Intrinsic voxel correlation in fMRI

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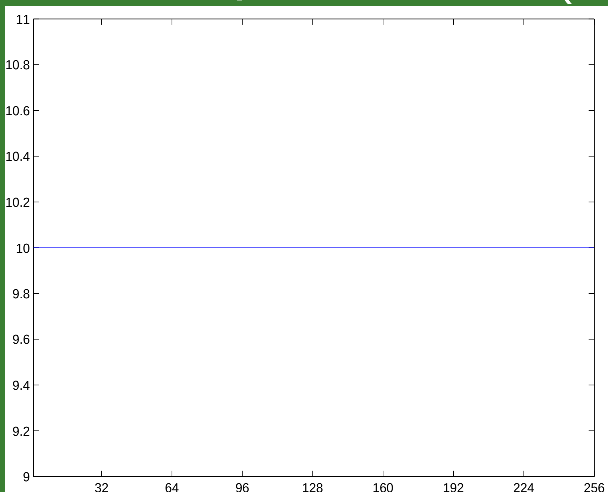
Outline

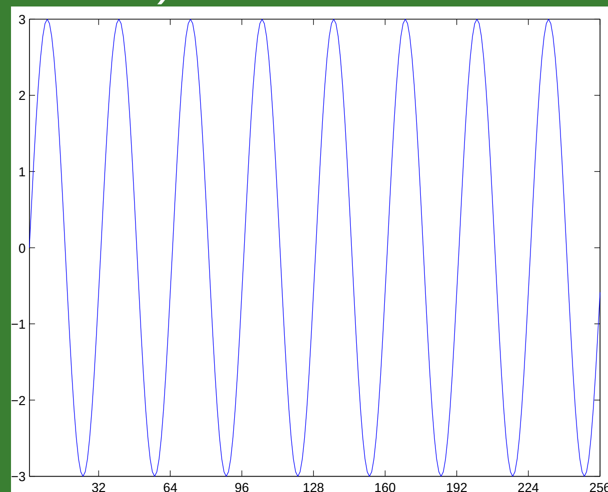
- ➢ Complex Image Reconstruction Part I
  - Complex voxels from complex k-space measurements
- ➢ Complex Statistical Activation Method Part I
  - Real-Valued: Magnitude-Only, Phase-Only
  - Complex-Valued: Magnitude & Phase
- ➢ Complex Image Reconstruction Part II
  - Relating voxels to k-space measurements
  - Statistical Implications
- ➢ Complex Statistical Activation Methods Part II
  - fMRI activation directly from k-space measurements
- ➢ Discussion

Complex Image Reconstruction Part I

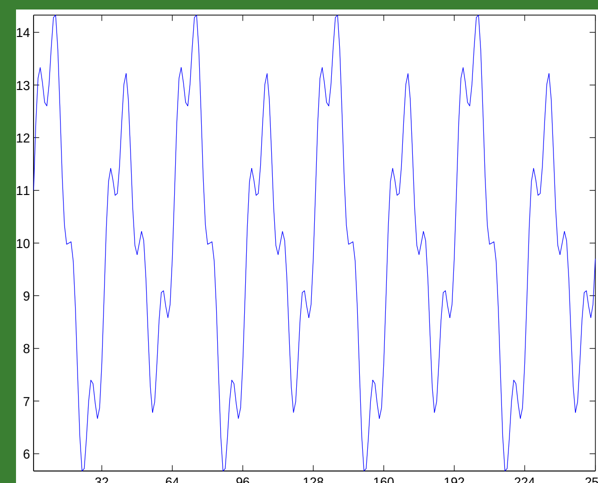
The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)



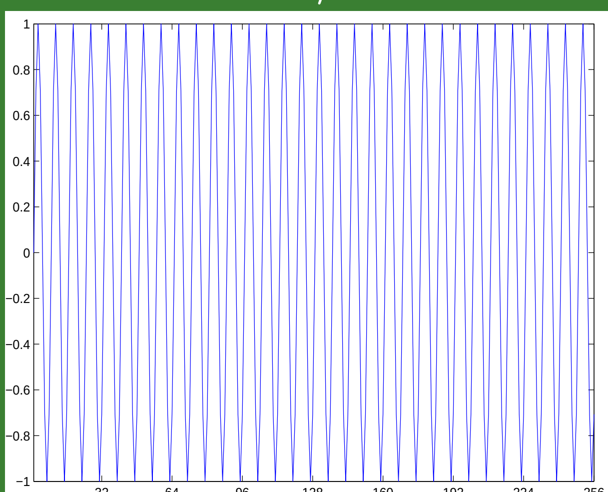
$10, \cos 0/512$ Hz



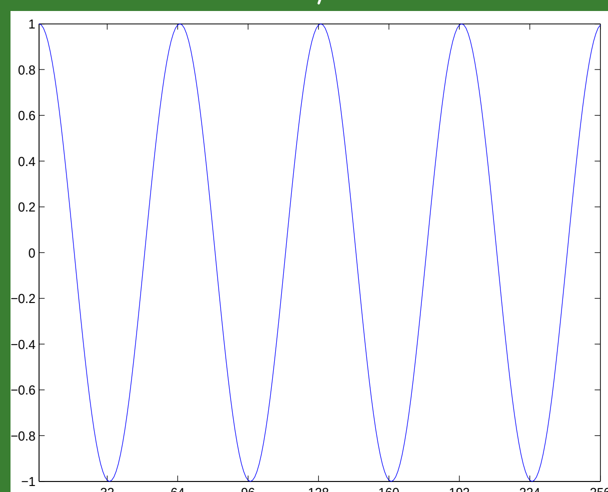
$3 * \sin 8/512$ Hz



Sum



$\sin 32/512$ Hz



$\cos 4/512$ Hz

Complex Image Reconstruction Part I

The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)
The DFT process is to make the data a vector $y = (y_1, ..., y_n)'$
Form the forward Fourier matrix $\bar{\Omega} = \bar{\Omega}_R + i\bar{\Omega}_I$

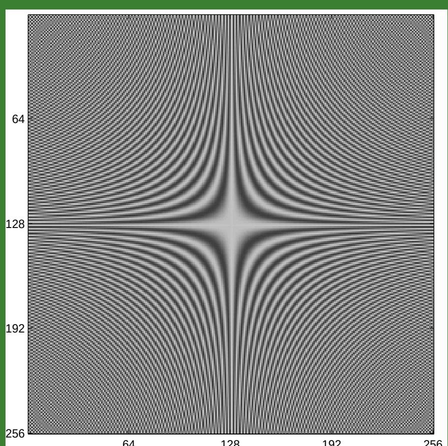
$$\underset{\substack{n \times 1 \\ \text{Complex}}}{\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix}} = \underset{\substack{n \times n \\ \text{Complex}}}{\left(\bar{\Omega}_R + i\,\bar{\Omega}_I\right)} \underset{\substack{n \times 1 \\ \text{Real}}}{\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}}$$

$$(f_R + if_I) = \left(\bar{\Omega}_R + i\,\bar{\Omega}_I\right)(y_R + iy_I)$$

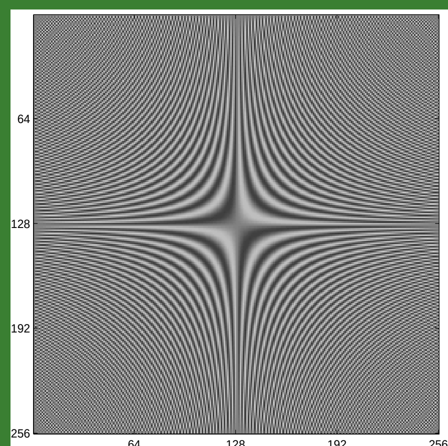
Complex Image Reconstruction Part I

The Complex-Valued (Discrete) Fourier Transform (TR=2s)

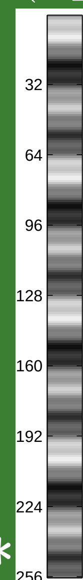
$$(\bar{\Omega}_R \quad + i \quad \bar{\Omega}_I) * (y_R \; + i \; y_I) = (f_R \; + i \; f_I)$$



$+ i$

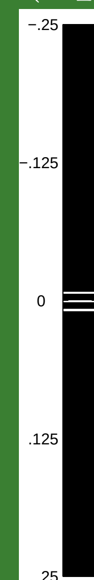


$*$

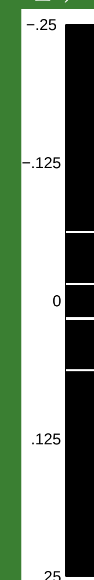


$+ i$

$=$



$+ i$



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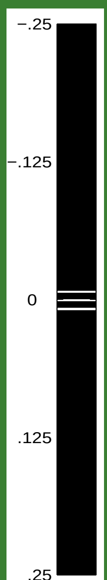
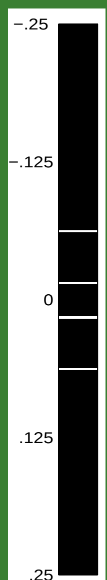
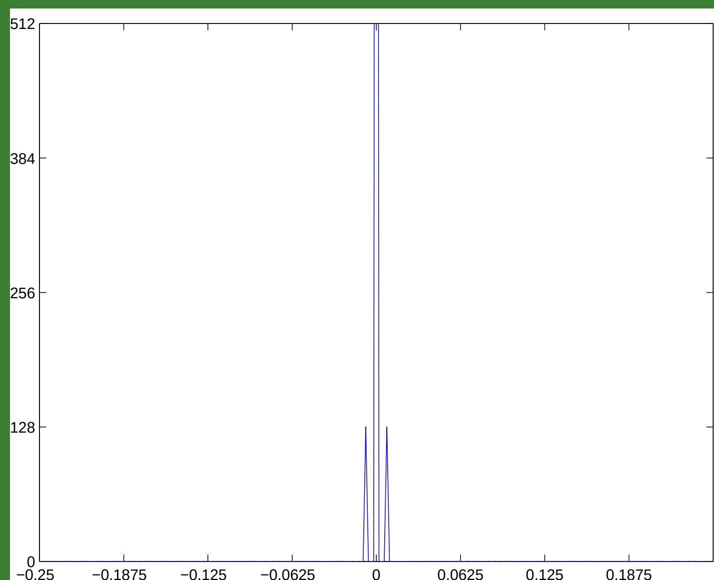
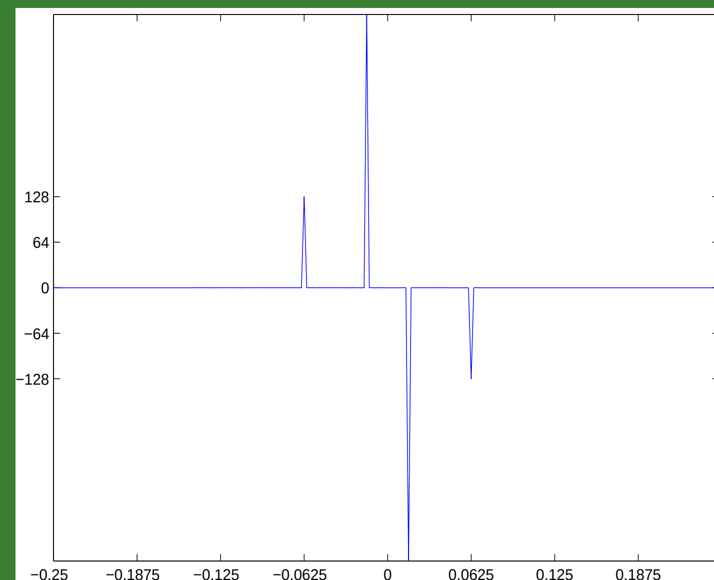




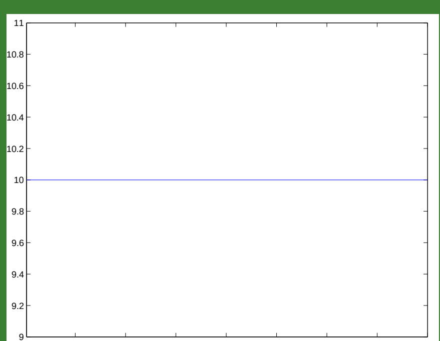




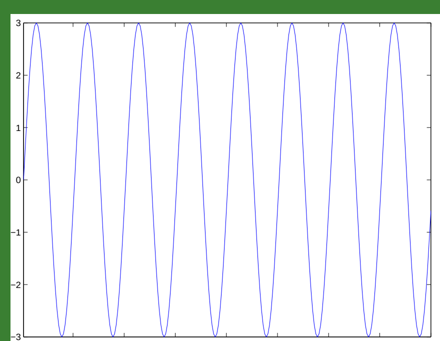
Cosines Sines

Cosines

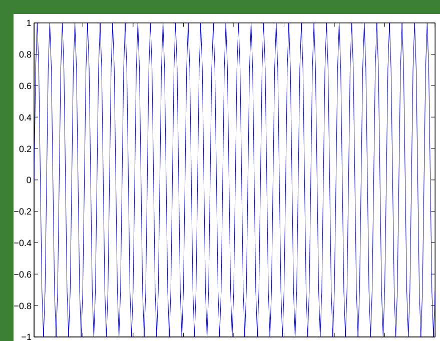
Sines



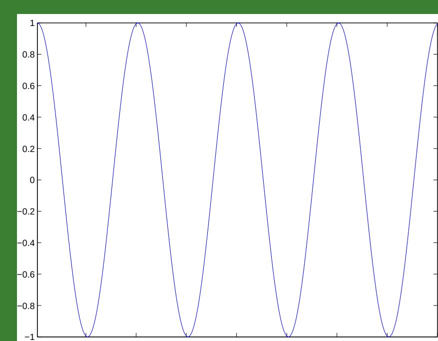
10, $\cos 0/512$ Hz



$3 * \sin 8/512$ Hz



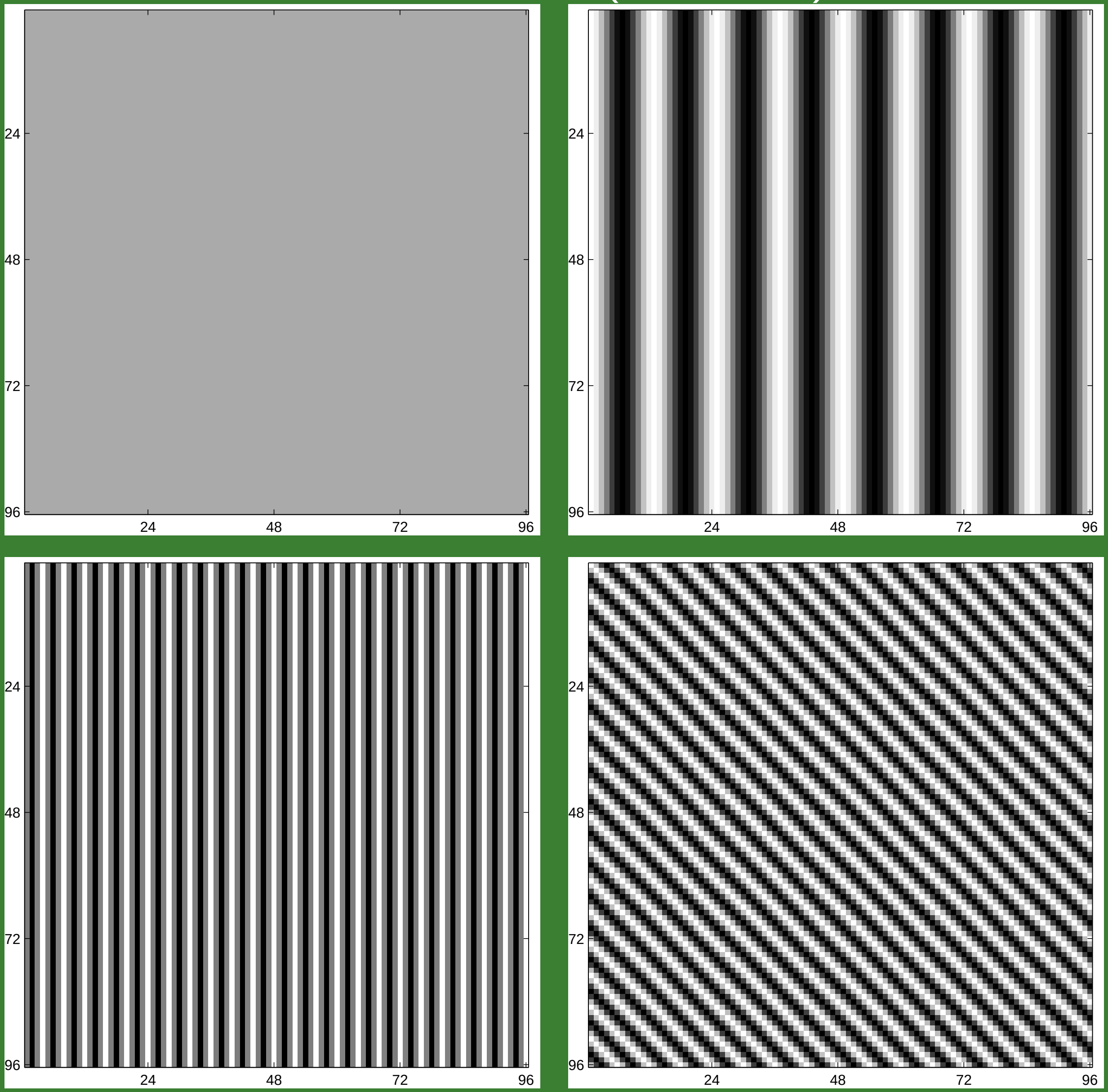
$\sin 32/512$ Hz

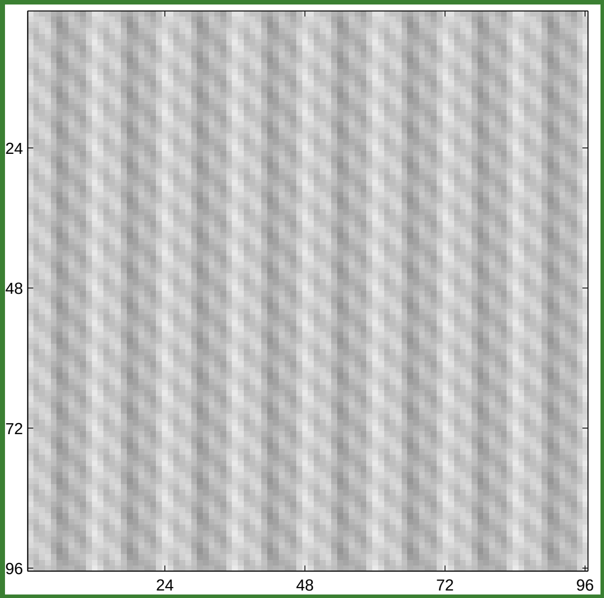


$\cos 4/512$ Hz

Complex Image Reconstruction Part I

The Complex-Valued 2D (Discrete) Fourier Transform





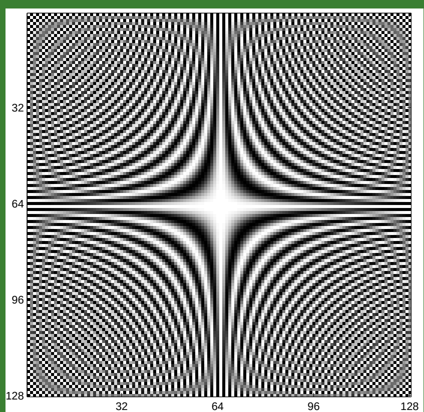
$$\sum = \cos + \cos + \sin + \cos$$

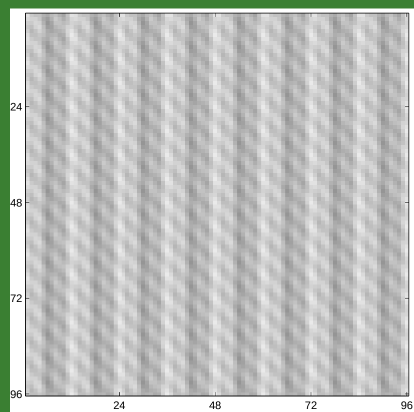
FOV=192 mm, mat=96×96, vox=2 mm^3

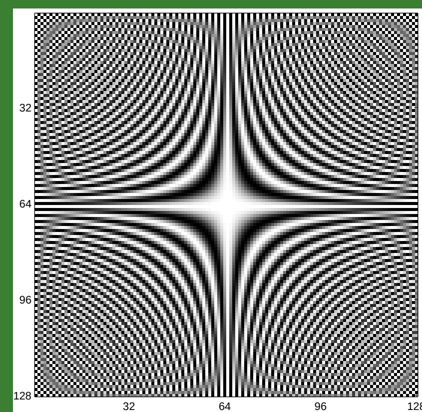
Complex Image Reconstruction Part I

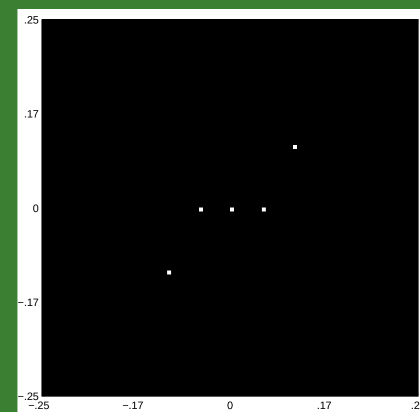
The Complex-Valued 2D (Discrete) Fourier Transform

$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) \quad * \quad (R_R + iR_I) \quad * \quad (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T \quad = \quad (S_R + iS_I)$$

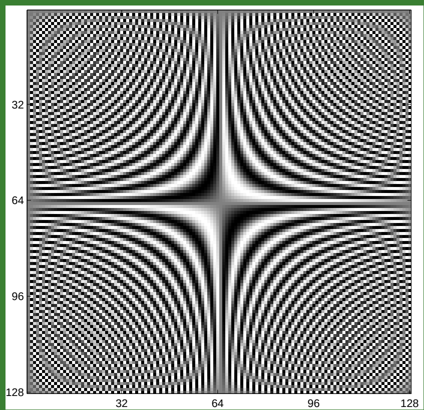


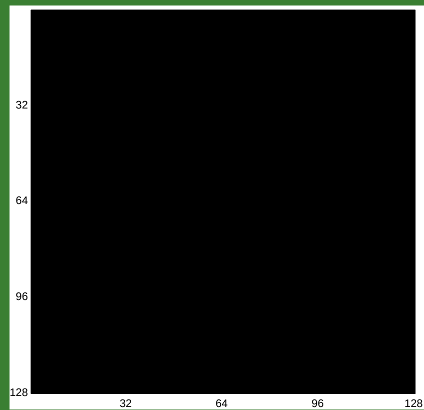


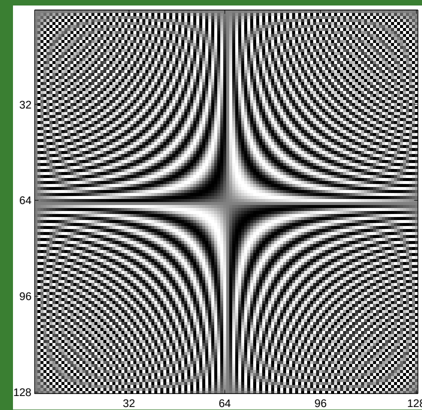


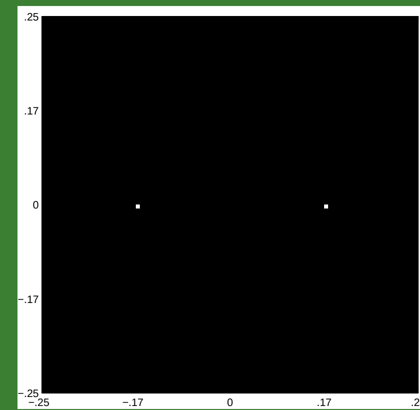


$$+\,i \quad * \quad +\,i \quad * \quad +\,i \quad = \quad +\,i$$





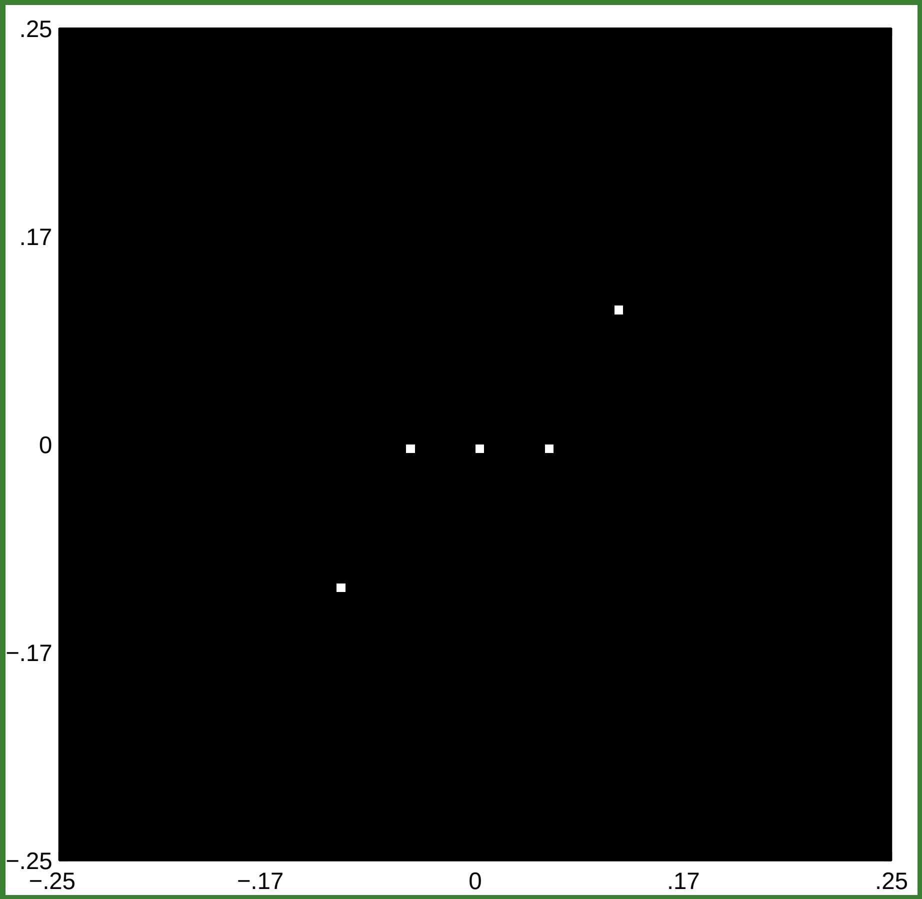




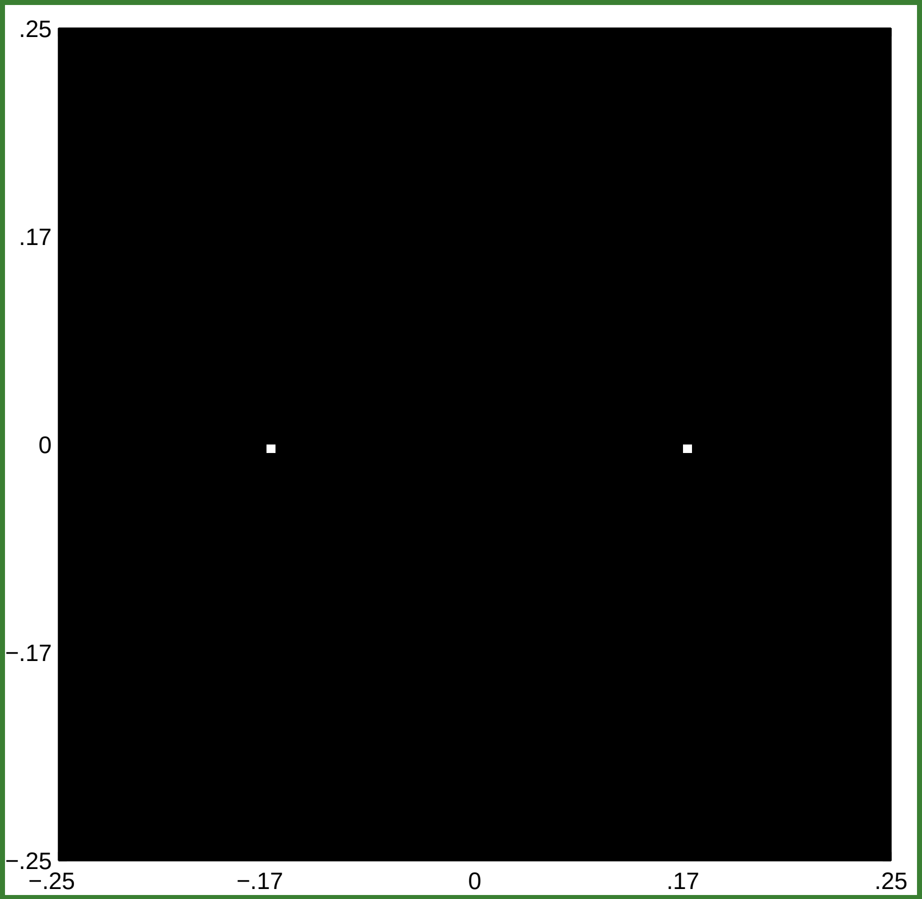
FOV=192 mm, mat=96×96, vox=2 mm^3

Complex Image Reconstruction Part I

The Complex-Valued 2D (Discrete) Fourier Transform



Real k-space



Imaginary k-space

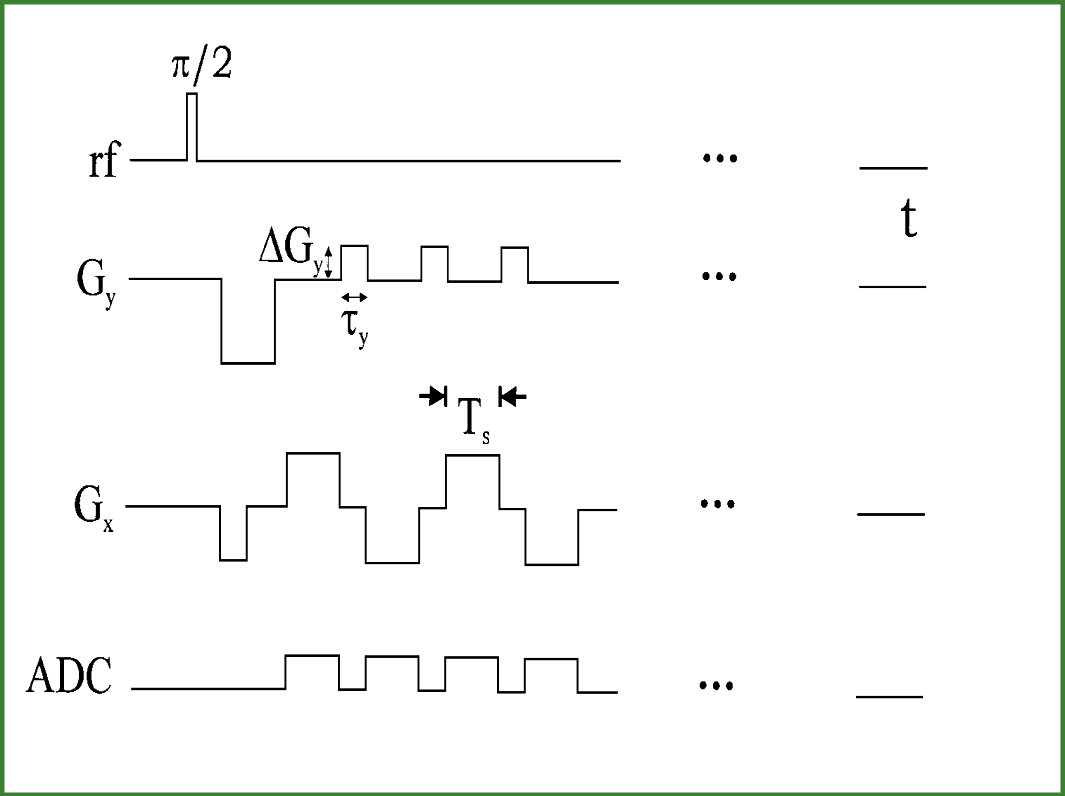
Note: Rotate bottom half up to get top!

FOV=192 mm, mat=96×96, vox=2 mm^3

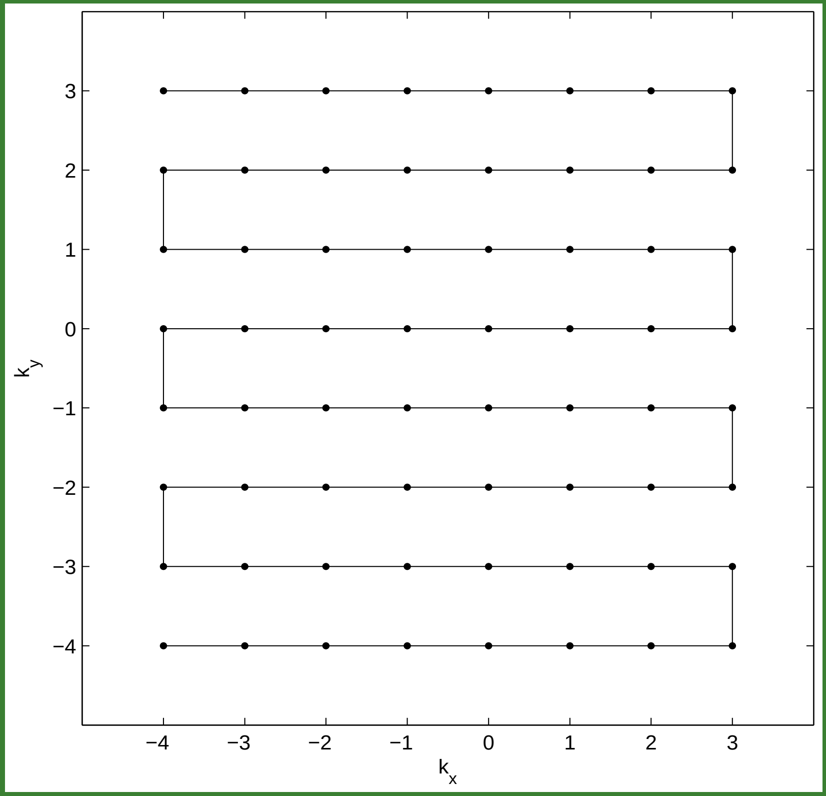
Complex Image Reconstruction Part I

G_x & G_y to encode, measure the complex-valued FT of the object.

$$S(k_x, k_y) = \int R(x,y) e^{-i2\pi(xk_x + yk_y)} dx\, dy,$$



(a) EPI Pulse Sequence



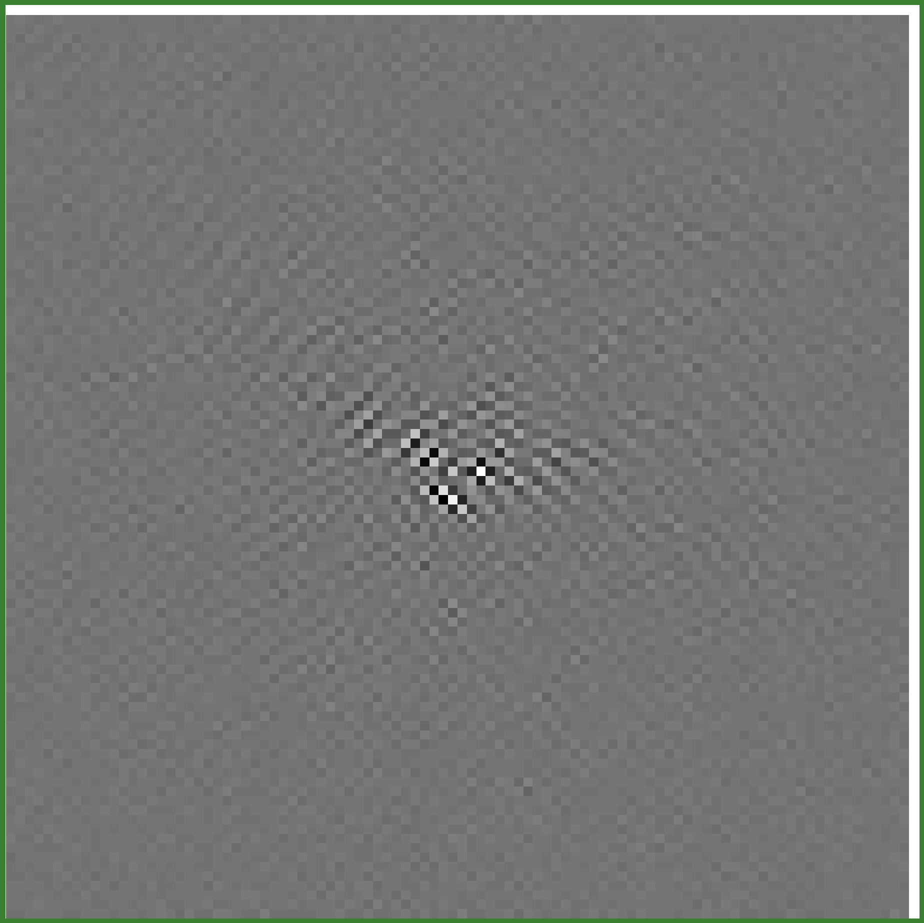
(b) k-Space Trajectory

Kumar, Welti and Ernst: NMR Fourier Zeugmatography, J. Magn. Reson. 1975

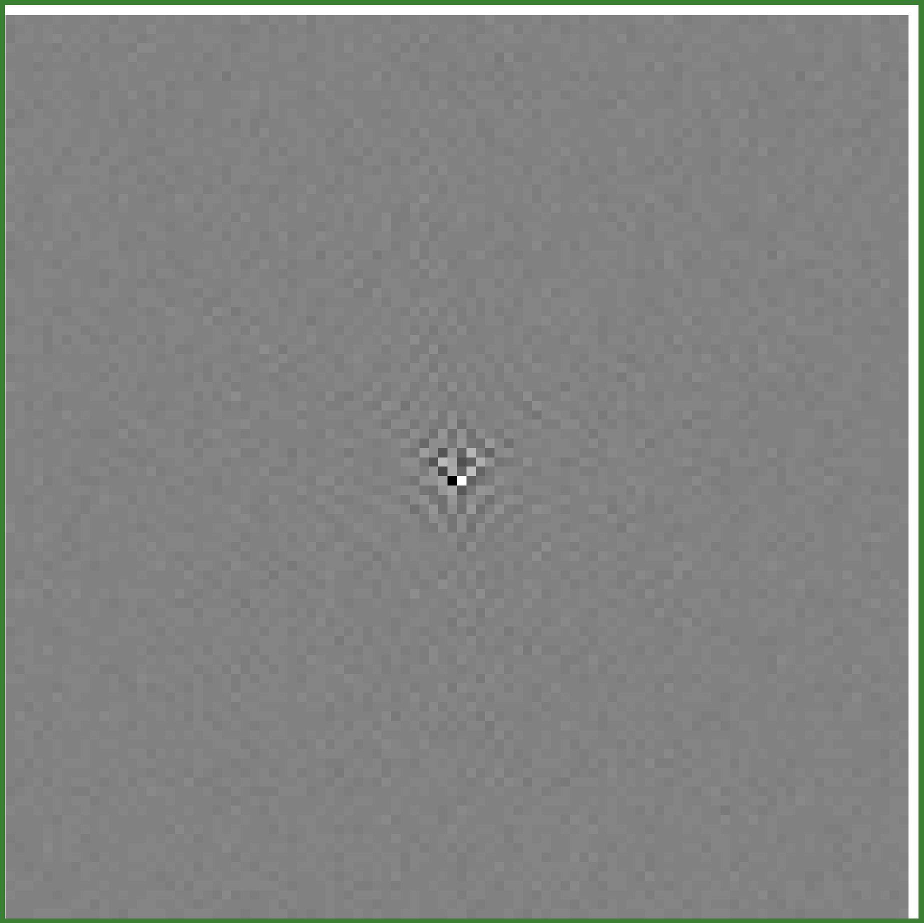
Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

Complex Image Reconstruction Part I

$S(k_x, k_y) = S_R(k_x, k_y) + iS_I(k_x, k_y)$, complex-valued FT of the object.



(a) real: 96×96



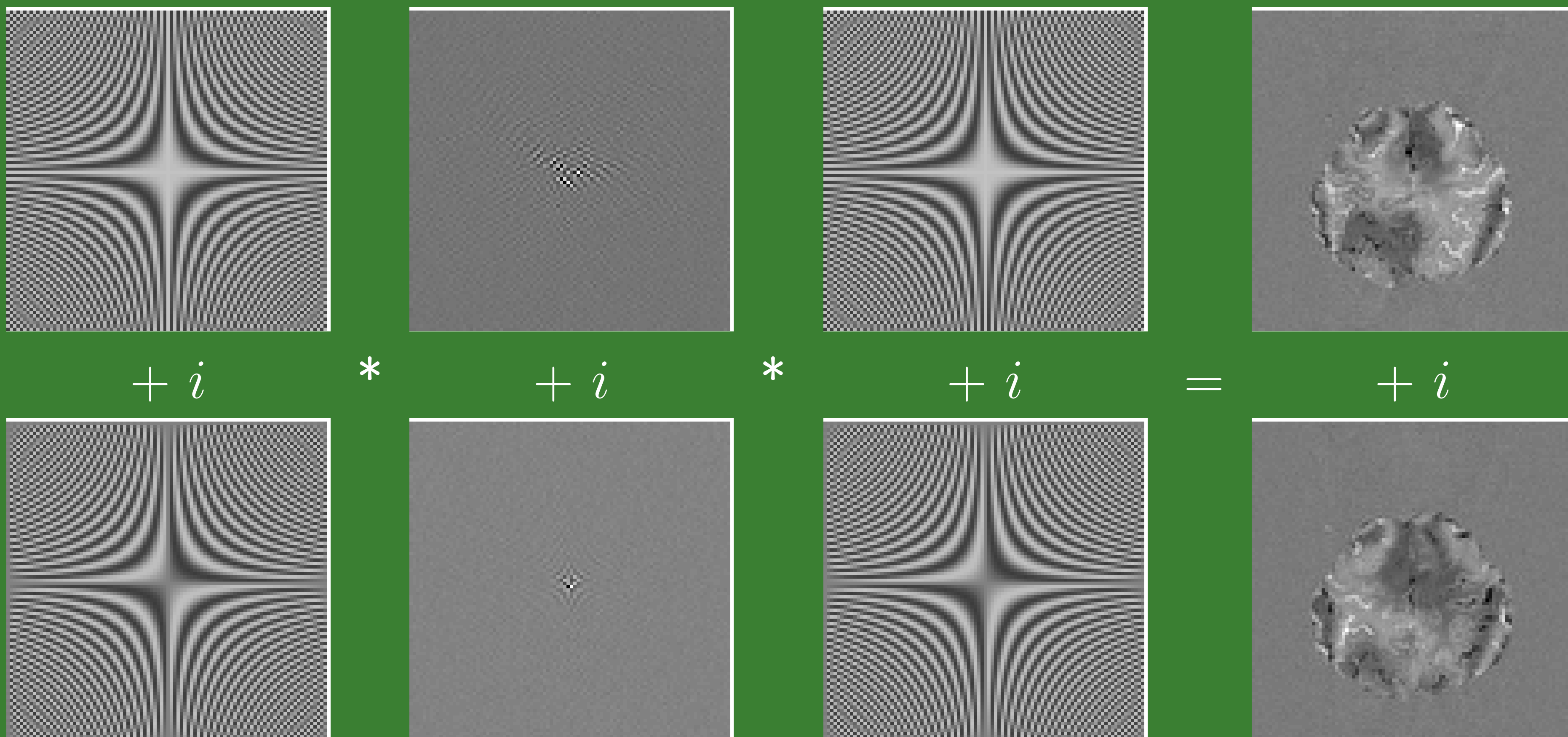
(b) imaginary: 96×96

Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

Complex Image Reconstruction Part I

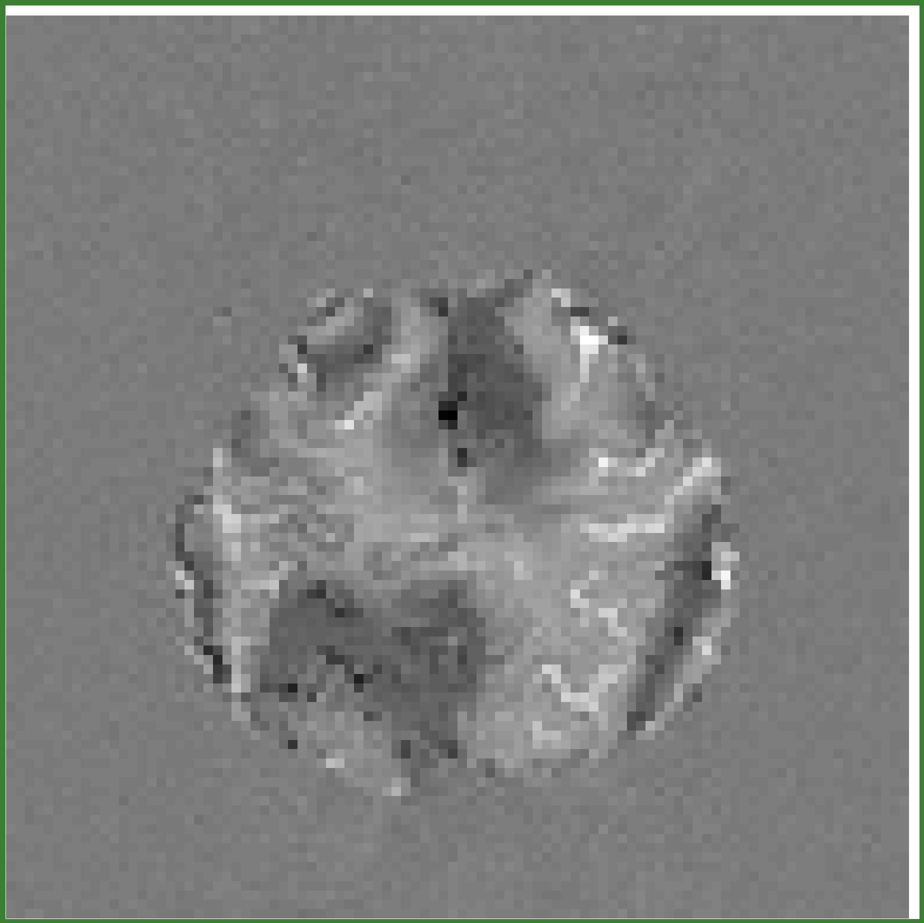
complex-valued 2D IFT

$$(\Omega_{yR} + i\Omega_{yI}) \quad * \quad (S_R + iS_I) \quad * \quad (\Omega_{xR} + i\Omega_{xI})^T \quad = \quad (R_R + iR_I)$$

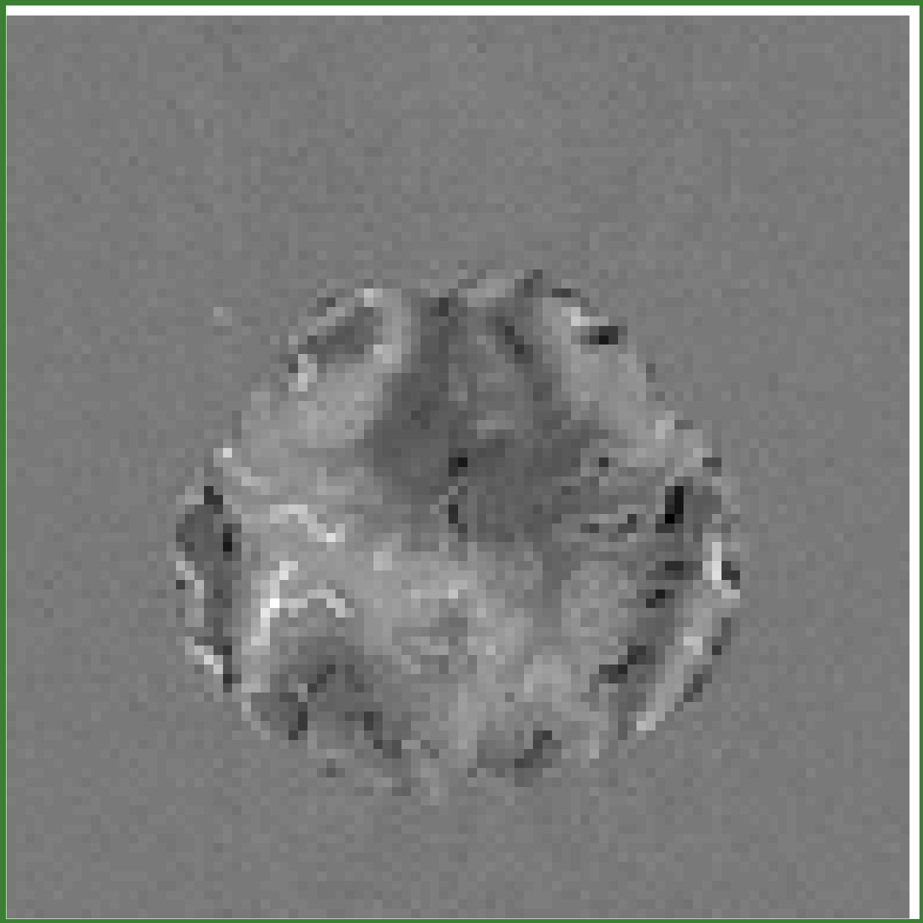


Complex Image Reconstruction Part I

Due to the imperfect Fourier encoding, the IFT reconstructed object is complex-valued, $R(x, y) = R_R(x, y) + iR_I(x, y)$.



(a) Real image, y_{Rt}

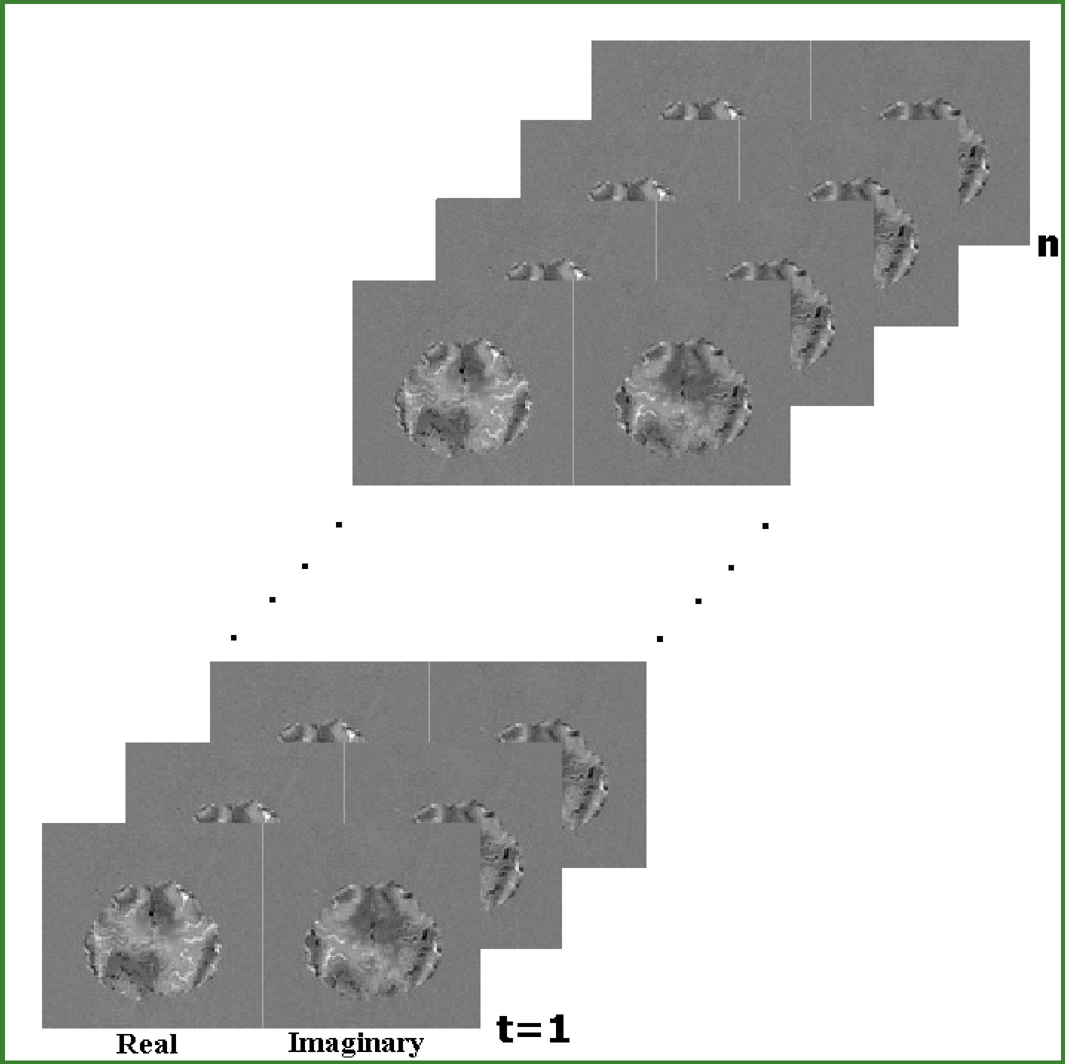


(b) Imaginary image, y_{It}

Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

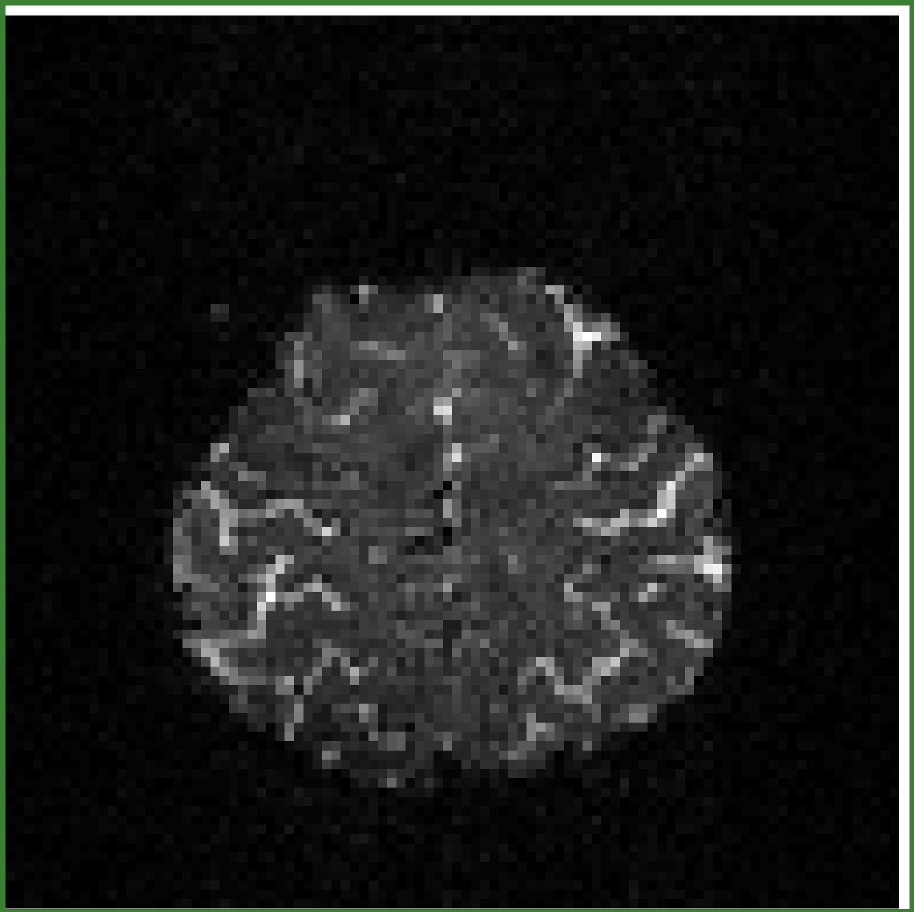
Complex Image Reconstruction Part I

This occurs over time in fMRI and results in complex-valued images and voxel time course observations, $y_t = y_{Rt} + iy_{It}$.



Complex Image Reconstruction Part I

Most fMRI studies transform from real-imaginary rectangular coordinates to magnitude-phase polar coordinates, $\rho(x,y) = m(x,y)e^{i\phi(x,y)}$.



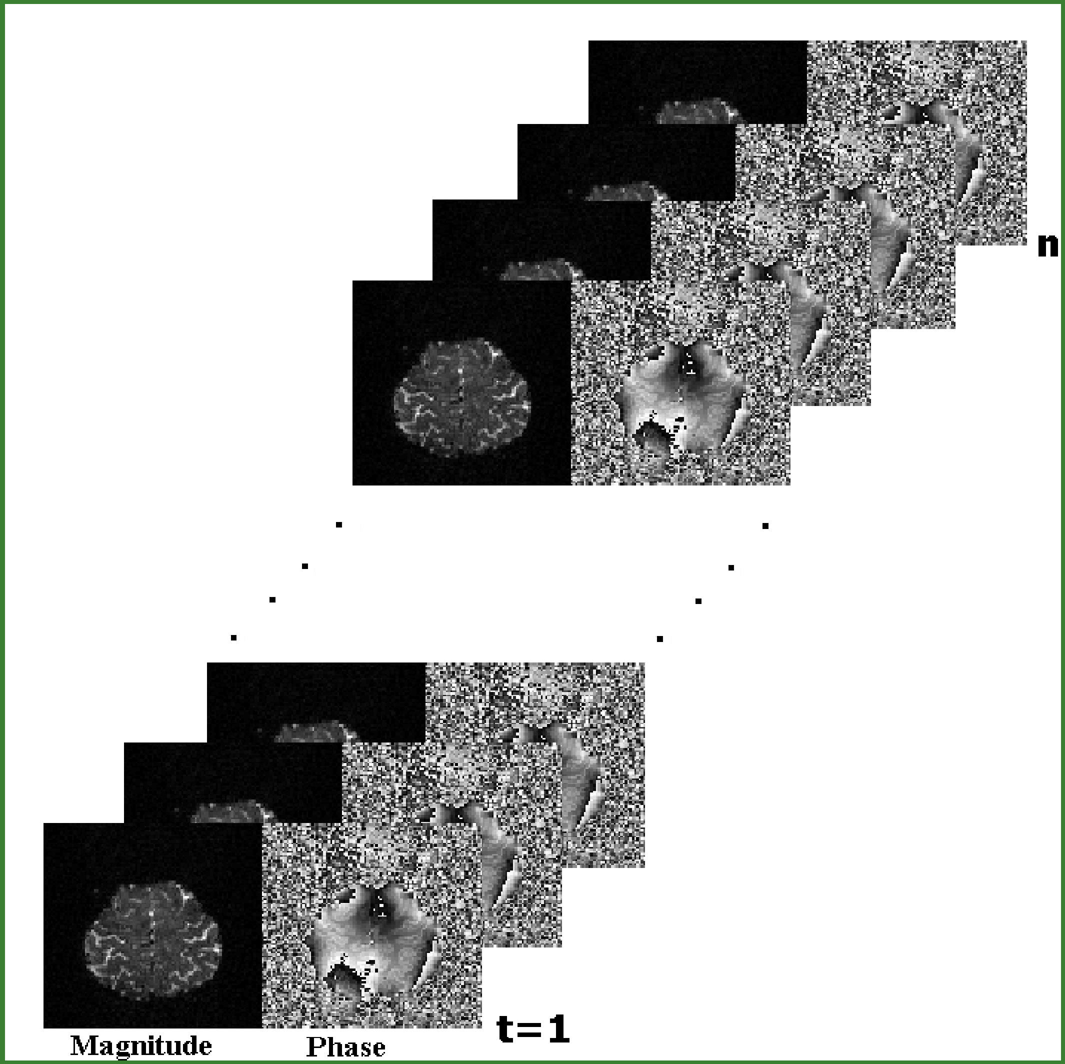
(a) Magnitude, $m_t = \sqrt{y_{Rt}^2 + y_{It}^2}$

(b) Phase, $\phi_t = \text{atan}_4(y_{It}/y_{Rt})$

Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

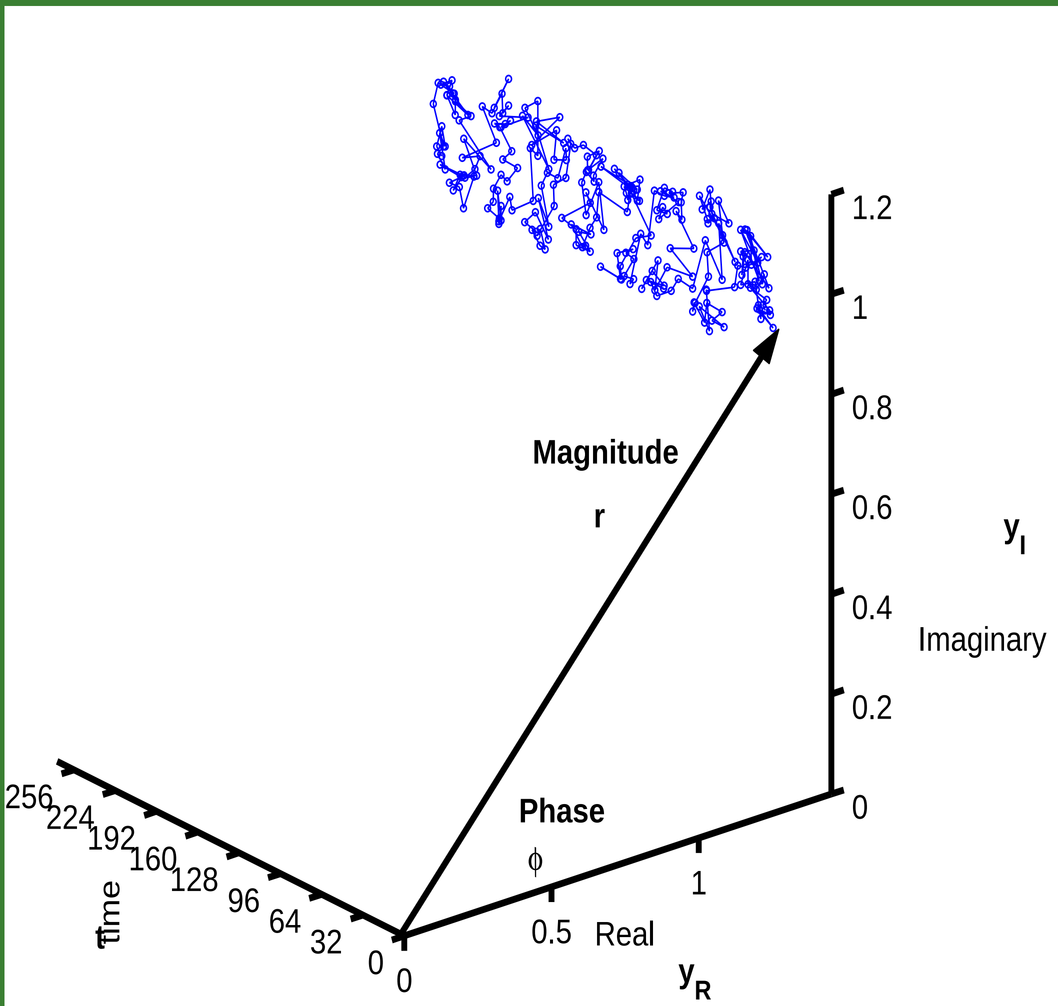
Complex Image Reconstruction Part I

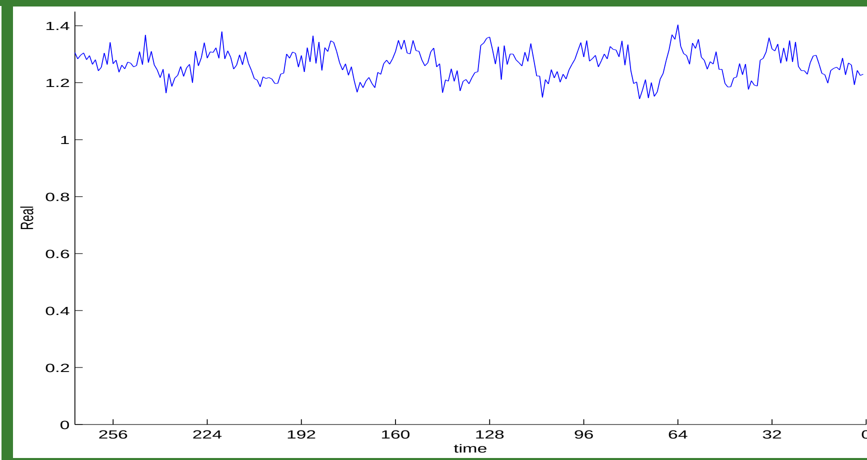
Collect a sequence of these reconstructed images over time.
Form voxel time courses, $y_t = r_t e^{i\phi_t}$.



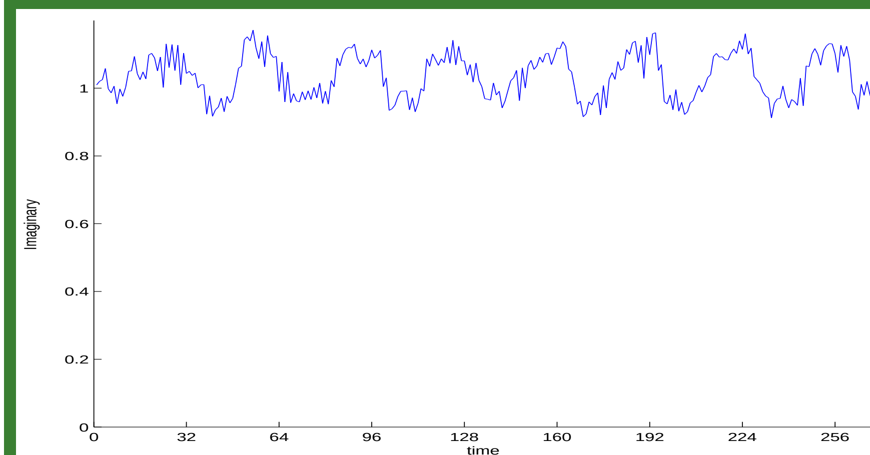
Complex Statistical Activation Method Part I

Time series are complex-valued or bivariate with phase coupled means.





Real



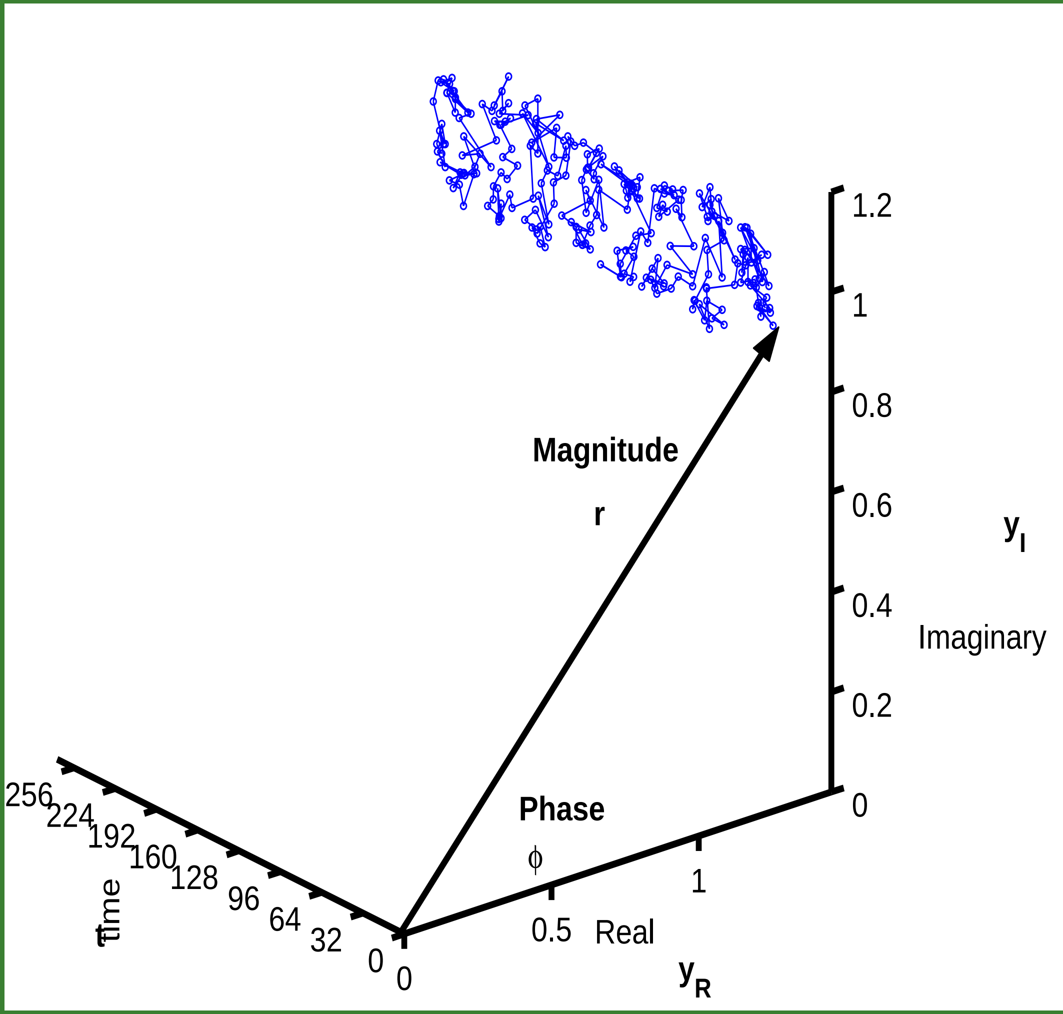
Imaginary

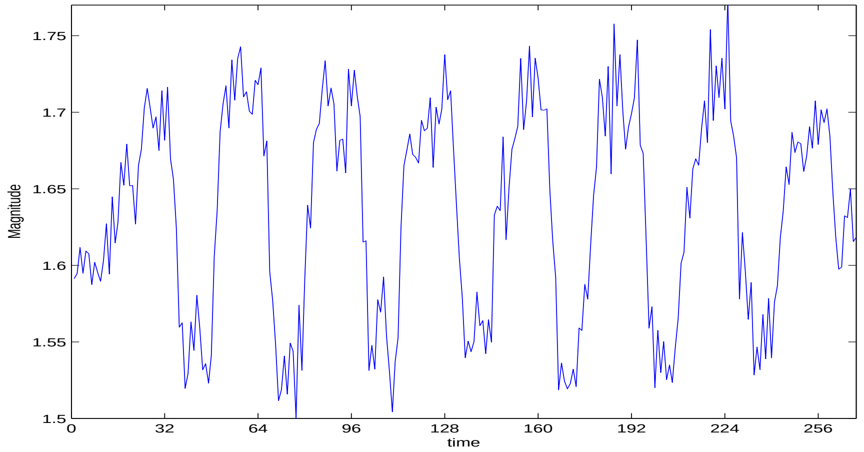
The y_R and y_I time courses have related vector length info!

This is a time series from a actual human experimental data!

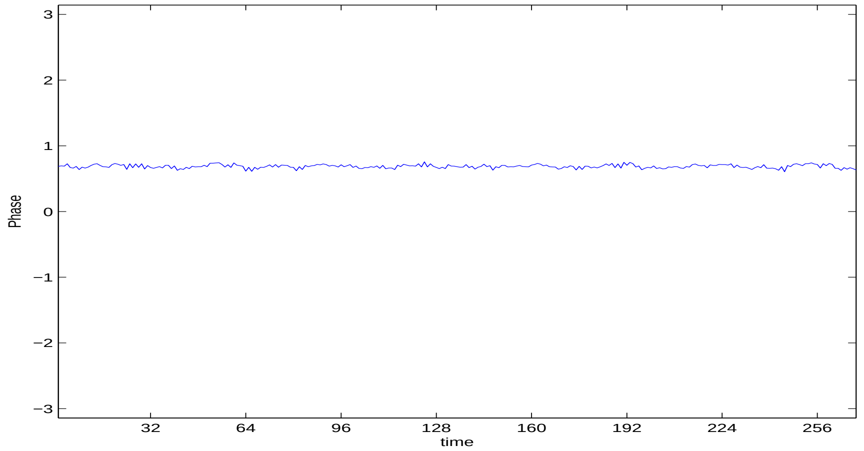
Complex Statistical Activation Method Part I

Time series are complex-valued or bivariate with phase coupled means.





Magnitude



Phase

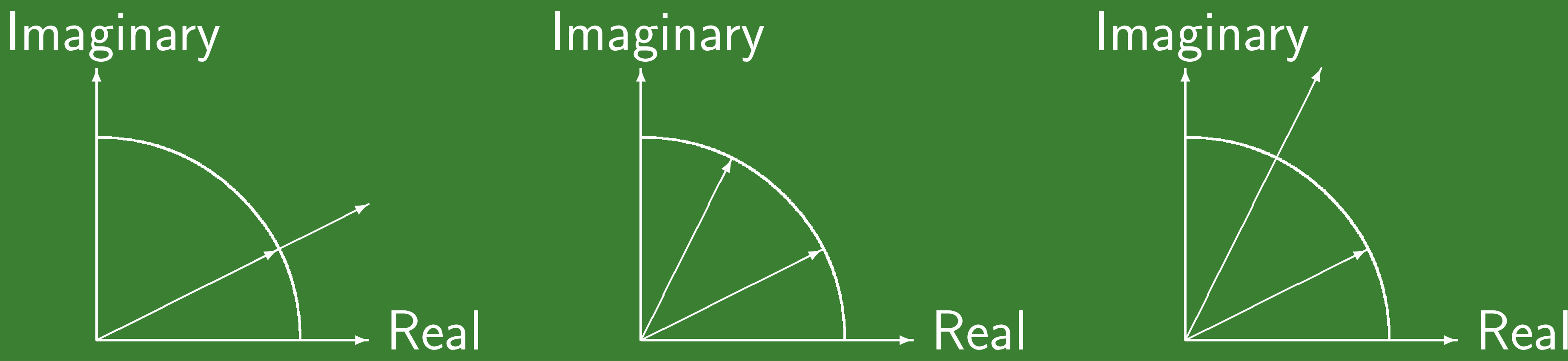
MO time courses only have vector length info!

PO time courses only has vector angle info!

Real-Imaginary or Magnitude-Phase time courses have all info!

Complex Statistical Activation Method Part I

Block-designed experiment: Off-On-Off-...-On-Off task

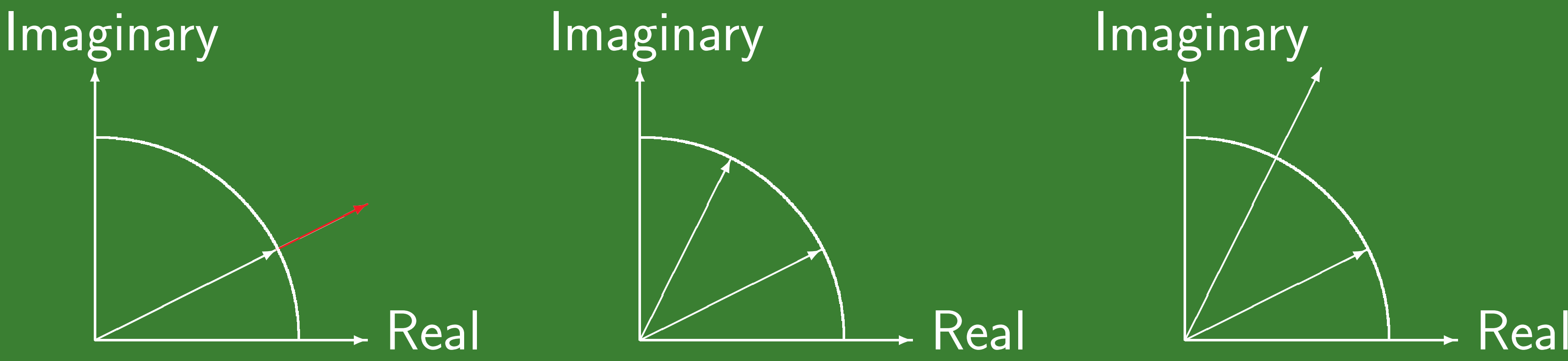


- ➢ Real Magnitude-Only (MO/UP) Activation[1,2,3]
- ➢ Real Phase-Only (PO) Activation[4]
- ➢ Complex Magnitude w/ Constant Phase (CP) Activation[5]
- ➢ Complex Magnitude &/or Phase (CM) Activation[6]
- ➢ Complex Magnitude w/ Phase Regressor Activation[7,8]

[1]Bandettini et al.: MRM, 30:161-173, 1993.
[2]Friston et al.: HBM, 2:189-210, 1995.
[3]Rowe and Logan: NeuroImage, 24:603-606, 2005.
[4]Rowe, Meller, and Hoffmann: Submitted, 2006
[5]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[6]Rowe: NeuroImage, 25:1310-1324, 2005b.
[7]Menon, MRM, 47:1-9, 2002.
[8]Nencka and Rowe: Submitted, 2006.

Complex Statistical Activation Method Part I

Block-designed experiment: Off-On-Off-...-On-Off task



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Complex Statistical Activation Method Part I

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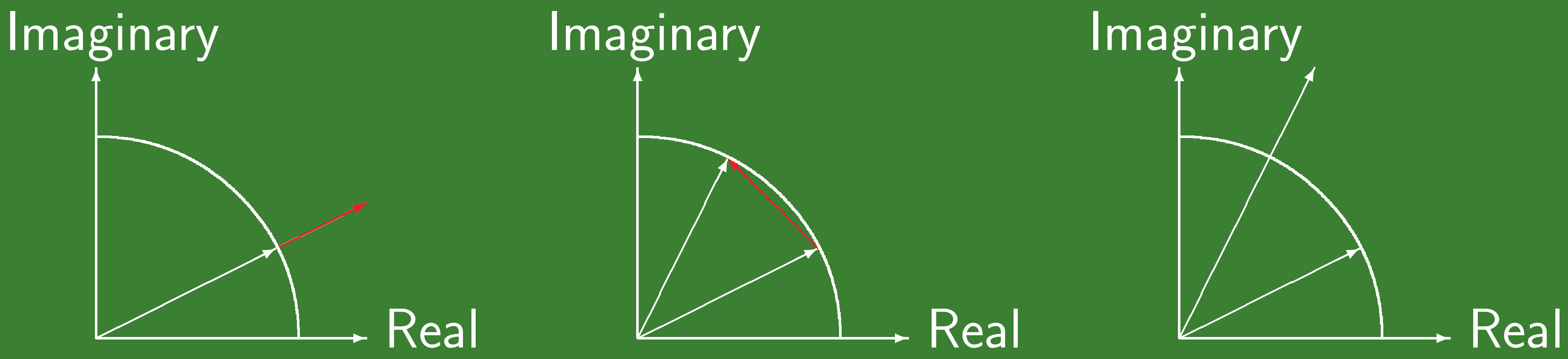

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Complex Statistical Activation Method Part I

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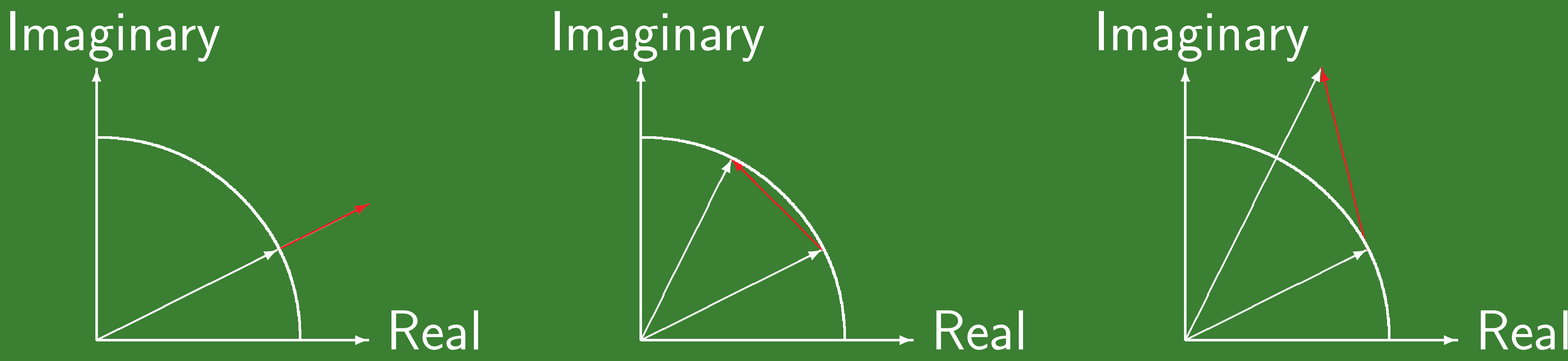

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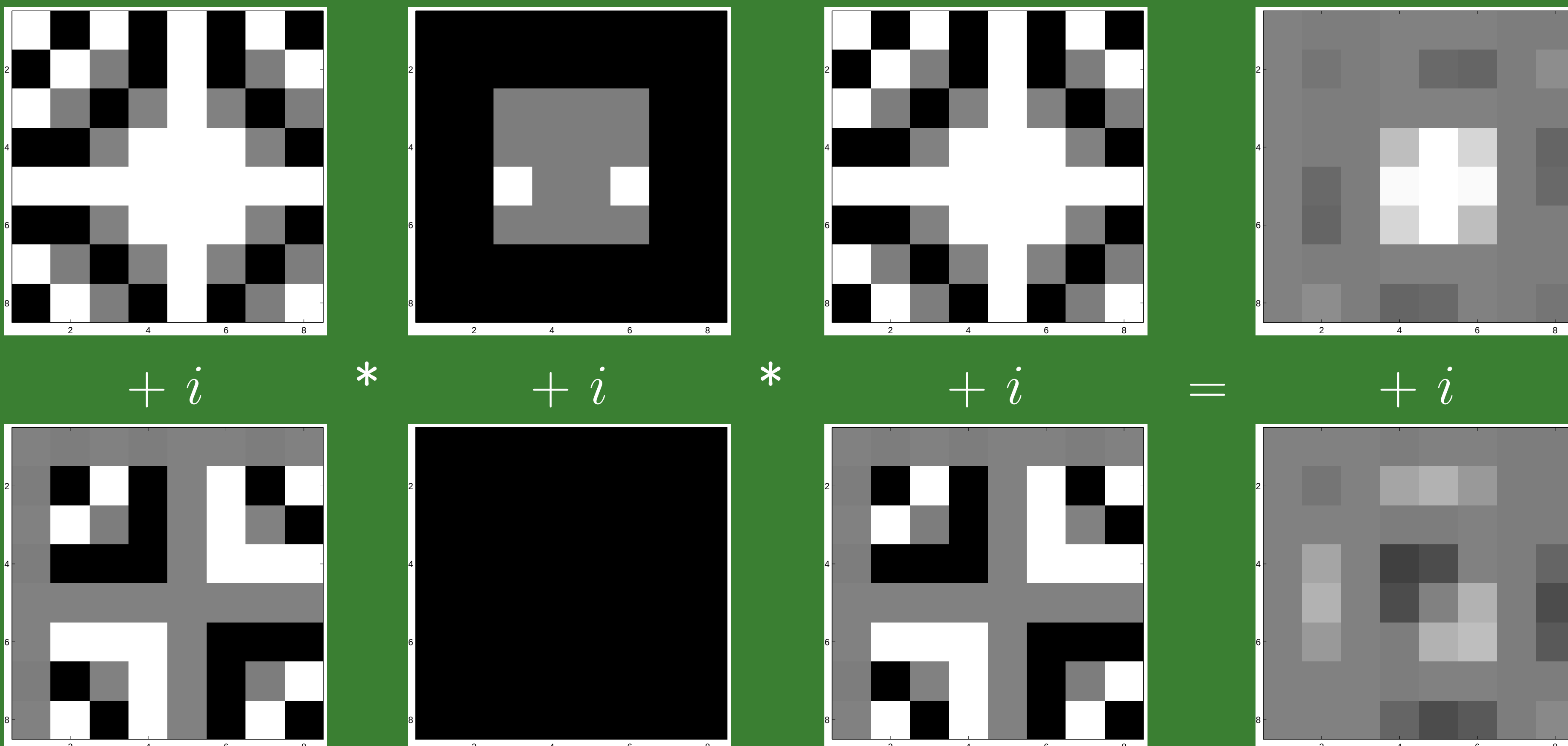
Complex Image Reconstruction Part II

complex-valued 2D F FT

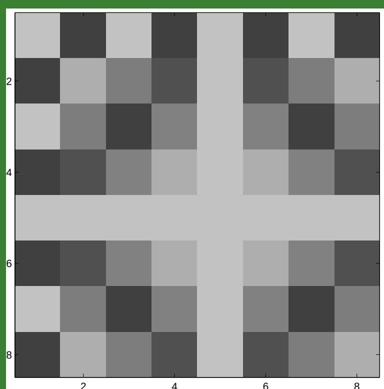
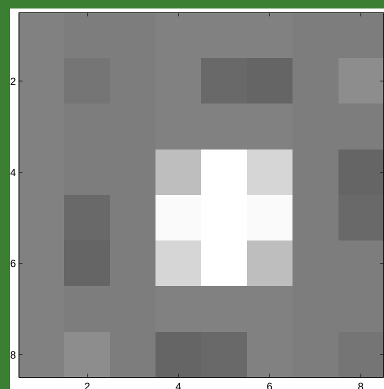
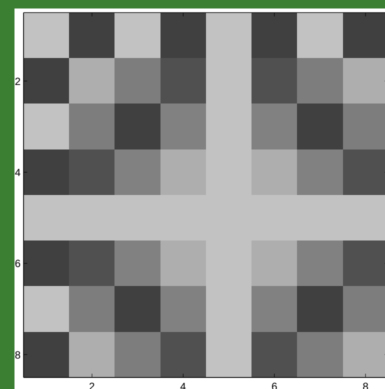
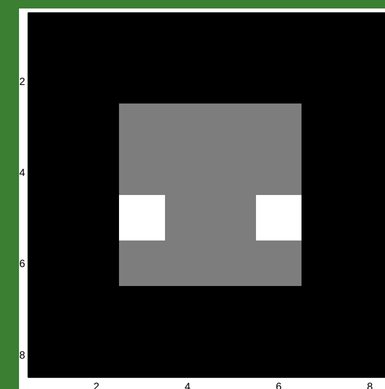
$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) \; * \; (R_R + iR_I) \; * \; (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T = (S_R + iS_I)$$



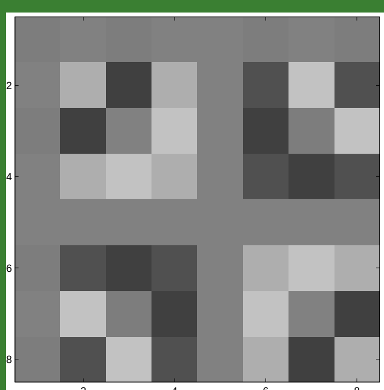
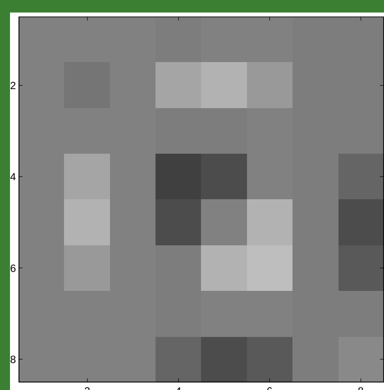
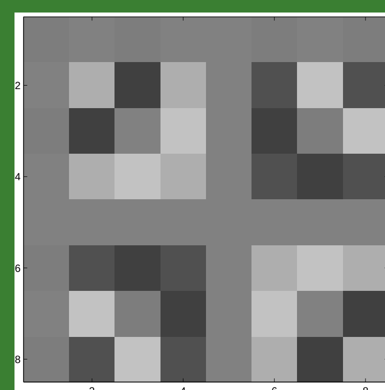
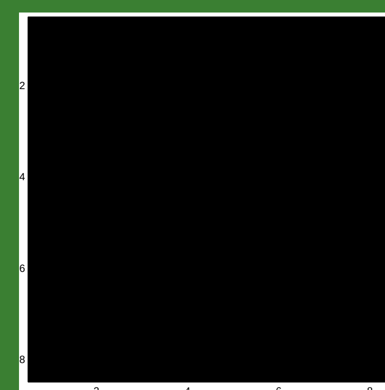
Complex Image Reconstruction Part II

complex-valued 2D I FT

$$(\Omega_{yR} + i\Omega_{yI}) \;*\; (S_R + iS_I) \;*\; (\Omega_{xR} + i\Omega_{xI})^T = (R_R + iR_I)$$

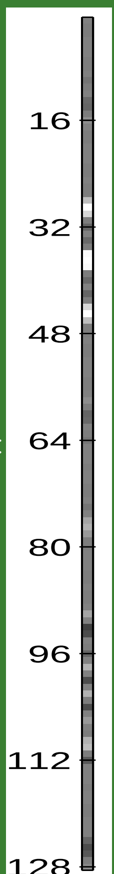
$+\, i$ * $+\, i$ * $+\, i$ = $+\, i$

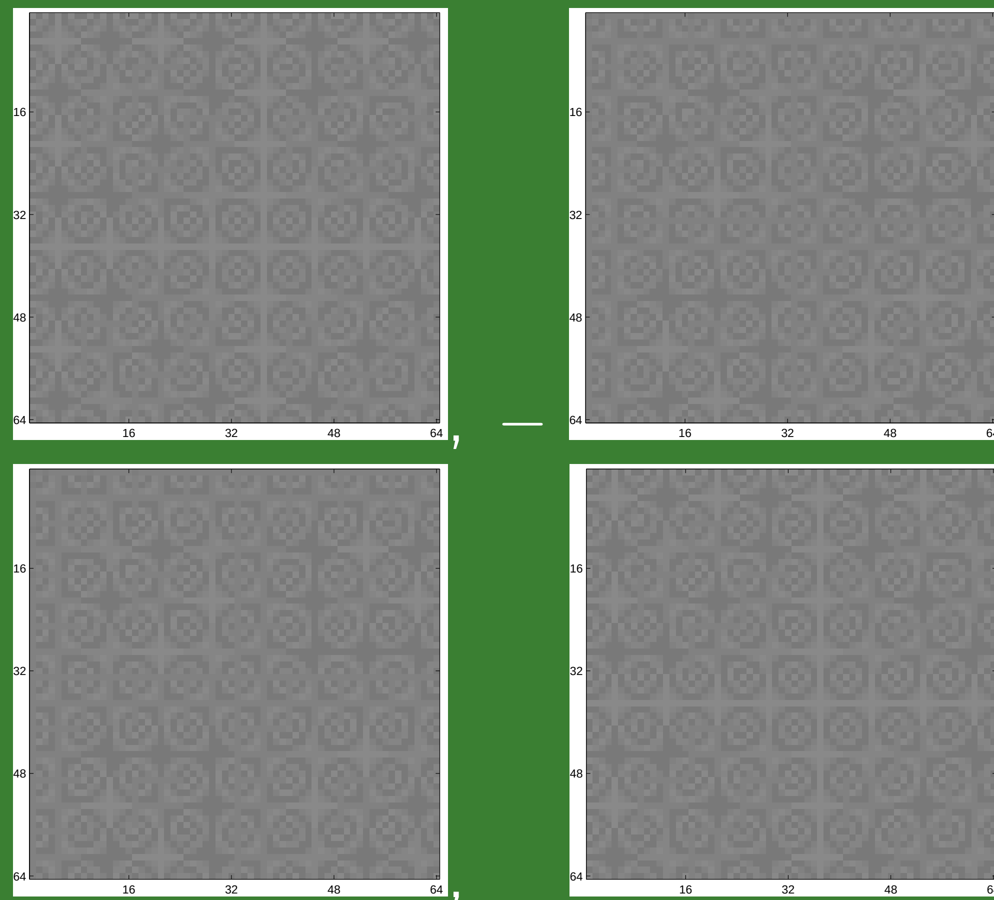
Complex Image Reconstruction Part II

$$s = \text{vec}(S_R^T, S_I^T) \qquad \Omega = \begin{bmatrix} \Omega_R & -\Omega_I \\ \Omega_I & \Omega_R \end{bmatrix}$$

$s =$



$\Omega =$

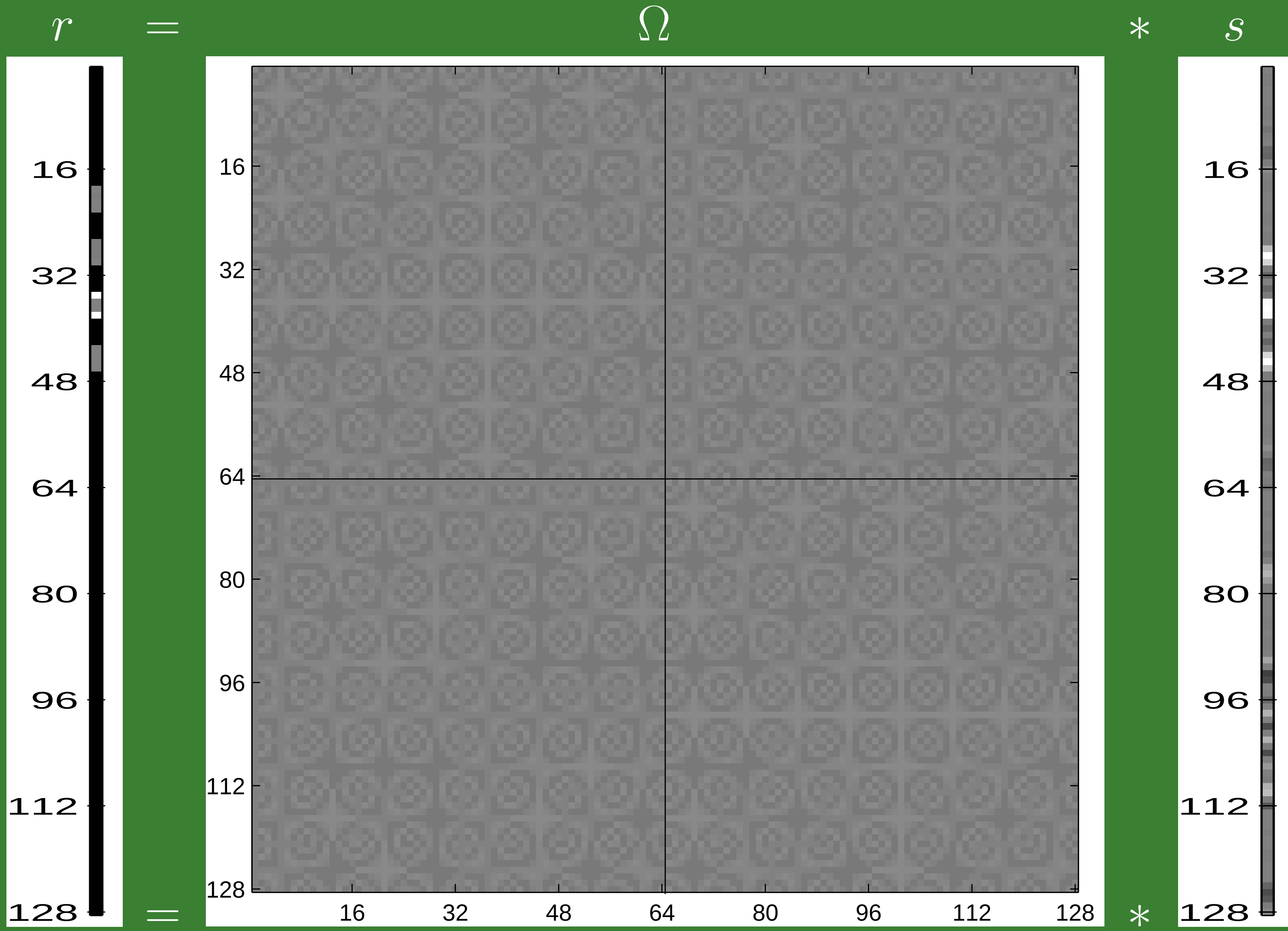


$$s = \begin{pmatrix} s_R \\ s_I \end{pmatrix}$$

$$\begin{aligned} \Omega_R &= [(\Omega_{yR} \otimes \Omega_{xR}) - (\Omega_{yI} \otimes \Omega_{xI})] \\ \Omega_I &= [(\Omega_{yR} \otimes \Omega_{xI}) + (\Omega_{yI} \otimes \Omega_{xR})] \end{aligned}$$

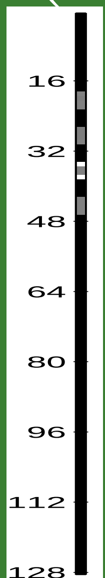
$\otimes$ is Kronecker product that multiplies each element of 1^{st} matrix argument with entire 2^{nd} matrix argument

Complex Image Reconstruction Part II

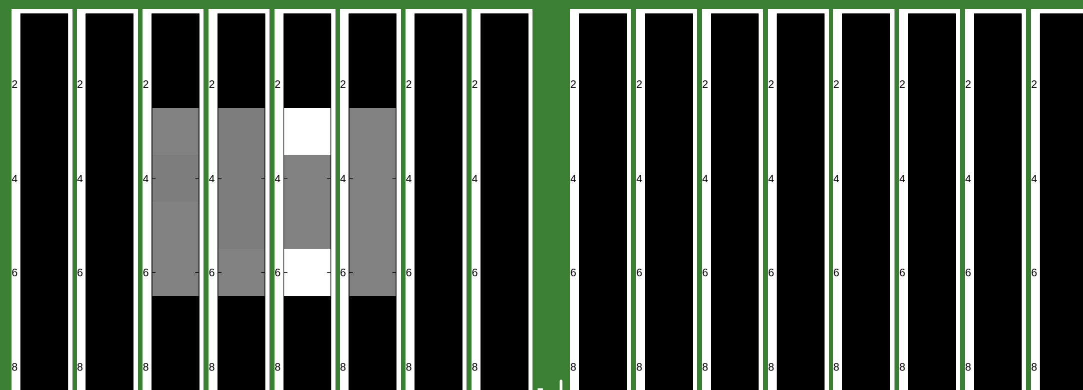


Complex Image Reconstruction Part II

$$r = \begin{pmatrix} r_R \\ r_I \end{pmatrix} \quad \longrightarrow \quad R^T = \overline{\text{vec}}(r) = (R_R^T, R_I^T)$$



$\longrightarrow$

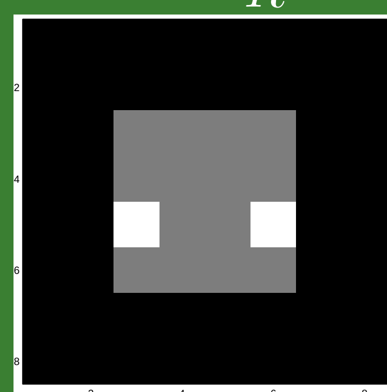


$, \downarrow$

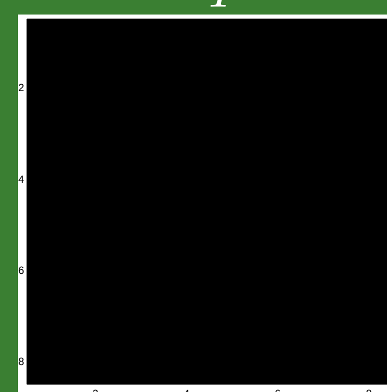
$\longleftarrow \quad \longleftarrow$

$$\downarrow \quad R_C = R_R + i R_I$$

$\longrightarrow$



$+ i$



Statistical Implications

Showed that $r = \Omega s$.

$$\begin{array}{llll} \text{If} & \mathrm{E}(s) = s_0 & \text{and} & \mathrm{var}(s) = \Gamma \\ \text{then} & \mathrm{E}(r) = \Omega s_0 & \text{and} & \mathrm{var}(r) = \Omega \Gamma \Omega^T \end{array}$$

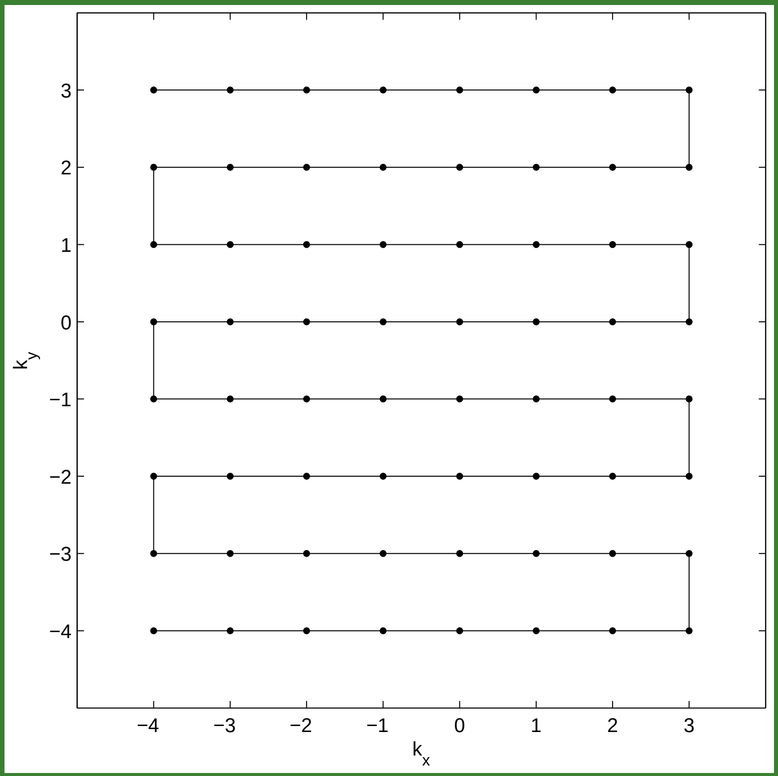
And vise versa.

We know we have spatially correlated voxels (the r's).

This can only result from correlated k-space measurements (the s's)

Statistical Implications

Could part of the correlation between voxels be from temporally correlated k-space measurements along the EPI trajectory?

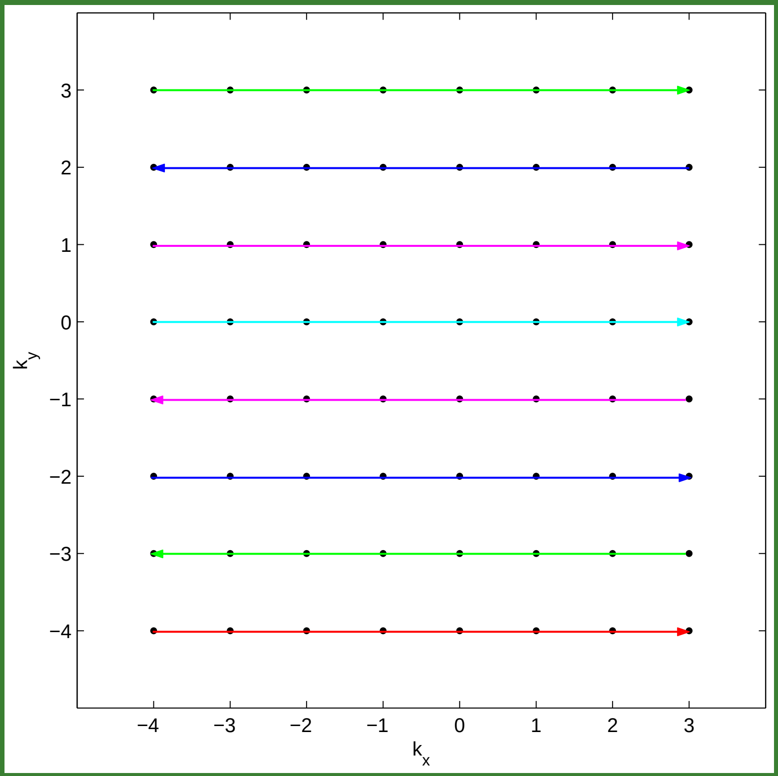


k-Space Trajectory

Is $(-3, -4)$ correlated with $(-4, -4)$?
Is $(-2, -4)$ correlated with $(-3, -4)$?

Statistical Implications

Could part of the correlation between voxels be from k-space adjustments?



k-Space Trajectory

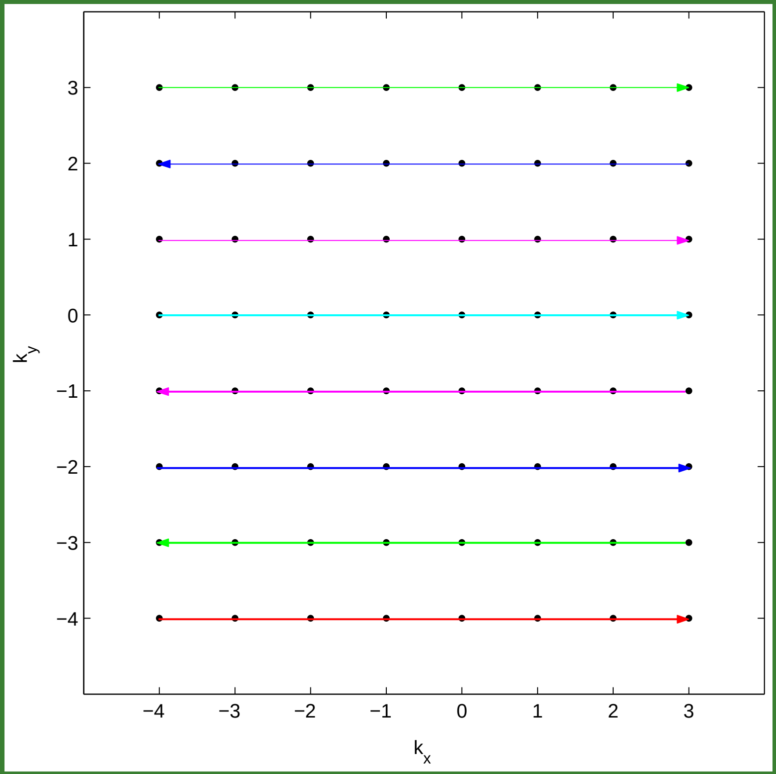
Shifting of k-space odd/even lines?

Evens left and odds right through time induces k-space correlation!

Effect on voxel spatial correlation?

Statistical Implications

Could part of the correlation between voxels be from partial k-space acquisition?



Partial k-Space Acquisition

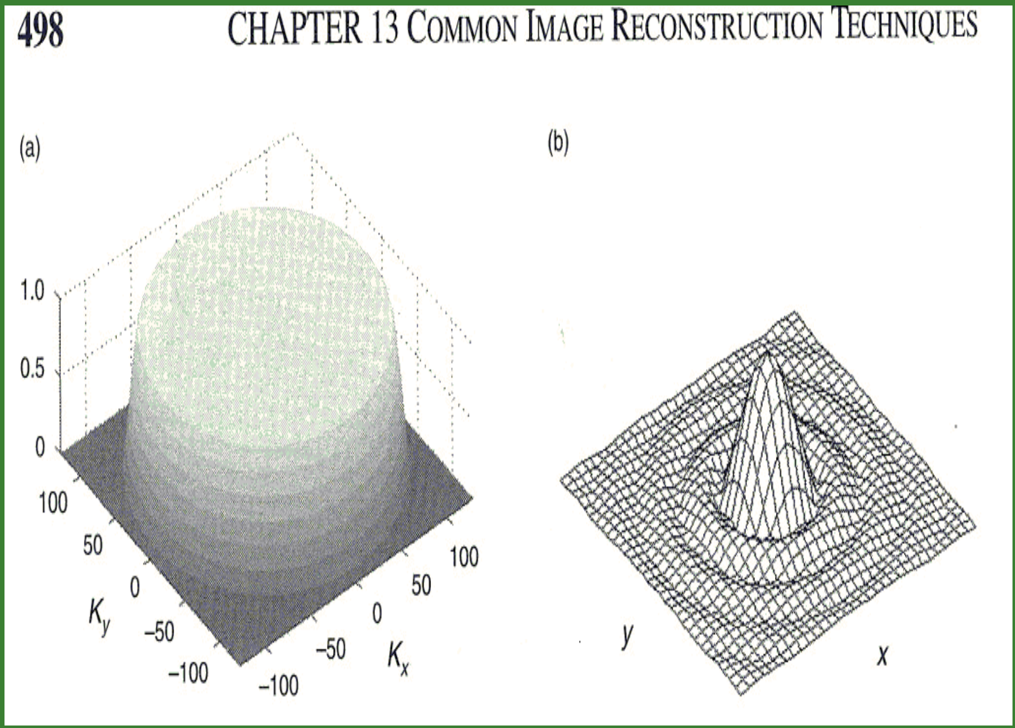
Acquistion of k-space botom $1/2$ lines?

Make up top $1/2$ from bottom $1/2$!

Maybe k-space correlation correlated since same numbers.
Effect on voxel spatial correlation?

Statistical Implications

What about Apodization, AKA k-space smoothing?



k-Space Apodization

Bernstein, King, Zhou: *Handbook of MRI Pulse Sequences*, Academic Press, 2004.

Effect on voxel spatial correlation?

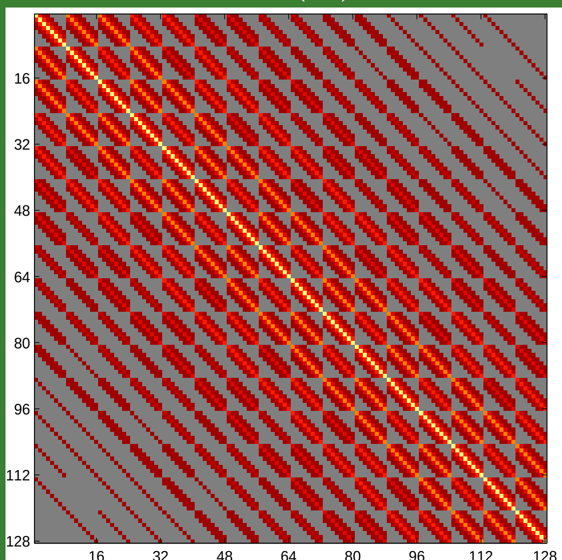
Increase correlation of higher spatial frequencies?

"an outstanding reference source that covers all the important aspects of pulse sequence design and implementation."
–G.H. Glover, in NMR IN BIOMEDICINE (2005)

Statistical Implications

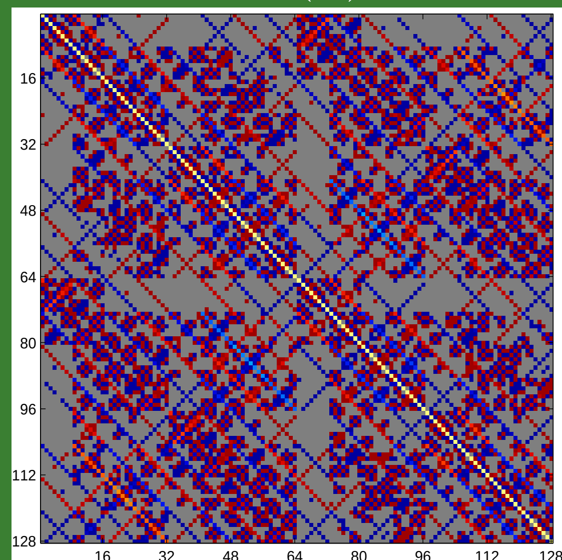
Sample k-space Correlation (along EPI trajectory) 8×8 image

var(s)



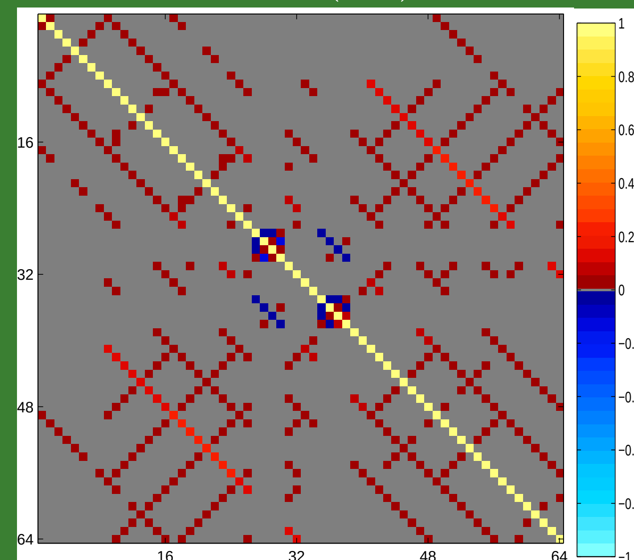
Real-Imaginary k-space

var(r)



Real-Imaginary Voxels

var(m)



Magnitude Voxels

Some good correlations $\sim .3$ in magnitude data.

Maybe mising out on correlations seen in complex?

Should decompose covariance into constituent parts.

var$(s) = \Gamma = \Gamma_P + \Gamma_N + \Gamma_I$

var$(r) = \Omega\Gamma\Omega^T = \Omega\Gamma_P\Omega^T + \Omega\Gamma_N\Omega^T + \Omega\Gamma_I\Omega^T$

Discussion

Described Complex Image Reconstruction.

Related voxels to k-space measurements.

Not shown fMRI activation directly from k-space measurements!

Further research is needed.

To be continued ...

Thank You.

Could complex-valued activation be the future of fMRI analysis?
Compute activation directly from original k-space measurements!

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Colleagues:
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Dr. Shi-Jiang Li
Dr. Andrzej Jesmanowicz