Noise Assumptions in Complex-Valued SENSE MR Image Reconstruction

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August 4, 2010

OUTLINE

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Motivation

In MRI k-space for images is not measured instantaneously.

In parallel imaging, sub-sampled k-space points are measured in parallel and combined to form a single image.

Image and volume measurement time is decreased at the expense of increased image reconstruction difficulty and time.

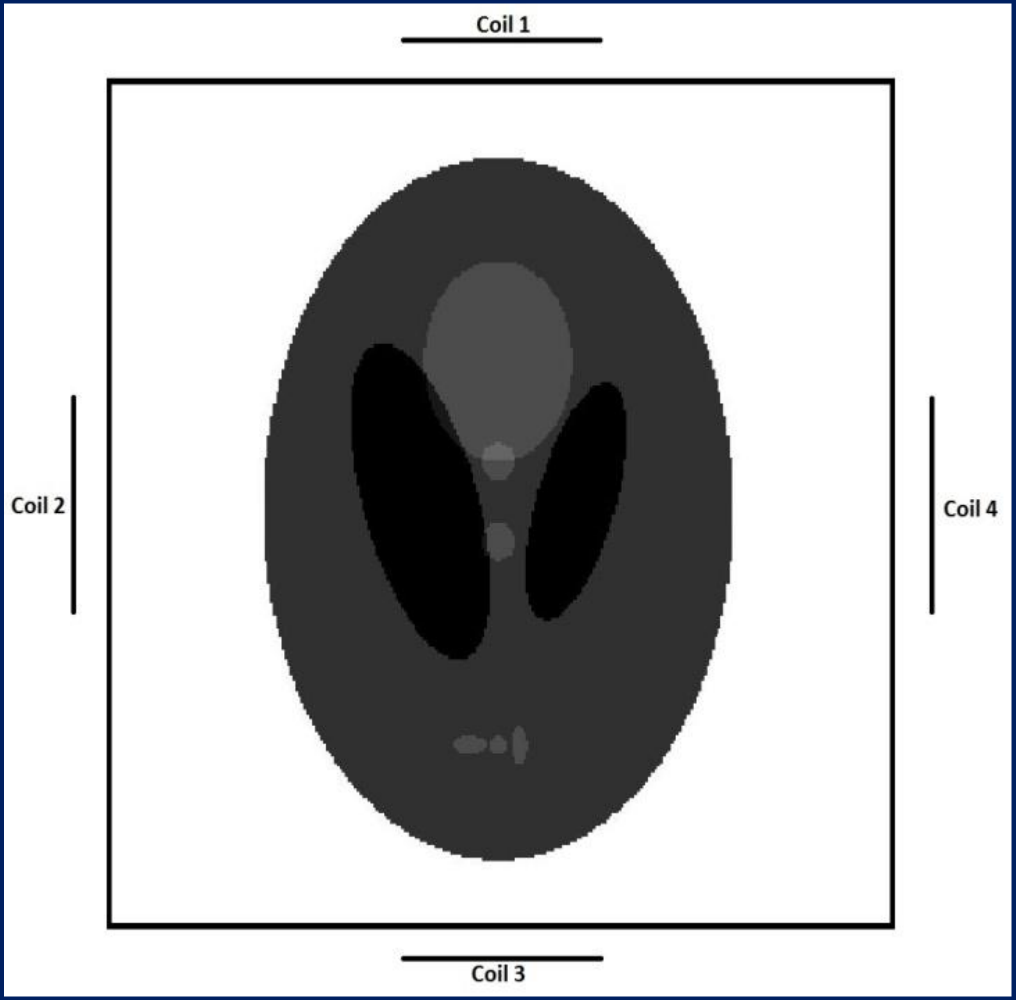
The SENSE parallel imaging reconstruction technique utilizes a complex-valued least squares estimation process.

However, in SENSE the covariance is not properly modeled.

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.

Background

In parallel imaging there is more than one receive coil.



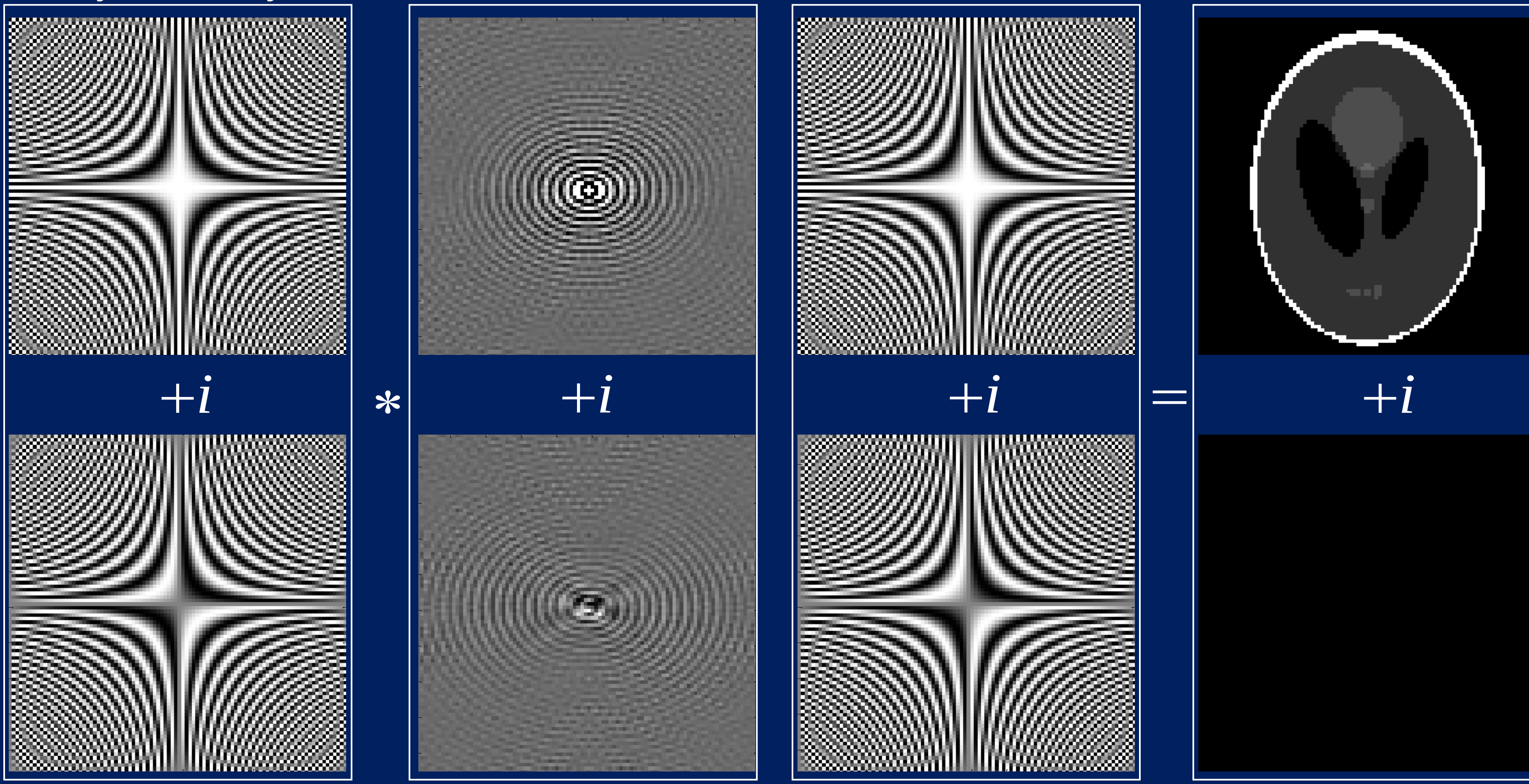
Each coil measures a k-space array that is reconstructed into an aliased image then combined to form a single unaliased image.

Bruce and Rowe: In progress.

Background

Image inverse Fourier Reconstruction for single coil.

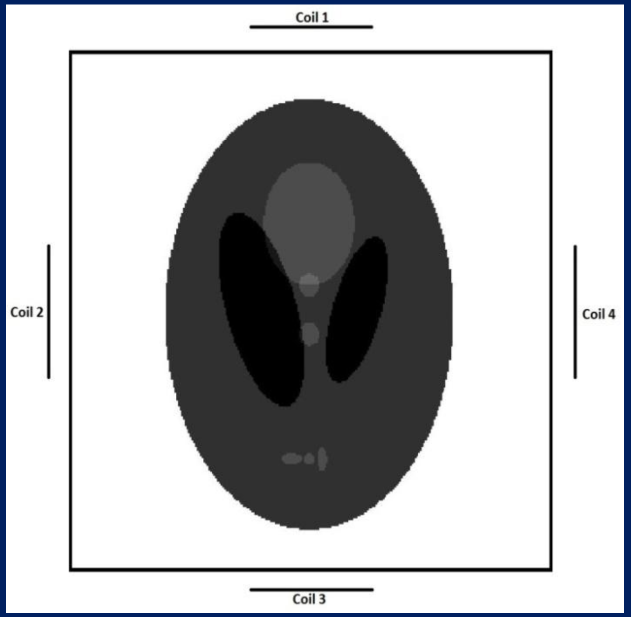
$$(\Omega_{yR}+i\Omega_{yI}) * (F_R+iF_I) * (\Omega_{xR}+i\Omega_{xI})^T = (V_R+iV_I)$$

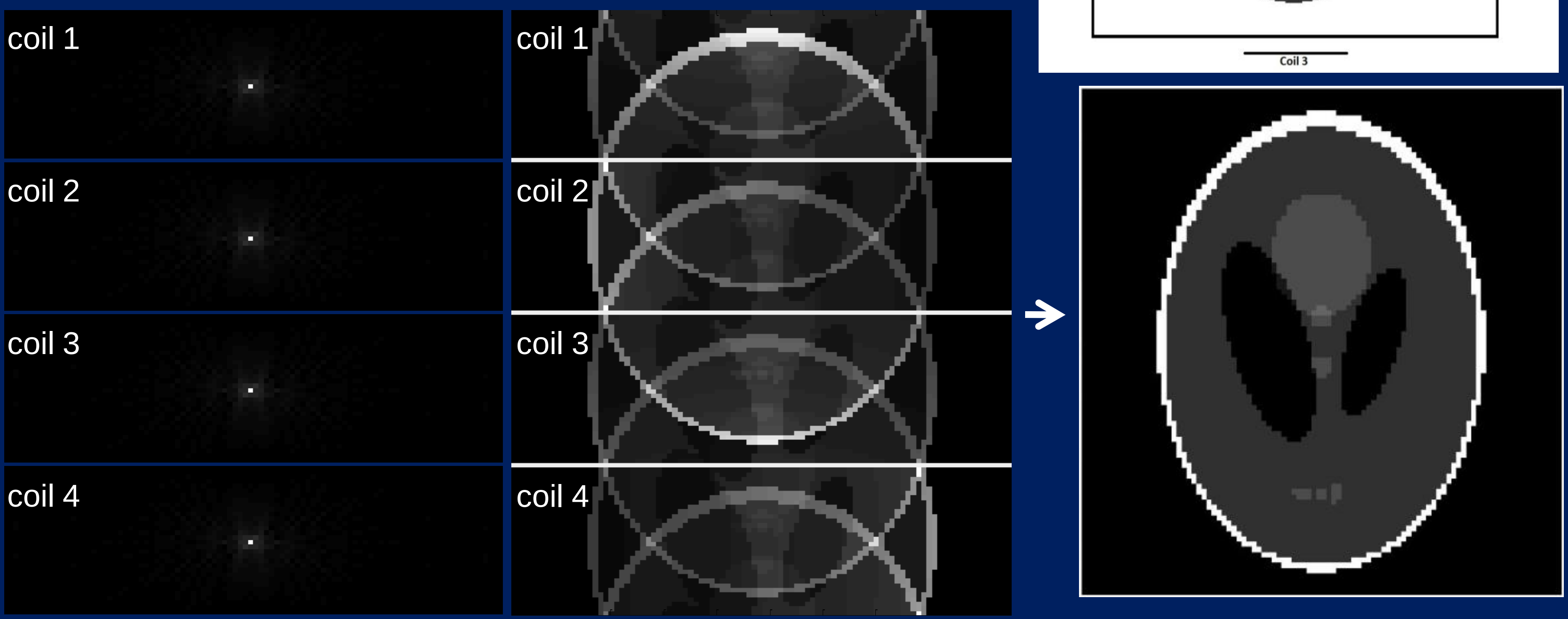


Bruce and Rowe: In progress.

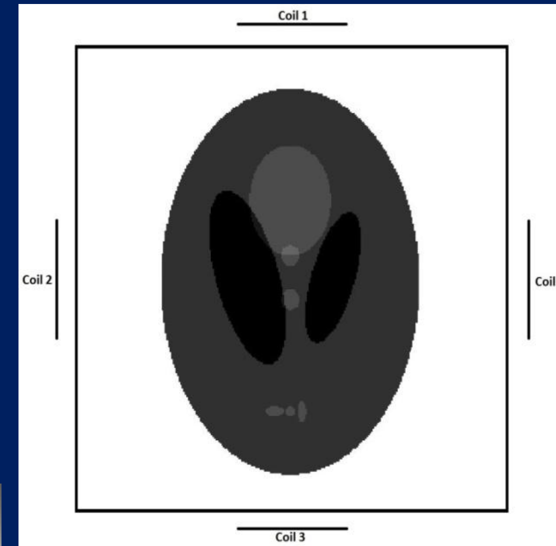
Background

Each coil measures a k-space array that is reconstructed into an aliased image then combined to form a single image.





Bruce and Rowe: In progress.



Background

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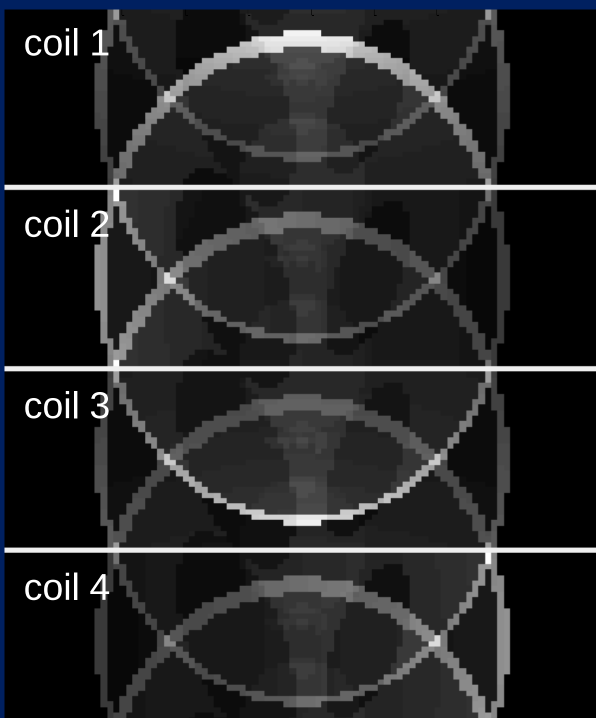
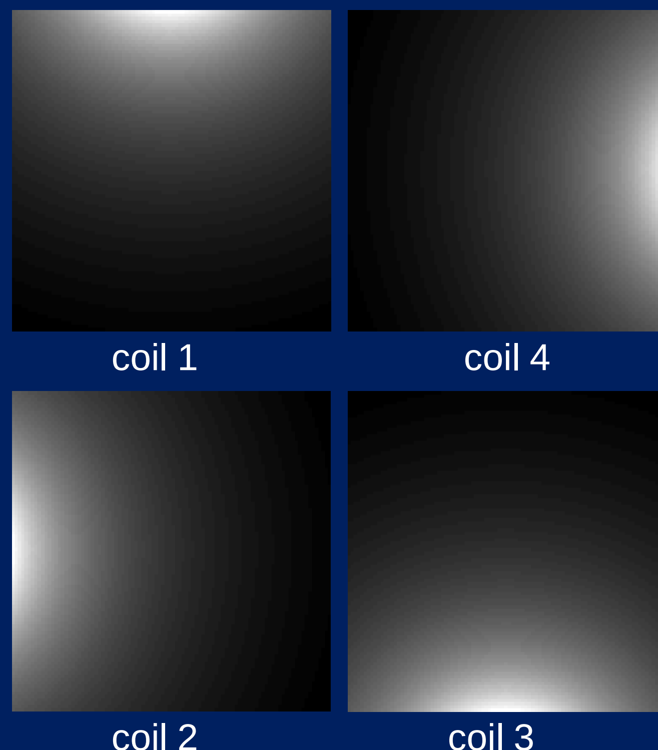
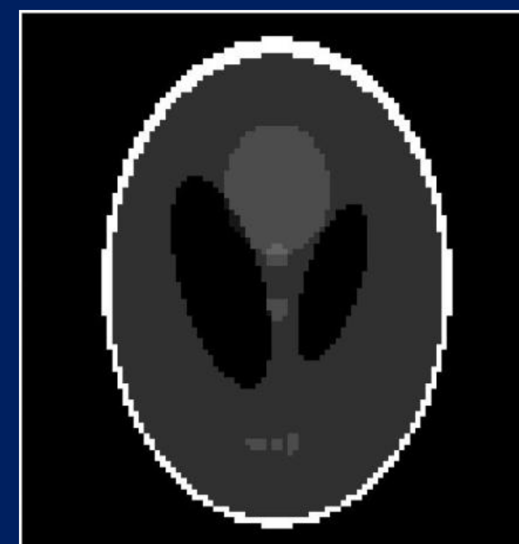


a_C



S_C



v_C

Bruce and Rowe: In progress.

Methods

The SENSE model for aliased voxel values from n coils is

$$\underset{n\times 1}{a_C} = \underset{n\times A}{S_C} \underset{A\times 1}{v_C} + \underset{n\times 1}{\varepsilon_C} \quad , \quad \varepsilon_C \sim CN(0,\Psi_C)$$

where for each voxel

$$\Psi_C = \Psi_R + i\Psi_I$$

a_C is a vector of the n complex-valued aliased voxel values

$$a_C = a_R + ia_I$$

v_C is a vector of the A unaliased voxel values

$$v_C = v_R + iv_I$$

S_C is an nxA matrix of complex-valued coil sensitivities

$$S_C = S_R + iS_I$$

ε_C is a vector of the n complex-valued error values

$$\varepsilon_C = \varepsilon_R + i\varepsilon_I$$

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.

Methods

The SENSE process

$$\underset{n\times 1}{a_C} = \underset{n\times A}{S_C} \; \underset{A\times 1}{v_C} + \underset{n\times 1}{\varepsilon_C} \;, \qquad \varepsilon_C \sim CN(0, \Psi_C)$$

$$\Psi_C = \Psi_R + i\Psi_I$$

uses the complex normal distribution

$$f(\varepsilon_C) = (2\pi)^{-n} \left|\Psi_C\right|^{-1} e^{-1/2\varepsilon_C^H \Psi_C^{-1} \varepsilon_C},$$

H is the conjugate transpose (Hermetian)

and for N_C coil measurements

$$f(a_C) = (2\pi)^{-n} \left|\Psi_C\right|^{-1} e^{-1/2(a_C - S_C v_C)^H \Psi_C^{-1} (a_C - S_C v_C)}$$

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.
Wooding The multivariate distribution of complex normal variables. Biometrika 43:212–215, 1956.
Bruce and Rowe: In progress.

Methods

From the distribution for the n coil measurements

$$f(a_C) = (2\pi)^{-n} \left|\Psi_C\right|^{-1} e^{-1/2(a_C - S_C v_C)^H \Psi_C^{-1}(a_C - S_C v_C)}$$

the voxel values can be estimated as

$$v_C = (S_C^H \Psi_C^{-1} S_C)^{-1} S_C^H \Psi_C^{-1} a_C$$

with knowledge of S_C and Ψ_C.

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.
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Bruce and Rowe: In progress.

Methods

Instead of writing the model with complex numbers as

$$\underset{n\times 1}{a_C} = \underset{n\times A}{S_C} \underset{A\times 1}{v_C} + \underset{n\times 1}{\varepsilon_C} ,$$

$$a_C = a_R + i a_I \quad S_C = S_R + i S_I \quad v_C = v_R + i v_I \quad \varepsilon_C = \varepsilon_R + i \varepsilon_I$$

we can write the model using an isomorphism as

$$\underset{2n\text{x}1}{a} = \underset{2n\text{x}2A}{S} \underset{2A\text{x}1}{v} + \underset{2n\text{x}1}{\varepsilon}$$

$$a = \begin{pmatrix} a_R \\ a_I \end{pmatrix} \quad S = \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix} \quad v = \begin{pmatrix} v_R \\ v_I \end{pmatrix} \quad \varepsilon = \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.
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Bruce and Rowe: In progress.

Methods

Then the distribution for n coil measurements is

$$f(a) = (2\pi)^{-n} \left|\Psi_{SE}\right|^{-1/2} \mathrm{e}^{-1/2(a-Sv)'\Psi_{SE}^{-1}(a-Sv)},$$

with

$$a = \begin{pmatrix} a_R \\ a_I \end{pmatrix} \qquad S = \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix} \quad v = \begin{pmatrix} v_R \\ v_I \end{pmatrix} \qquad \varepsilon = \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

and the complex normal distribution imposes skew-symmetric

$$\Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}$$

Wooding The multivariate distribution of complex normal variables. Biometrika 43:212–215, 1956.
Bruce and Rowe: In progress.

Methods

The skew-symmetric covariance structure

$$\Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}$$ is incorrect.

What this says is that $\mathrm{cov}(I,I) = \mathrm{cov}(R,R)$

and that $\mathrm{cov}(I,R) = -\mathrm{cov}(R,I)$.

The proper covariance structure should be

$$\Psi_{SI} = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi'_{RI} & \Psi_I \end{pmatrix}$$

(SE for SENSE and SI for new covariance model SENSE-ITIVE)

Bruce and Rowe: In progress.

Methods

Examine the difference between the two covariance structures

$$\Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix} \qquad \Psi_{SI} = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi'_{RI} & \Psi_I \end{pmatrix}$$

in the distribution

$$f(a) = (2\pi)^{-n} \left|\Psi_{SE/SI}\right|^{-1/2} \mathrm{e}^{-1/2(a-Sv)'\Psi_{SE/SI}^{-1}(a-Sv)}$$

through estimates

$$v_{SE} = (S'\Psi_{SE}^{-1}S)^{-1}S'\Psi_{SE}^{-1}a$$

$$v_{SI} = (S'\Psi_{SI}^{-1}S)^{-1}S'\Psi_{SI}^{-1}a$$

Bruce and Rowe: In progress.

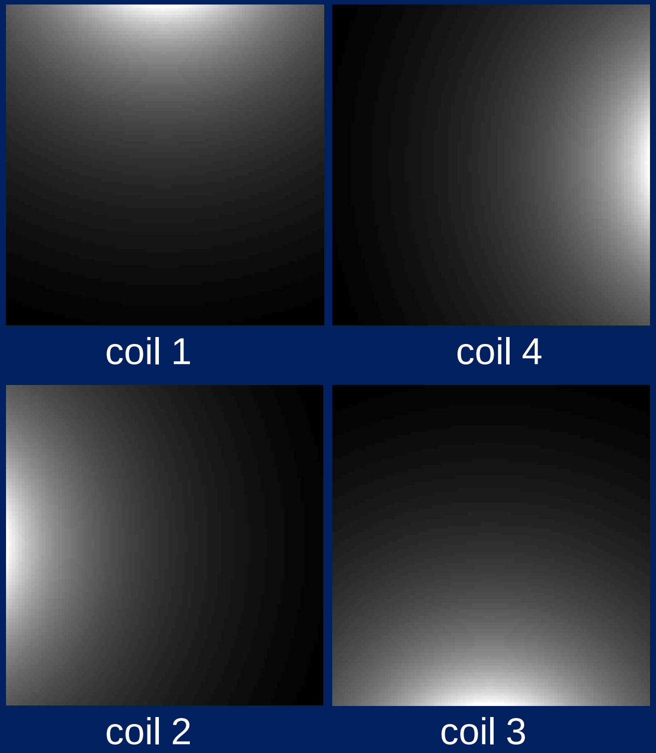
Results

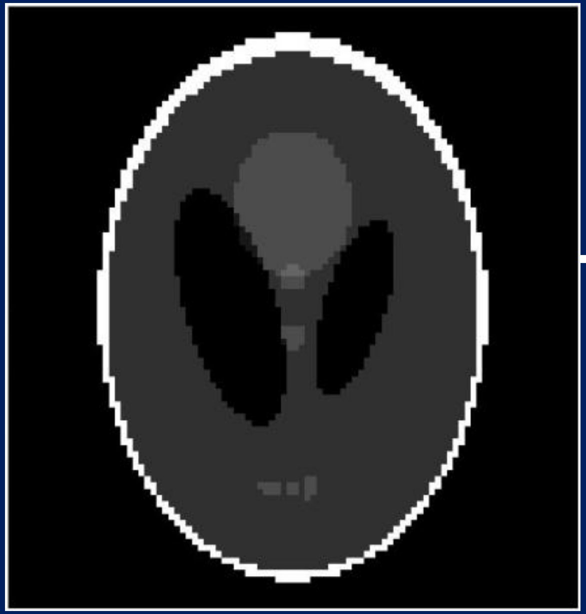
Noiseless multi-coil spatial frequency arrays are with

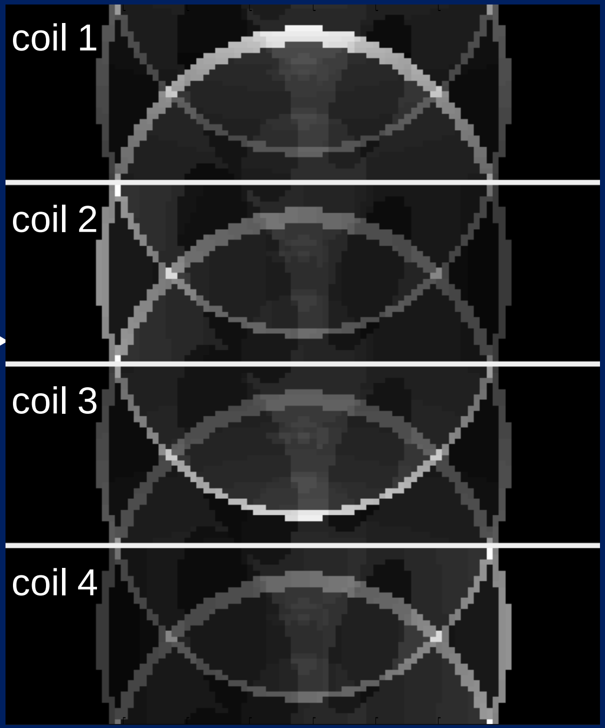
$$\Psi_{SI} = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi'_{RI} & \Psi_I \end{pmatrix}$$

$$\Psi_I = \Psi_R$$

$$\Psi_R = \begin{pmatrix} 1 & .33 & .11 & .33 \\ .33 & 1 & .33 & .11 \\ .11 & .33 & 1 & .33 \\ .33 & .11 & .33 & 1 \end{pmatrix} \quad \Psi_{RI} = \begin{pmatrix} 0 & -.11 & -.07 & -.11 \\ .33 & 0 & -.11 & -.07 \\ .42 & .26 & 0 & -.11 \\ .26 & .42 & .26 & 0 \end{pmatrix}$$

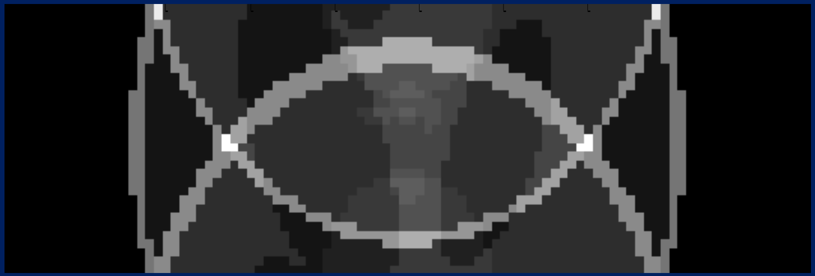






Bruce and Rowe: In progress.

Results
Magnitude



SENSE

SENSE-ITIVE

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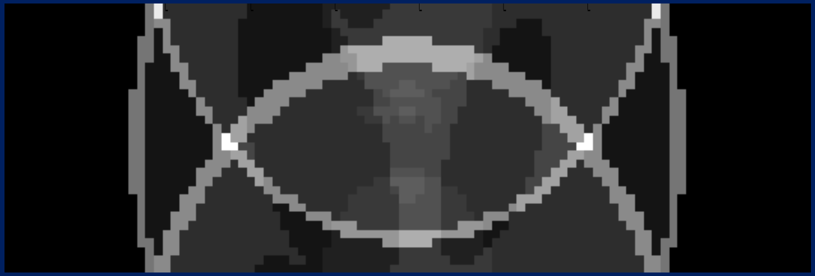
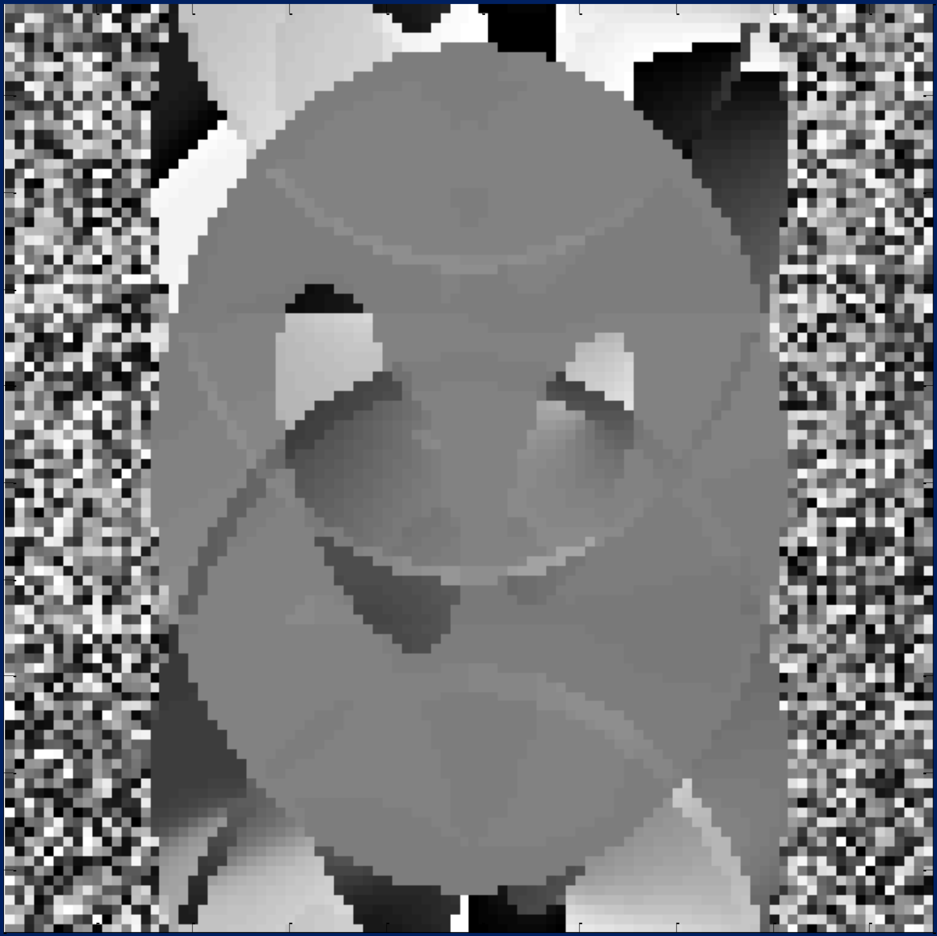
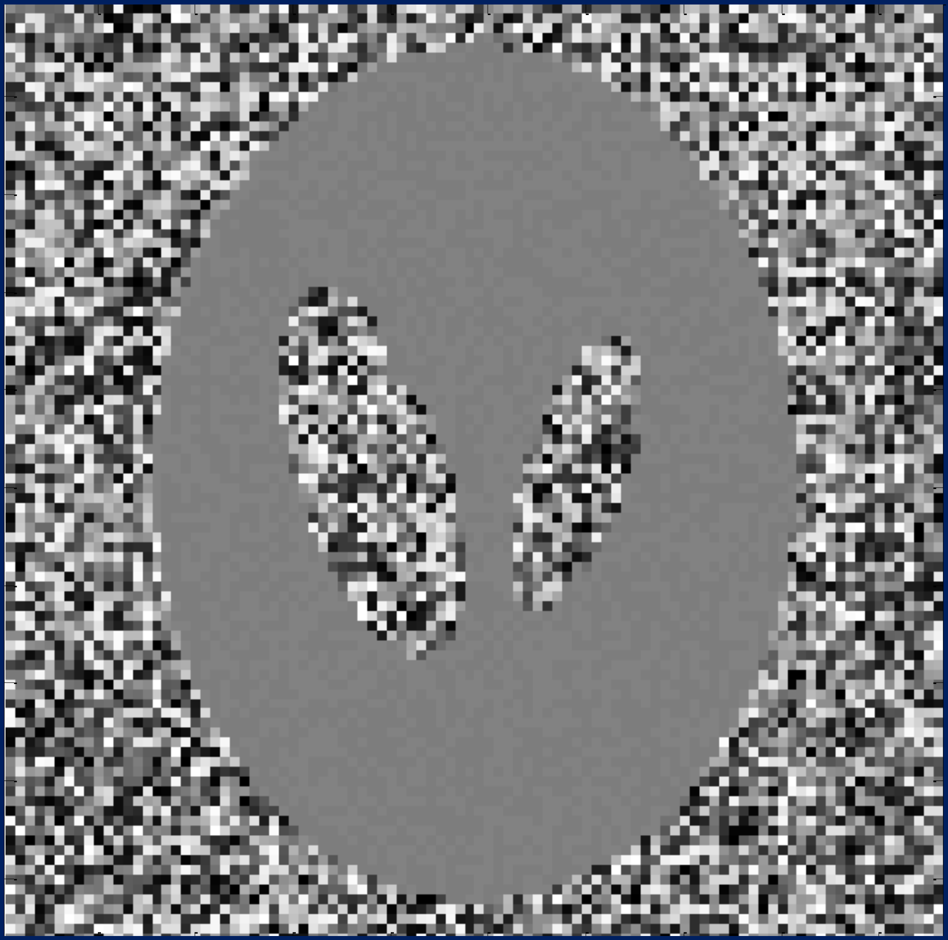
Results

Phase





SENSE



SENSE-ITIVE

Discussion

The SENSE image reconstruction method was described.

The SENSE reconstruction written with an isomorphism.

The covariance structure of complex SENSE described.

New SENSE-ITIVE method described with proper covariance.

Results of SENSE & SENSE-ITIVE reconstruction presented.

Ghosting present in SENSE magnitude and phase images.

Better reconstruction in SENSE-ITIVE reconstruction especially phase used for complex-valued time series activation.

Thank You

Acknowledgements:
Iain Bruce, Marquette University
Muge Karaman, Marquette University