2.5: Did intermediate added slices but simulated phase.

Data 3: Sim/Exp Add Experimental Images

Acquired 720 full volumes and added slices to simulate aliasing, used first $m=2$ for reference images.

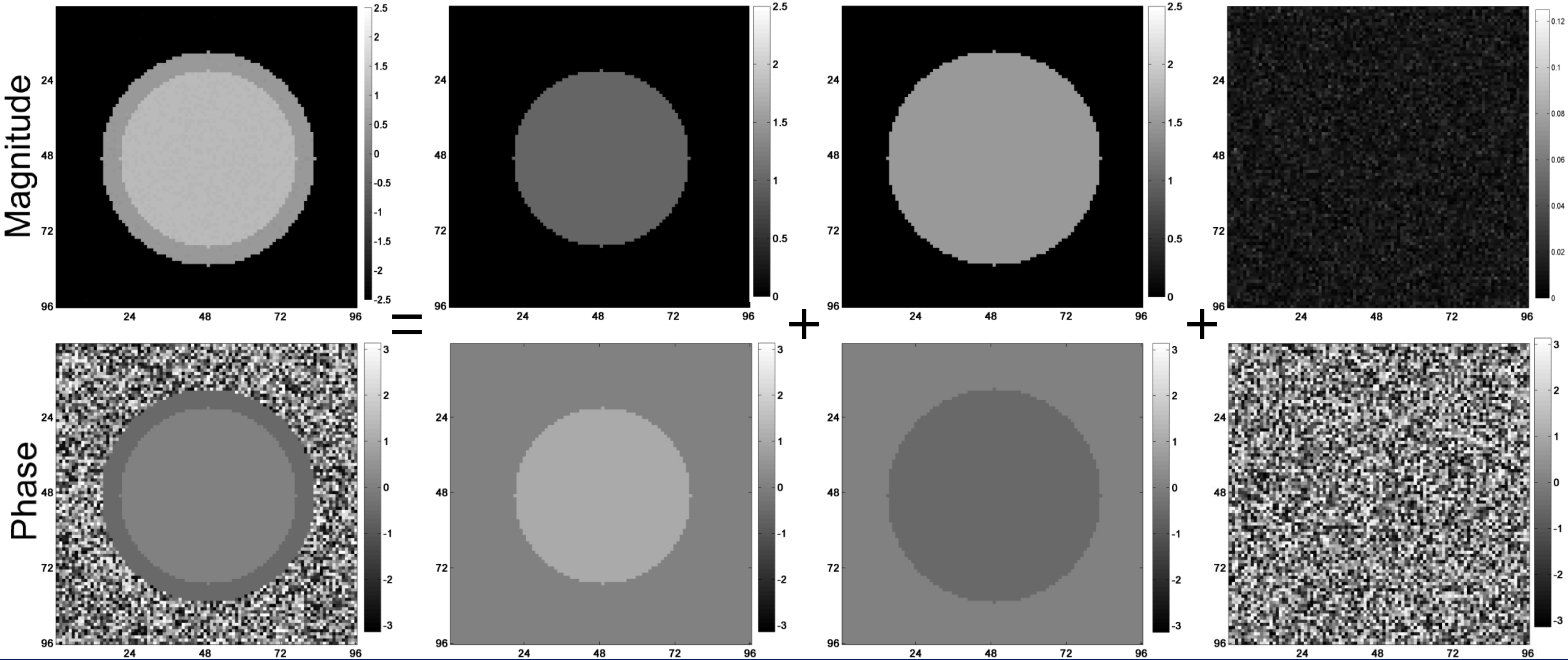
Data 4: Exp Data Magnitude and Phase

Acquired 20 aliased full images used $m=2$ for reference images then acquired $n=720$ aliased images.

5. Results from the Approaches

Data 1: Sim Constant Magnitude and Constant Phase

$\rho_1 = S_1 = 1$, $\rho_2 = S_2 = 1.5$, $\theta_1 = \phi_1 = \pi/2 - \pi/6$, $\theta_2 = \phi_2 = -\pi/6$, $m=2$, $n=720$, $\sigma = .01$

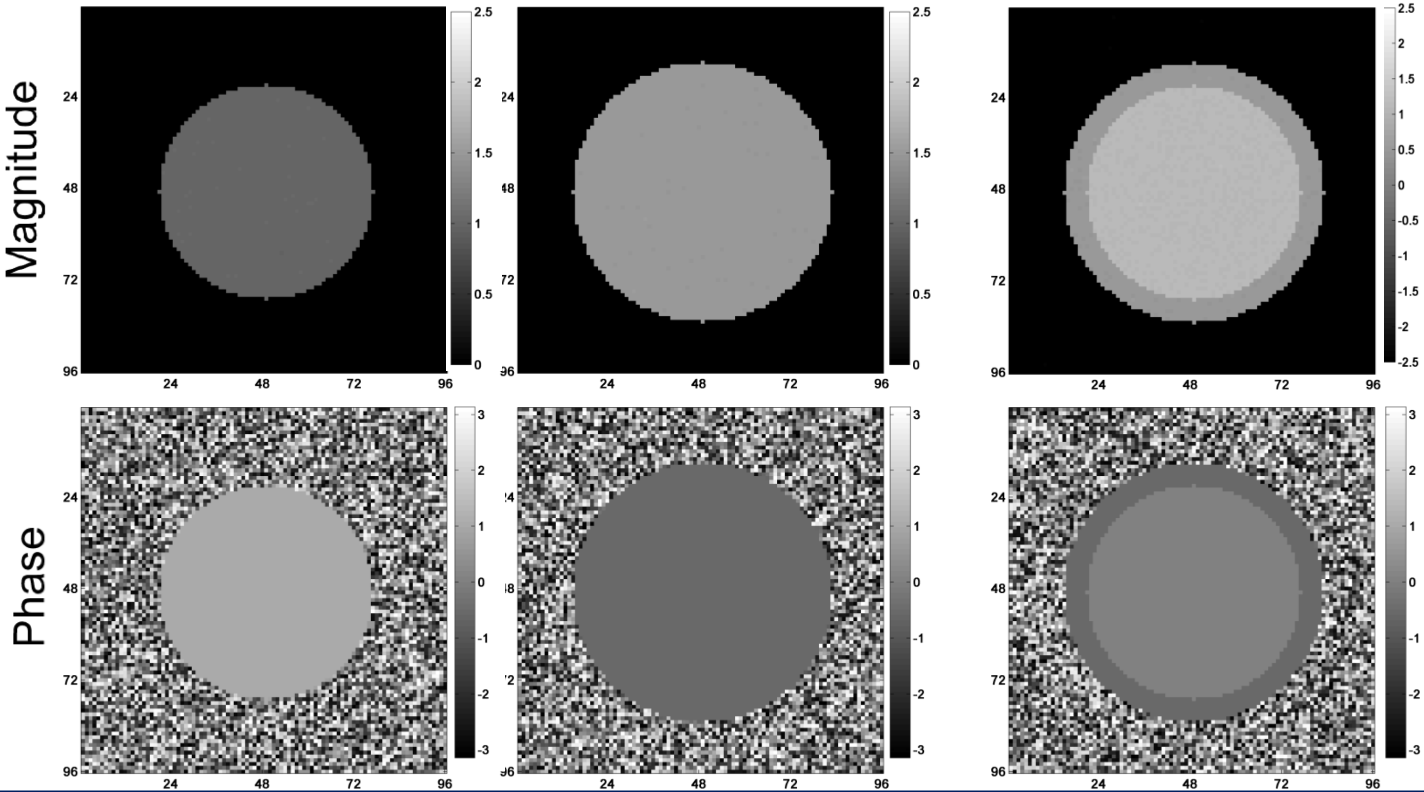


$$re^{i\varphi} = \rho_1 e^{i\theta_1} + \rho_2 e^{i\theta_2} + \varepsilon_M e^{i\varepsilon_P}$$

5. Results from the Approaches

Data 1: Sim Constant Magnitude and Constant Phase

Reference Images | Aliased Image



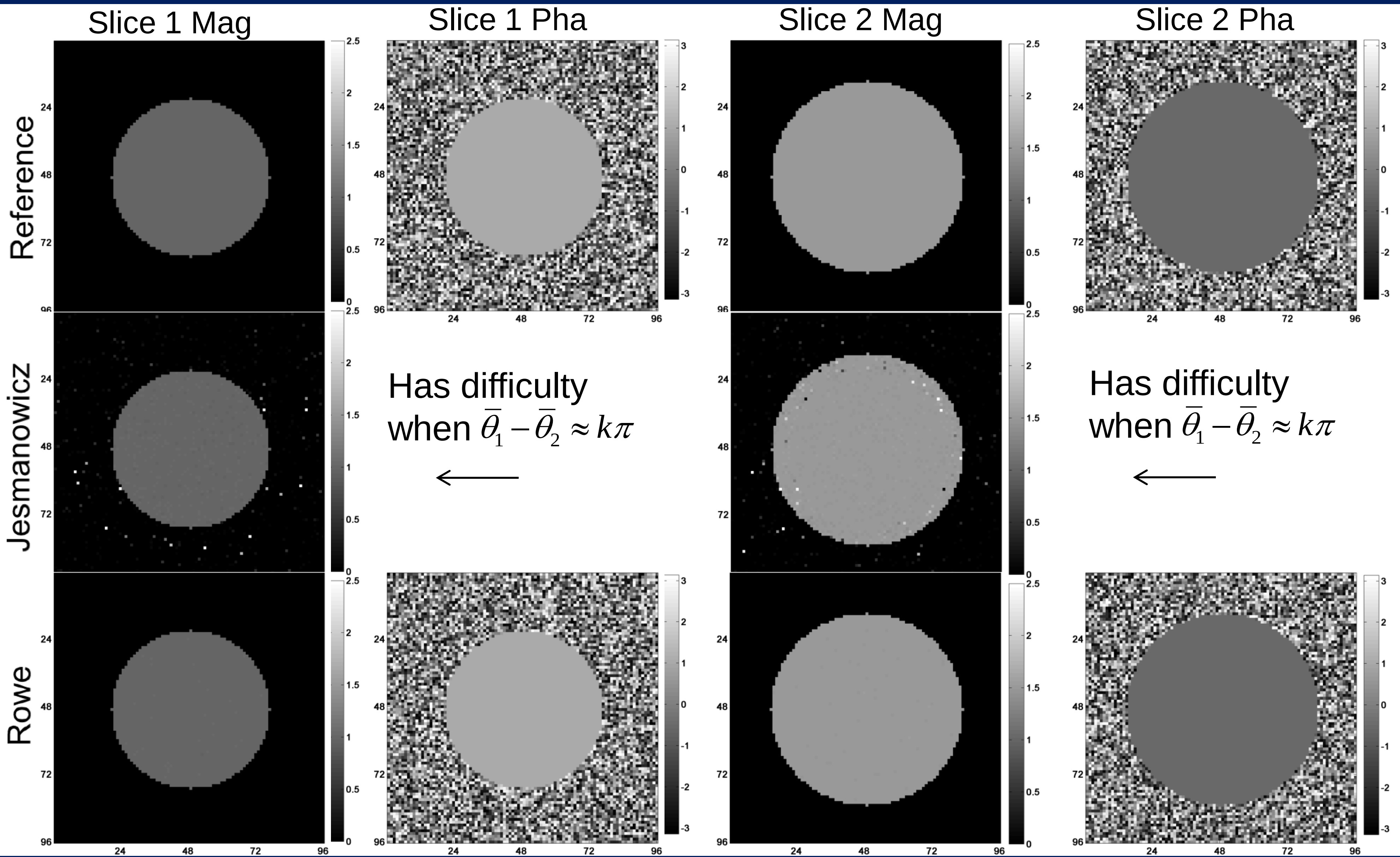
Jesmanowicz M-O

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = X_J^{-1} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

Rowe C-V

$$\begin{pmatrix} \hat{\rho}_1 \cos \hat{\theta}_1 \\ \hat{\rho}_1 \sin \hat{\theta}_1 \\ \hat{\rho}_2 \cos \hat{\theta}_2 \\ \hat{\rho}_2 \sin \hat{\theta}_2 \end{pmatrix} = X_R^{-1} \begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix}$$

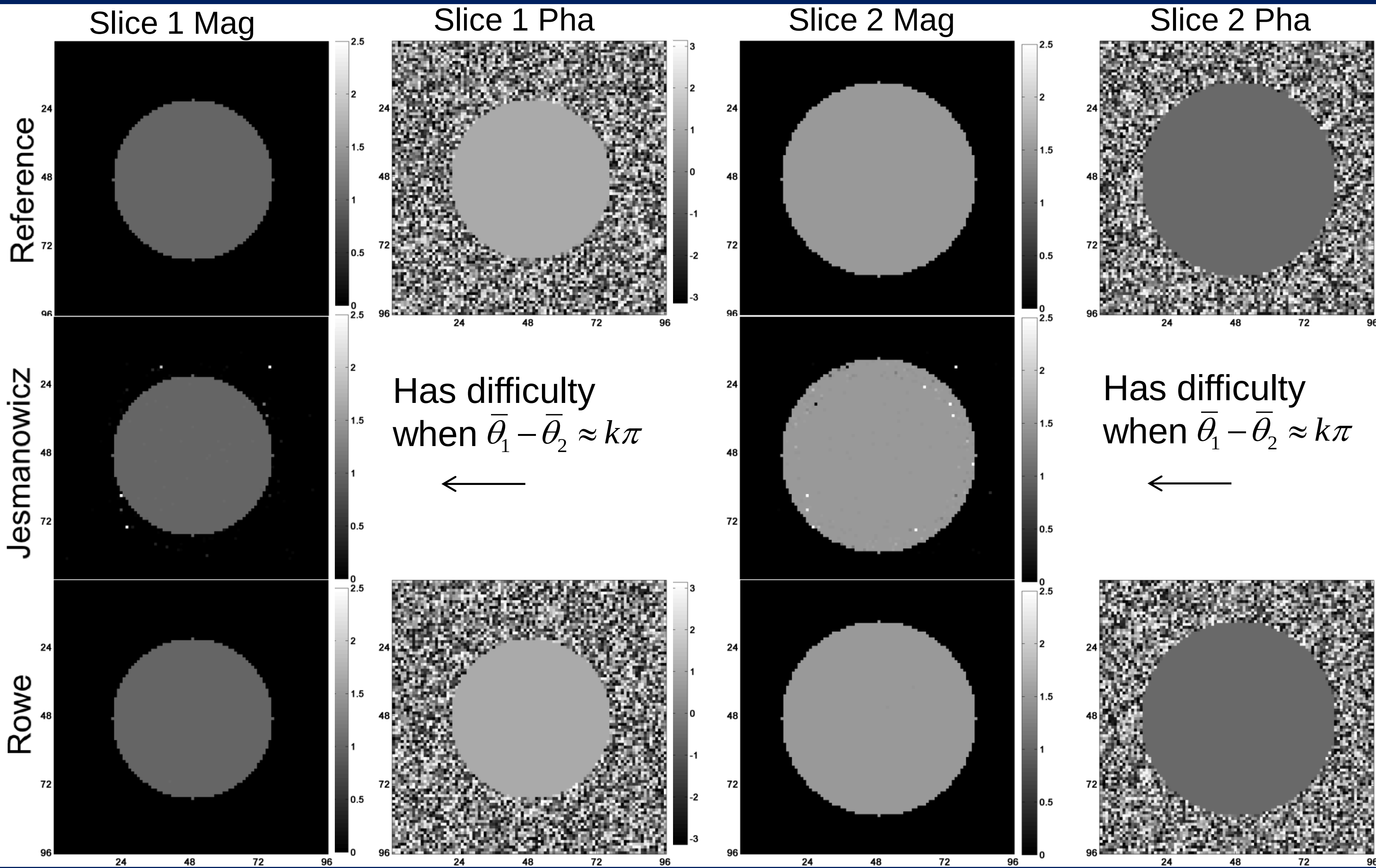
Data 1: Reconstructed Image



Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

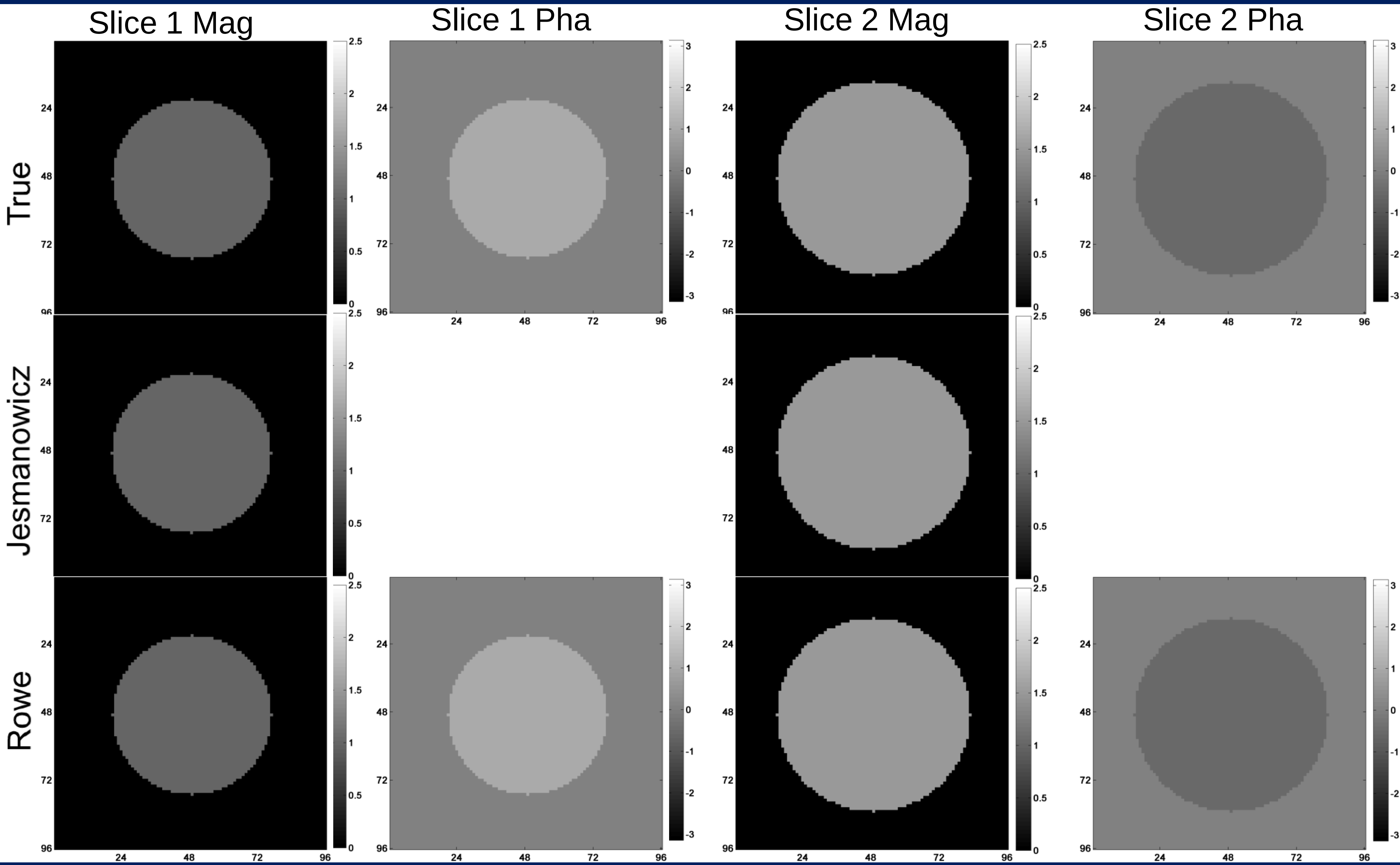
Data 1: Reconstructed Means



Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Data 1: Reconstructed Expectations

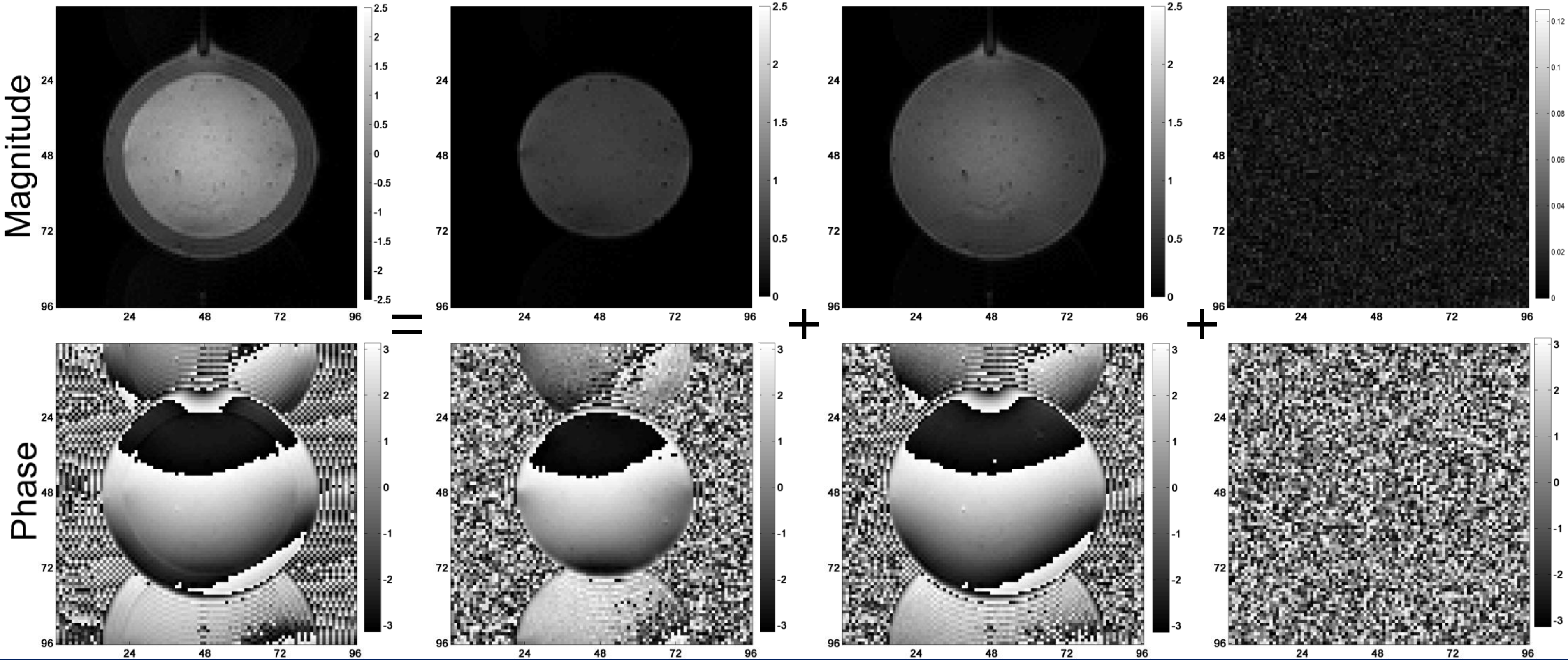


Acquired full volumes, averaged, then added slices with noise.

5. Results from the Approaches

Data 2: Sim from Exp Magnitude and Phase Avg

$\rho_1 = S_1$, $\rho_2 = S_2$, $\theta_1 = \phi_1$, $\theta_2 = \phi_2$, $m=2$, $n=720$, $\sigma=.01$

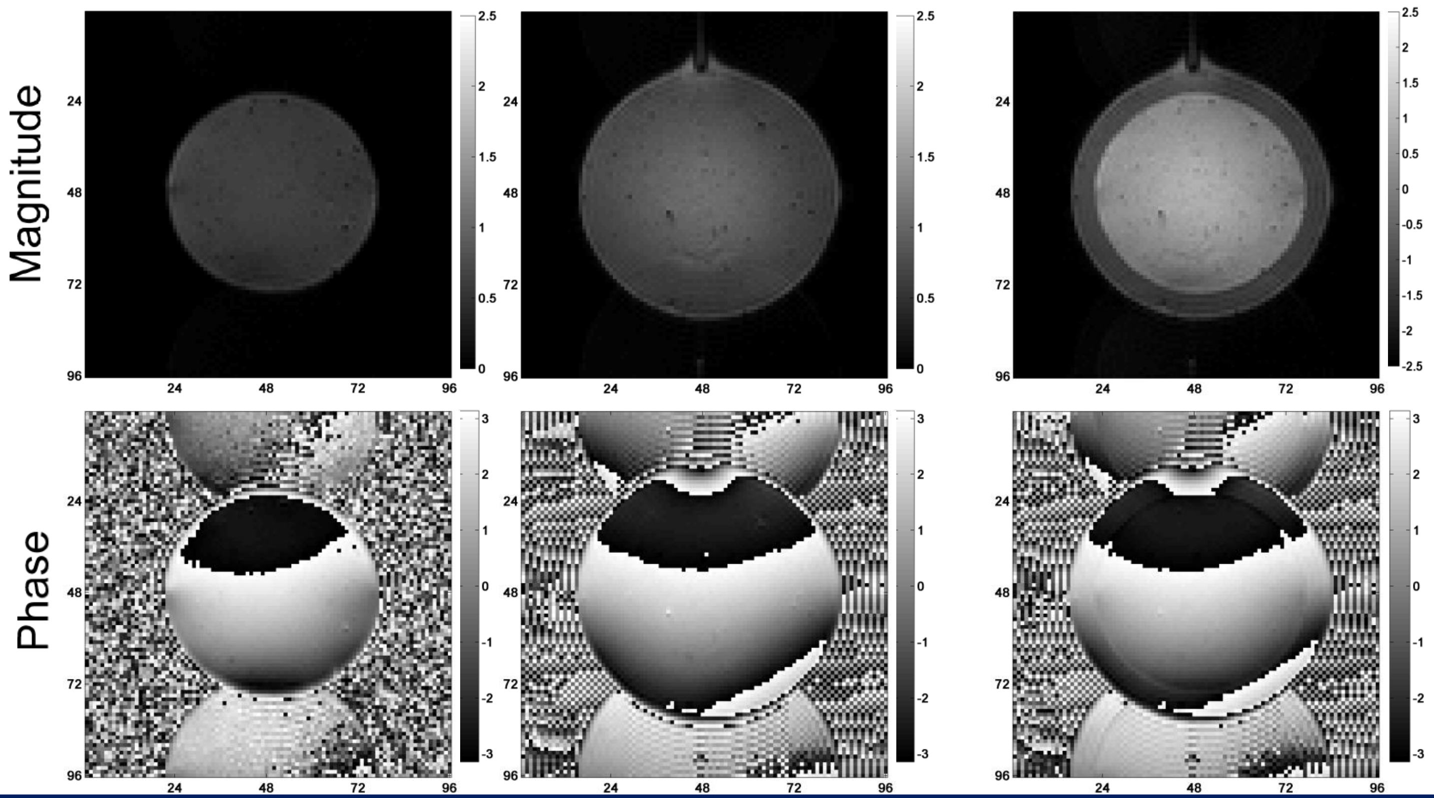


$$re^{i\varphi} = \rho_1 e^{i\theta_1} + \rho_2 e^{i\theta_2} + \varepsilon_M e^{i\varepsilon_P}$$

5. Results from the Approaches

Data 2: Sim Constant Magnitude and Constant Phase

Reference Images | Aliased Image



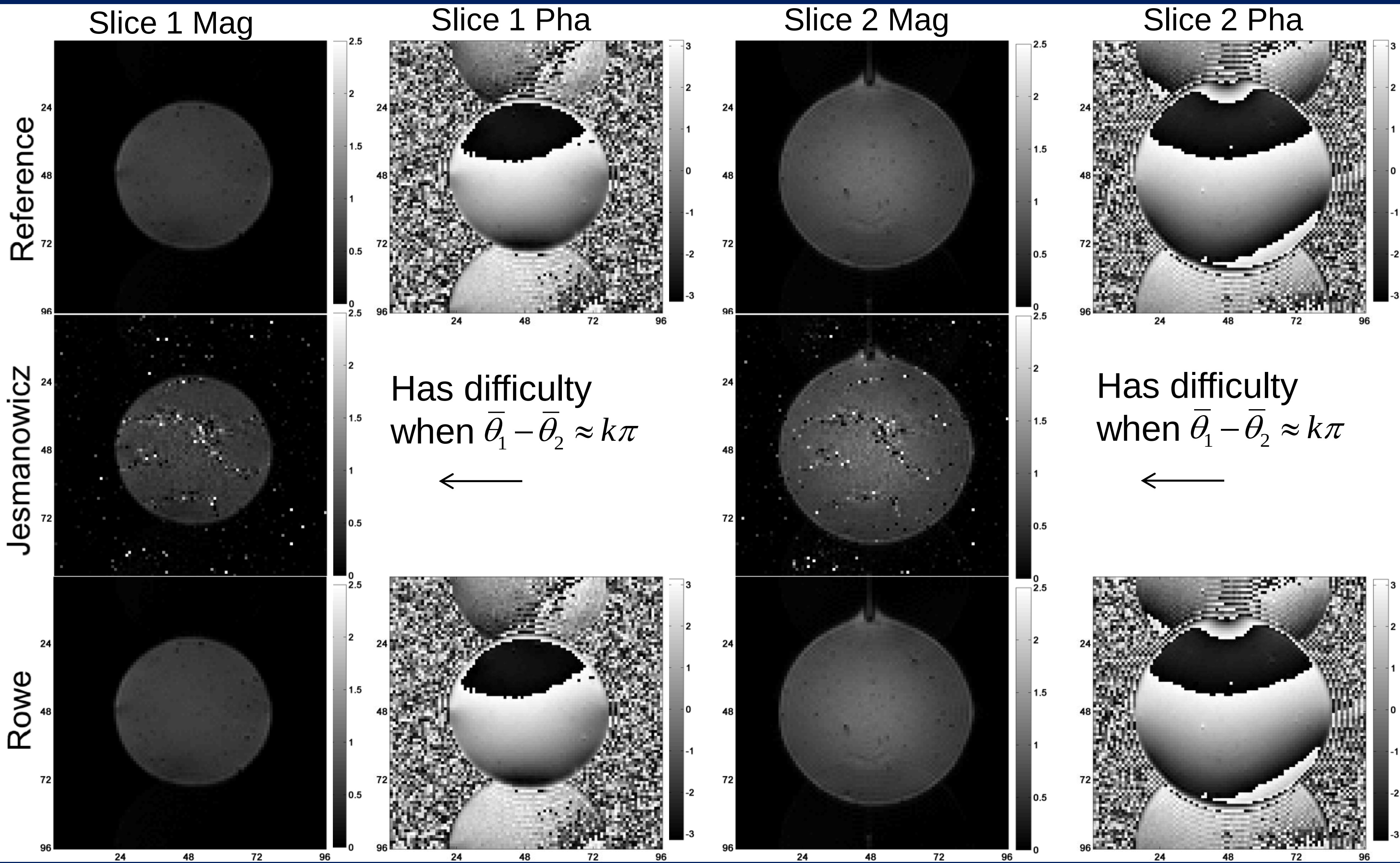
Jesmanowicz M-O

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = X_J^{-1} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

Rowe C-V

$$\begin{pmatrix} \hat{\rho}_1 \cos \hat{\theta}_1 \\ \hat{\rho}_1 \sin \hat{\theta}_1 \\ \hat{\rho}_2 \cos \hat{\theta}_2 \\ \hat{\rho}_2 \sin \hat{\theta}_2 \end{pmatrix} = X_R^{-1} \begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix}$$

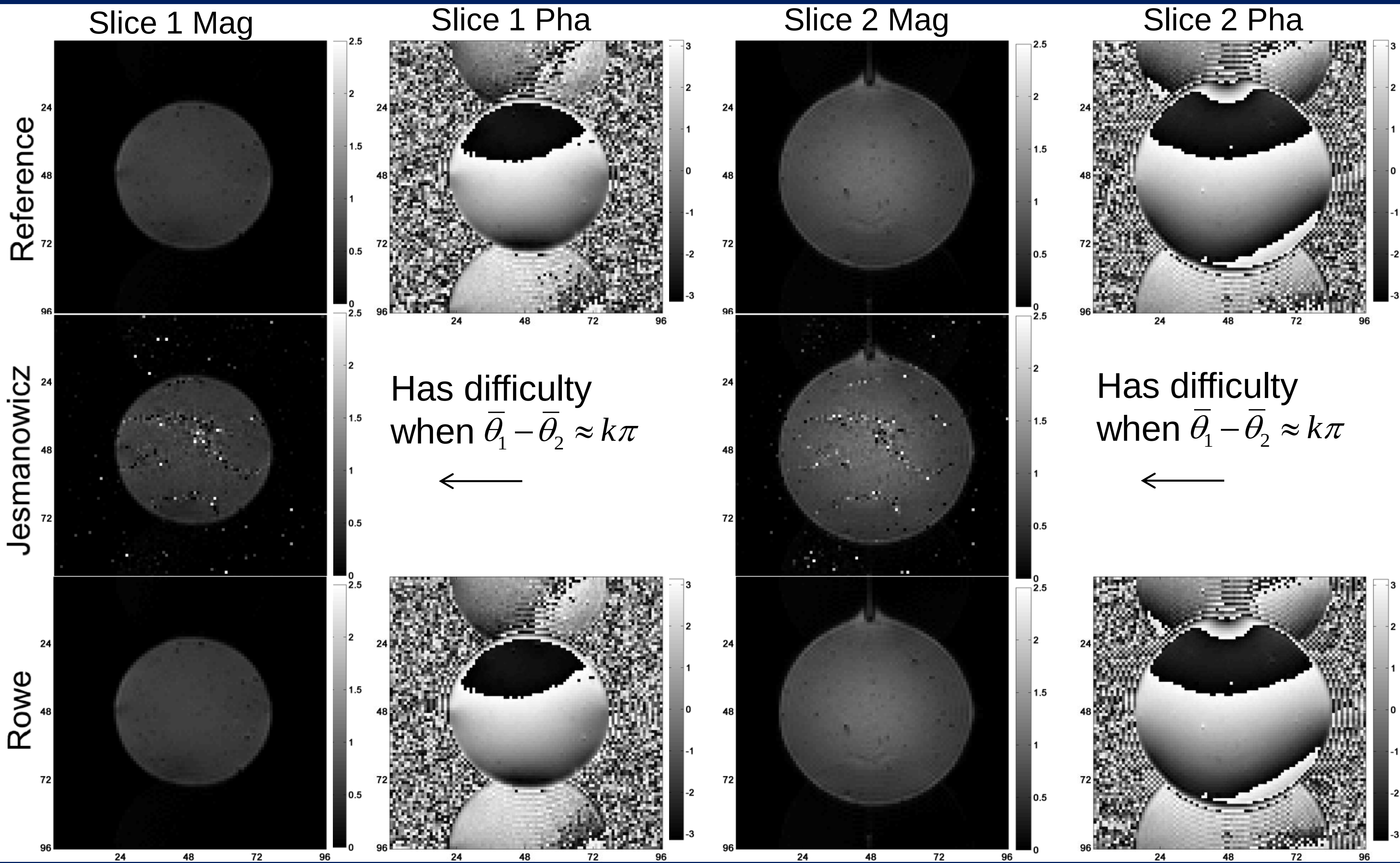
Data 2: Reconstructed Image



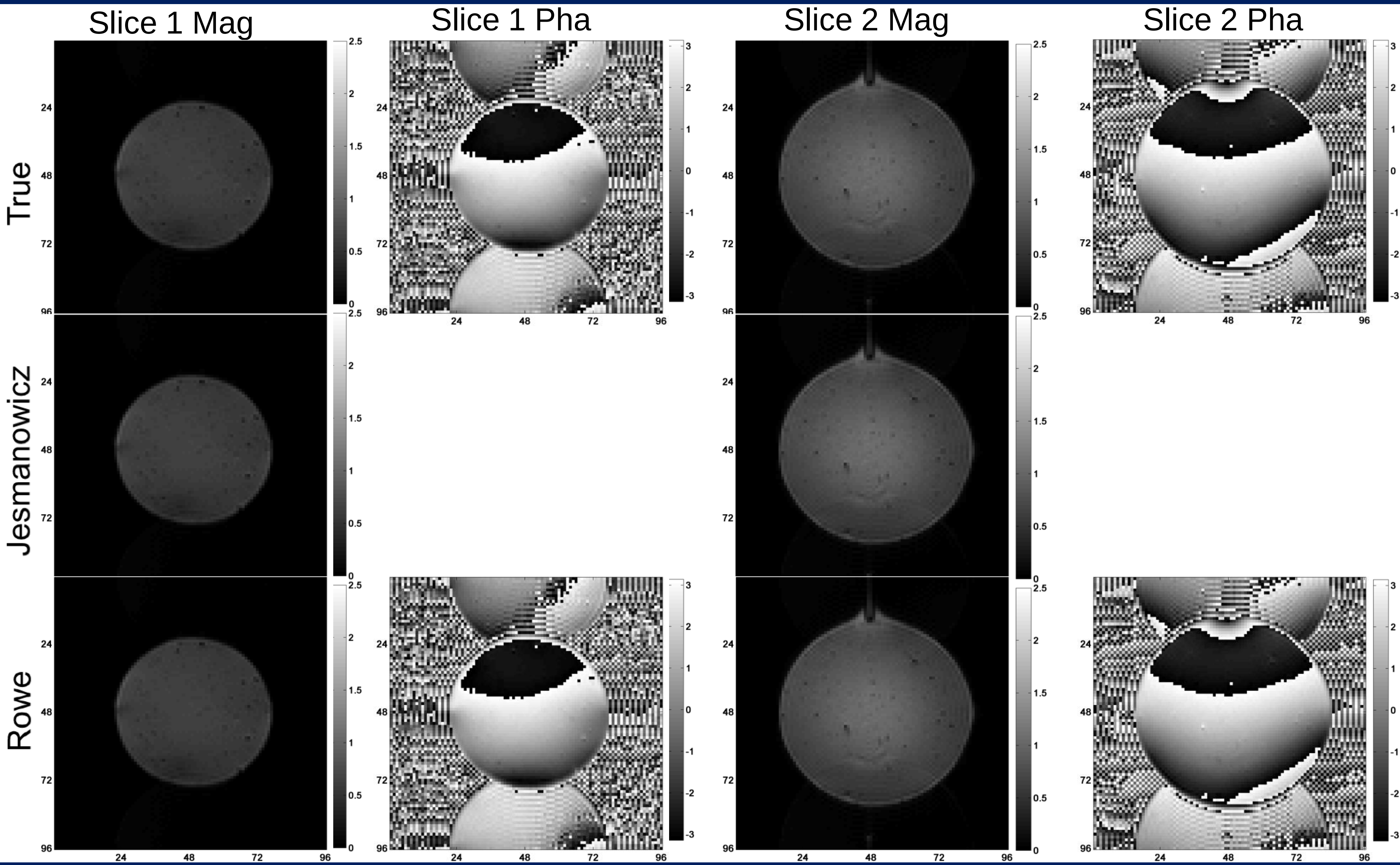
Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Data 2: Reconstructed Means



Data 2: Reconstructed Expectations

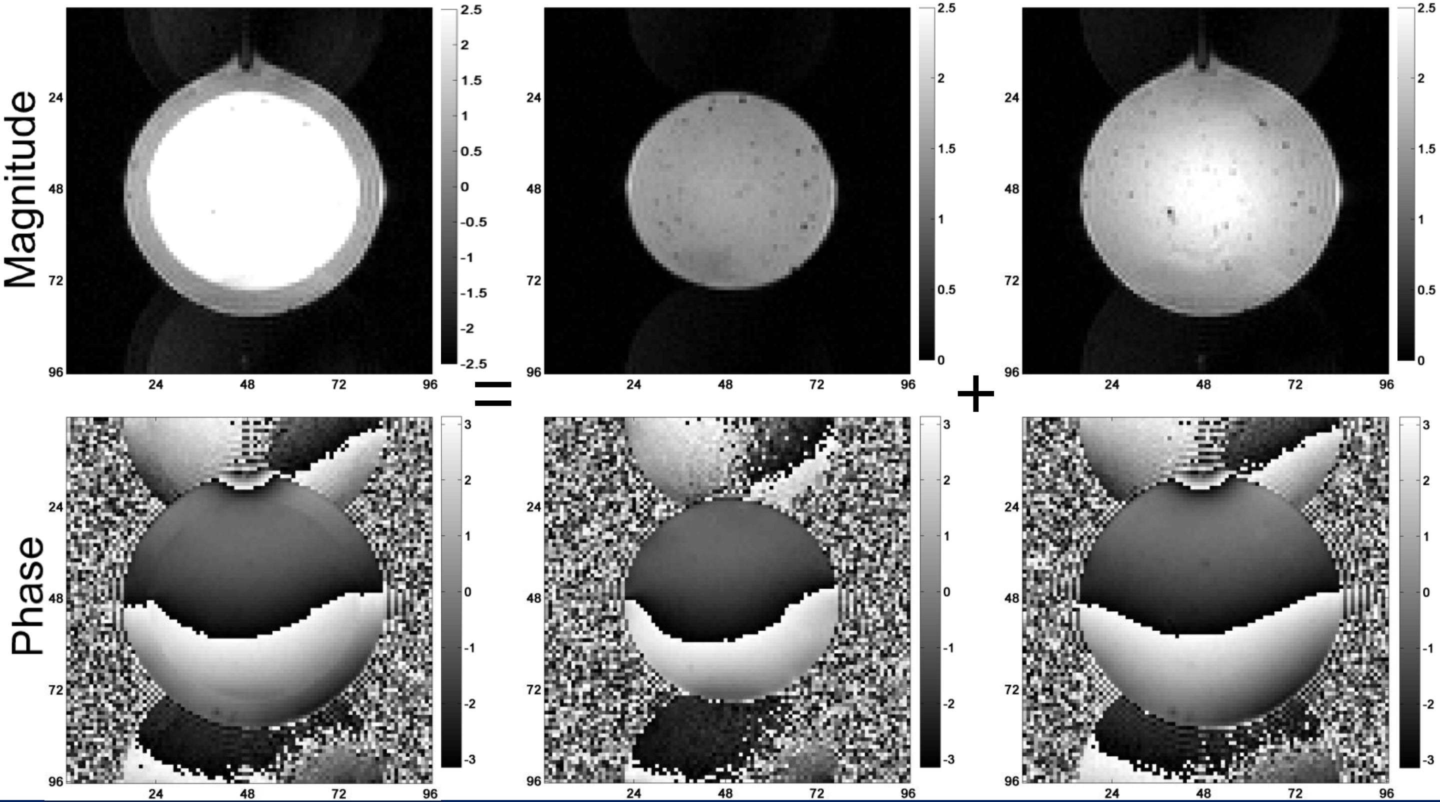


5. Results from the Approaches

Data 3: Sim/Exp Add Experimental Images

Acquired full images and added slices to simulate aliasing.

m=2, n=720



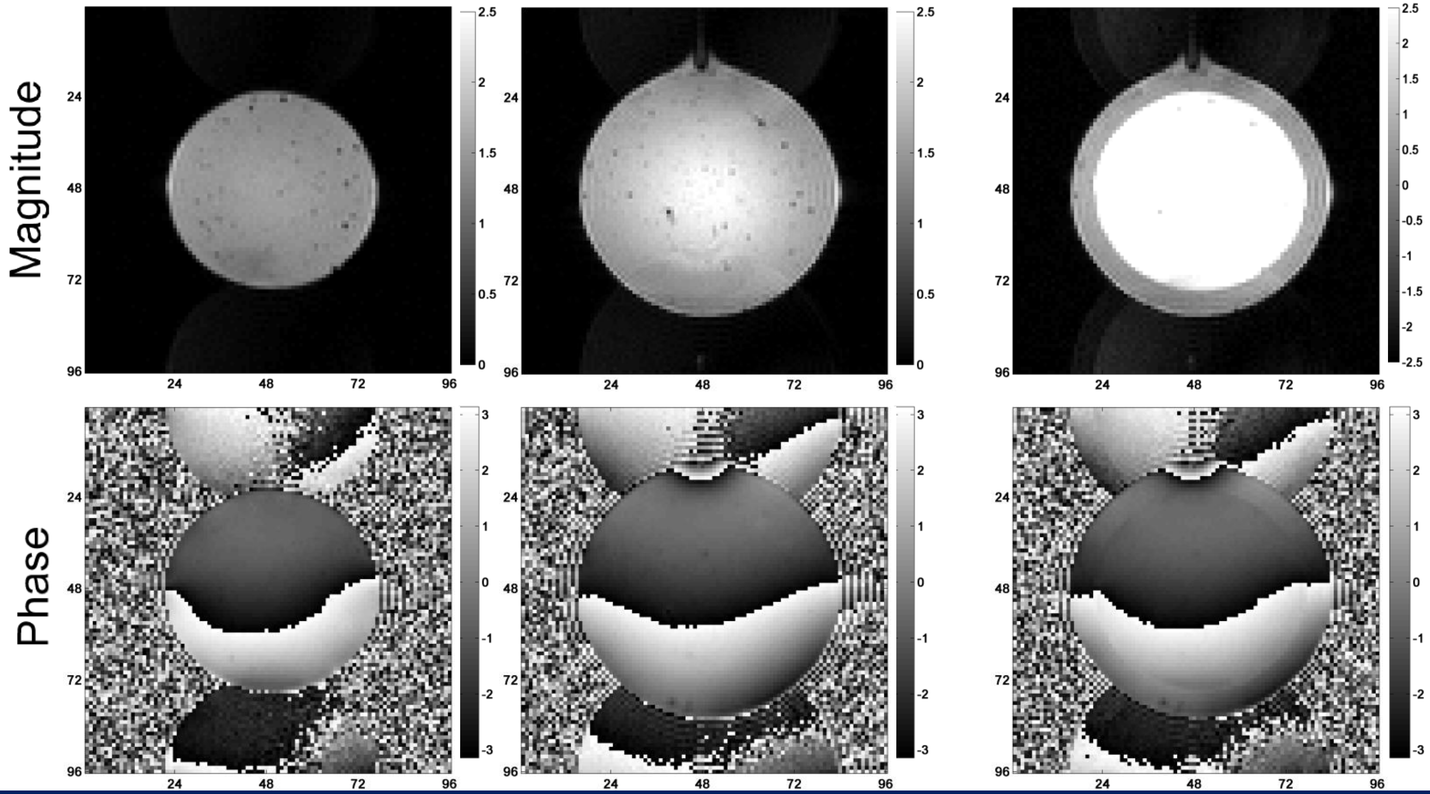
GE 3.0 T
10 slices
TRs=720
TR=1000 ms
TE=42.5 ms
BW=208.3 kHz
FOV=24 cm
SLTH=2.5 mm
FA=90 degrees
EESP=752 ms
96×96 k-space

$$re^{i\varphi} = \rho_1 e^{i\theta_1} + \rho_2 e^{i\theta_2}$$

5. Results from the Approaches

Data 3: Sim/Exp Add Experimental Image

Reference Images Aliased Image



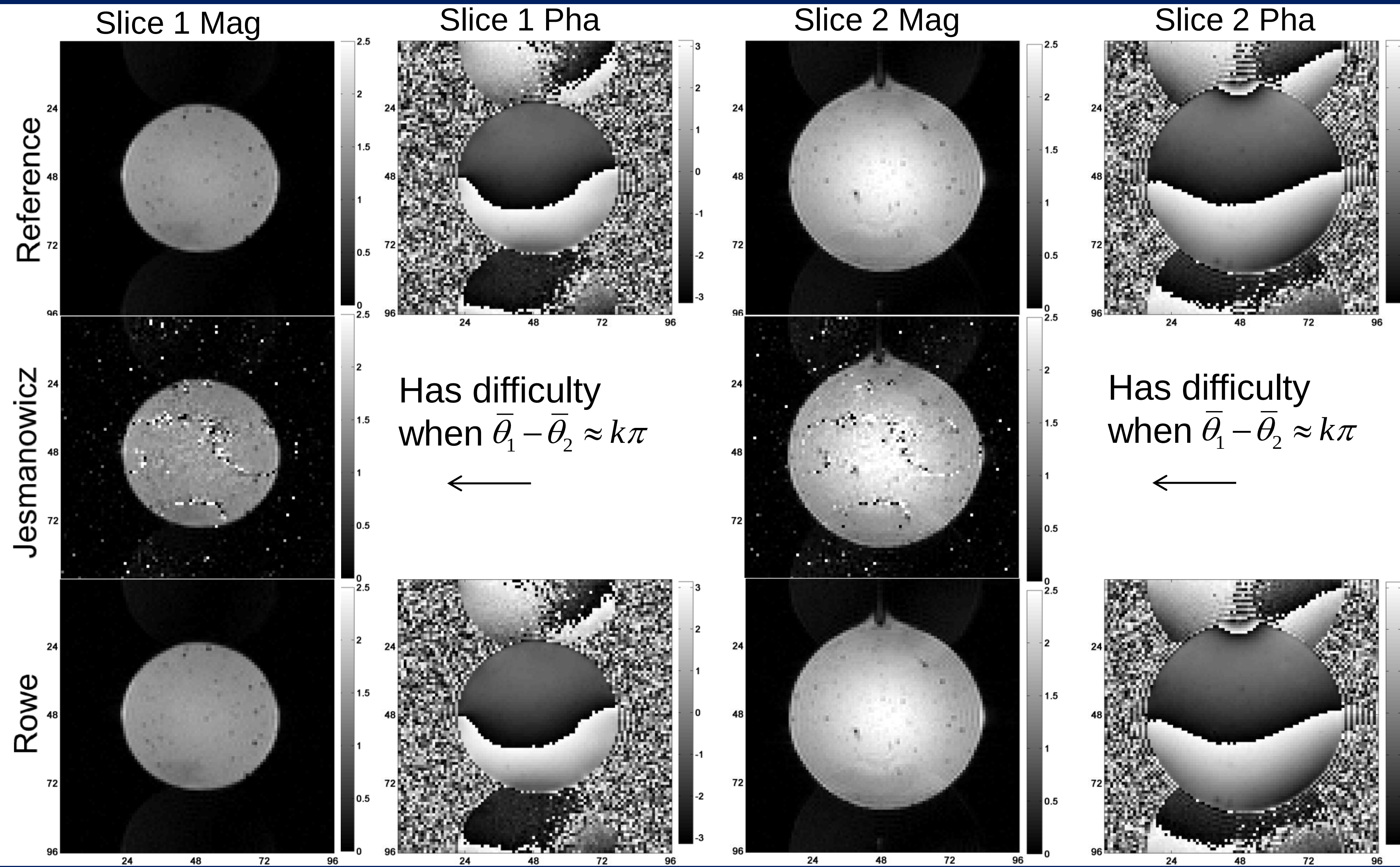
Jesmanowicz M-O

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = X_J^{-1} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

Rowe C-V

$$\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = X_R^{-1} \begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix}$$

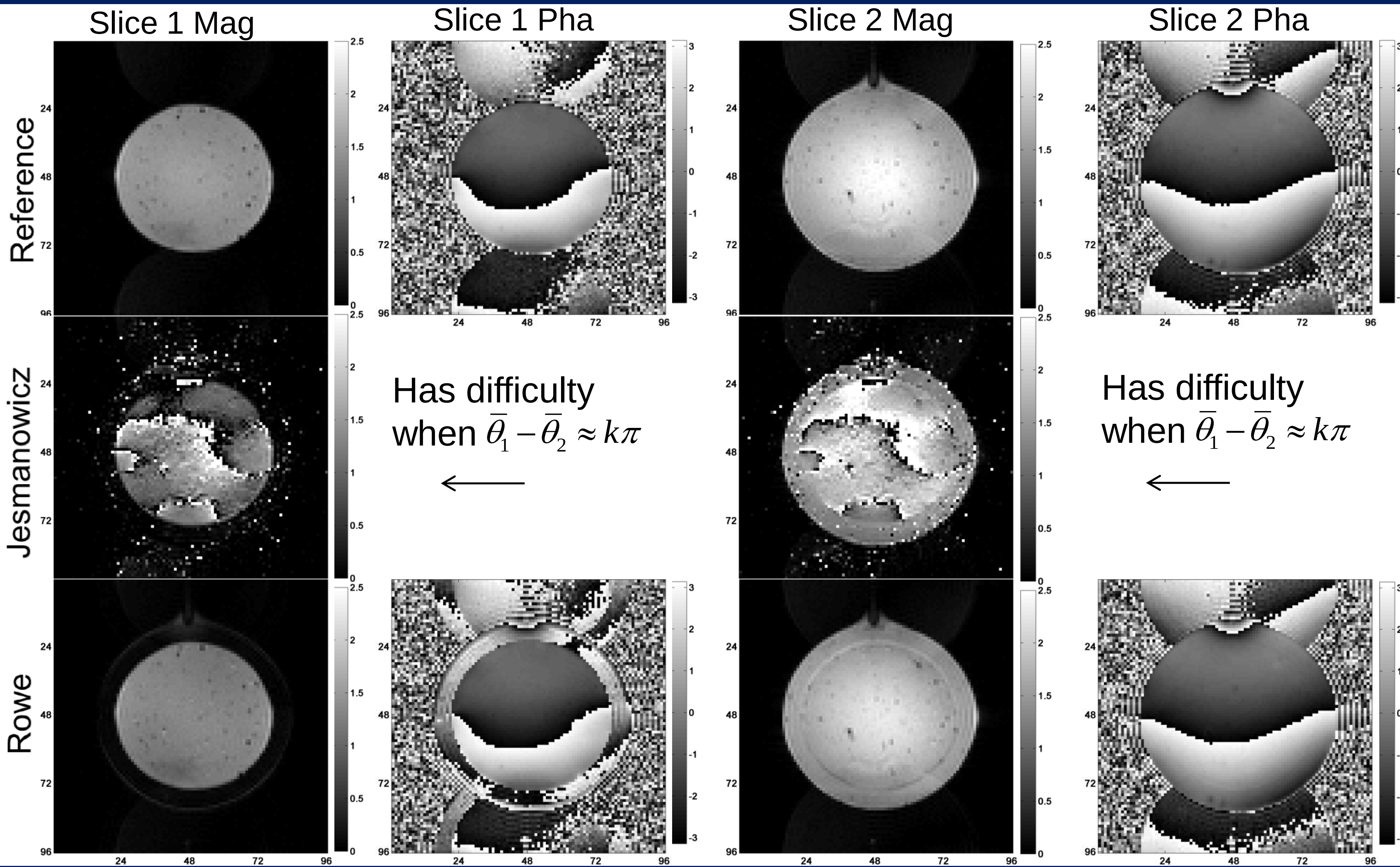
Data 3: Reconstructed Image



Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Data 3: Reconstructed Means



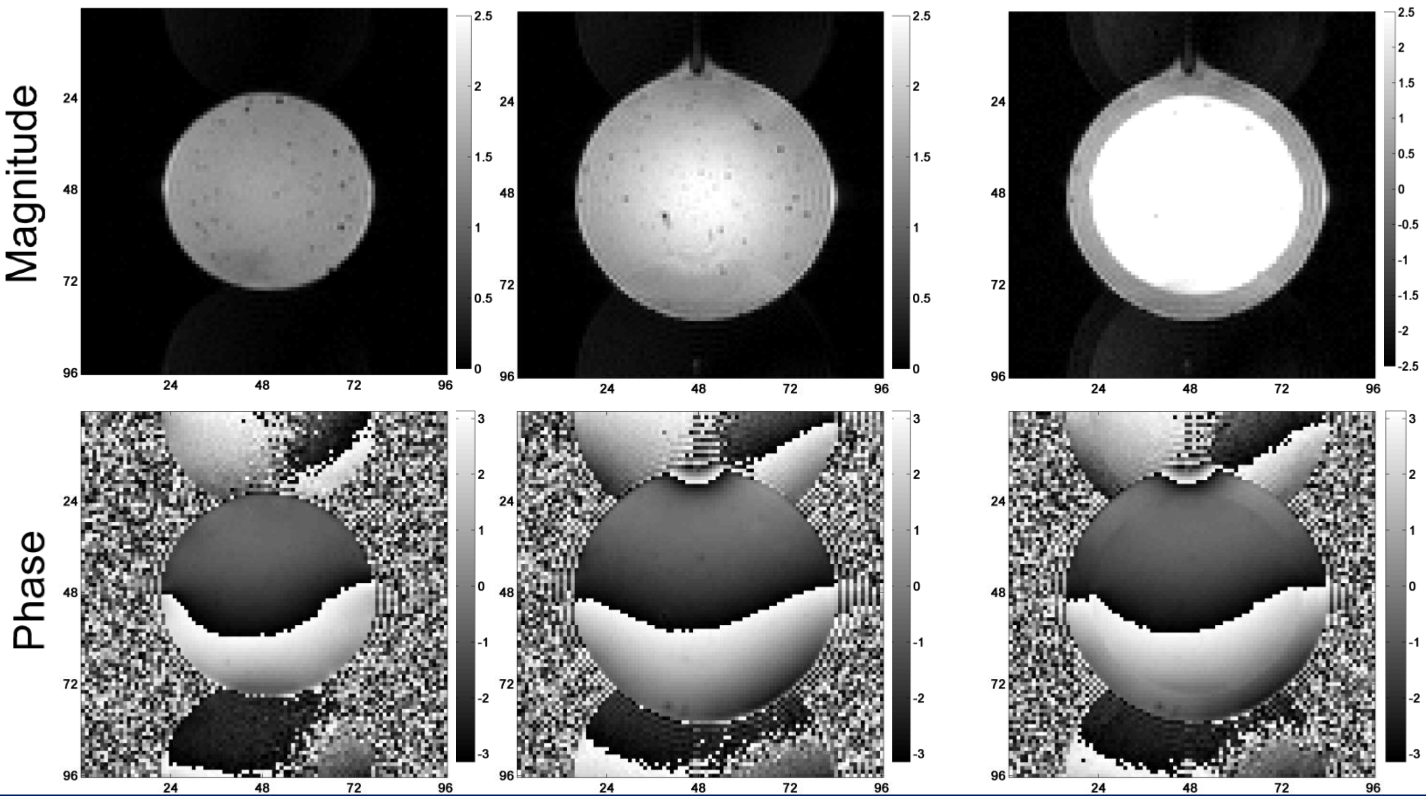
5. Results from the Approaches

Data 4: Exp Data Magnitude and Phase

Acquired aliased images.

Reference Images

Aliased Image m=2, n=720



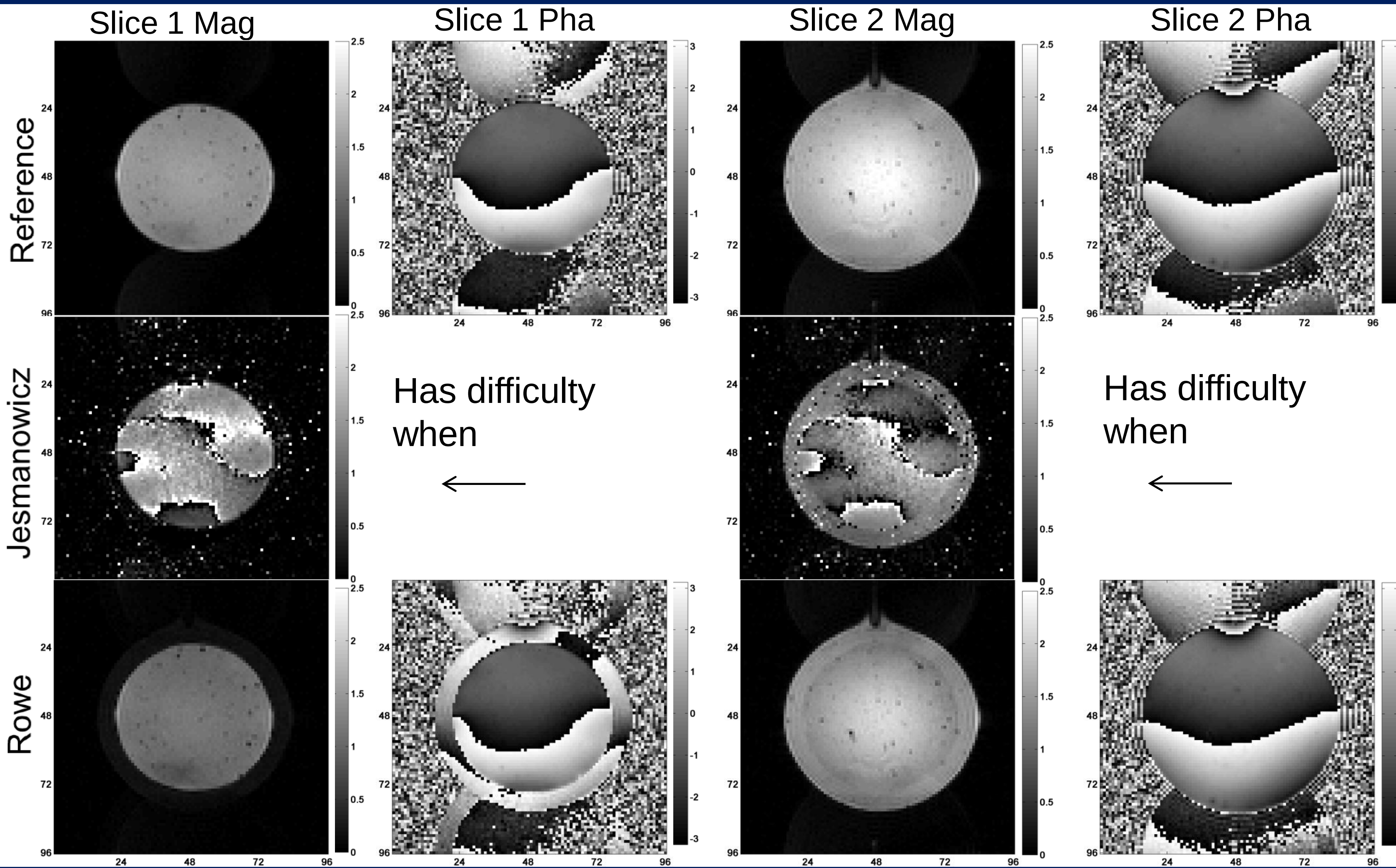
Jesmanowicz M-O

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = X_J^{-1} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

Rowe C-V

$$\begin{pmatrix} \hat{\rho}_1 \cos \hat{\theta}_1 \\ \hat{\rho}_1 \sin \hat{\theta}_1 \\ \hat{\rho}_2 \cos \hat{\theta}_2 \\ \hat{\rho}_2 \sin \hat{\theta}_2 \end{pmatrix} = X_R^{-1} \begin{pmatrix} y_R \\ y_I \\ \nu_R \\ \nu_I \end{pmatrix}$$

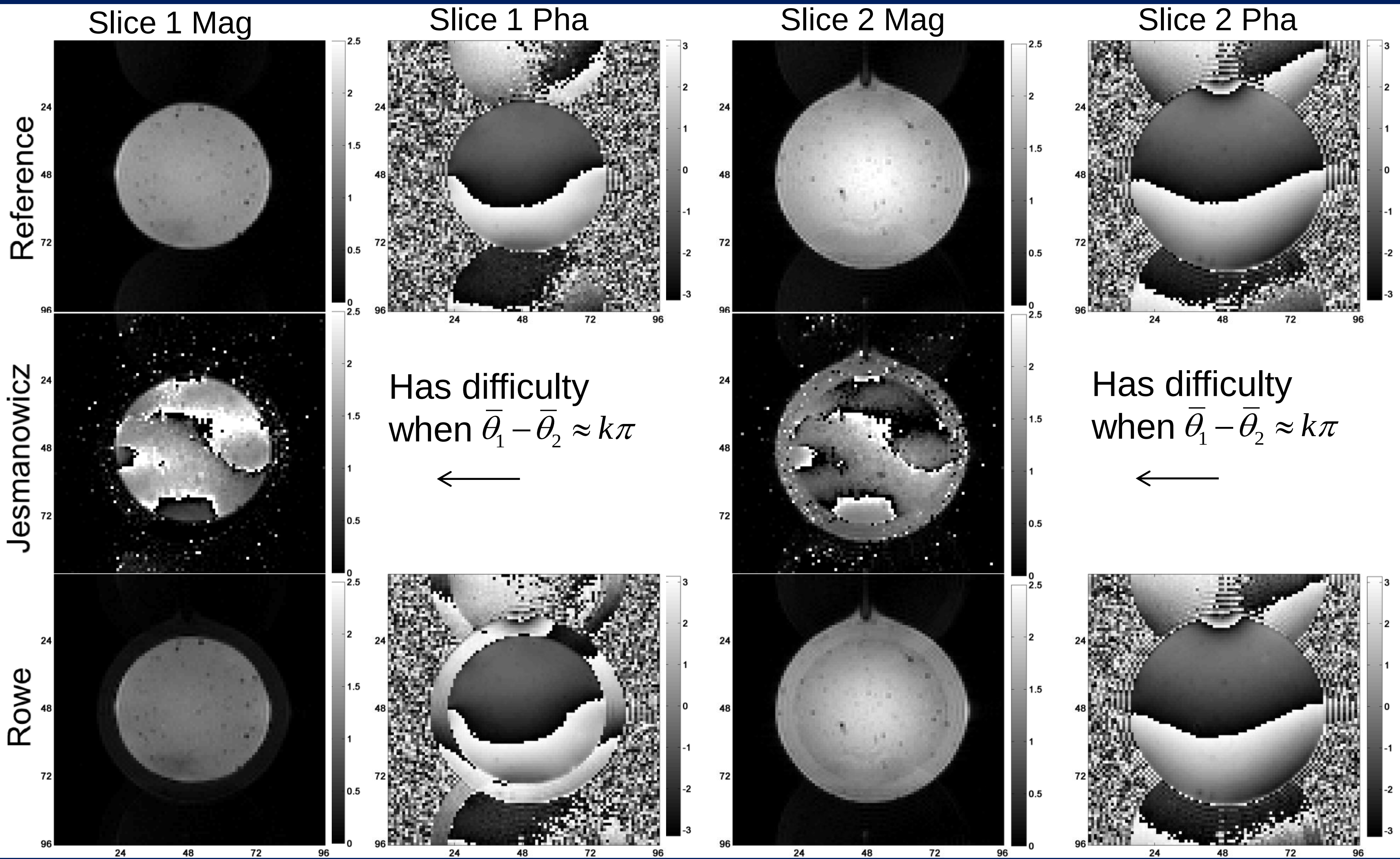
Data 4: Reconstructed Image



Has difficulty when ←

Has difficulty when ←

Data 4: Reconstructed Means



Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

Has difficulty when $\bar{\theta}_1 - \bar{\theta}_2 \approx k\pi$ ←

6. Discussion

Mathematical description of the 2 slice 1 coil aliasing process.

Mathematical description of Jesmanowicz approach.

Mathematical description of New Rowe approach.

Expectation (of two-ish) and Covariance of one of aproaches.

Results of the two different approaches.

Caution: Two reconstructions may lead to correlated voxels!

Apply to real human data.

Generalize to more than 1 coil and higher accelerations.

Thank You!