Utilizing Induced Voxel Correlation in fMRI Analysis

Daniel B. Rowe, Ph.D.
Joint with Andrew S. Nencka

Associate Professor
Computational Sciences Program
Department of Mathematics,
Statistics, and Computer Science

Adjunct Associate Professor
Department of Biophysics

August 3, 2011

OUTLINE

1. Reconstruction-Preprocessing

2. Induced Correlation

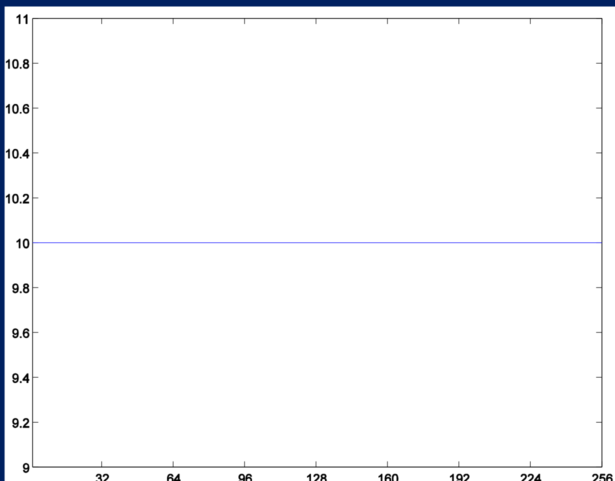
3. Utilizing Induced Correlation

4. Results

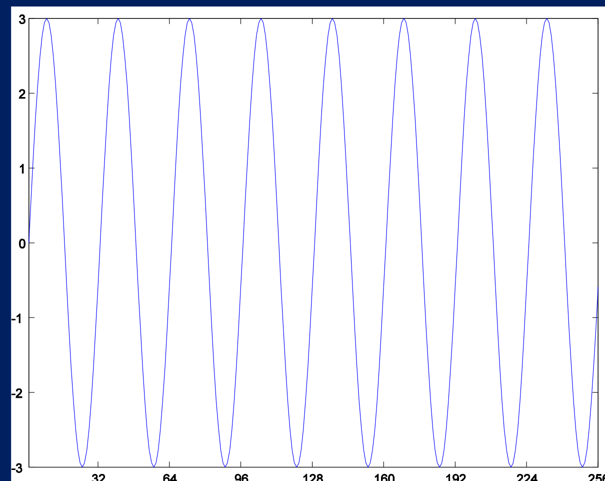
5. Discussion

Reconstruction: 1D FT

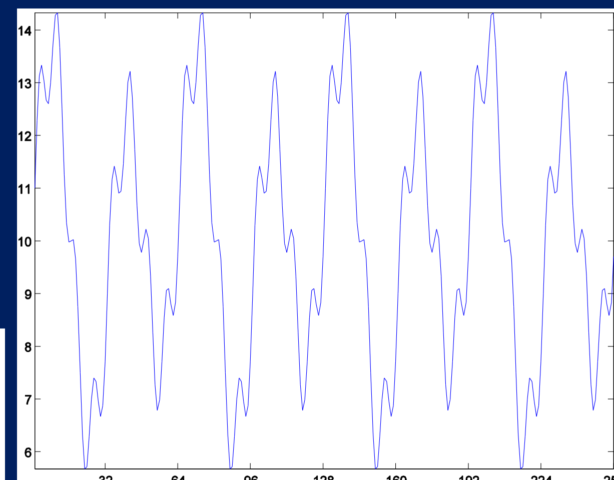
(n=256, Δt=2 s)



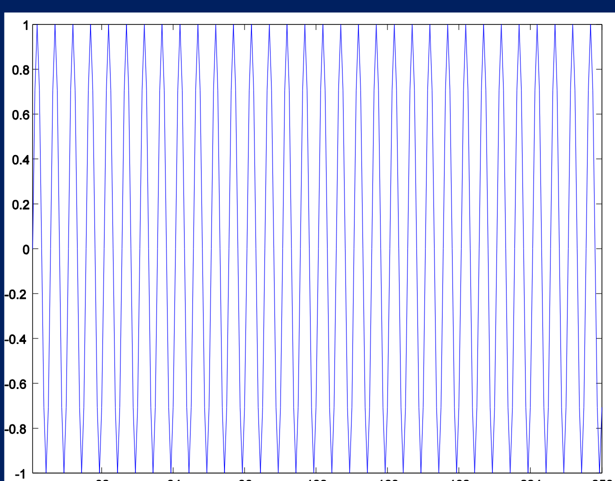
$10\cos(2\pi 0/512t)$



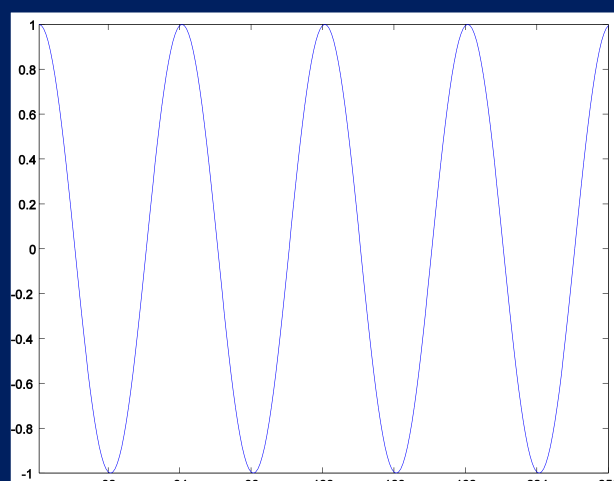
$3\sin(2\pi 8/512t)$



sum



$\cos(2\pi 32/512t)$

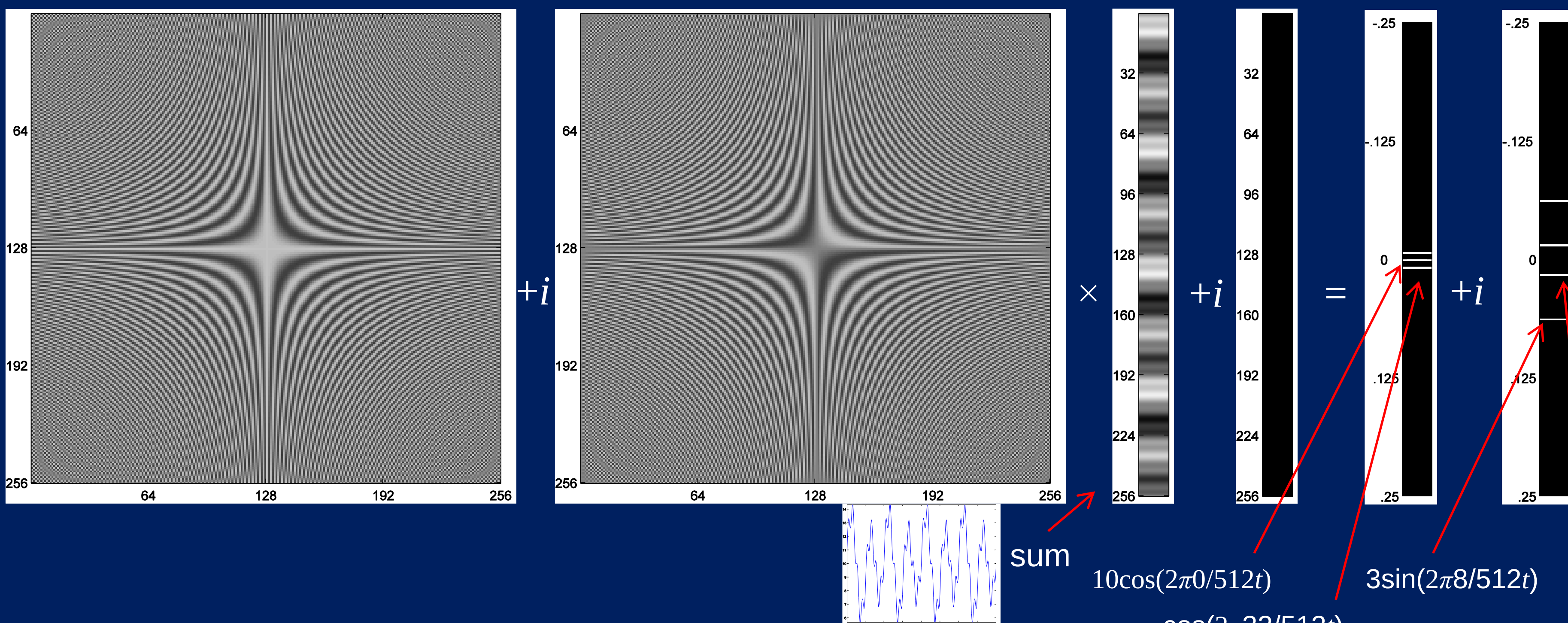


$\sin(2\pi 4/512t)$

Reconstruction: 1D FT

$(n{=}256,\ \Delta t{=}2\ \text{s})$

$$(\overline{\Omega}_R + i\,\overline{\Omega}_I) \times (y_R + i\, y_I) = (f_R + i\, f_I)$$

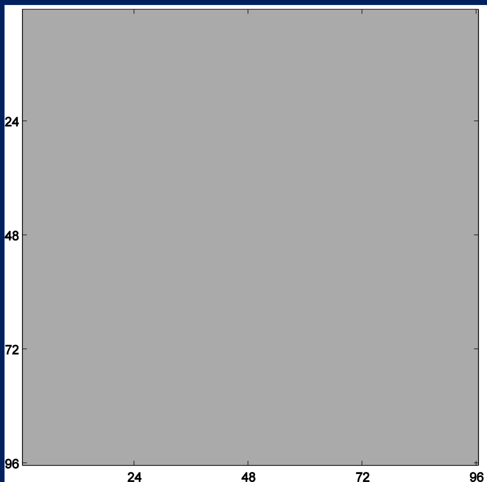


There are lines at the frequency locations.
Real part (image) represents constituent cosine frequencies.
Imaginary part (image) represents constituent sine frequencies.
Intensity of the lines represents amplitude of that frequency.

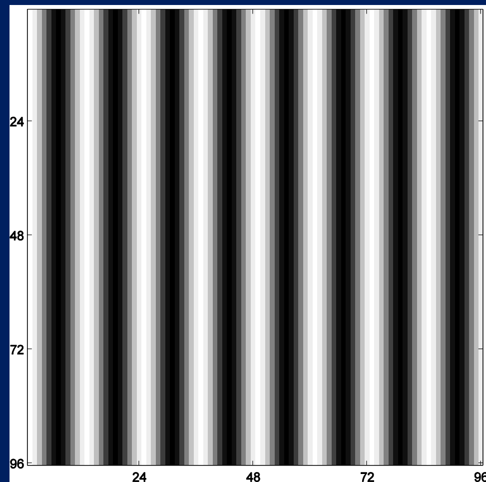
Reconstruction: 2D FT

(FOV=192 mm)

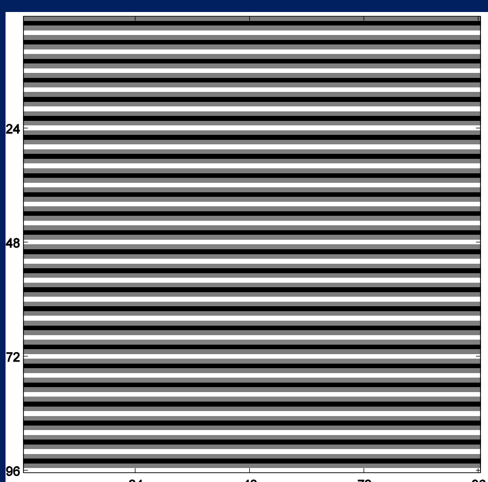
(n_x=n_y=96, Δx=Δy=2 mm)



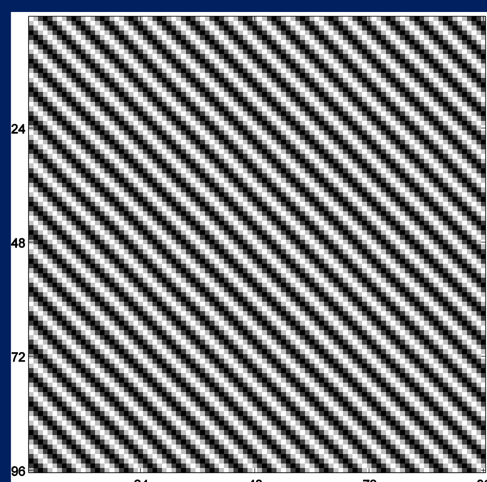
$10\cos(2\pi 0/96x)$



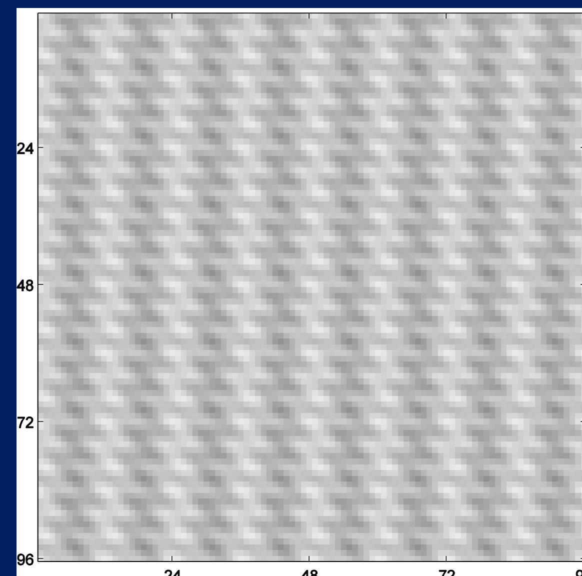
$1.5\cos(2\pi 8/96x)$



$\sin(2\pi 24/96y)$



$\cos(2\pi 4/96x+2\pi 4/96y)$



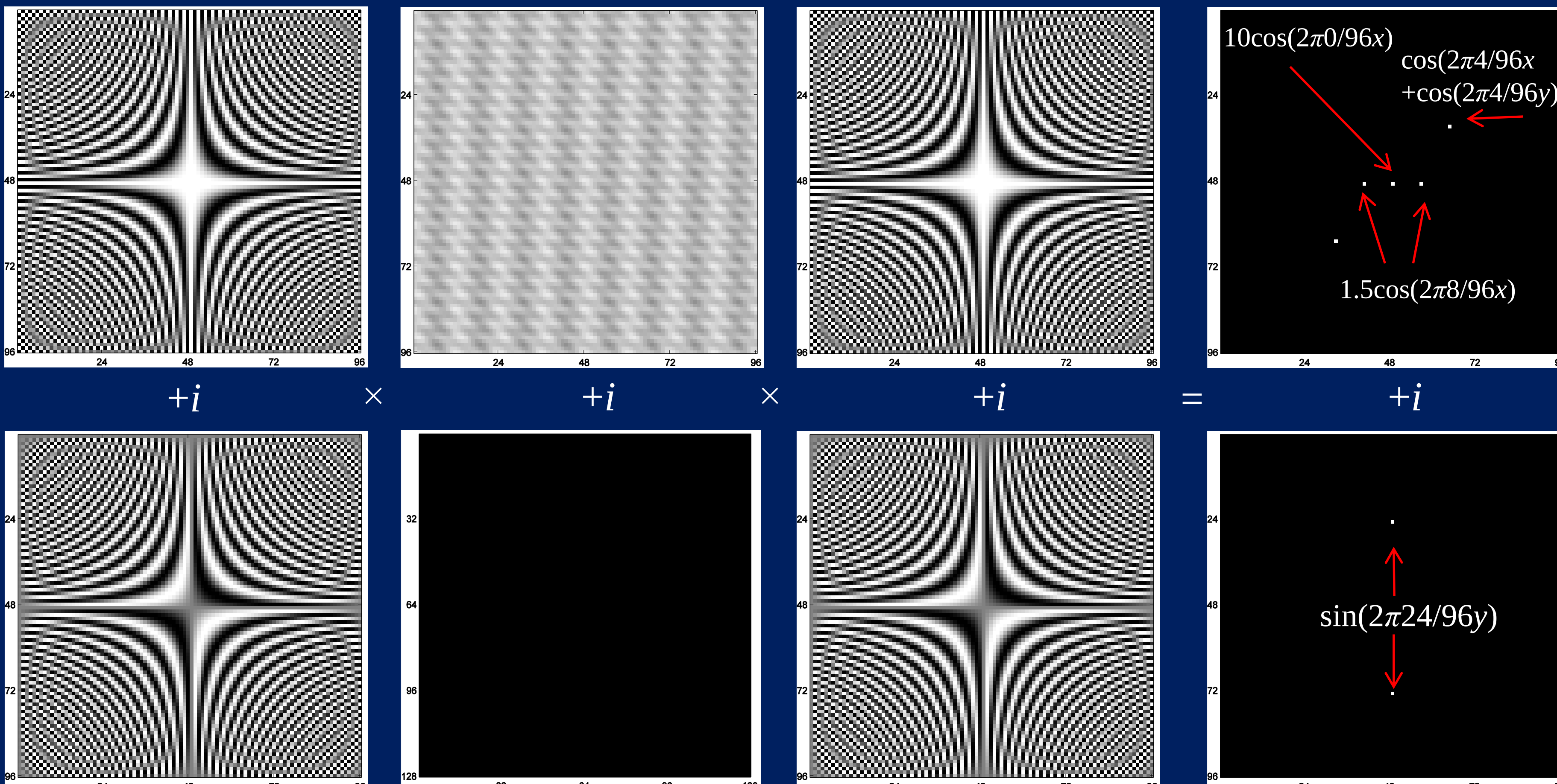
sum

Reconstruction: 2D FT

(FOV=192 mm)

(n_x=n_y=96, Δx=Δy=2 mm)

$$(\overline{\Omega}_{yR} + i\,\overline{\Omega}_{yI}) \times (V_R + i\,V_I) \times (\overline{\Omega}_{xR} + i\,\overline{\Omega}_{xI})^T = (F_R + i\,F_I)$$



sum

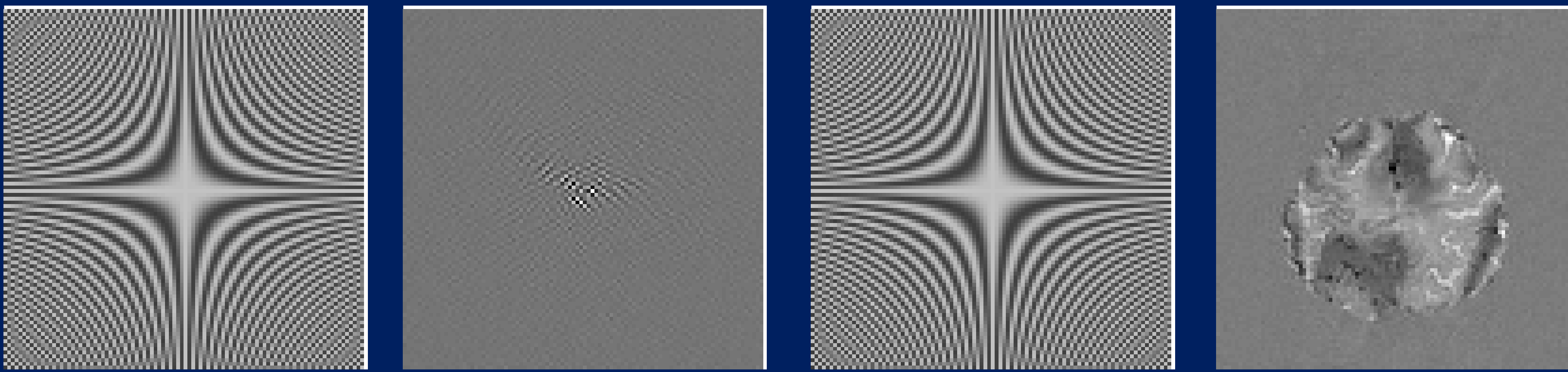
Measure

(FOV=192 mm)

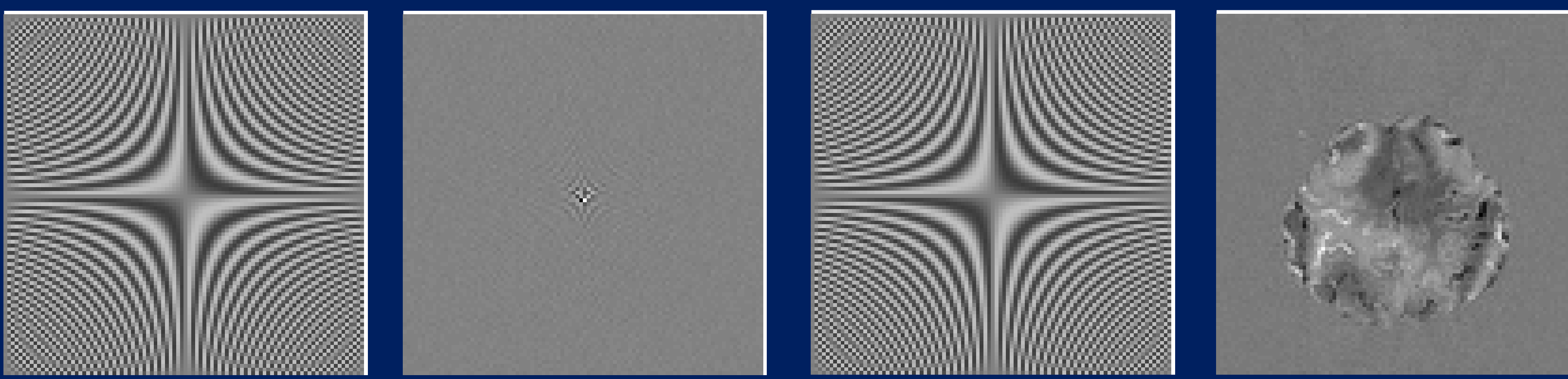
($n_x = n_y = 96$, $\Delta x = \Delta y = 2$ mm)

Reconstruction: 2D IFT

$$(\Omega_{yR} + i\,\Omega_{yI}) \times (F_R + i\,F_I) \times (\Omega_{xR} + i\,\Omega_{xI})^T = (V_R + i\,V_I)$$



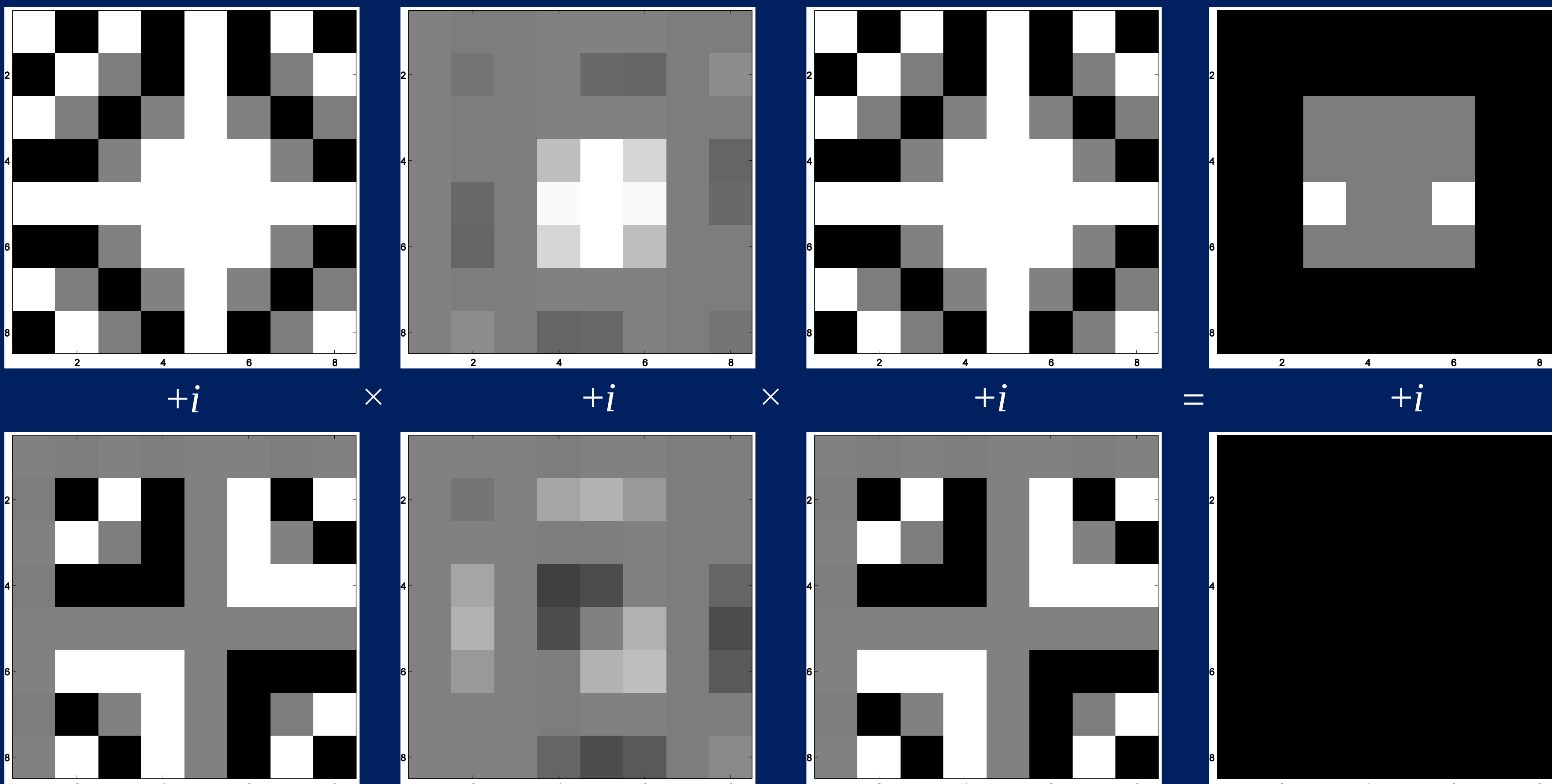
$+i \quad \times \quad +i \quad \times \quad +i \quad = \quad +i$



Spatial Frequencies

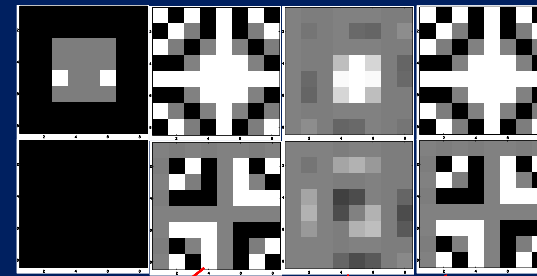
Reconstruction: 2D IFT

$$(\Omega_{yR} + i\,\Omega_{yI}) \quad \times \quad (F_R + i\,F_I) \quad \times \quad (\Omega_{xR} + i\,\Omega_{xI})^T \quad = \quad (V_R + i\,V_I)$$

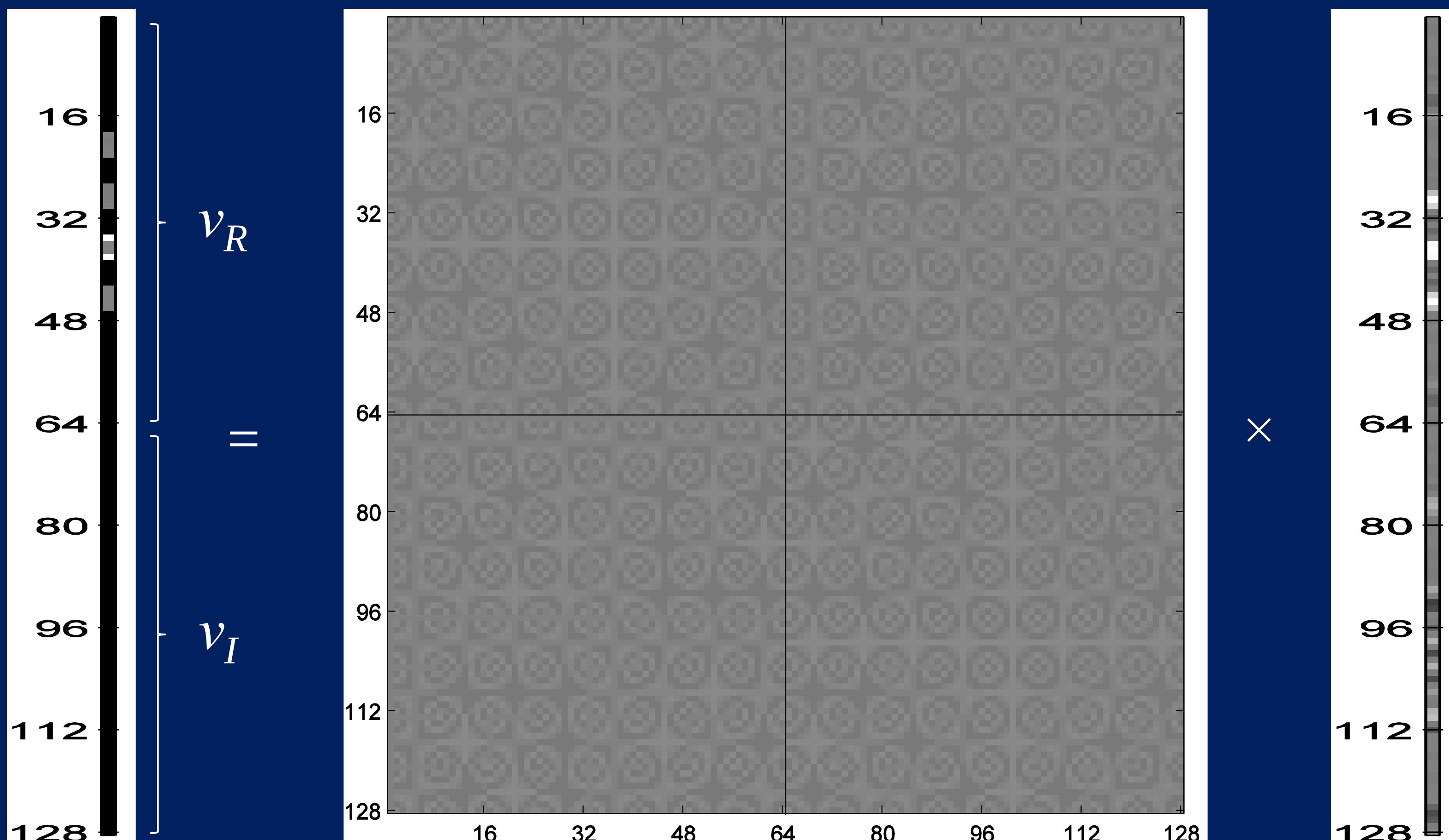


Spatial Frequencies

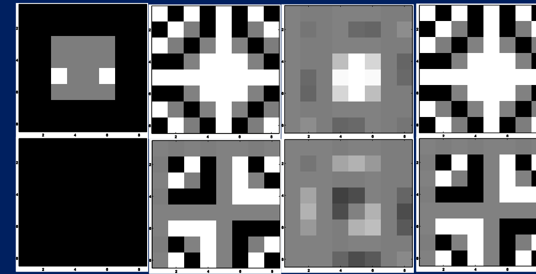
Reconstruction: 2D IFT Isomorphism



$$v = \Omega \times f$$



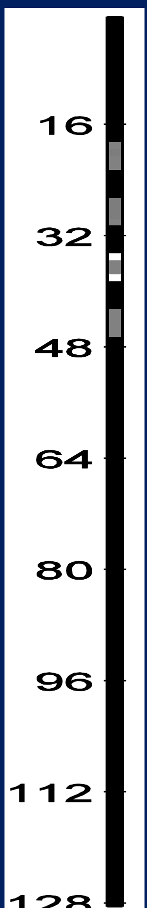
Rowe, Nencka, Hoffmann: JNSM, 159:361-369, 2007.



Reconstruction: Processing Image

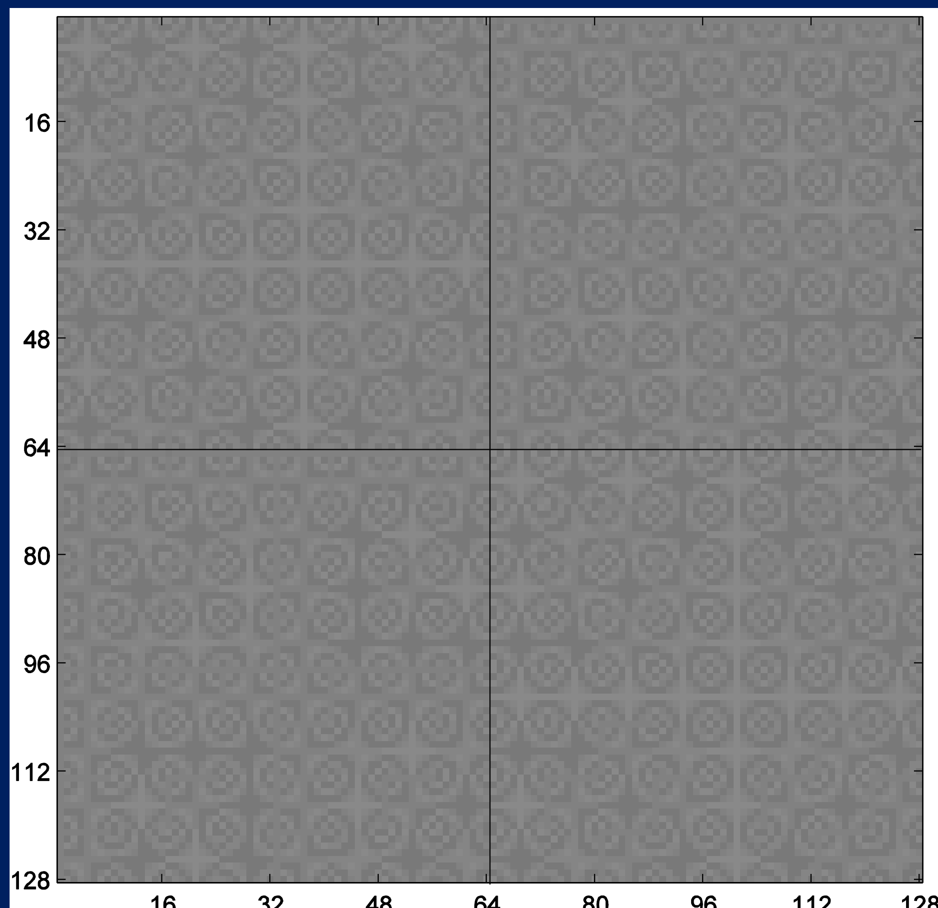
$$v = O_I \times \Omega_a \times O_I \times f$$

Ω_a ← adjusted



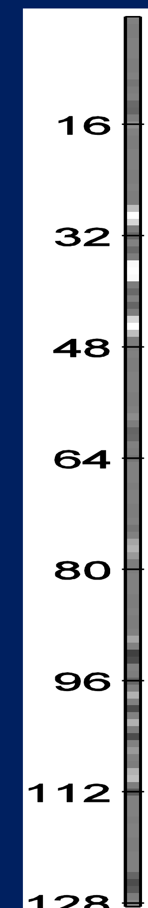
$= O_I \times$

image processing



$\times O_I \times$

k-space processing



Nencka,Hahn,Rowe:JNSM,181:268-82,2009.

Reconstruction: Processing Image

$$\nu \quad = \quad O_I \times \Omega_a \times O_k \times f$$

These operators are:

$$f = P_C \, \mathcal{R} \, C \, \mathcal{F}$$

Permute RI…RI→RR…II

Reverse rows

Censor uturns

k-space vector

$$O_k = \mathcal{A} \, \mathcal{Z} \, \mathcal{H} \, \underbrace{P_R^{-1} \Omega_{row}^{-1} \Phi \Omega_{row} P_R}_{\text{Nyquist}}$$

Apodize

Zero-fill

Homodyne

unpermute rows

IFT rows

shift rows

FT rows

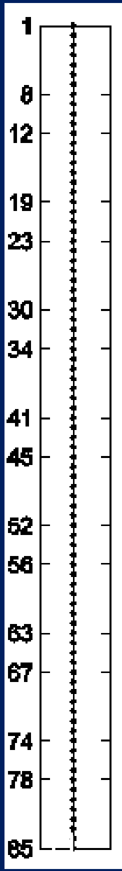
permute rows

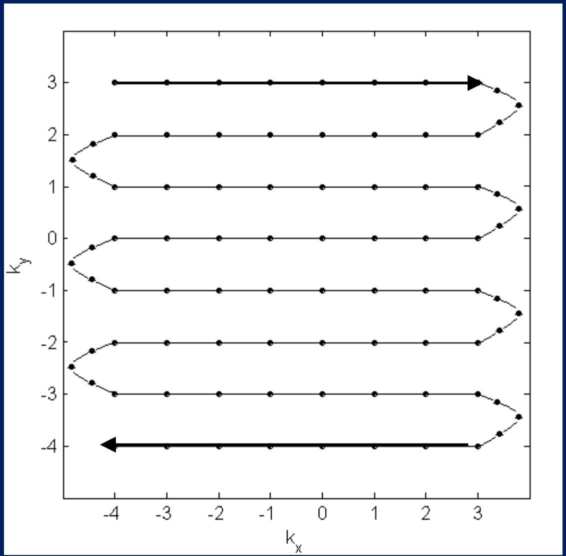
$\Omega_a = \Omega$ adjusted for ΔB and for T_2^*.

$O_I = I_2 \otimes S_m$

Image smoothing

$\mathcal{F}$





Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Induced Correlation: Mean and Covariance

If $E(f)=f_0$, then for Of, $E(Of)=Of_0$.

If $\mathrm{cov}(f)=\Gamma$, then for Of, $\mathrm{cov}(Of)=O\Gamma O^T$.

This means that with $v=\underbrace{O_I\Omega_a O_k}_{O} f$.

$$E(\nu) = O_I \Omega_a O_k f_0$$

$$\mathrm{cov}(\nu) = (O_I \Omega_a O_k)\Gamma(O_k^T \Omega_a^T O_I^T) = \underset{2p\times 2p}{\Sigma}$$ ← Spatial Covariance

$$\mathrm{cor}(\nu) = R_\Sigma$$ ← Spatial Correlation

So even if $\Gamma = \sigma_k^2 I$, it is not necessarily true that $\Sigma = \sigma_I^2 I$!

This has H_0 fMRI noise and fcMRI connectivity implications!

Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

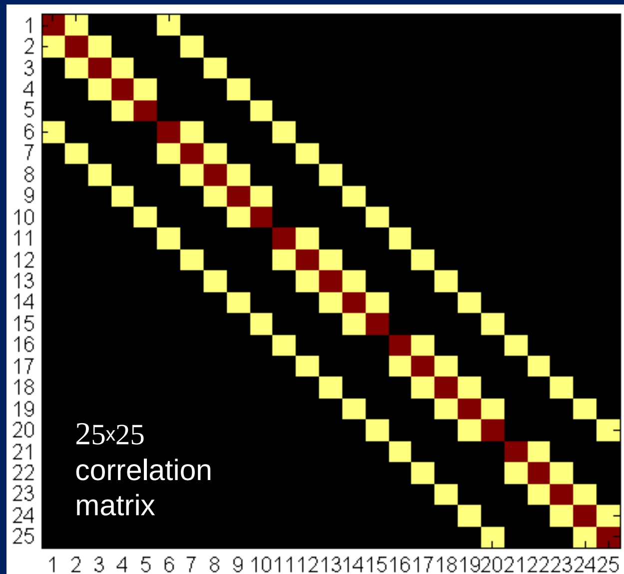
Nencka, Rowe: OHBM, 2007.

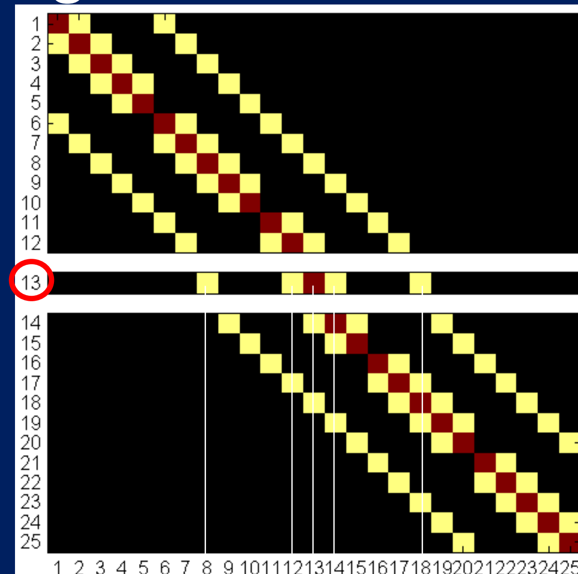
Induced Correlation: Matrix to Image

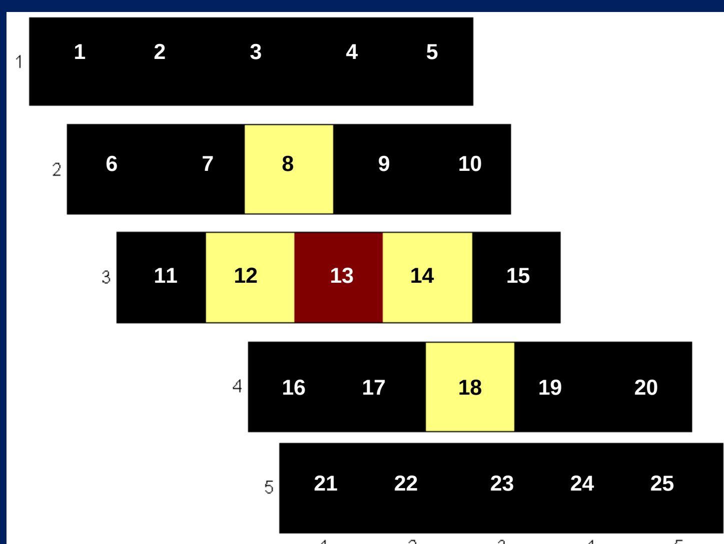
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

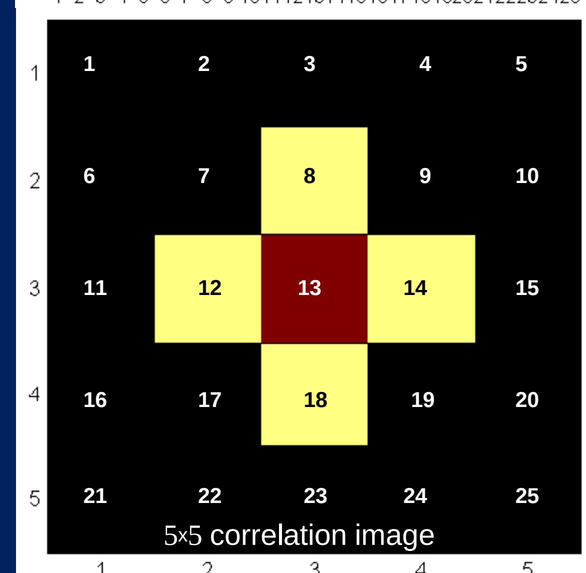
5×5 image

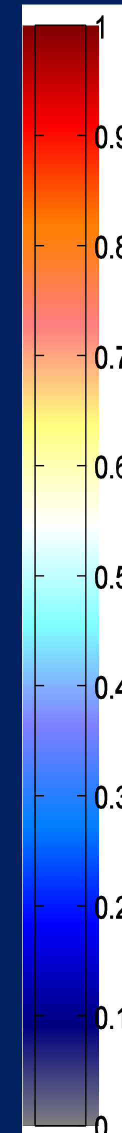
cor =







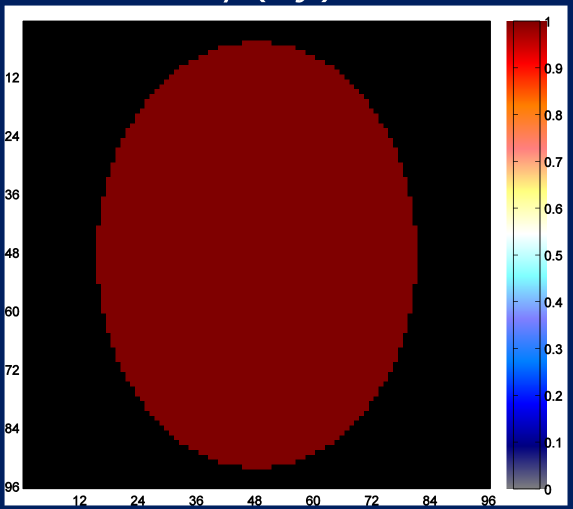




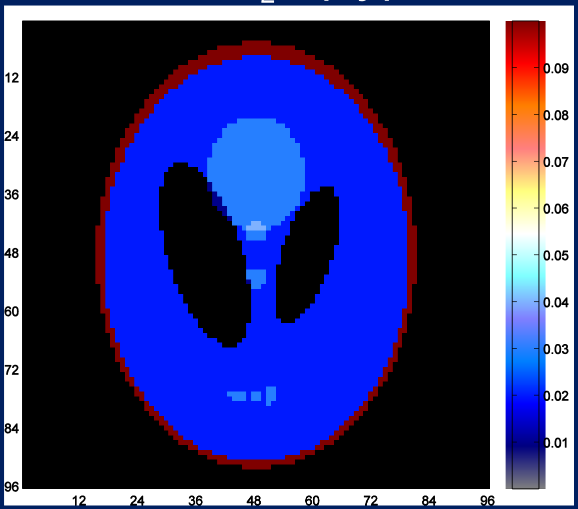
$$f(t) = \int\int \rho(x,y) e^{-t/T_2^*(x,y)} e^{-i\gamma\Delta B(x,y)t} e^{-i2\pi(k_x x + k_y y)} dxdy$$

Induced Correlation: Simulation Parameters

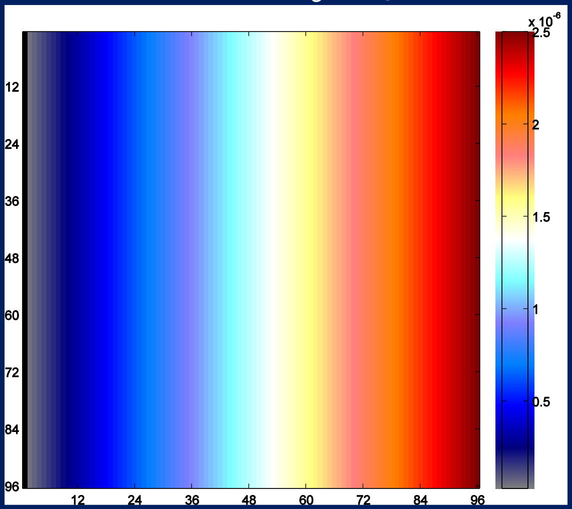
$\rho(x,y)$



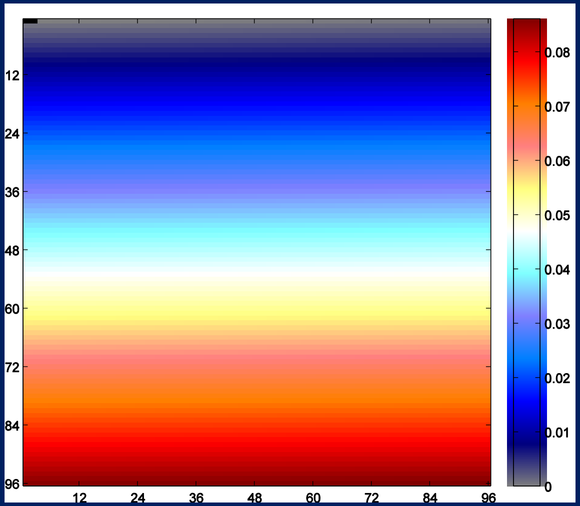
T_2* (x,y)



$\boldsymbol{\Delta B_0}$ (x,y)



Δt



$\mathcal{H} \neq \mathrm{I}$

16 over scan lines

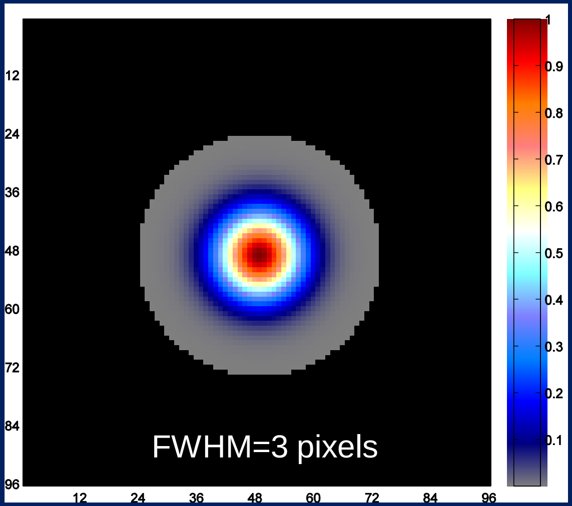
BW=250 kHz

Δt =4 μs

EES=0.96 ms

TE=50.0 ms

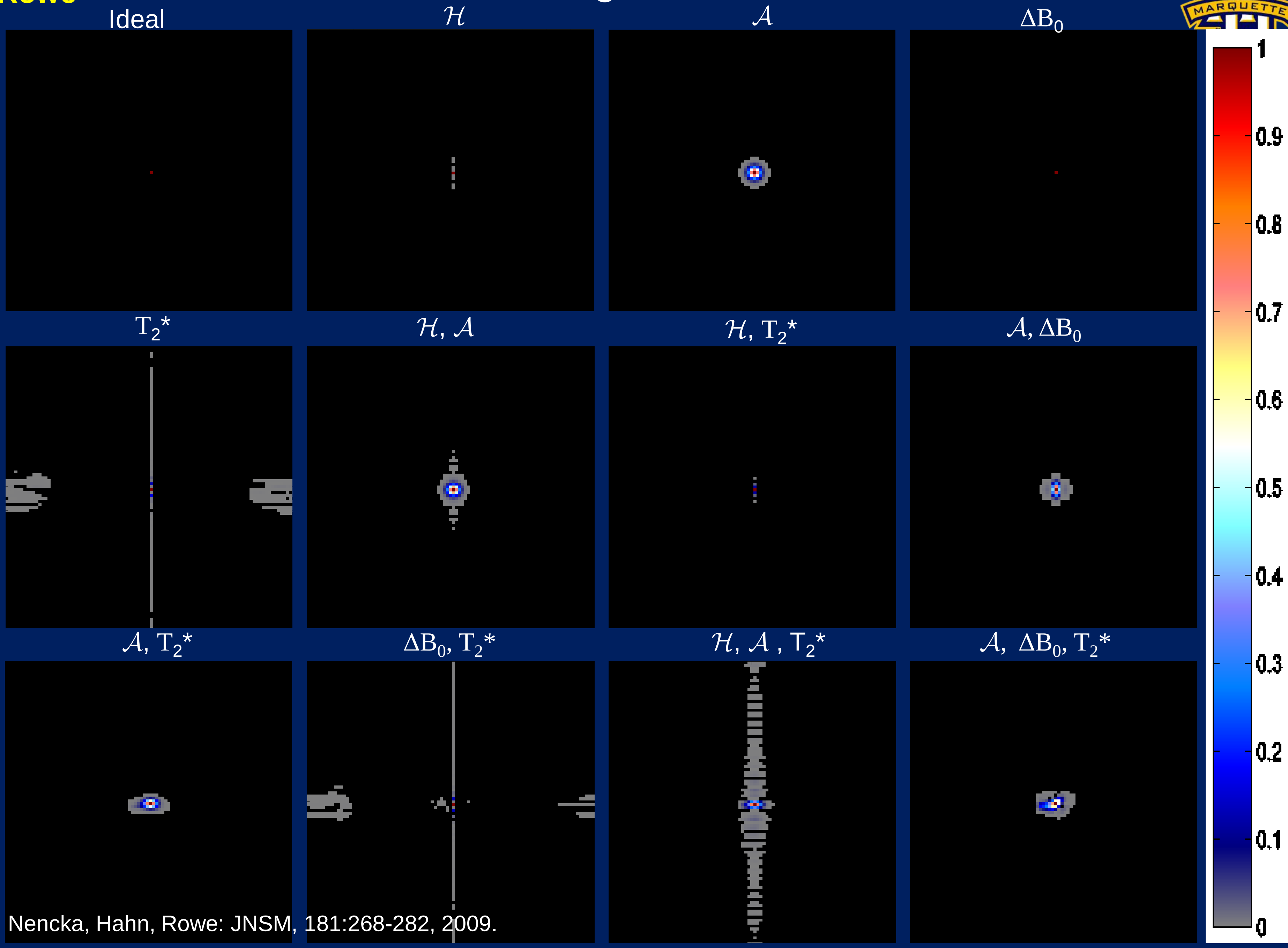
$\mathcal{A} \neq \mathrm{I}$



Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Rowe

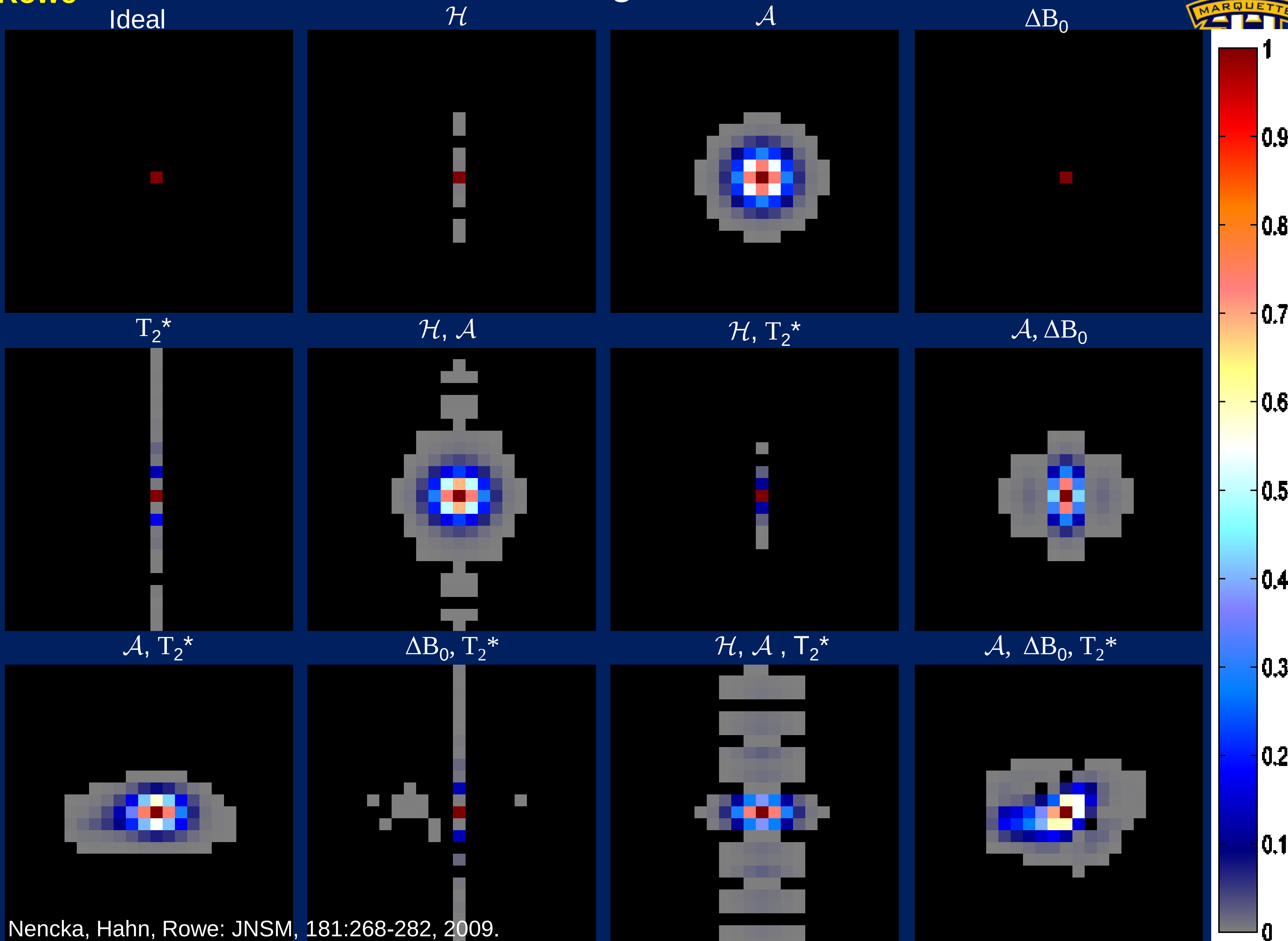
Induced Correlation: Magnitude²



Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Induced Corrrelation: Magnitude² Zoomed

Rowe



Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

here is a for each voxel

a

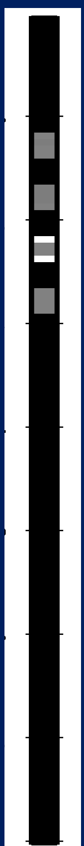
Induced Correlation: SENSE Multi Coil Combine

processing on unfolded image vector

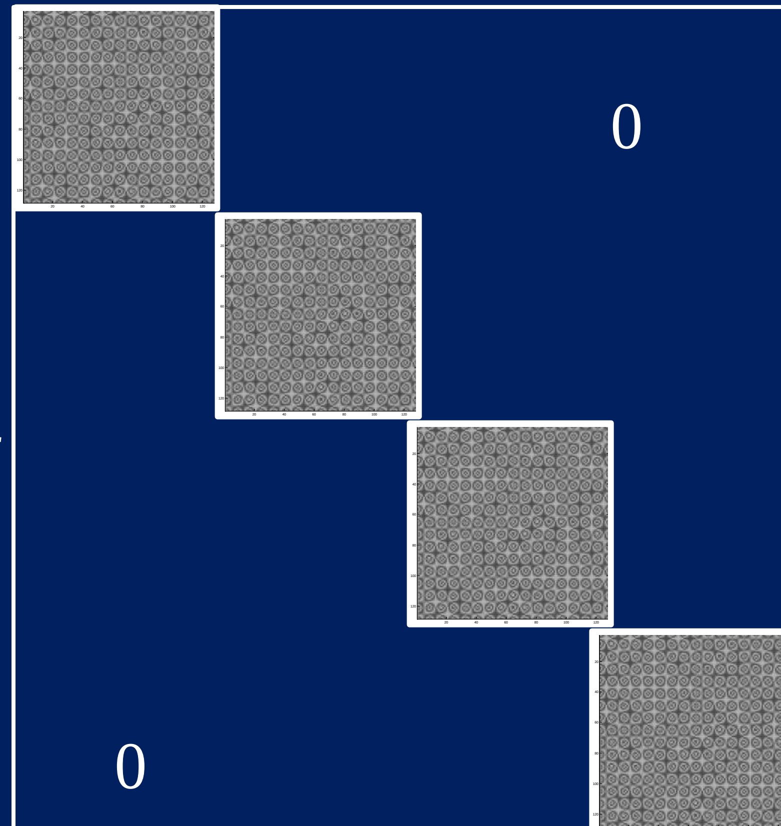
processing on each coil image vector

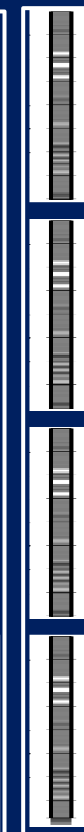
insert processing on each coil k-space vector

$$y = P_u \quad U \quad P_S P_C \quad (I_n \otimes \Omega) \quad f$$



$$= P_U \begin{bmatrix} U_1 & & 0 \\ & \ddots & \\ 0 & & U_p \end{bmatrix} P_S P_C \begin{bmatrix} \Omega & & & 0 \\ & \Omega & & \\ & & \Omega & \\ 0 & & & \Omega \end{bmatrix}$$





Single image vector

permute

unfold matrix U's have S and Ψ

permute to by folded voxel

reconstruct n=4 images

k-space vector of n images

Bruce, Karaman, Rowe: MRI, Accepted, 2011.

Induced Correlation: SENSE Multi Coil Combine

$$y = \underbrace{O_I \; P_u \; U \; P_S P_C \; (I_n \otimes \Omega_a O_k)}_{O} \; f$$

where

$f = (f_1, \ldots, f_n)'$ are coil k-space

O_k is k-space preprocessing

Ω_a is adj. inverse Fourier matrix $\Omega_a = \Omega$

P_u, P_S, P_C, permutation matrices

U SENSE unfolding matrix

O_I is image space preprocessing

k-space vector
censor uturns
row reverse
permute

$f = P_C \mathcal{R} C \mathcal{F}$

$O_k = \mathcal{A}\,\mathcal{Z}\,\mathcal{H}\,\underbrace{P_R^{-1}\Omega_{row}^{-1}\Phi\Omega_{row}P_R}$

adjusted for ΔB
and for T_2^*

$O_I = I_2 \otimes S_m$

Image smoothing

Bruce, Karaman, Rowe: MRI, Accepted, 2011.

Induced Correlation: SENSE Multi Coil Combine

Statistical Expectation and Covariance.

If $E(f)=f_0$, then for Mf, $E(Mf)=Mf_0$.

If $\text{cov}(f)=\Gamma$, then for Mf, $\text{cov}(Mf)=M\Gamma M'$.

This means that with $y = Of$,

$$E(y) = Of_0 \quad \text{and} \quad \text{cov}(y) = O\Gamma O' = \underset{2p\times 2p}{\Sigma}$$

$$\rightarrow \text{cor}(\nu) = D_\Sigma^{-1/2}\Sigma D_\Sigma^{-1/2}$$

So even if $\Gamma = \sigma^2 I$, it is not necessarily true that $\Sigma = \sigma^2 I$!

This has H_0 fMRI noise and fcMRI connectivity implications!

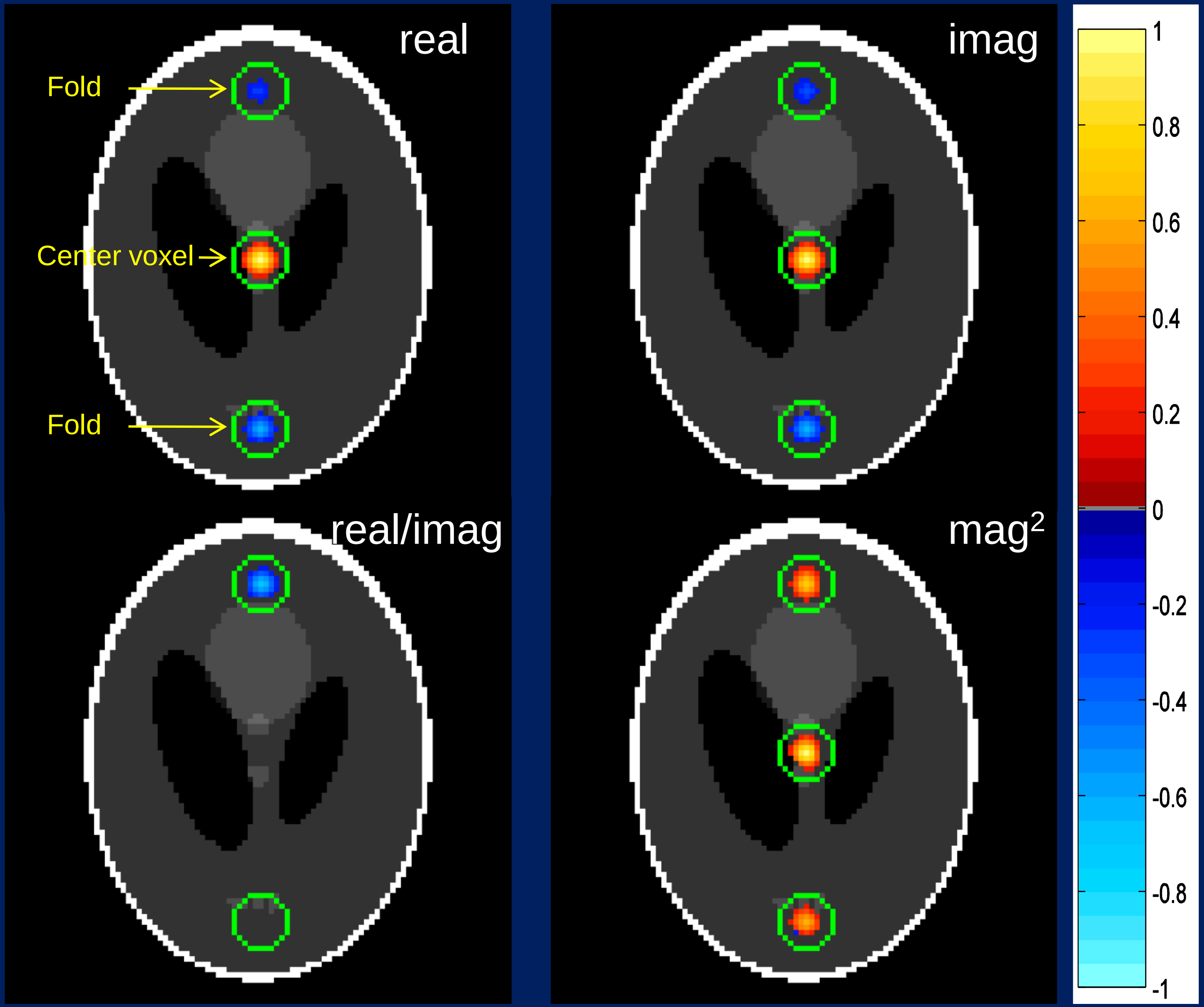
Bruce, Karaman, Rowe: MRI, Accepted, 2011. Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Correlations induced about the center voxel.

Induced Correlation: SENSE Multi Coil Combine

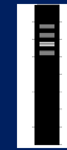
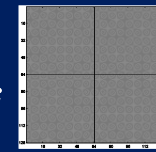
N_X=96
N_Y=96
n = 4
A = 3
FWHM=3

Functional connectivity implications



TH=0.01

Bruce, Karaman, Rowe: MRI, Accepted, 2011.

 $= O_{Ij}$  O_{kj} 

Induced Correlation: Extend to Time Series

Reconstruction of n images described as:

$$\underset{2np\times 1}{\nu} = I \quad R \quad K \quad f$$

diagonal

$$K = BlkDiag(O_{Kt})$$
$$R = BlkDiag(\Omega_{at})$$
$$I = BlkDiag(O_{It})$$

$$\underset{2np\times 1}{\begin{pmatrix} \nu_1 \\ \vdots \\ \nu_n \end{pmatrix}} = \underset{2np\times 2np}{\begin{pmatrix} O_{I1}\Omega_{a1}O_{k1} & & 0 \\ & \ddots & \\ 0 & & O_{In}\Omega_{an}O_{kn} \end{pmatrix}} \underset{2np\times 1}{\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}}$$

image 1 k-space vector

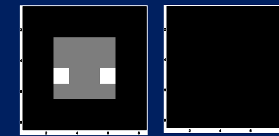
image n k-space vector

image-space vector of n images

Reconstruction k-space operation matrix for n images

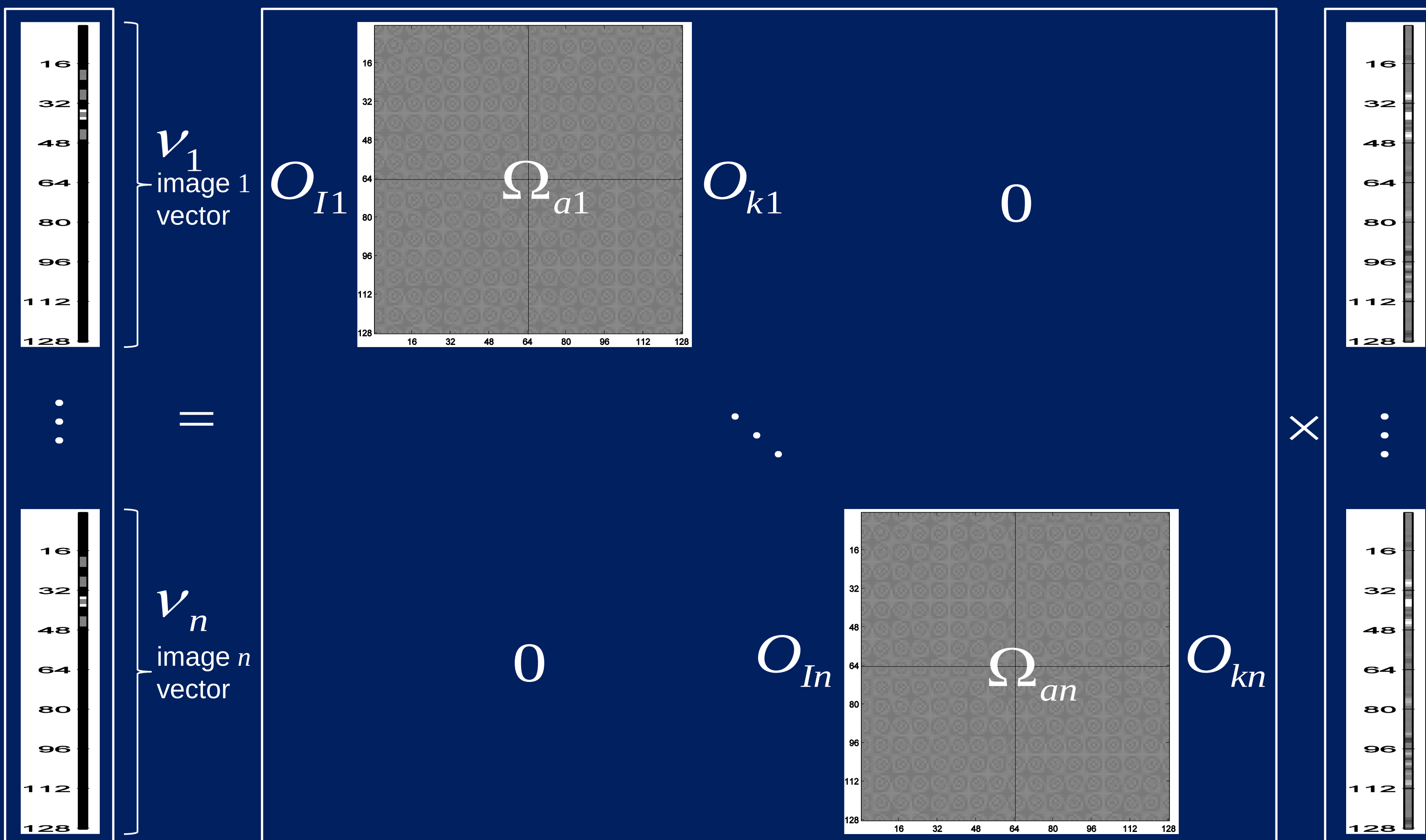
k-space vector of n images

Nencka, Rowe: In Progress.



Induced Correlation: Extend to Time Series

$$\nu = IRK \times f$$



Nencka, Rowe: In Progress.

Induced Correlation: Extend to Time Series

(dyn ΔB_0, Δx, Δt, freq filt)

$$y = T \cdot P \cdot IRK \cdot f$$

voxel 1 temporal processing

$$\underset{2np\times 1}{\begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix}} = \underset{2np\times 2np}{\begin{pmatrix} O_{T1} & & 0 \\ & \ddots & \\ 0 & & O_{Tp} \end{pmatrix}} P \underset{2np\times 2np}{\begin{pmatrix} O_{I1}\Omega_{a1}O_{k1} & & 0 \\ & \ddots & \\ 0 & & O_{In}\Omega_{an}O_{kn} \end{pmatrix}} \underset{2np\times 1}{\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}}$$

voxel p series filter

permute from measurements by image to by voxel

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

voxel 1 temporally processed series

n reals

$$y_j = \begin{pmatrix} y_{Rj} \\ y_{Ij} \end{pmatrix} \begin{matrix} \}\ n\times 1 \\ \}\ n\times 1 \end{matrix}$$

voxel j temporally processed series

$$j = 1, \ldots, p$$

Nencka, Rowe: In Progress.

Induced Correlation: Mean and Covariance

If $E(f)=f_0$, then for $E(Of)=Of_0$.

If $\mathrm{cov}(f)=\Gamma$, then for $\mathrm{cov}(Of)=O\Gamma O^T$.

This means that with $y=\underbrace{TPIRK}_{O}f$.

$$E(y)=TPIRKf_0$$

$$\mathrm{cov}(y)=(TPIRK)\Gamma(K^TR^TI^TP^TT^T)=\underset{2np\times 2np}{\Sigma}$$

Spatio-Temporal Covariance

HUGE

$$\mathrm{cor}(y)=R_\Sigma$$

Spatio-Temporal Correlation

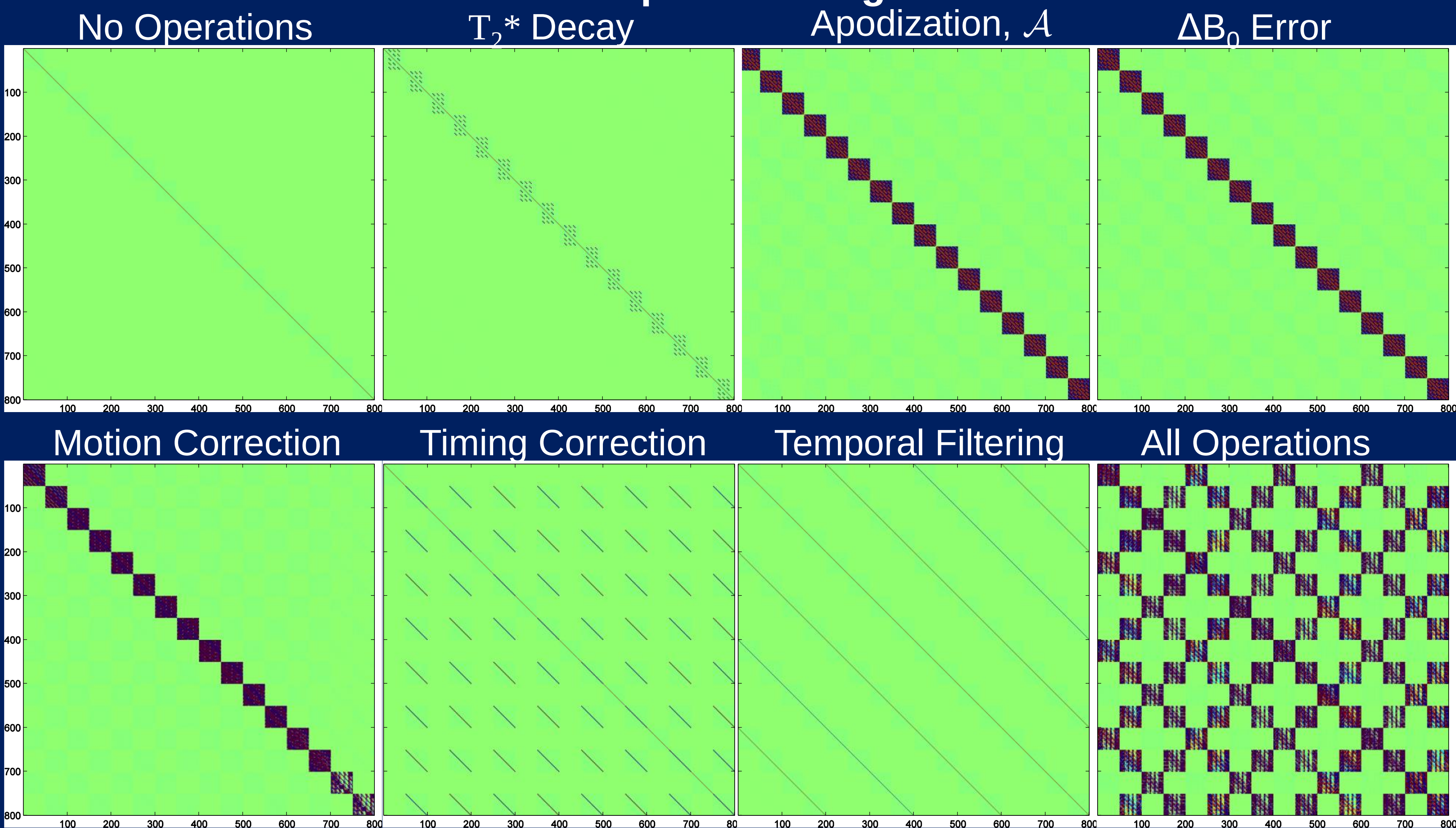
So even if $\Gamma=\sigma_k^2 I$, it is not necessarily true that $\Sigma=\sigma_I^2 I$!

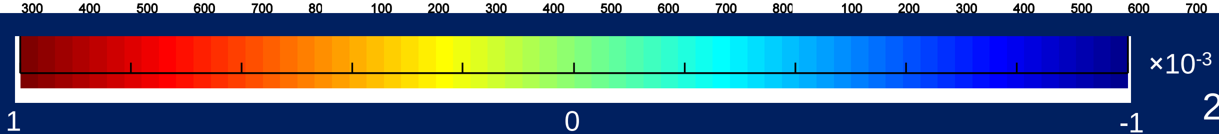
This has H_0 fMRI noise and fcMRI connectivity implications!

Nencka, Rowe: In Progress.

$$\underset{800\times 800}{O} = TPIRK$$

Induced Correlation: Example 5×5 image 8 TRs 2 slices

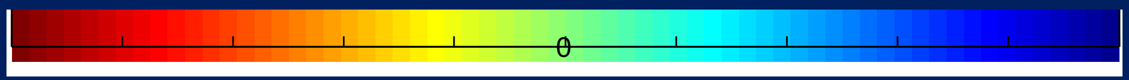




Nencka, Rowe: In Progress.

$$\underset{800\times800}{\Sigma} = OIO^T \rightarrow \underset{800\times800}{R_\Sigma}$$

Induced Correlation: Example 5×5 image 8 TRs 2 slices



Nencka, Rowe: In Progress.

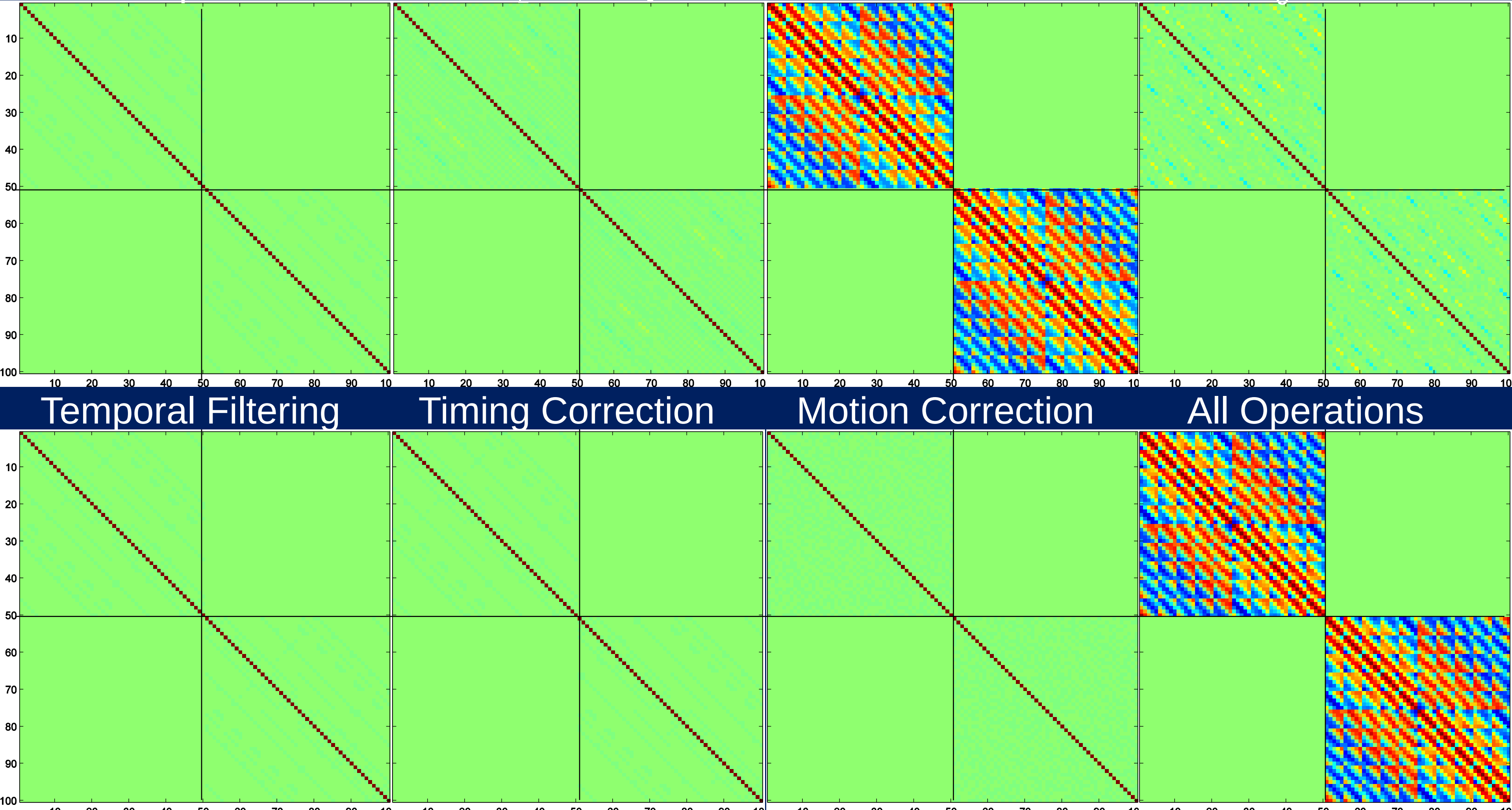
Rowe

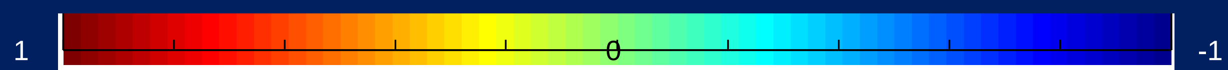
$$\underset{100\times100}{\Sigma} = OIO^T \rightarrow \underset{100\times100}{R_S}$$

Induced Correlation: Example 5×5 image 8 TRs 2 slices

No Operations | T_2* Decay | Apodization, $\mathcal{A}$ | ΔB_0 Error

Temporal Filtering | Timing Correction | Motion Correction | All Operations



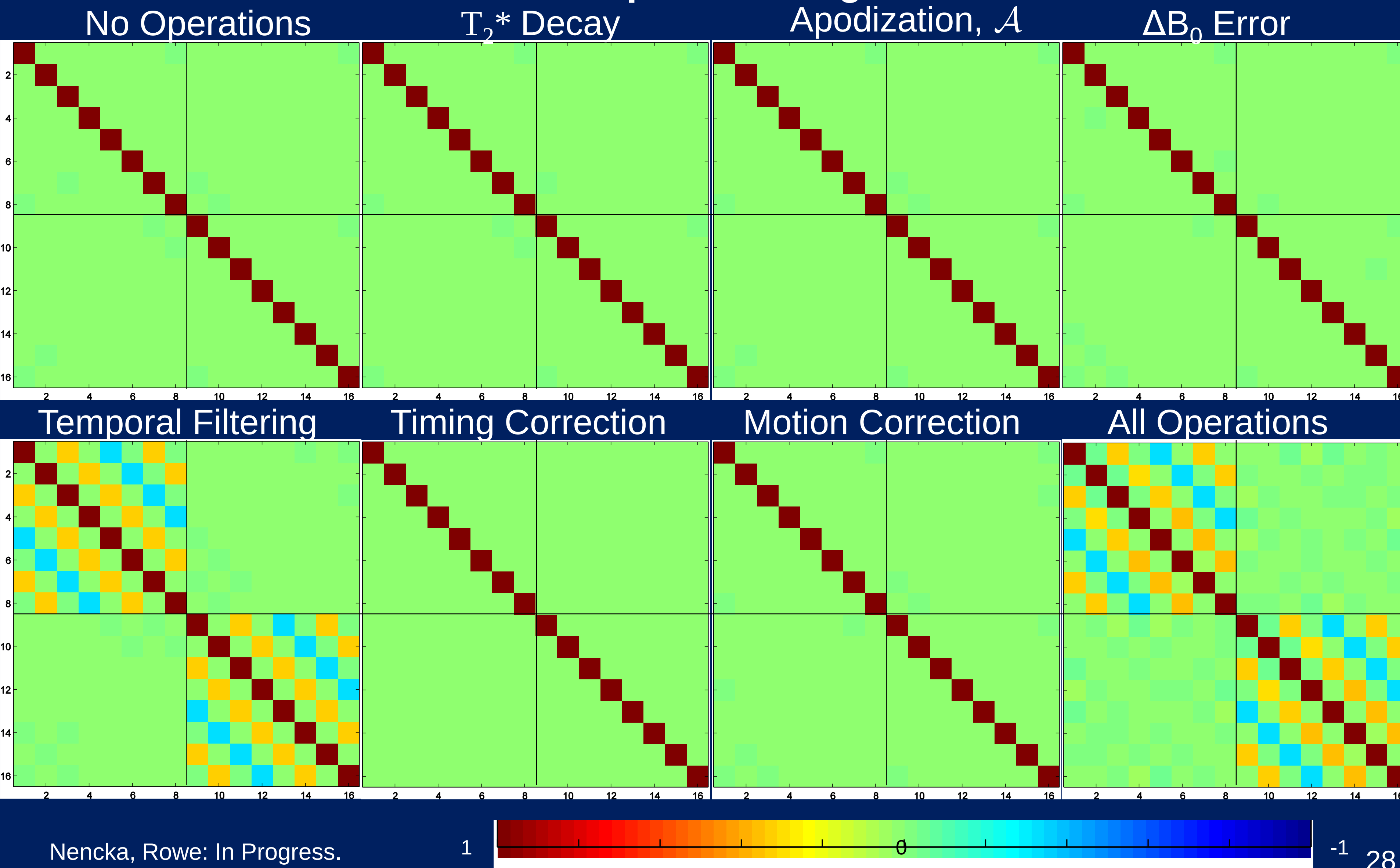


Nencka, Rowe: In Progress.

Rowe

$$\underset{16\times 16}{\Sigma} = OIO^T \rightarrow \underset{16\times 16}{R_T}$$

Induced Correlation: Example 5×5 image 8 TRs 2 slices



Nencka, Rowe: In Progress.

Utilizing Induced Correlation:

Since for all voxels is

$$\underset{2np\times 2np}{\Sigma} = \begin{pmatrix} \begin{array}{c|} \Sigma_1 \\ \hline \end{array} & & \Sigma_{jk} \\ & \ddots & \\ \Sigma'_{jk} & & \begin{array}{|c} \hline \Sigma_p \end{array} \end{pmatrix} \Rightarrow$$

Then for voxel j, the R-I cov

$$\underset{2n\times 2n}{\Sigma_j} = \begin{pmatrix} \Sigma_{jR} & \Sigma_{jRI} \\ \Sigma'_{jRI} & \Sigma_{jI} \end{pmatrix}$$

and the magnitude[2] covariance is

$$\begin{aligned} \delta_j &= tr(\Sigma_j) + \mu'_j \mu_j \\ \Lambda_{jj} &= 2tr(\Sigma'_j \Sigma_j) + 4\mu'_j \Sigma_j \mu_j \quad , \\ \Lambda_{jk} &= 2tr(\Sigma'_{jk} \Sigma_{jk}) + 4\mu'_j \Sigma_{jk} \mu_k \end{aligned}$$

Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Utilizing Induced Correlation:

Complex-Valued

$$C_j = \begin{pmatrix} \cos\theta_{j1} & & 0 \\ & \ddots & \\ 0 & & \cos\theta_{jn} \end{pmatrix} \quad S_j = \begin{pmatrix} \sin\theta_{j1} & & 0 \\ & \ddots & \\ 0 & & \sin\theta_{jn} \end{pmatrix}$$

$$\begin{pmatrix} y_{jR} \\ y_{jI} \end{pmatrix} = \begin{pmatrix} C_j X \beta_j \\ S_j X \beta_j \end{pmatrix} + \begin{pmatrix} \eta_{jR} \\ \eta_{jI} \end{pmatrix}, \qquad \eta_j \sim N(0, \Sigma_j)$$

Compute activation individually for each voxel.

Incorporate Induced Covariance

Magnitude-Only (assuming high SNR)

$$m_j = X\beta_j + \varepsilon_j, \qquad \varepsilon_j \sim N(0, \Lambda_j)$$

Compute activation individually for each voxel.

Can form larger spatio-tempporal model.

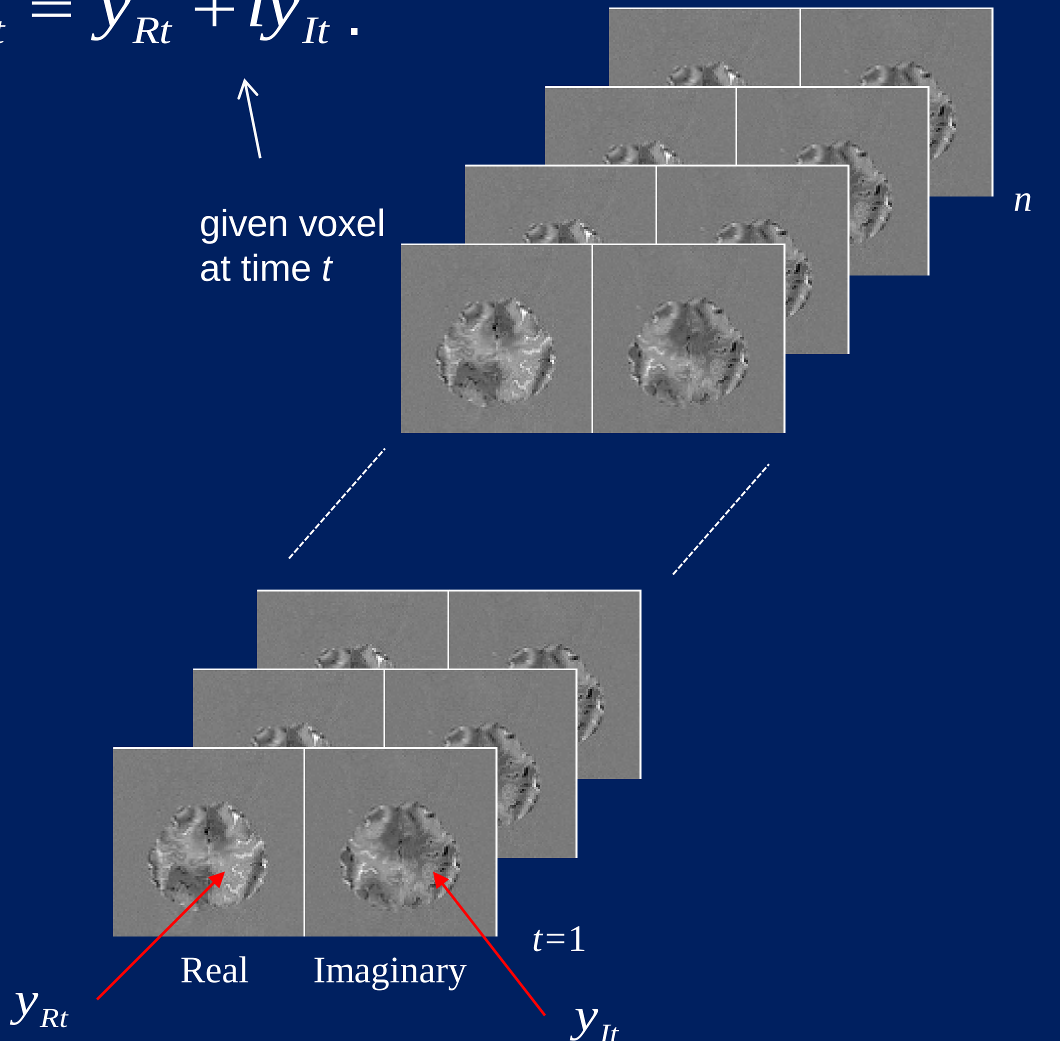
Nencka, Hahn, Rowe: JNSM, 181:268-282, 2009.

Rowe: NIMG, 25:1310-1324, 2005b.

Utilizing Induced Correlation: Complex fMRI

The fMRI data is truly complex-valued images and voxel time series, $y_t = y_{Rt} + iy_{It}$.

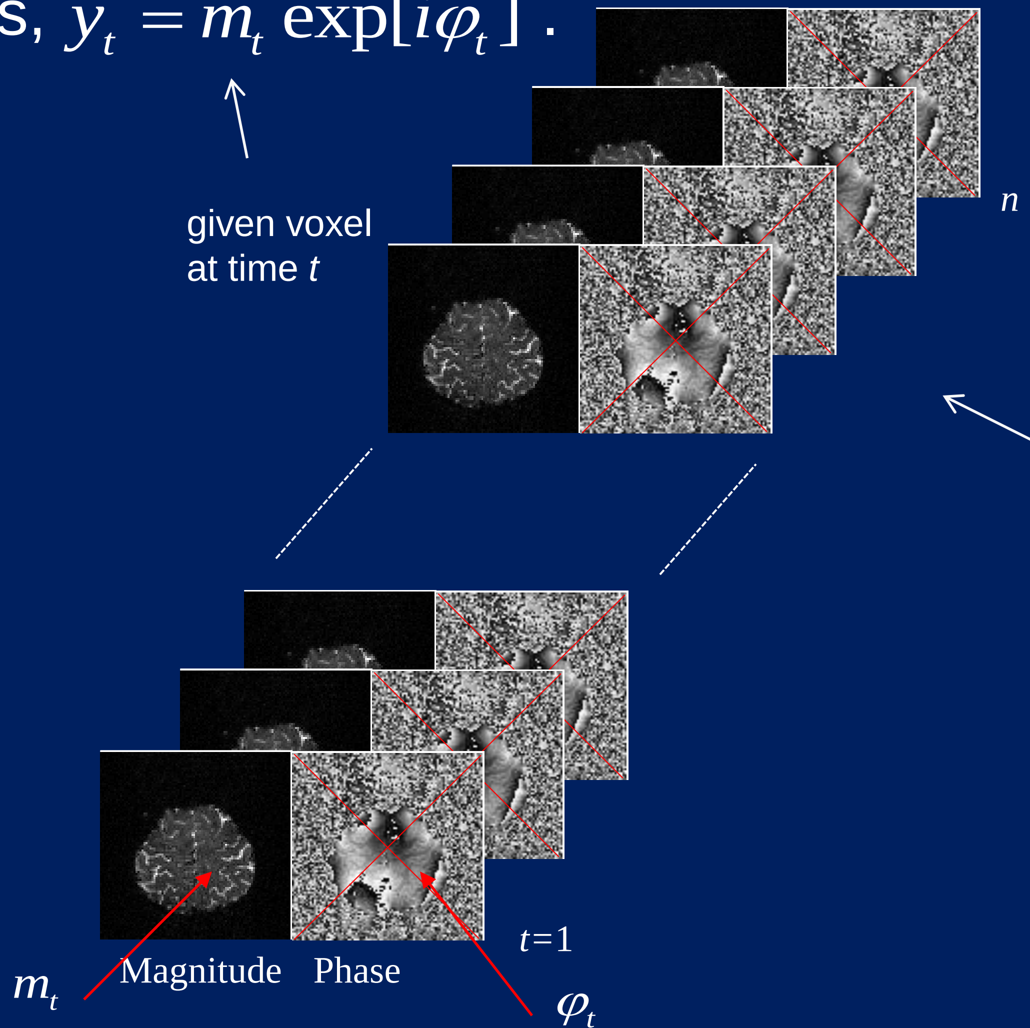
Cartesian coordinates
Real-Imaginary



Utilizing Induced Correlation: Magnitude-Only fMRI

Complex-valued images to magnitude and phase images and time series, $y_t = m_t \exp[i\varphi_t]$.

Polar Coordinates
Magnitude-Phase



Phase discarded!
(in nearly all fMRI)

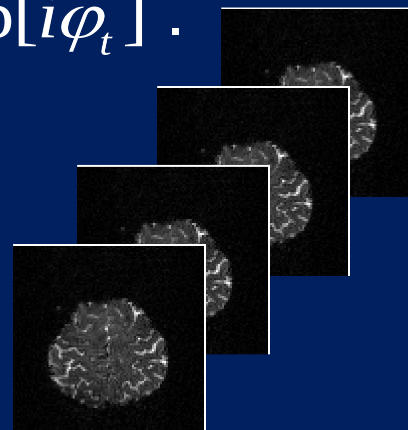
½ of numbers
are discarded
(and processed)

Biological information
in phase through space!
And also through time.

Utilizing Induced Correlation: Magnitude-Only fMRI

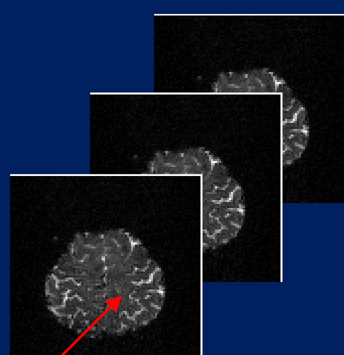
Complex-valued images to magnitude and phase images and time series, $y_t = m_t \exp[i\varphi_t]$.

given voxel
at time *t*



Magnitude-Only

m_t



m_t Magnitude Phase

Phase discarded!
(in nearly all fMRI)

½ of numbers
are discarded
(and processed)

Biological information
in phase through space!
And also through time.

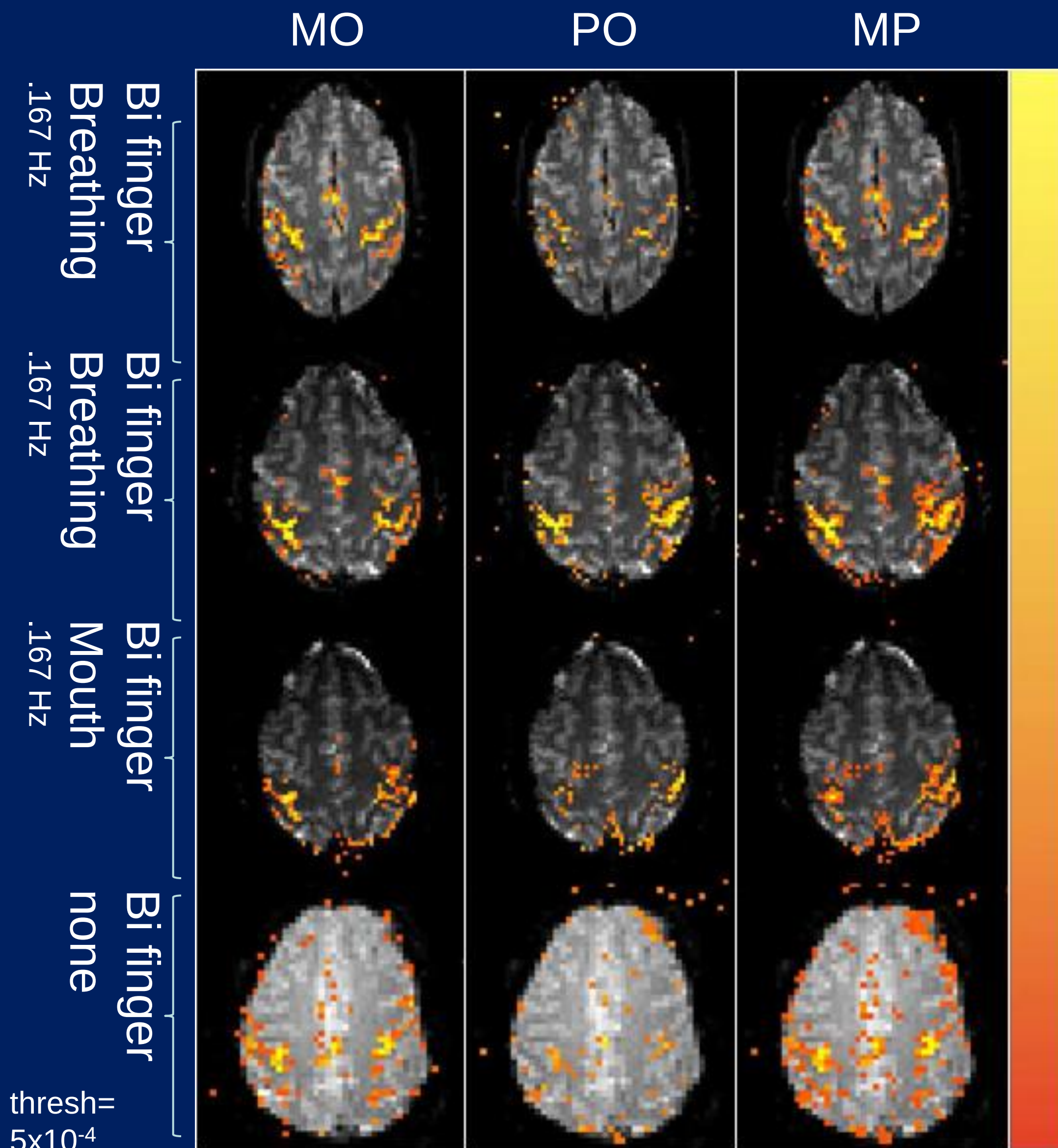
Results: Independent

20s off+16×(8 s on 8 s off), 276 TRs
12 axial slices, 96 × 96, FOV = 24 cm
TH = 2.5 mm, TR = 1 s, TE = 34.6 ms
FA = 45◦, BW = 125 kHz, ES = .708 ms

20s off+16×(8 s on 8 s off), 276 TRs
10 axial slices, 96 × 96, FOV = 24 cm
TH = 2.5 mm, TR = 1 s, TE = 42.8 ms
FA = 45◦, BW = 125 kHz, ES = .768 ms

20s off+16×(8 s on 8 s off), 276 TRs
10 axial slices, 96 × 96, FOV = 24 cm,
TH = 2.5 mm, TR = 1 s, TE = 42.8 ms
FA = 45◦, BW = 125 kHz, ES = . 768 ms

20s off+10×(8 s on 8 s off), 180 TRs
9 axial slices, 64 × 64, FOV = 24 cm
TH = 3.8 mm, TR = 1 s, TE = 26.0 ms
FA = 45◦, BW = 125 kHz, ES = .680 ms



Rowe: NIMG, 25:1310-1324, 2005.
Rowe: MRM, to appear, 2009.

Hahn, Nencka, Rowe: NIMG, 742-752, 2009.
Hahn, Nencka, Rowe: HBM, Online, 2011.

$$\Delta B_t = \frac{\arg\left(I_t \sum_{j=1}^{n} \left(\frac{I_j^*}{|I_j|} \right) \right)}{\gamma \mathrm{TE}}$$

Discussion:

When DATA ANLYSTS preprocess RESEARCHERS data,

THEY change the mean and covariance structure.

Many preprocessing operations have been shown

to modify or induce a correlation.

WE need to utilize this correlation in OUR analysis model!

Thank You

Acknowledgements:

Marquette University
Iain P. Bruce, M.S.
M. Muge Karaman, M.S.

Medical College of Wisconsin
Andrew S. Nencka, Ph.D
Andrew D. Hahn, Ph.D.