FMRI Statistical Brain Activation From k-Space Data

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MR Physics

Bloch Equation

$$\frac{d\mathbf{M}}{dt} = \gamma\mathbf{M} \times \mathbf{B} + \frac{1}{T_1}(\mathbf{M}_0 - \mathbf{M}_z) - \frac{1}{T_2}\left(\mathbf{M}_x + \mathbf{M}_y\right)$$

Solving the Bloch equation yields transverse magnetization

$$M_{xy}(x,y,z,t)=M_{xy0}(x,y,z)e^{-t/T_2}e^{-i\gamma B_0 t}e^{-i\gamma\int[G_x(t)x+G_y(t)y+G_z(t)z]dt}$$

By Faraday's law of induction, the signal at time t is

$$s(t) = \int_x\int_y M(x,y)e^{-i2\pi k_x(t)x}e^{-i2\pi k_y(t)y}\; dx\; dy$$

Take the IFT of the spatial frequencies to get our magnetzation image.

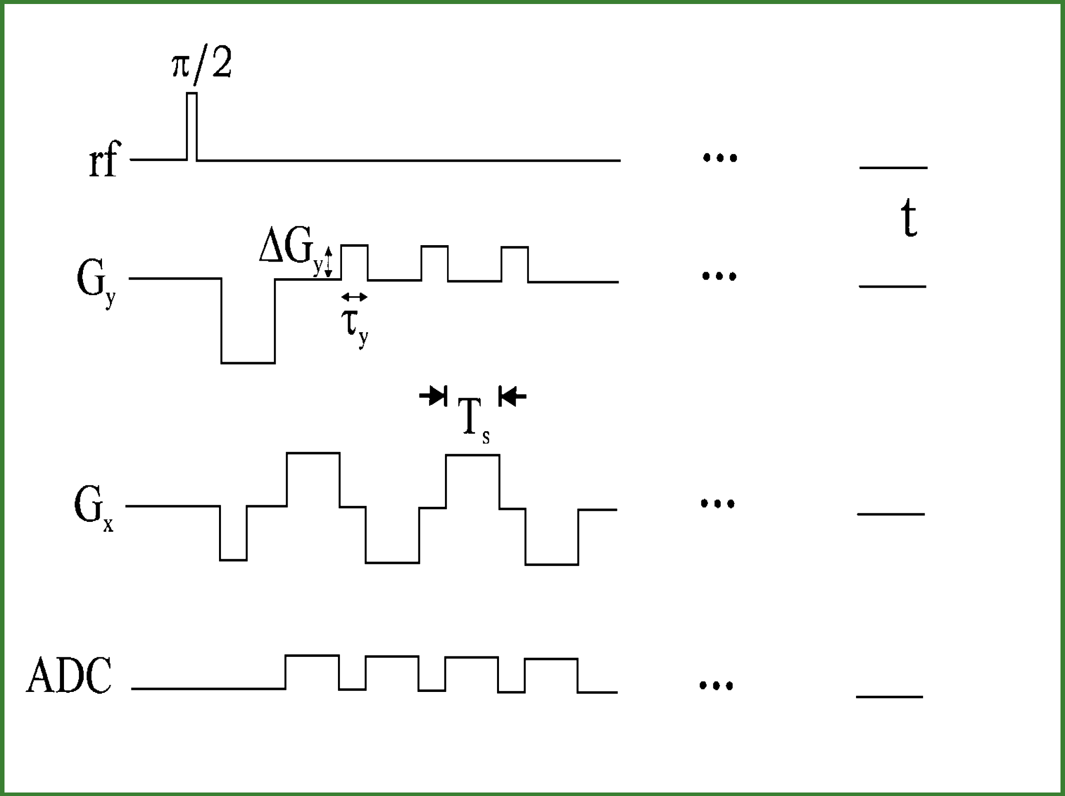
$$\hat{M}(x,y) = \sum_{k_x}\sum_{k_y}\hat{s}(k_x,k_y)e^{i2\pi k_x x}e^{i2\pi k_y y}$$

The magnetzation is related to our PSD and is complex valued.

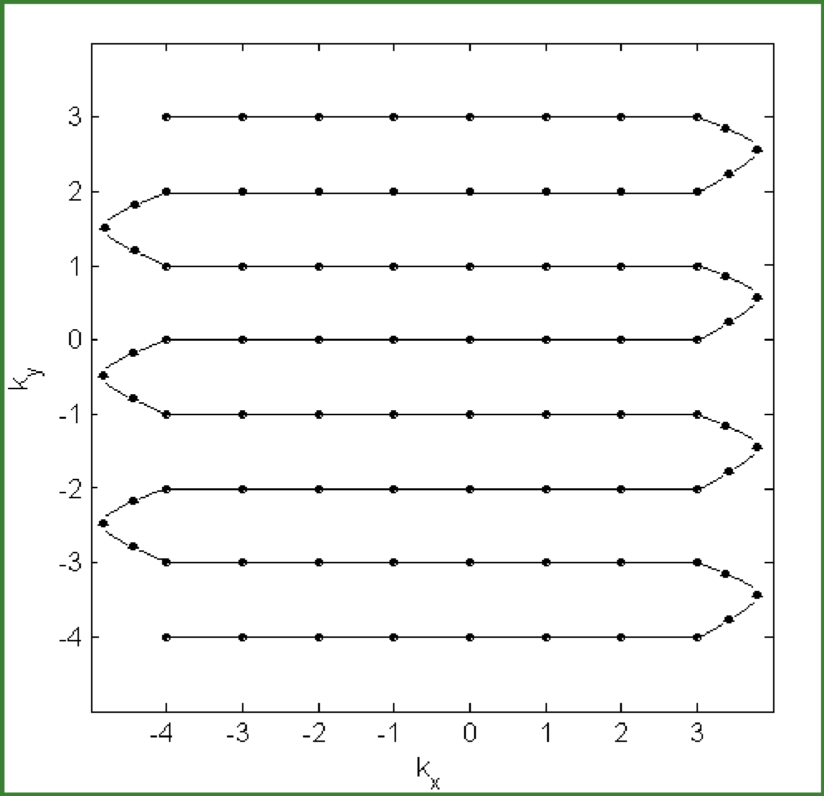
Pulse Sequence

How do we get signal that is spatial frequencies?

We apply G_x & G_y magnetic field gradients to encode then we measure the complex-valued DFT of the object.



(a) Gradient Echo-EPI Pulse Sequence



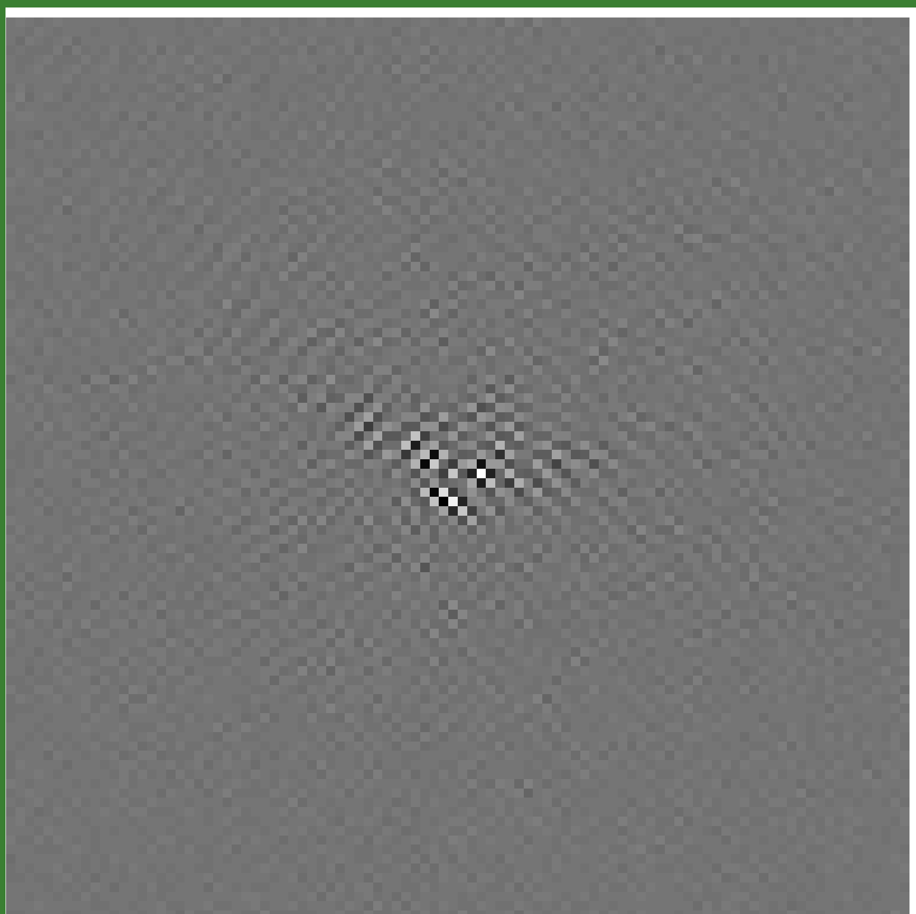
(b) Corresponding k-Space Trajectory

Kumar, Welti and Ernst: NMR Fourier Zeugmatography, J. Magn. Reson. 1975

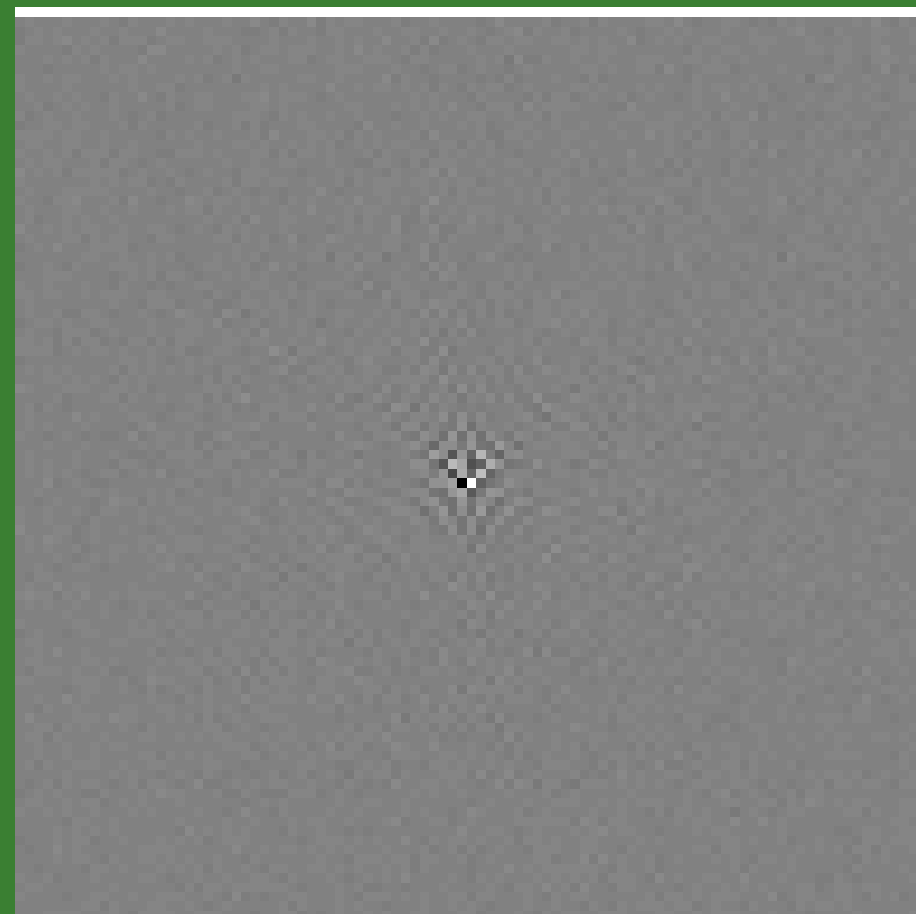
Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

Complex Image Reconstruction I

$S(k_x,k_y) = S_R(k_x,k_y) + iS_I(k_x,k_y)$, the complex-valued DFT of object



(a) real: 96×96



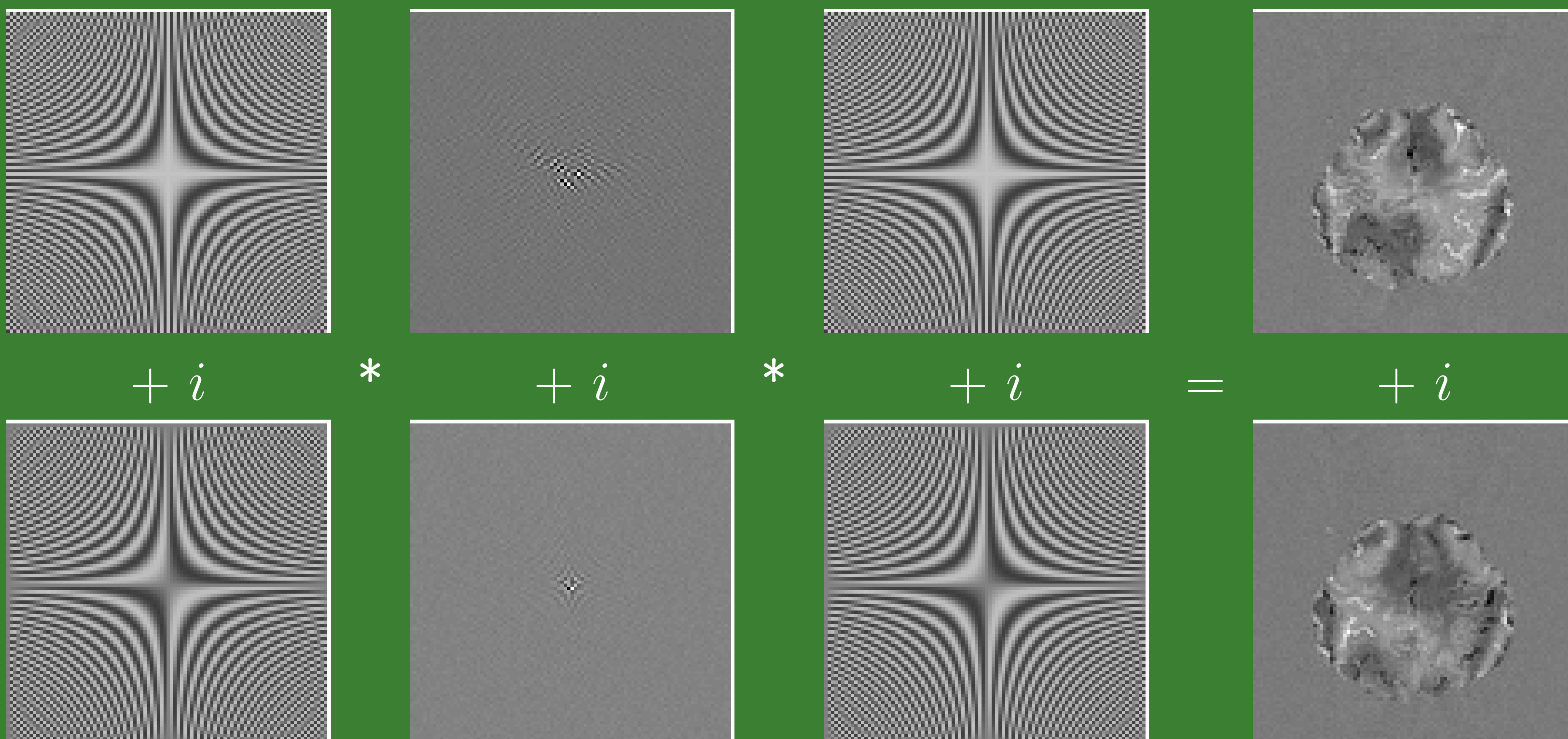
(b) imaginary: 96×96

FOV=192 mm, mat=96×96, vox=2 mm^3

Complex Image Reconstruction I

complex-valued 2D IFT

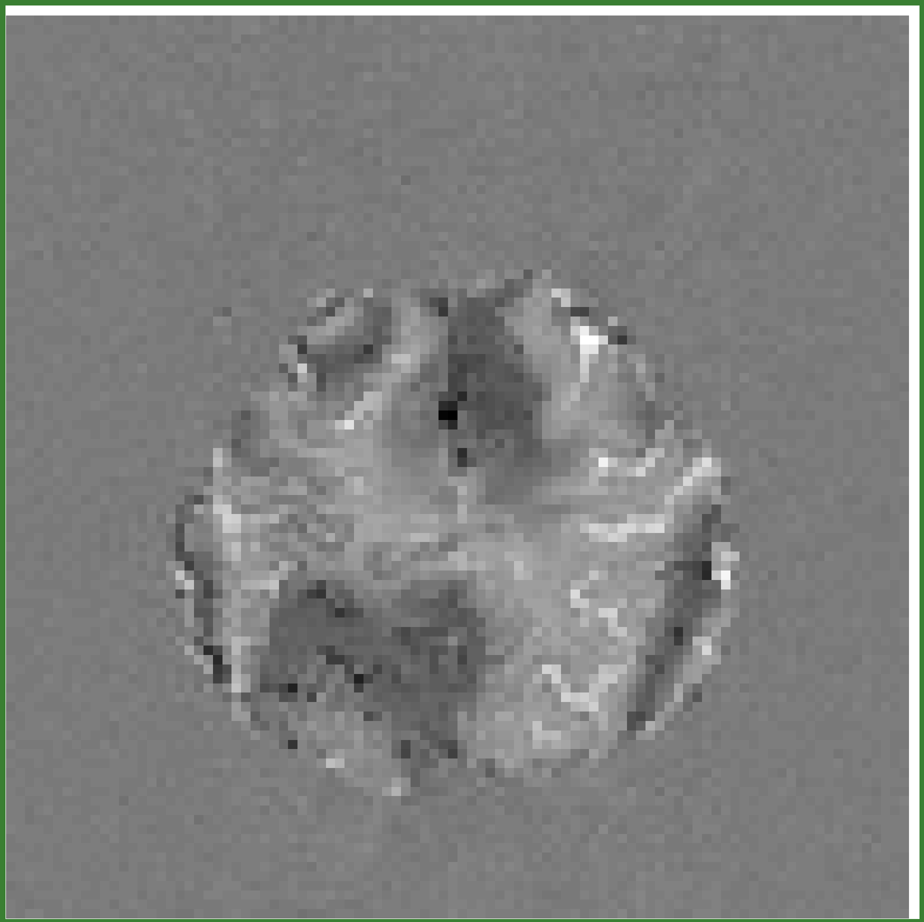
$$(\Omega_{yR} + i\Omega_{yI}) \quad * \quad (S_R + iS_I) \quad * \quad (\Omega_{xR} + i\Omega_{xI})^T \quad = \quad (M_R + iM_I)$$



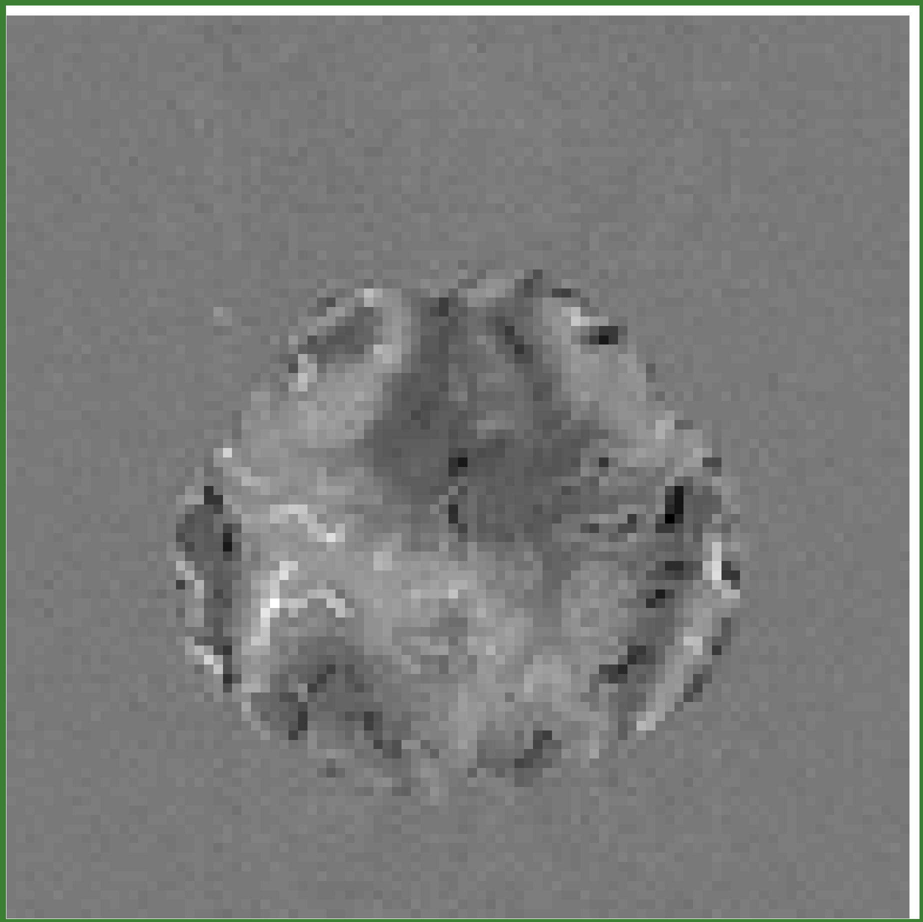
FOV=192 mm, mat=96×96, vox=2 mm^3

Complex Image Reconstruction I

Due to the imperfect Fourier encoding, the IFT reconstructed object is complex-valued, $M(x,y) = M_R(x,y) + iM_I(x,y)$.



(a) Real image, y_R

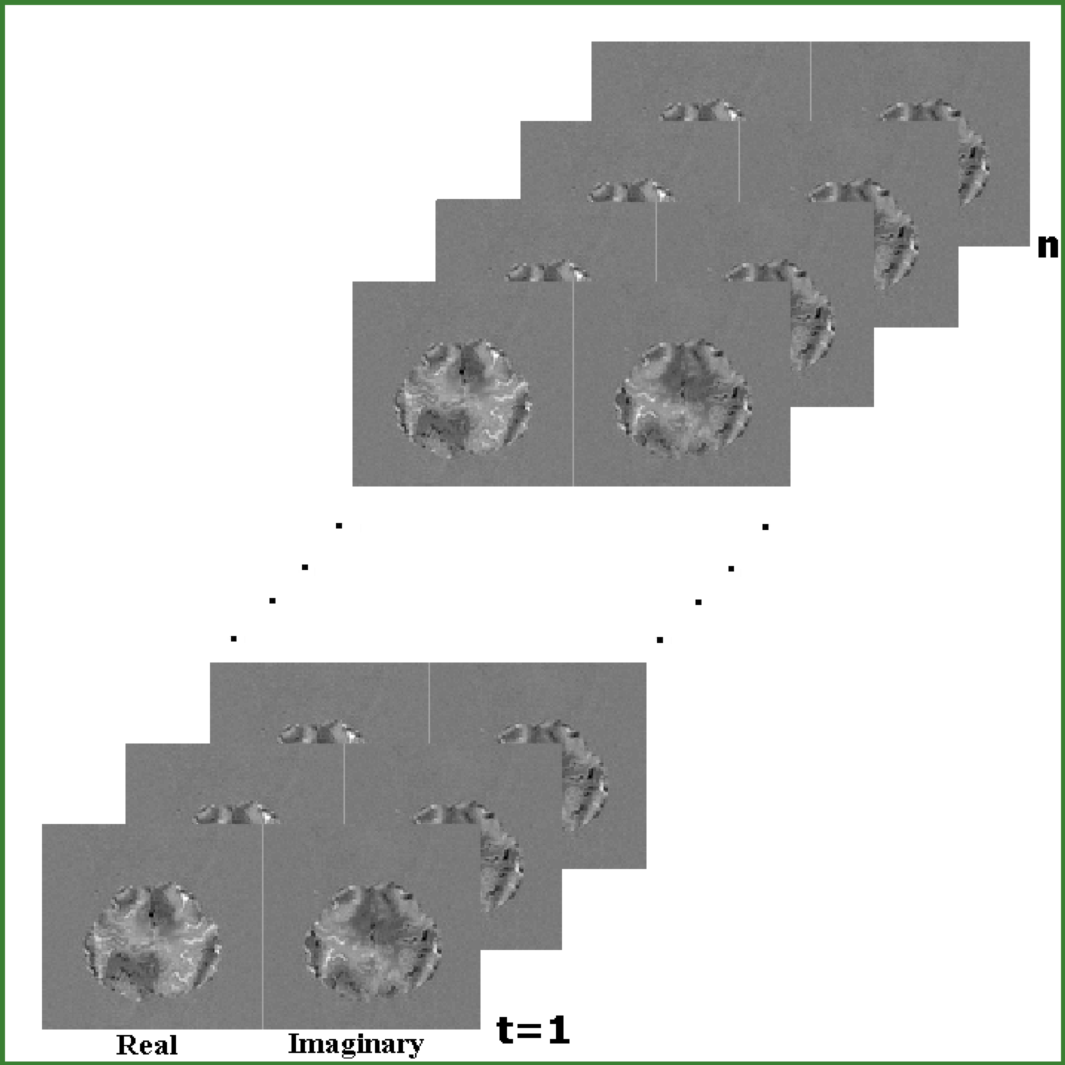


(b) Imaginary image, y_I

FOV=192 mm, mat=96×96, vox=2 mm^3

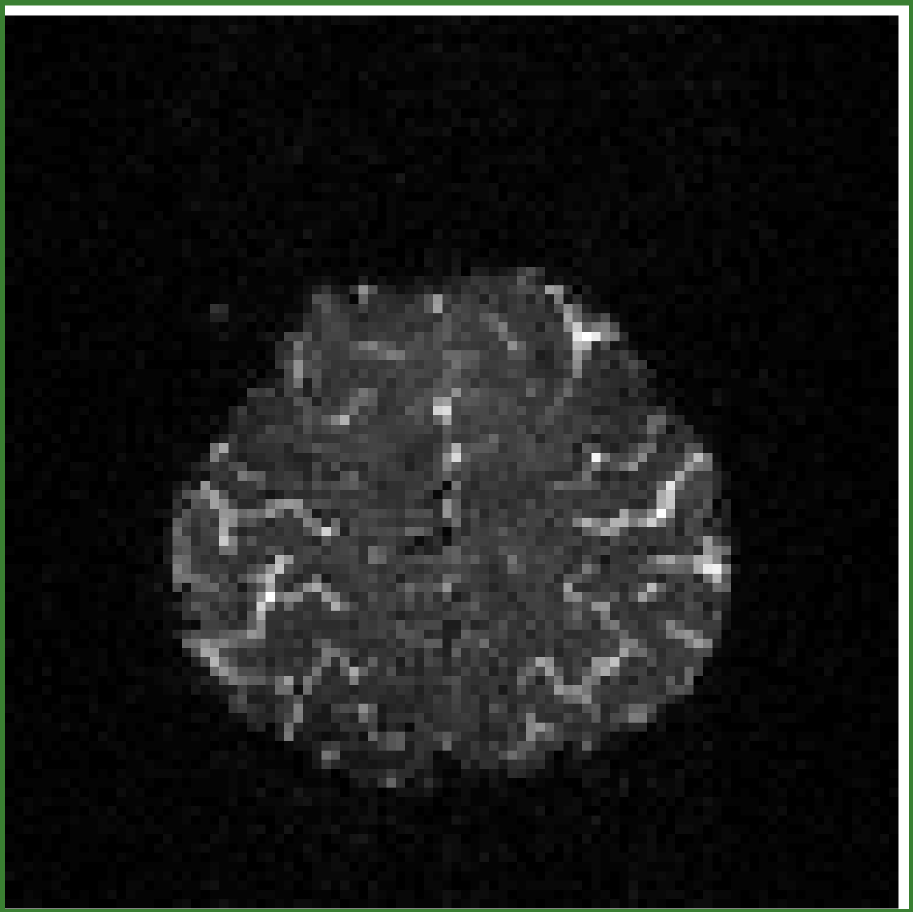
Complex Time Series Formation

This occurs over time in fMRI and results in complex-valued images and voxel time course observations, $y_t = y_{Rt} + iy_{It}$.



Complex Time Series Formation

Most fMRI studies transform from real-imaginary rectangular coordinates to magnitude-phase polar coordinates, $M(x,y) = R(x,y)e^{i\Phi(x,y)}$.

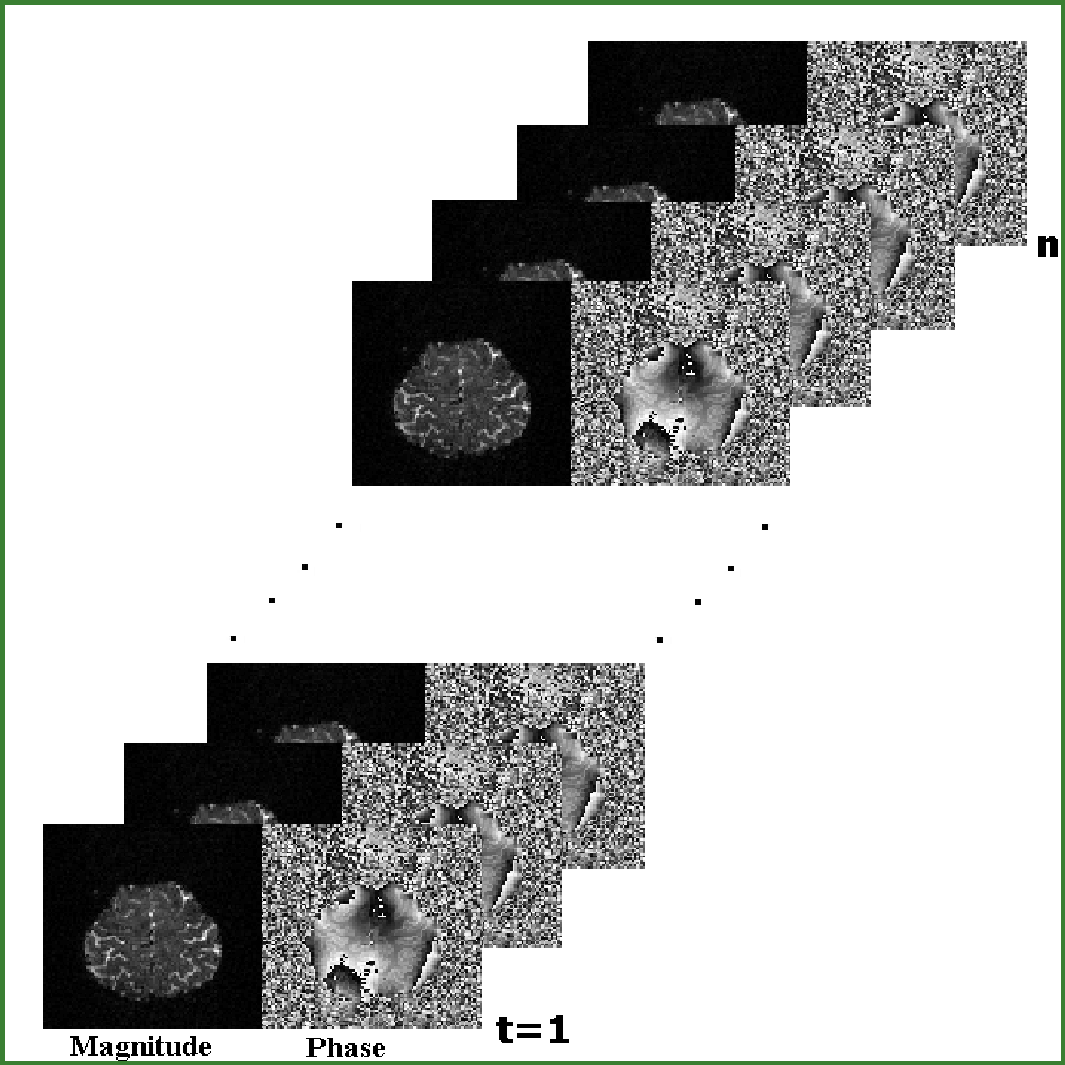


(a) Magnitude, $r_t = \sqrt{y_{Rt}^2 + y_{It}^2}$

(b) Phase, $\phi_t = \mathrm{atan}_4(y_{It}/y_{Rt})$

Complex Time Series Formation

Collect a sequence of these reconstructed images over time.
Form voxel time courses, $y_t = r_t e^{i\phi_t}$.



Complex Time Series Formation

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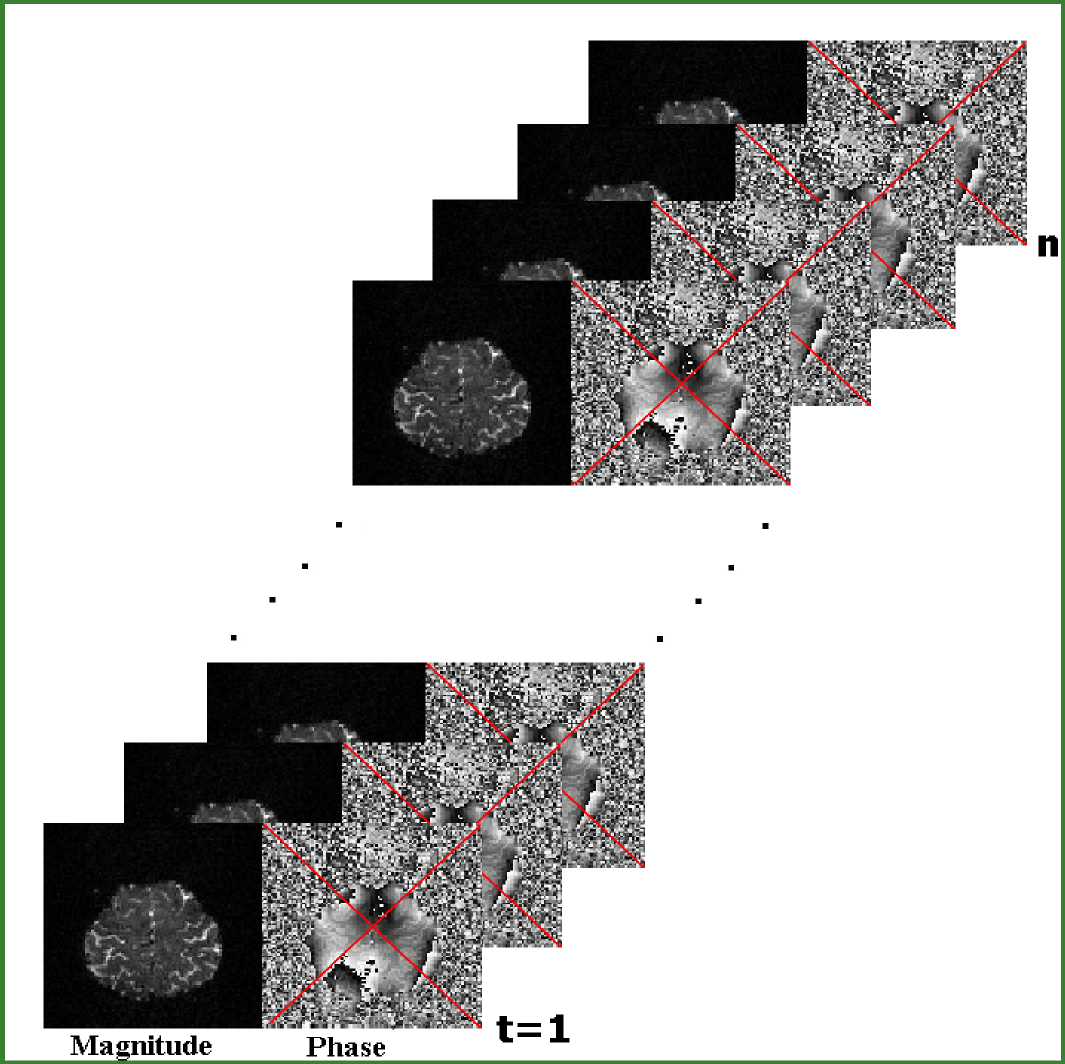
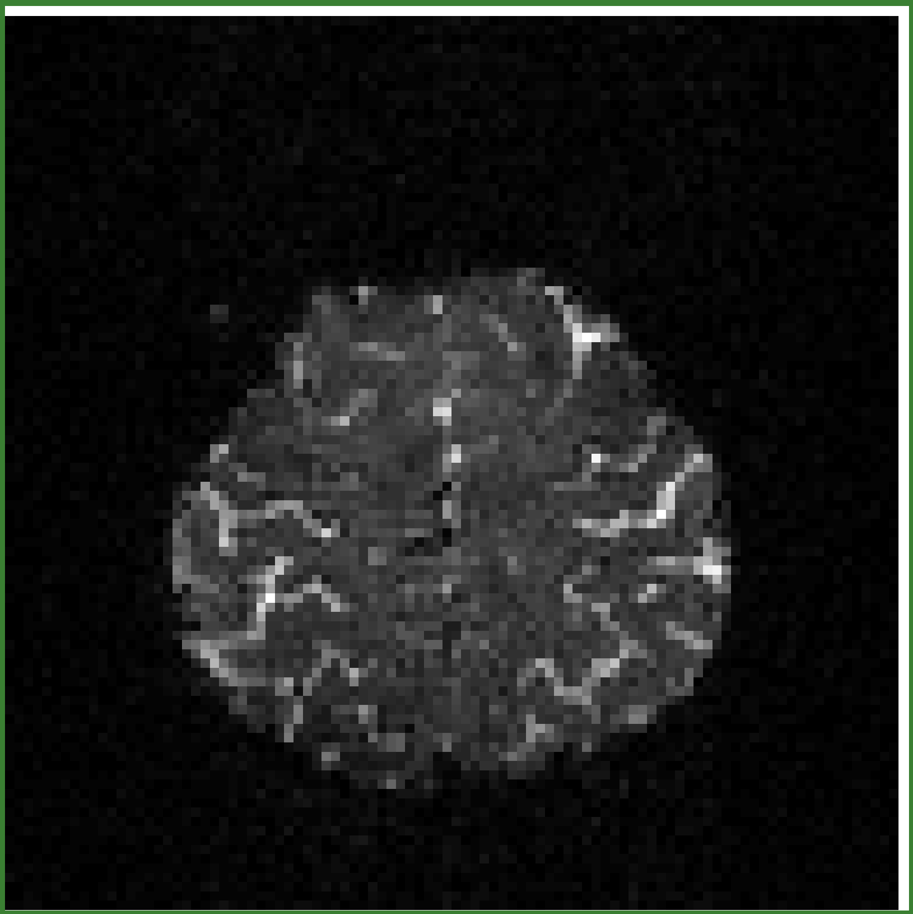
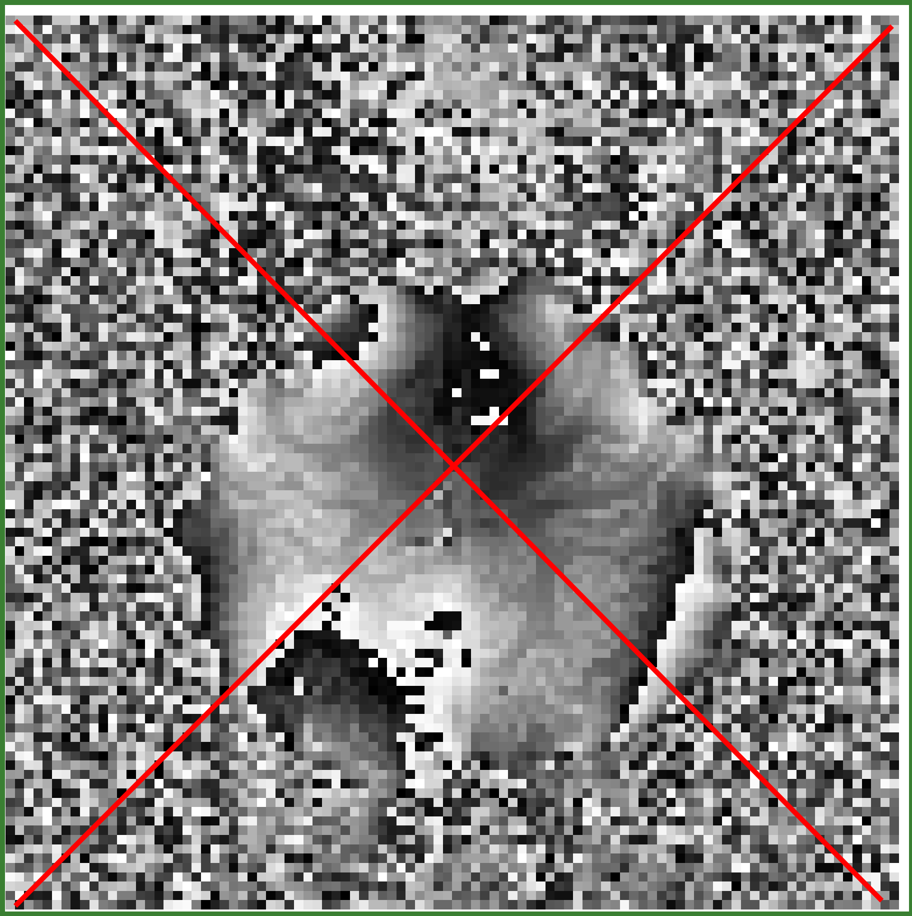


Complex Time Series Formation

Most fMRI studies transform from real-imaginary rectangular coordinates to magnitude-phase polar coordinates, $M(x,y) = R(x,y)e^{i\Phi(x,y)}$.



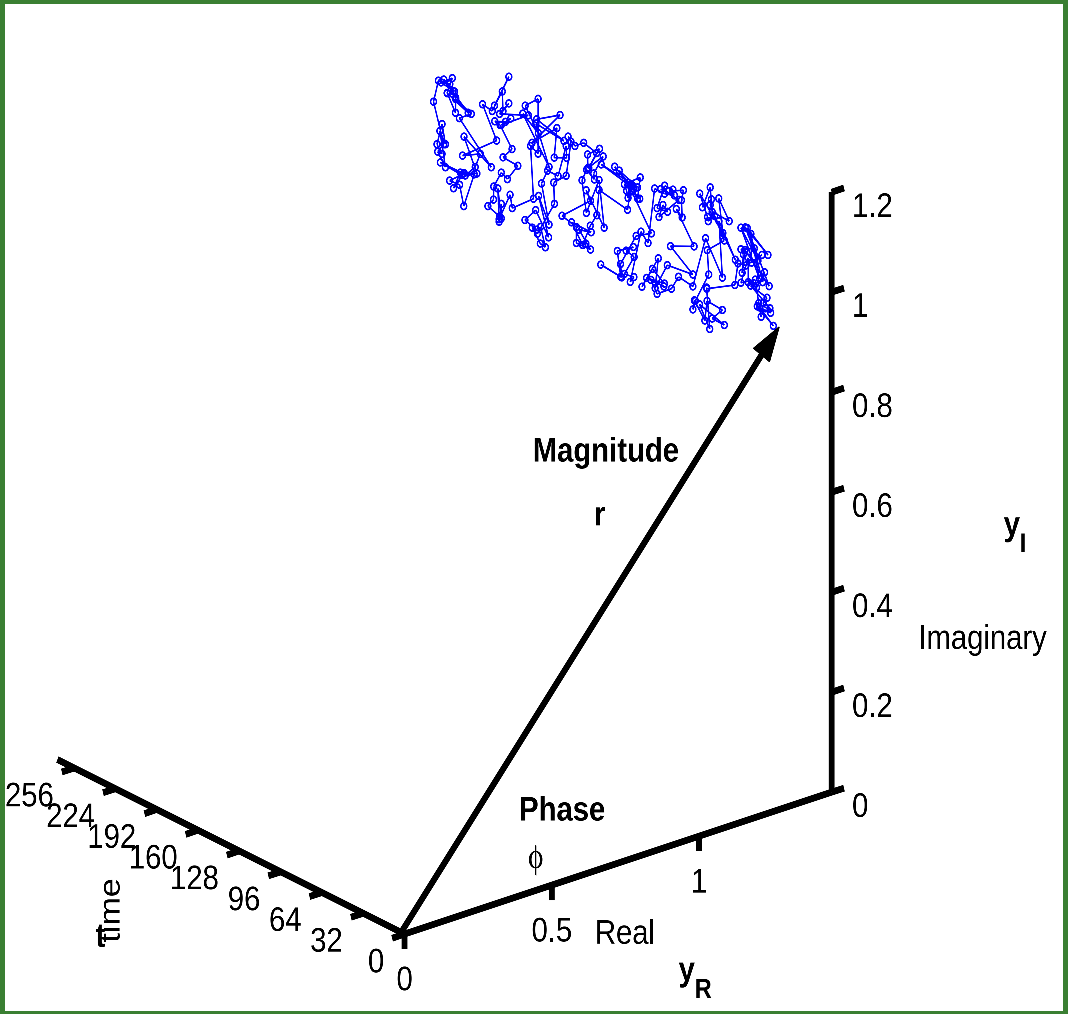
(a) Magnitude, $r_t = \sqrt{y_{Rt}^2 + y_{It}^2}$

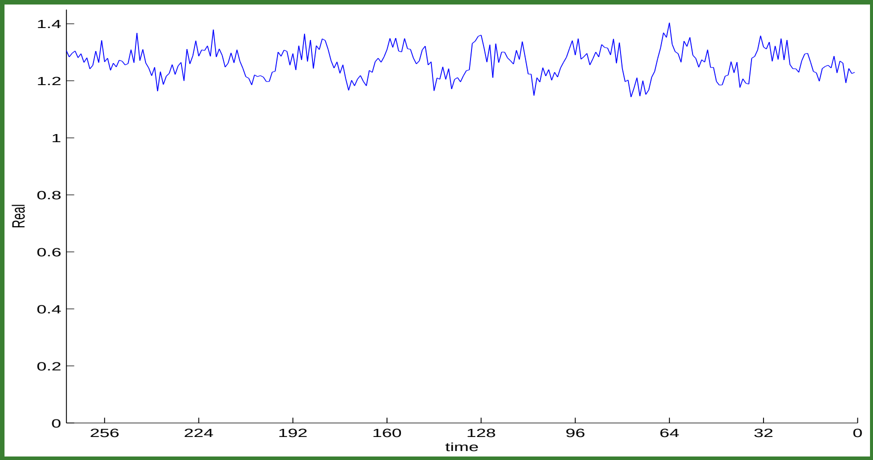


(b) Phase, $\phi_t = \text{atan}_4(y_{It}/y_{Rt})$

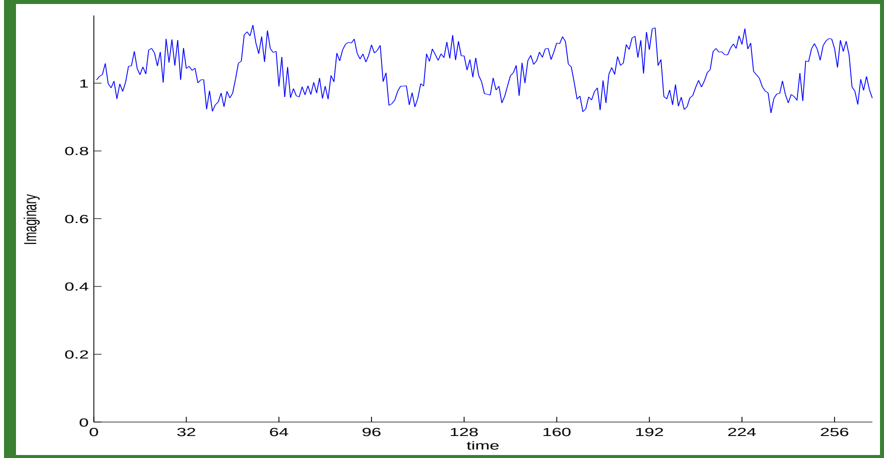
Complex Time Series Formation

Time series are complex-valued or bivariate with phase coupled means.





Real: Task related changes!



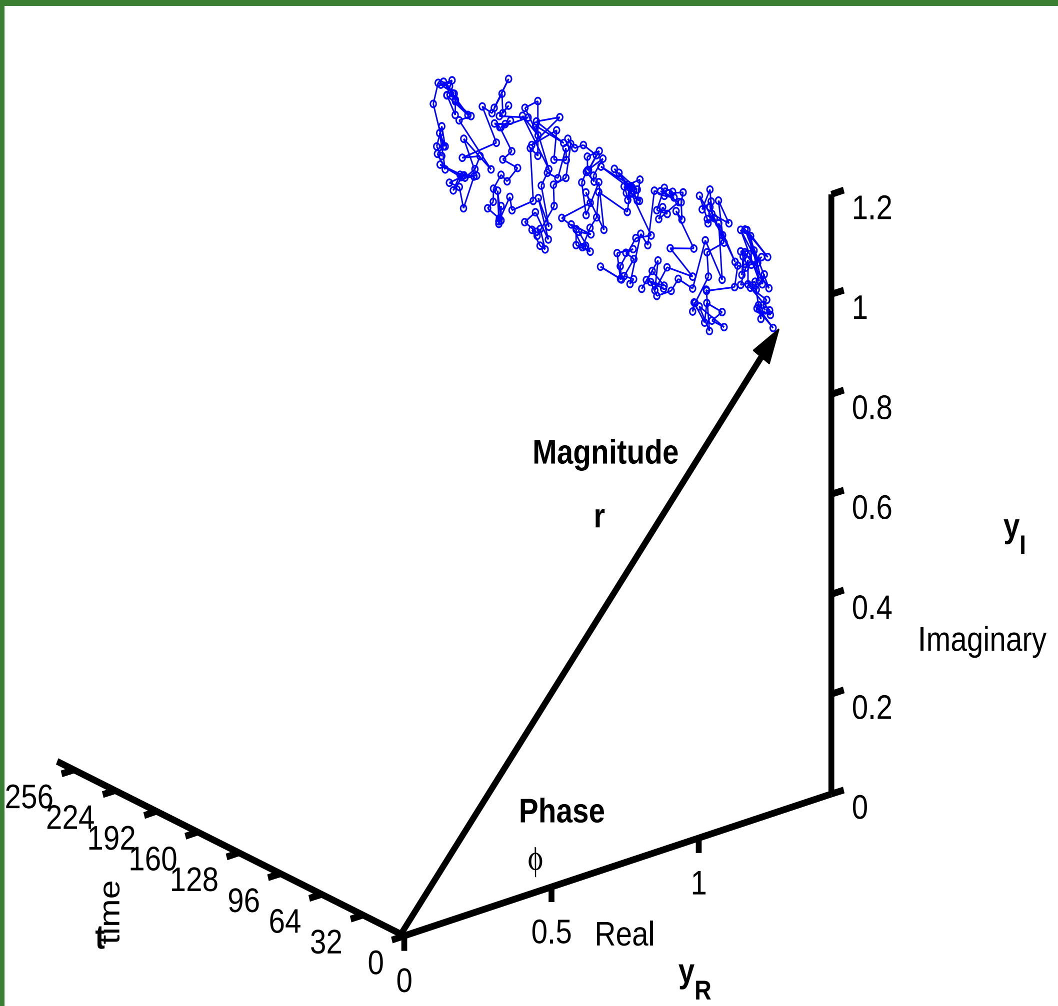
Imaginary: Task related changes!

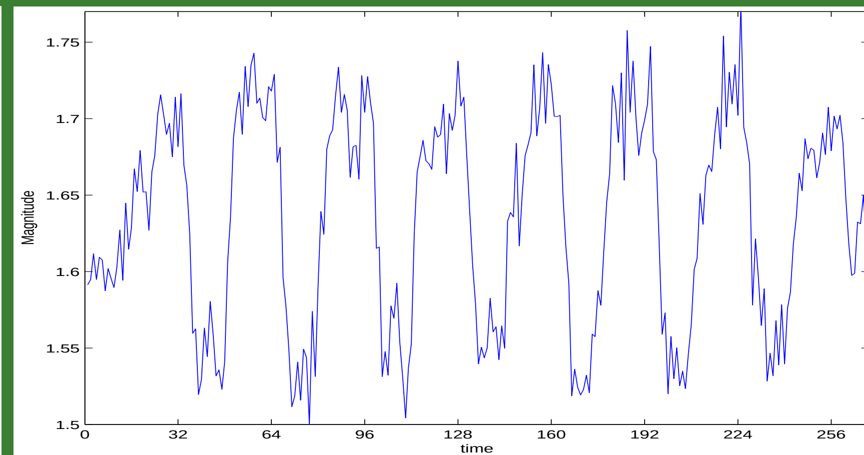
The y_R and y_I time courses have related vector length info!

This is a time series from actual human experimental data!

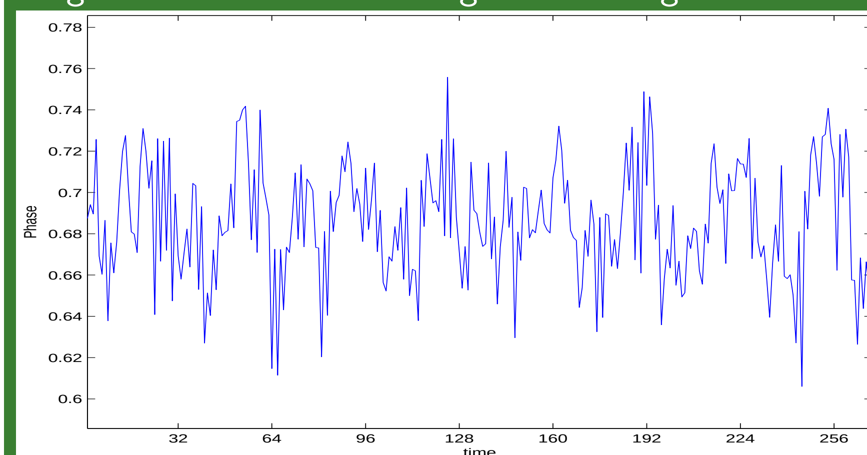
Complex Time Series Formation

Time series are complex-valued or bivariate with phase coupled means.





Magnitude: Task related magnitude changes!



Phase: Task related phase changes!

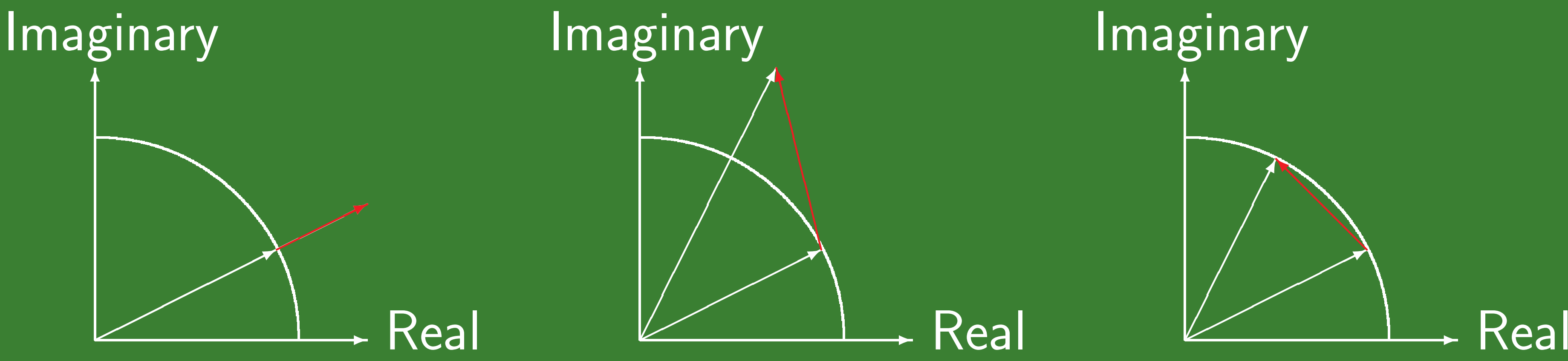
Real-Imaginary or Magnitude-Phase time courses have all info!

Recent work indicates that phase time courses may exhibit TRPCs

Menon, 2002; Hoogenrad et al., 1998; Borduka et al., 1999; Chow et al., 2006;

Complex Time Series Activation

Block-designed experiment: Off-On-Off-...-On-Off task



- ➢ Complex Magnitude w/ Constant Phase (CP) Activation[1,2]
- ➢ Complex Magnitude &/or Phase (CM) Activation[3]
- ➢ Real Magnitude-Only (MO/UP) Activation (Discard Phase)[4,5]
- ➢ Real Phase-Only (PO) Activation (Discard Magnitude)[6]

[1]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[2]Rowe: NeuroImage 25:1124-1132, 2005a.
[3]Rowe: NeuroImage, 25:1310-1324, 2005b.
[4]Bandettini et al.: Magn Reson Med, 30:161-173, 1993.
[5]Rowe and Logan: NeuroImage, 24:603-606, 2005.
[6]Rowe, Meller, Hoffmann: J Neurosci Meth, 161:331-341, 2007.

Complex Time Series Activation

$$\begin{pmatrix} y_{Rt} \\ y_{It} \end{pmatrix} = \begin{pmatrix} \rho_t \cos\theta_t \\ \rho_t \sin\theta_t \end{pmatrix} + \begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} , \qquad \begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} \sim N(0, \Sigma) \ .$$

y_{Rt} and y_{It} are the real and imaginary observations at time t

η_{Rt} and η_{It} are the real and imaginary noise terms at time t

➢ Magnitude activation in complex data
$\rho_t = x_t'\beta = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t}$

I:	Magnitude-only[4,5]	$\theta_t = \theta_{t'}$
II:	Constant Phase[1,2]	$\theta_t = \theta$
III:	Linear Phase[3]	$\theta_t = \gamma_0 + \gamma_1 u_{1t} + \cdots + \gamma_{q_2} u_{q_2 t}$

➢ Phase-only Activation in complex data, $\rho_t = \rho_{t'}$
$\theta_t = \gamma_0 + \gamma_1 u_{1t} + \cdots + \gamma_{q_2} u_{q_2 t}$

[1] Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[2] Rowe: NeuroImage 25:1124-1132, 2005a.
[3] Rowe: NeuroImage, 25:1310-1324, 2005b.
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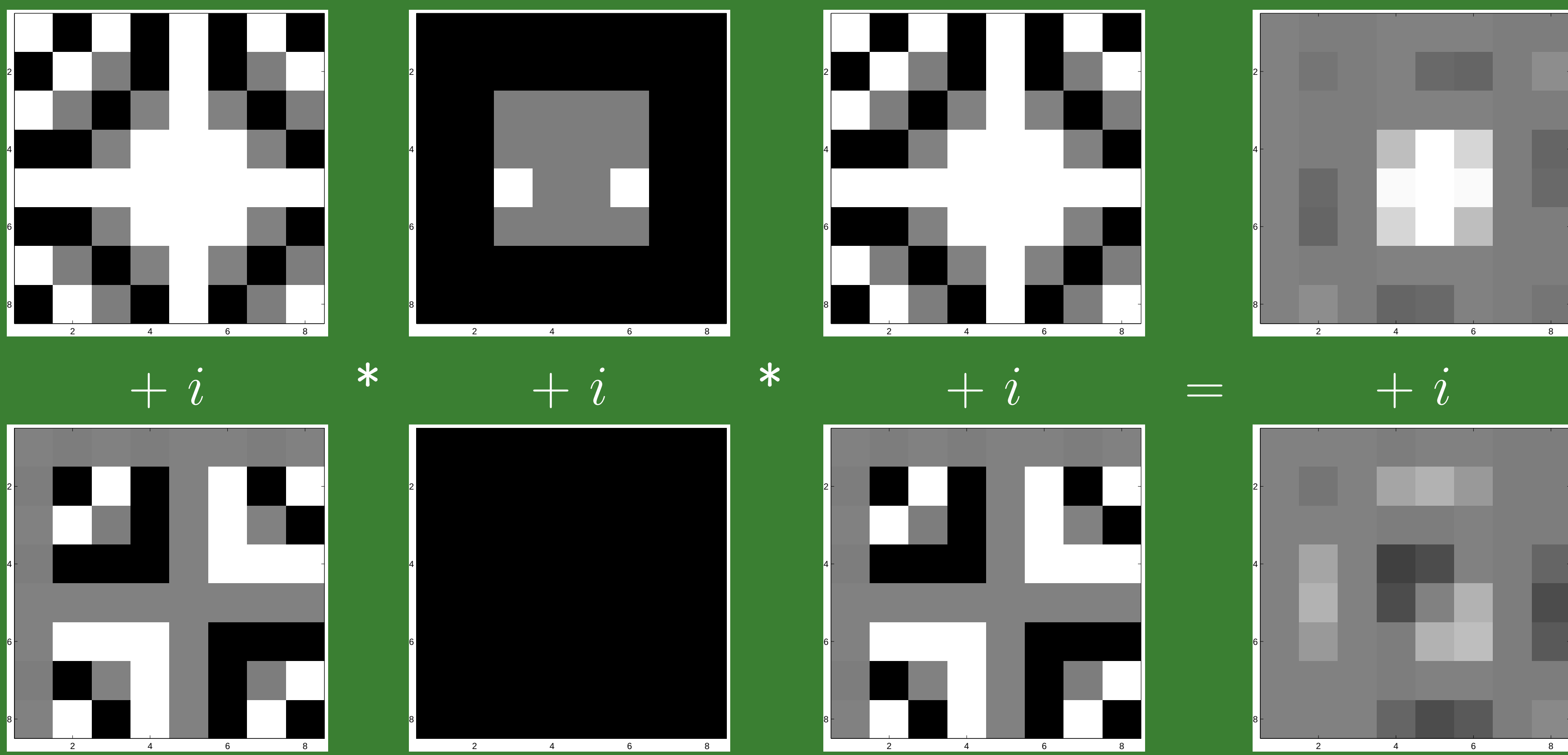
Complex Image Reconstruction II

complex-valued 2D F FT

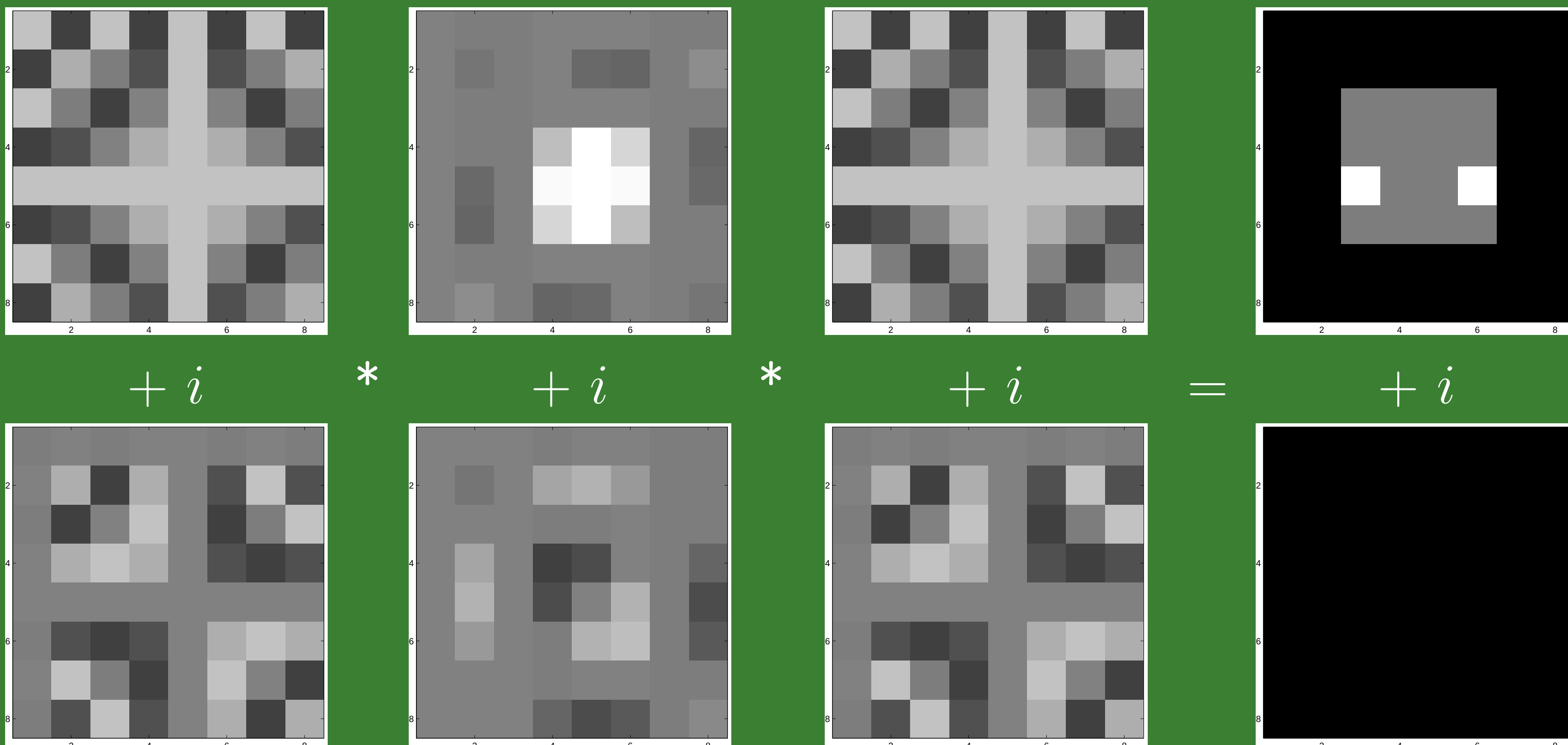
$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) \; * \; (M_R + iM_I) \; * \; (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T \; = \; (S_R + iS_I)$$



Complex Image Reconstruction II

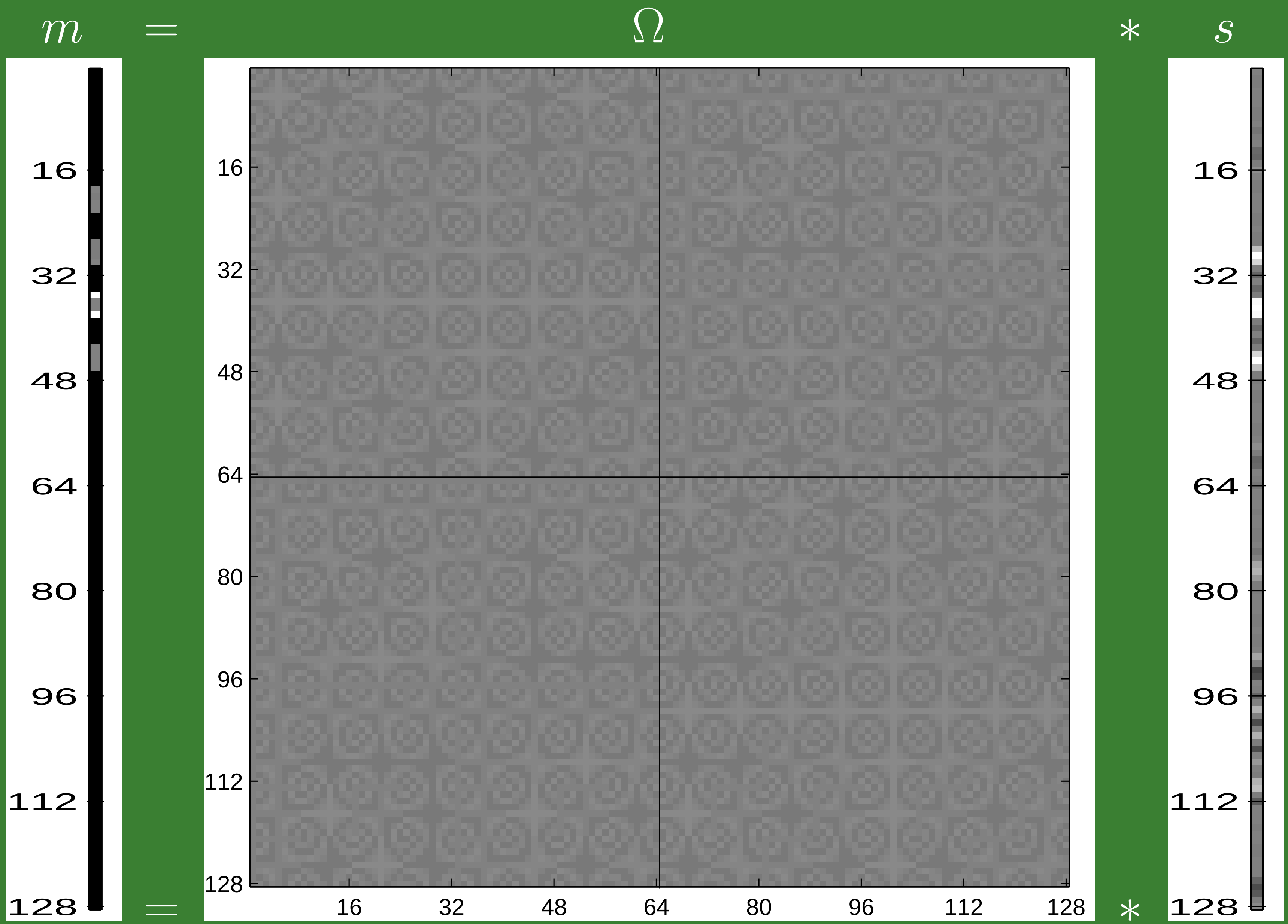
complex-valued 2D I FT

$$(\Omega_{yR} + i\Omega_{yI}) \;\; * \;\; (S_R + iS_I) \;\; * \;\; (\Omega_{xR} + i\Omega_{xI})^T \;\; = \;\; (M_R + iM_I)$$



Stack rows of reals then imaginaries for s. Ω from Ω_{xR}, Ω_{xI}, Ω_{yR}, Ω_{yI}.

Complex Image Reconstruction II



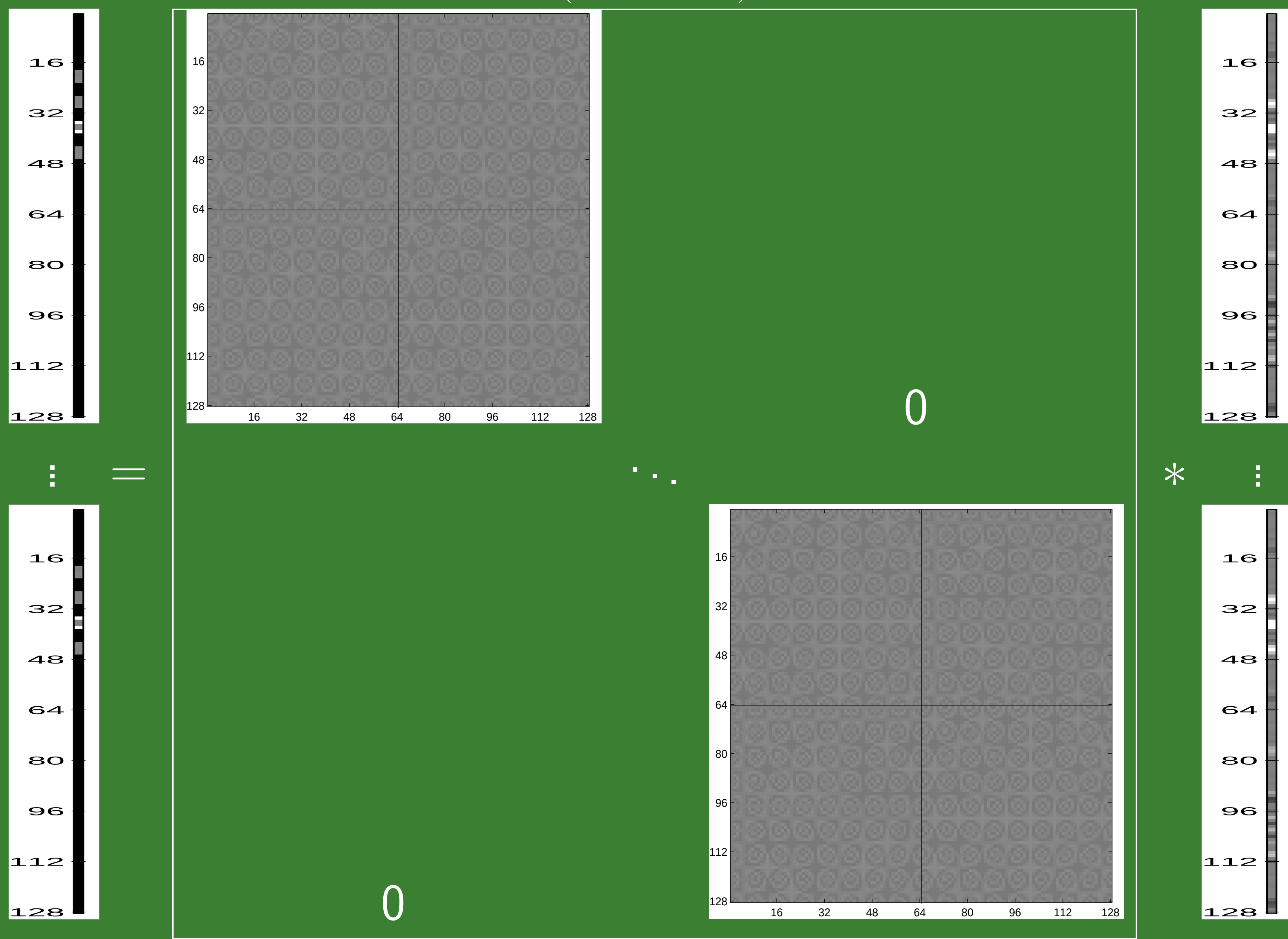
Unstack m to real then imaginary rows to form image.

Rowe, Nencka, Hoffmann: JNMeth, 2007.

Complex Statistical Activation Methods Part II

Can stack these spatial frequency vectors for each of the n TRs

$$m \quad = \quad (I_n \otimes \Omega\ ) \quad * \quad s$$



Complex Statistical Activation Methods Part II

Now convert the m that has Real/Imaginaries stacked for images to y that has the Real/Imaginaries stacked for voxels

$$y = P * m$$

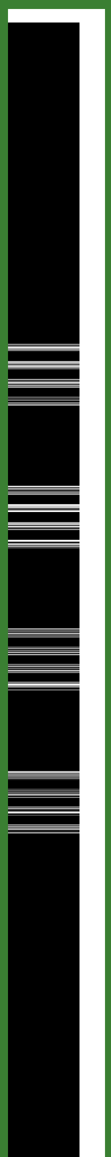
$$y = P * (I_n \otimes \Omega)s$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix} = P * \begin{pmatrix} \Omega & & 0 \\ & \ddots & \\ 0 & & \Omega \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix}$$

Complex Statistical Activation Methods Part II

Amazingly can convert from m to y via a permutation matrix P.

$$y \quad = \quad P \qquad (I \otimes \Omega)s$$



$=$

Complex Statistical Activation Methods Part II

But of course the y's are obs in each voxel for complex activation

$$\begin{pmatrix} y_{1R} \\ y_{1I} \\ \hline \vdots \\ \hline y_{pR} \\ y_{pI} \end{pmatrix} = \begin{pmatrix} A_1 X\beta_1 \\ B_1 X\beta_1 \\ \hline \vdots \\ \hline A_p X\beta_p \\ B_p X\beta_p \end{pmatrix} + \begin{pmatrix} \eta_{1R} \\ \eta_{1I} \\ \hline \vdots \\ \hline \eta_{pR} \\ \eta_{pI} \end{pmatrix}$$

Estimate β's,θ's, and σ^2's from my complex models
Compute activation from my complex activation models.

$\rho_t = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t}$

I:	Constant Phase	$A_j = I_n \cos\theta_j$ and $B_j = I_n \sin\theta_j$
II:	Unrestricted Phase	$A_{jt} = \cos\theta_{jt}$ and $B_{jt} = \sin\theta_{jt}$
III:	Linear Phase	$\theta_{jt} = \gamma_{0j} + \gamma_{1j} t + \cdots + \gamma_{q_2 j} u_{q_2 t}$

j is voxel, $j = 1, ..., p$
t is time, $t = 1, ..., n$

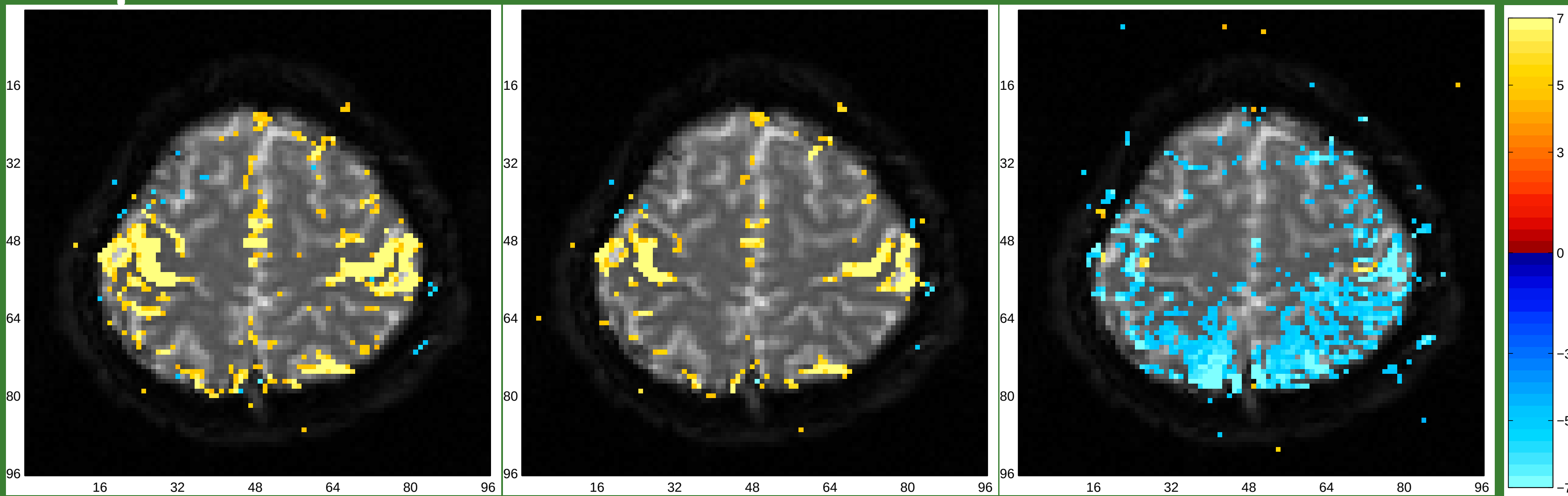
Activations in Human Experimental Data

Imaging Parameters:
1.5T GE Signa
5 axial slices of 128x128
96 acq.-2.0833mm^2
128 recon.-1.5625mm^2
FOV =20cm
TR=1000ms
TE=47ms
FA=90°

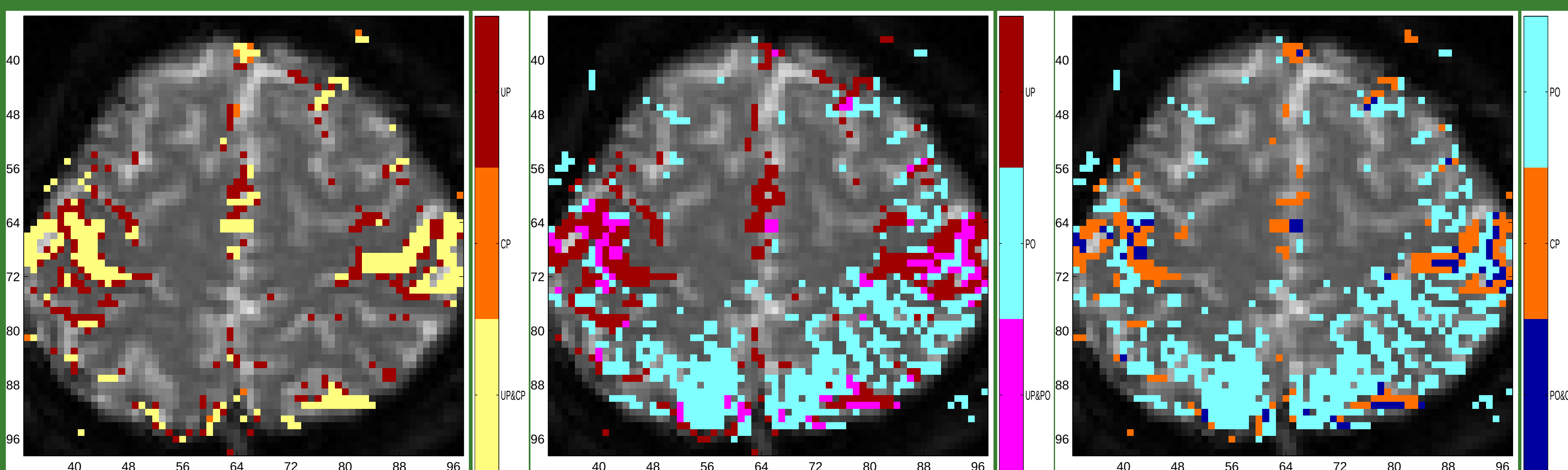
Task:
RH male Bilateral sequential finger tapping light triggered
Block design
16 off + 8$\times$(16on+16off);

Rowe and Logan: NeuroImage, 23:1078-1092, 2004.

z Maps: 5% Bonferroni Threshold



(a) UP/MO (b) CP (c) PO



(d) UP/MO & CP (e) UP/MO &PO (f) CP & PO

[1]Rowe and Logan: NeuroImage, 23:1078-1092, 2004. [2]Logan and Rowe: NeuroImage, 22:95-108, 2004.

Complex Statistical Activation Methods Part II

Can also do directly from k-space data

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = P^{-1}\left(I_n \otimes \Omega^{-1}\right) \begin{pmatrix} X\beta_1 \cos\theta_1 \\ X\beta_1 \sin\theta_1 \\ \vdots \\ X\beta_p \cos\theta_p \\ X\beta_p \sin\theta_p \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{pmatrix}$$

$$\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = \underbrace{P^{-1}\left[I_n \otimes \Omega^{-1}(I_2 \otimes X)\right]}_{\text{Known}} \begin{pmatrix} \beta_1 \cos\theta_1 \\ \beta_1 \sin\theta_1 \\ \vdots \\ \beta_p \cos\theta_p \\ \beta_p \sin\theta_p \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \vdots \\ \epsilon_n \end{pmatrix}$$

Estimate β's,θ's (same as before), and γ^2's for ϵ's

Compute activation from previous complex activation models.

Can model actual measurements s with original error structure ϵ.

Statistical Implications

Showed that $m = \Omega s$.

$$\begin{array}{llll} \text{If} & \text{E}(s) = s_0 & \text{and} & \text{var}(s) = \Gamma \\ \text{then} & \text{E}(m) = \Omega s_0 & \text{and} & \text{var}(m) = \Omega \Gamma \Omega^T \end{array}$$

And vise versa from multivariate statistics.

There are pre-processing adjustments performed on the data in k-space to correct the signal.

This is done pre-reconstruction and pre-magnitude-only time series.

This is done to your data and you may not know it.
-Activation and connectivity implications

Statistical Implications

These adjustments can be written as linear operations, A.

If	$\mathrm{E}(s) = s_0$	and	$\mathrm{var}(s) = \Gamma$
and	$s_A = As$		
then	$\mathrm{E}(s_A) = As_0$	and	$\mathrm{var}(As) = A\Gamma A^T$
and	$\mathrm{E}(m_A) = \Omega As_0$	and	$\mathrm{var}(m) = \Omega A\Gamma A^T\Omega^T$

These adjustments modify the signal and noise.

These adjustments induce correlations between voxels (connectivity).

We should account for these adjustments (simpler model in k-space).

Nencka & Rowe OHBM 2007.

Discussion

Described complex k-space measurement.

Described complex image reconstruction.

Described complex fMRI activation.

Described relationship between voxel and k-space measurements.

Showed activation from k-space measurements.

Postulated implications of adjustments to k-space measurements.

Further research is needed...

Thank You.