# Magnitude and Phase Modeling for fMRI Brain Activation

**Daniel B. Rowe, Ph.D.**
**dbrowe@mcw.edu**

Department of Biophysics
Division of Biostatistics
Graduate School of Biomedical Sciences

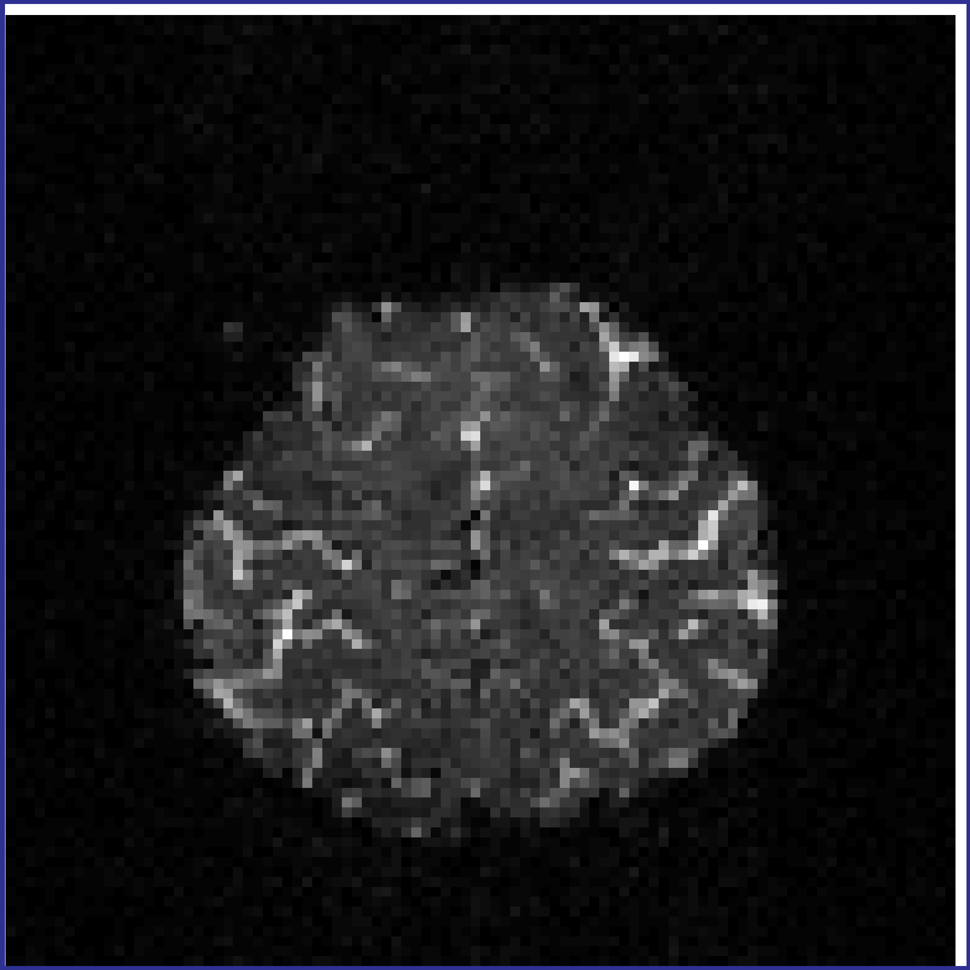
MEDICAL COLLEGE OF WISCONSIN
8701 Watertown Plank Rd.
Milwaukee, WI 53226

## Outline

- Complex Image Reconstruction
- Real Magnitude-Only (MO) Activation
- Real Phase-Only (PO) Activation
- Complex Magnitude w/ Constant Phase (CP) Activation
- Complex Magnitude and/or Phase (CM) Activation
- Activations in Human Experimental Data
- Discussion

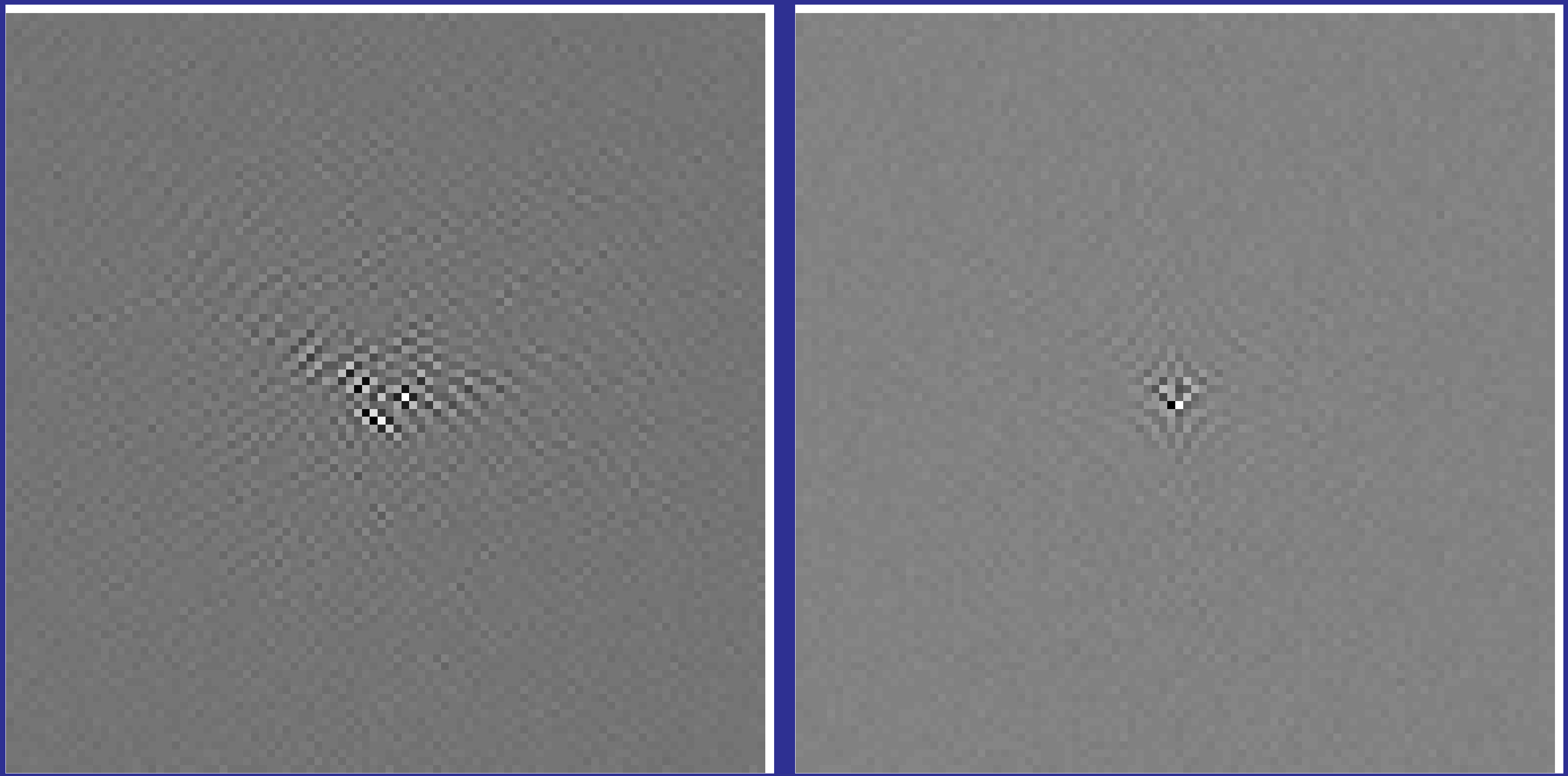
## Complex Image Reconstruction

In MRI we aim to image the PSD of a real-valued object, $\rho(x, y)$.



# Complex Image Reconstruction

Apply $G_x$ and $G_y$ gradients to encode then measure the complex-valued FT $s(k_x, k_y) = \int \rho(x,y) e^{-i2\pi(xk_x + yk_y)} dx\ xy$ of the PSD.

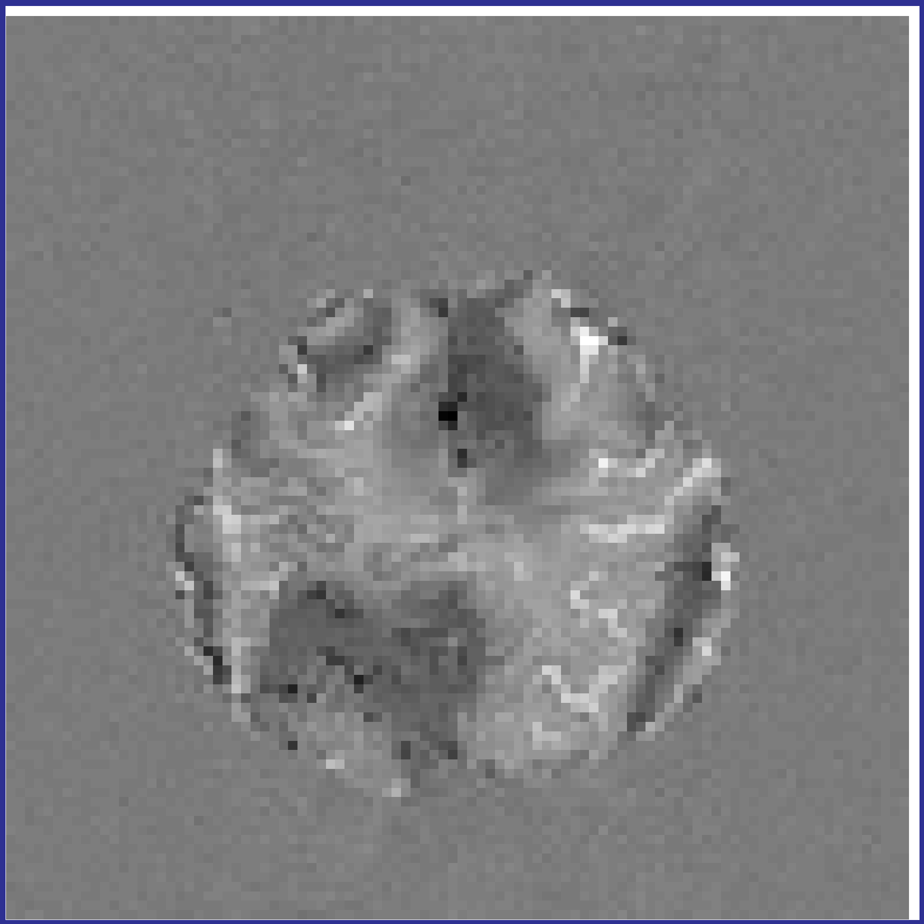


(a) real (b) imaginary

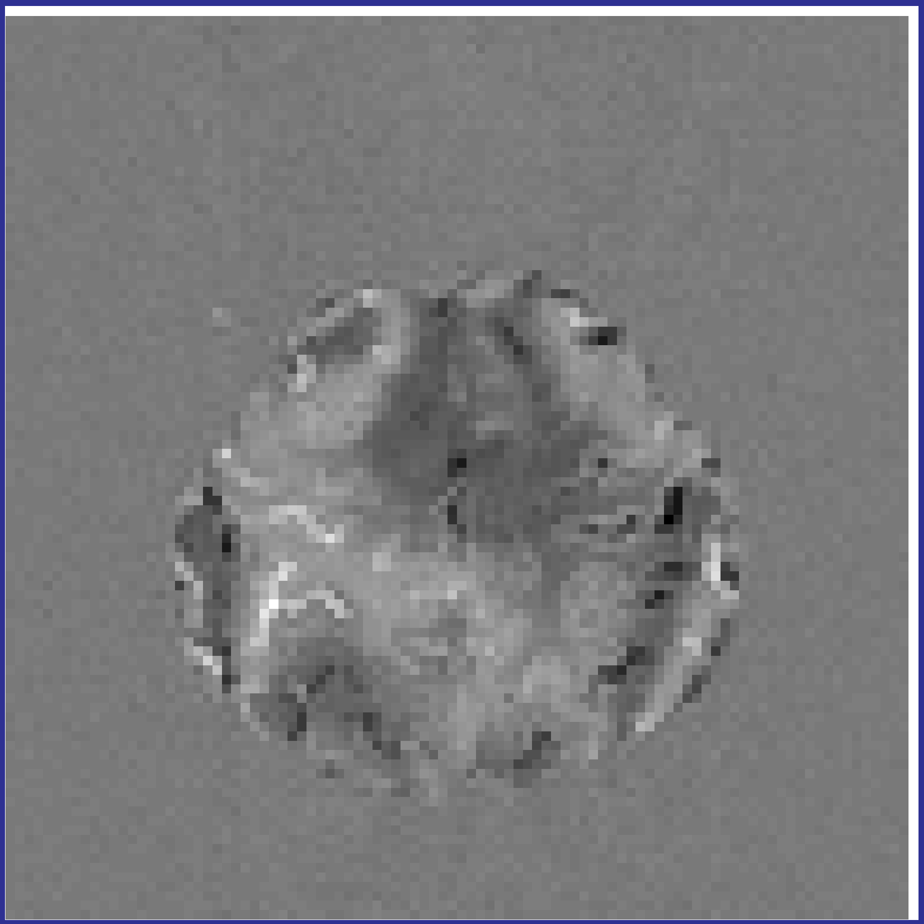
Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

# Complex Image Reconstruction

Due to the magnetic field "irregularities," the IFT reconstructed object is complex-valued, $\rho(x, y) = \rho_R(x, y) + i\rho_I(x, y)$.



(a) Real image, $y_{Rt}$

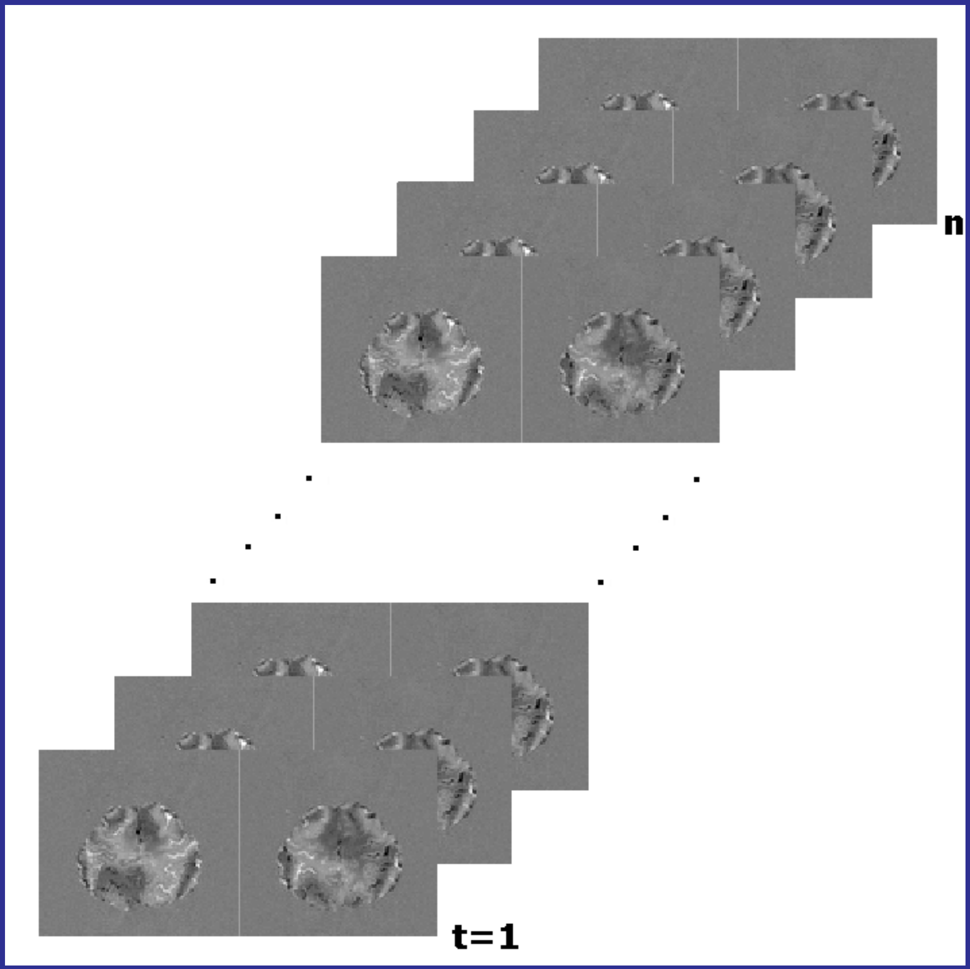


(b) Imaginary image, $y_{It}$

Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

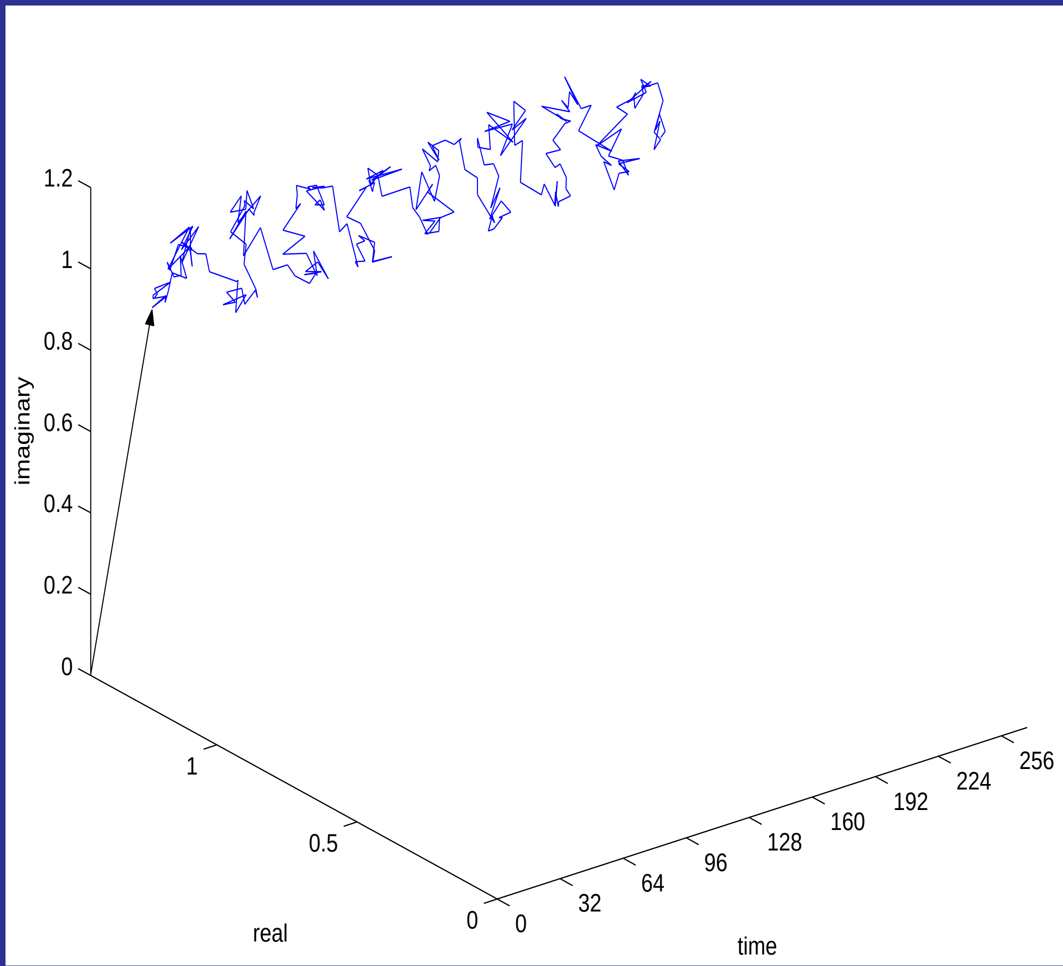
## Complex Image Reconstruction

This occurs over time in fMRI and results in complex-valued effective PSDs or voxel time course observations, $y_t = y_{Rt} + iy_{It}$.



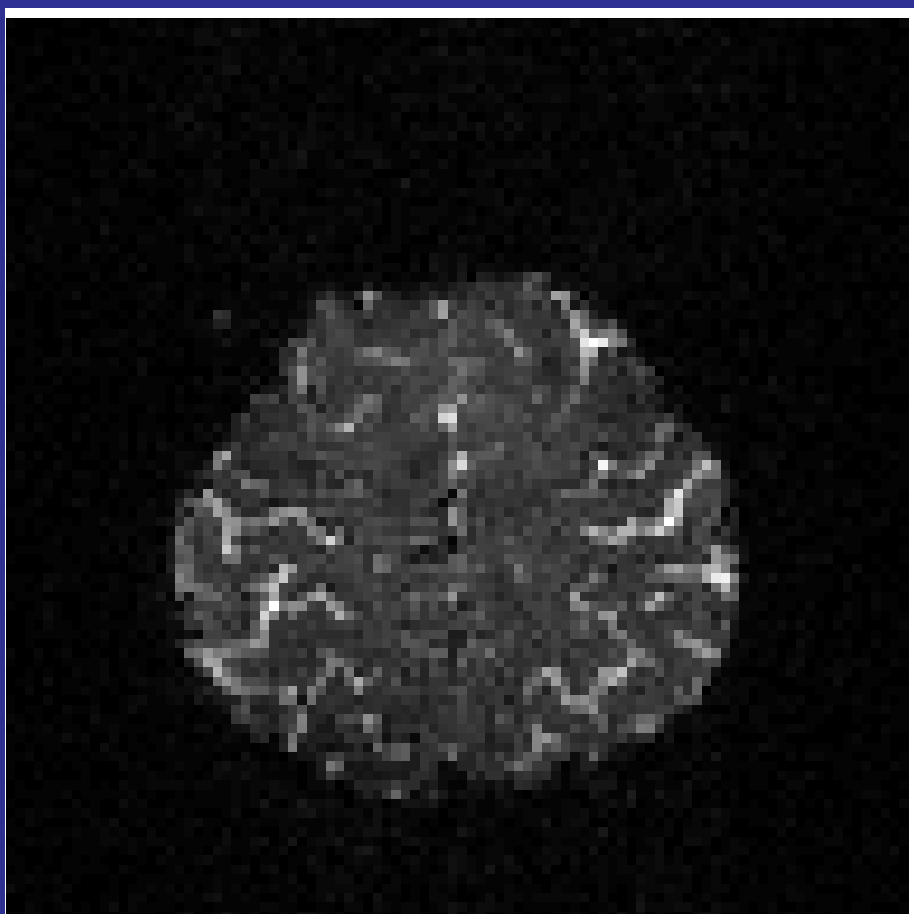
# Complex Image Reconstruction

Time series are complex-valued or bivariate with coupled means.

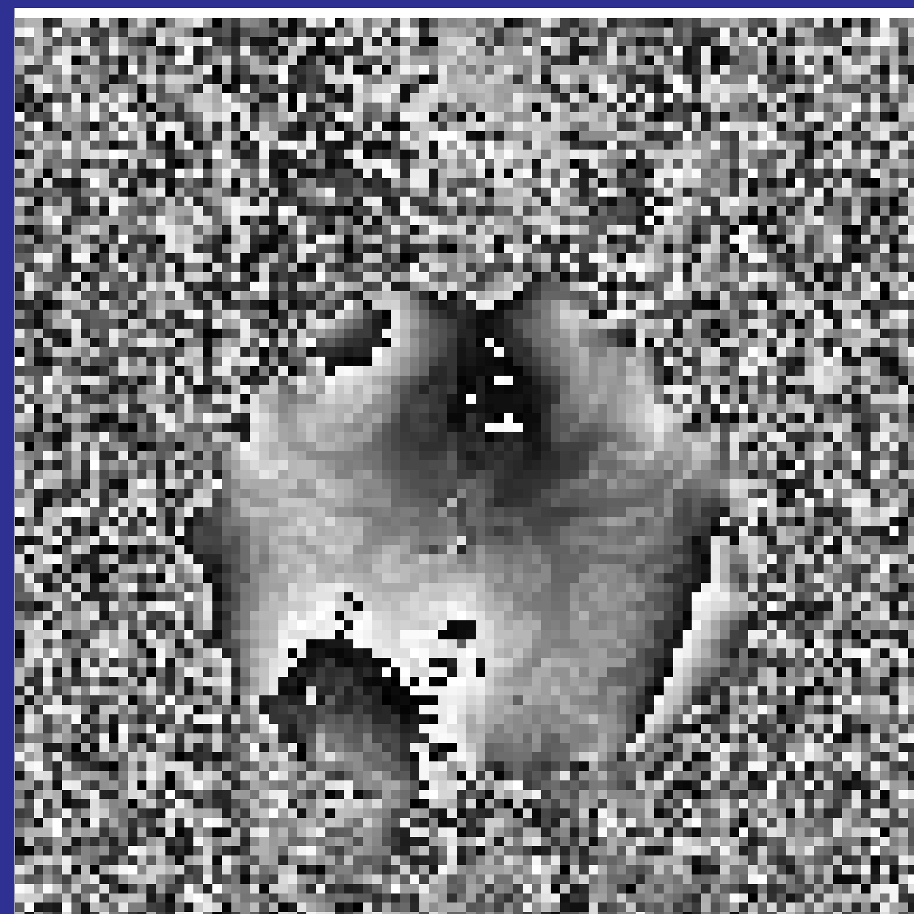


## Complex Image Reconstruction

Most fMRI studies transform from real-imaginary rectangular coordinates to magnitude-phase polar coordinates, $\rho(x,y) = m(x,y)e^{i\phi(x,y)}$.



(a) Magnitude, $m_t = \sqrt{y_{Rt}^2 + y_{It}^2}$

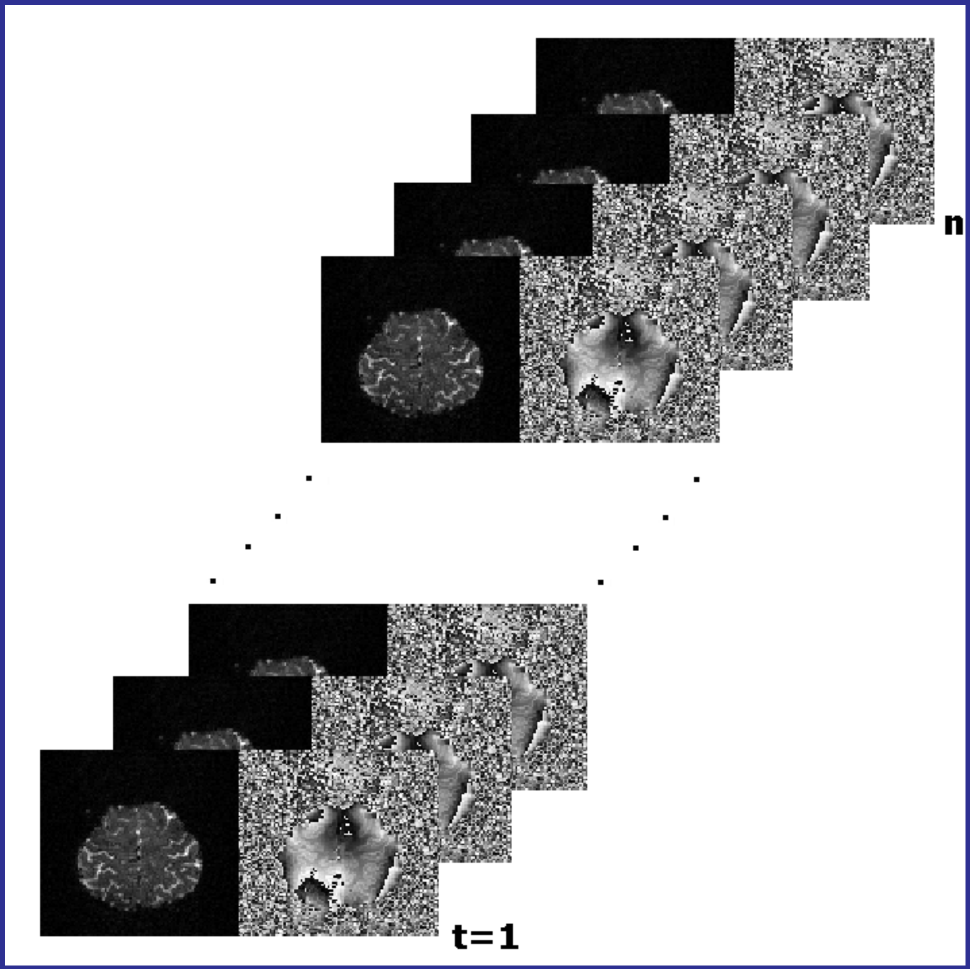


(b) Phase, $\phi_t = \text{atan}(y_{It}/y_{Rt})$

Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

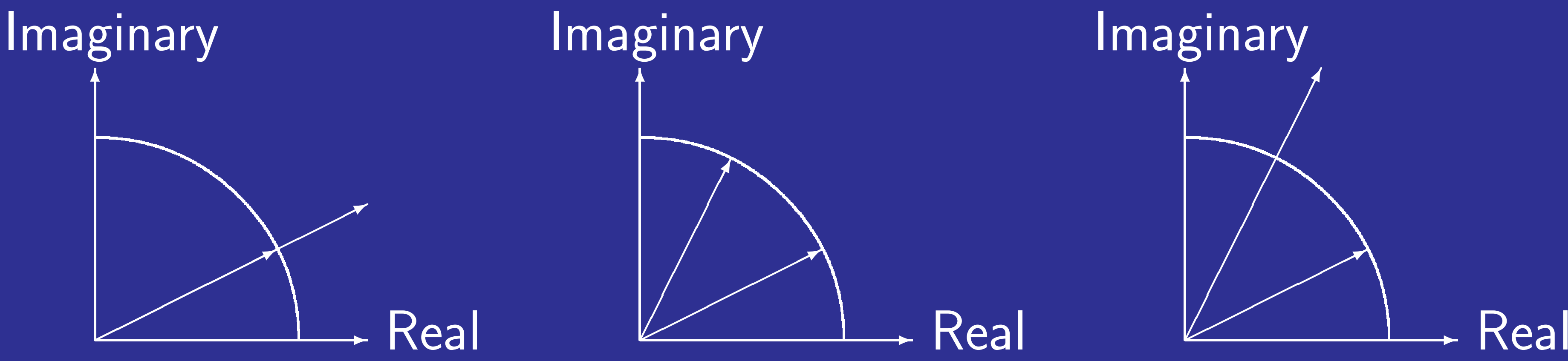
## Complex Image Reconstruction

Collect a sequence of these reconstructed images over time.
Form voxel time courses, $y_t = m_t e^{i\phi_t}$.



# Activation Methods

Block-designed experiment: Off-On-Off-...-On-Off task

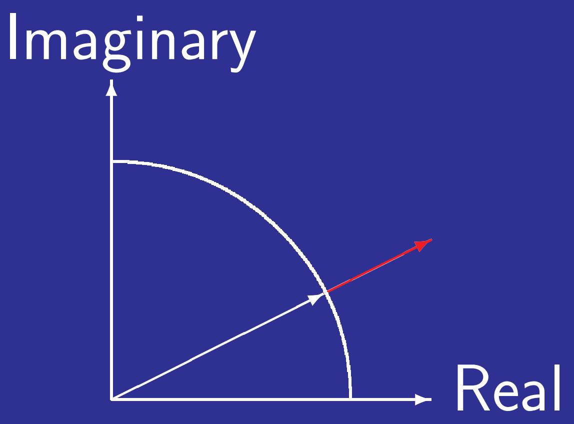


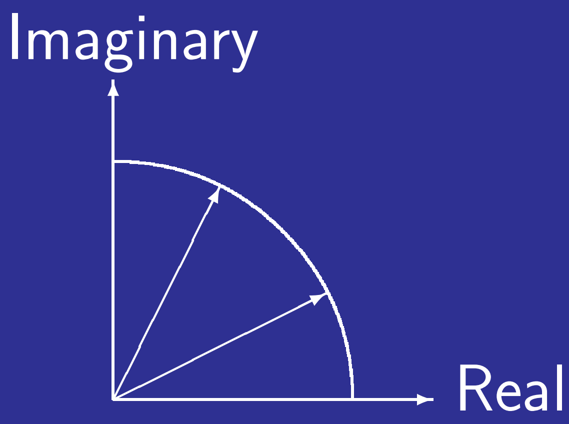

- Real Magnitude-Only (MO) Activation[1,2,3]
- Real Phase-Only (PO) Activation[4]
- Complex Magnitude w/ Constant Phase (CP) Activation[5]
- Complex Magnitude &/or Phase (CM) Activation[6]

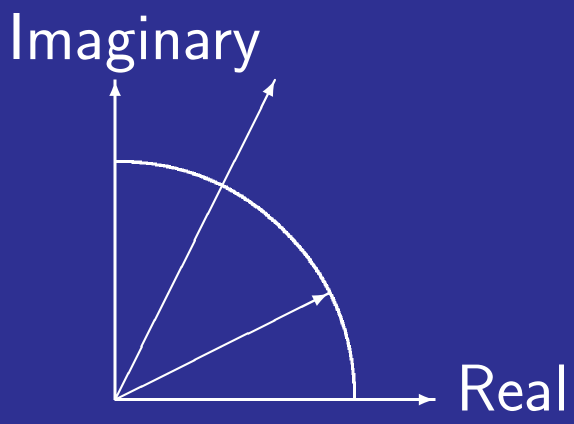
[1]Bandettini et al.: MRM, 30:161-173, 1993.
[2]Friston et al.: HBM, 2:189-210, 1995.
[3]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[4]Rowe and Meller: Submitted, 2005
[5]Rowe and Logan: NeuroImage, 24:603-606, 2005.
[6]Rowe: NeuroImage, 25:1310-1324, 2005b.

# Activation Methods

Block-designed experiment: Off-On-Off-...-On-Off task



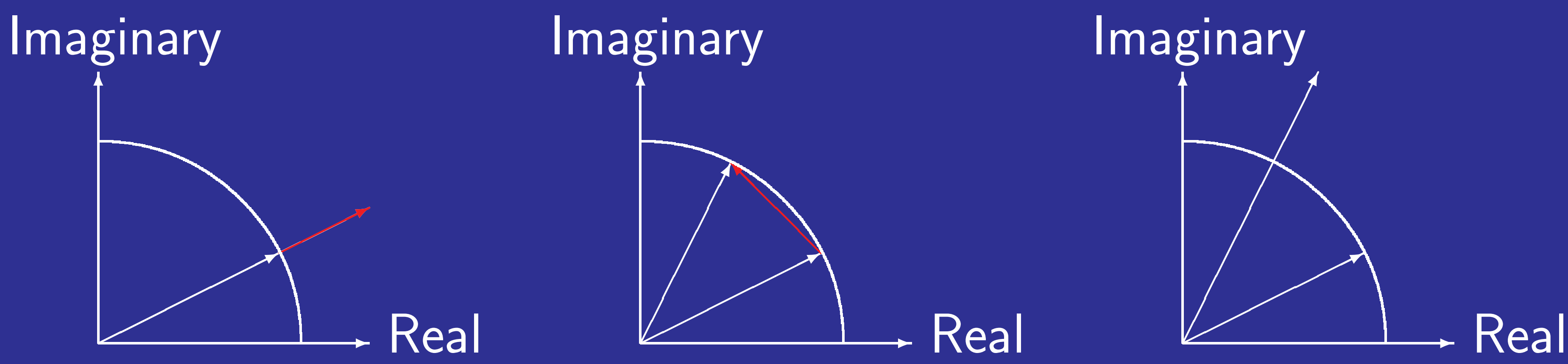




- Real Magnitude-Only (MO) Activation[1,2,3]
- Real Phase-Only (PO) Activation[4]
- Complex Magnitude w/ Constant Phase (CP) Activation[5]
- Complex Magnitude &/or Phase (CM) Activation[6]

[1]Bandettini et al.: MRM, 30:161-173, 1993.
[2]Friston et al.: HBM, 2:189-210, 1995.
[3]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[4]Rowe and Meller: Submitted, 2005
[5]Rowe and Logan: NeuroImage, 24:603-606, 2005.
[6]Rowe: NeuroImage, 25:1310-1324, 2005b.

## Activation Methods

Block-designed experiment: Off-On-Off-...-On-Off task

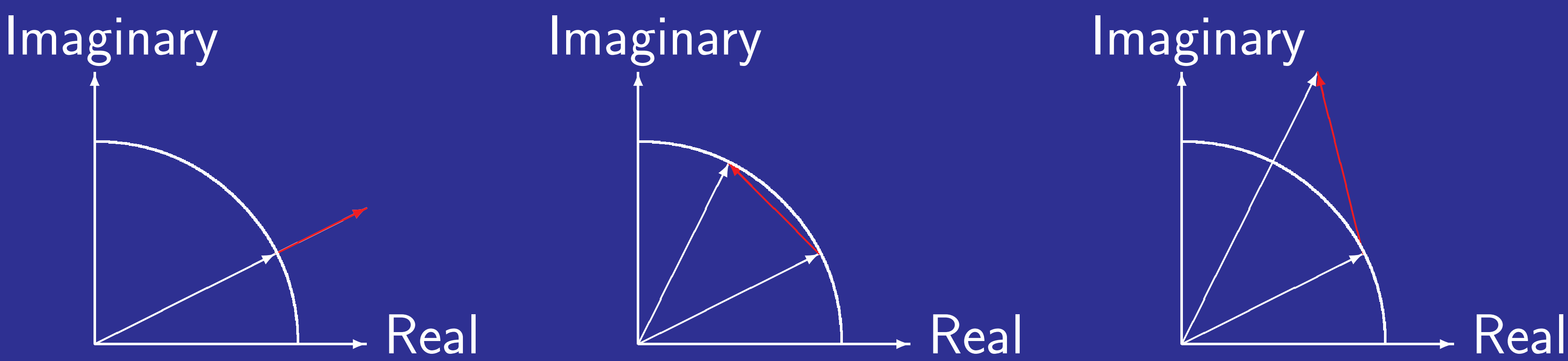



- Real Magnitude-Only (MO) Activation[1,2,3]
- Real Phase-Only (PO) Activation[4]
- Complex Magnitude w/ Constant Phase (CP) Activation[5]
- Complex Magnitude &/or Phase (CM) Activation[6]

[1]Bandettini et al.: MRM, 30:161-173, 1993.
[2]Friston et al.: HBM, 2:189-210, 1995.
[3]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[4]Rowe and Meller: Submitted, 2005
[5]Rowe and Logan: NeuroImage, 24:603-606, 2005.
[6]Rowe: NeuroImage, 25:1310-1324, 2005b.

## Activation Methods

Block-designed experiment: Off-On-Off-...-On-Off task




- ➢ Real Magnitude-Only (MO) Activation[1,2,3]
- ➢ Real Phase-Only (PO) Activation[4]
- ➢ Complex Magnitude w/ Constant Phase (CP) Activation[5]
- ➢ Complex Magnitude &/or Phase (CM) Activation Activation[6]

[1] Bandettini et al.: MRM, 30:161-173, 1993.
[2] Friston et al.: HBM, 2:189-210, 1995.
[3] Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[4] Rowe and Meller: Submitted, 2005
[5] Rowe and Logan: NeuroImage, 24:603-606, 2005.
[6] Rowe: NeuroImage, 25:1310-1324, 2005b.

# Real Magnitude-Only Activation

- Convert the complex valued images into magnitude images:

$$m_t = \sqrt{y_{Rt}^2 + y_{It}^2}$$

- Discard phase information
- Assume a high SNR for Ricean noise to be approximated by normal noise

$$\frac{m_t}{\sigma^2} e^{-\frac{(m_t^2+\rho_t^2)}{2\sigma^2}} \int_{-\pi}^{\pi} \frac{1}{2\pi} e^{\frac{\rho_t m_t}{\sigma^2} \cos(\phi_t - \theta_t)} d\phi_t \approx \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(m_t-\rho_t)^2}{2\sigma^2}}$$

- Assume normal noise on the magnitude data[1]

$$m = X\beta + \epsilon, \qquad \epsilon \sim N(0, \sigma^2 I_n)$$

$$m = \begin{pmatrix} m_1 \\ \vdots \\ m_n \end{pmatrix}, X = \begin{pmatrix} 1 & 1 & \pm 1 \\ \vdots & \vdots & \vdots \\ 1 & n & \pm 1 \end{pmatrix}, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$

[1] Gudbjartsson and Patz: MRM, 34:910-914, 1995.

# Real Magnitude-Only Activation

➢ Contrast hypotheses $C\beta = 0$ vs $C\beta \neq 0$, $C = (0, 0, 1)$, with MLEs:

∘ $\beta_2 \neq 0$:

$$\hat{\beta} = (X'X)^{-1}X'm$$
$$\hat{\sigma}^2 = \frac{1}{n}(m - X\hat{\beta})'(m - X\hat{\beta})$$

∘ $\beta_2 = 0$:

$$\tilde{\beta} = \Psi\hat{\beta}$$
$$\tilde{\sigma}^2 = \frac{1}{n}(m - X\tilde{\beta})'(m - X\tilde{\beta})$$
$$\Psi = (X'X)^{-1}\left[(X'X) - C'(C(X'X)^{-1}C')^{-1}C\right]$$

➢ A LRT[1] to evaluate significance with large sample $\chi^2$ activation statistics

$$-2\log(\lambda_{MO}) = n\log\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2}\right)$$

➢ The activation statistics thresholded[2] to determine "true" activation

[1] Rowe and Logan: NeuroImage, 23:603-606, 2005a. [2] Logan and Rowe: NeuroImage, 22:95-108, 2004.

## Real Phase-Only Activation

- Convert the complex-valued images into phase images:

$$\phi_t = \operatorname{atan}\left(\frac{y_{It}}{y_{Rt}}\right)$$

- Discard magnitude information
- Generally assume a high SNR for phase dist. to be normal[1]
- The same GLM as MO method used with phase time series:

$$\phi = U\gamma + \delta, \qquad \delta \sim N(0, \tau^2 I_n)$$

- The same MLEs as the MO method can be derived.
  Replace $m$ with $\phi$, $X$ with $U$, and $\beta$ with $\gamma$.
- The same LRT used for activation maps then thresholded

[1] Rowe and Meller: Submitted, 2005.

# Complex Magnitude with Constant Phase Activation

- A nonlinear regression model[1] on the complex-valued time series with normal noise on the real and imaginary observations:

$$\begin{pmatrix} y_{R1} \\ \vdots \\ y_{Rn} \\ y_{I1} \\ \vdots \\ y_{In} \end{pmatrix} = \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix} \begin{pmatrix} \beta\cos\theta \\ \beta\sin\theta \end{pmatrix} + \eta, \qquad \eta \sim N(0, \sigma^2 I_{2n})$$

- $X$ and $\beta$ are the same as the MO model
- The LHS is the real series stacked on the imaginary series
- $\theta$ is the temporally constant voxel-wise phase angle.
  If the magnetic field is stable and in grey matter, this is true.

[1]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.

# Complex Magnitude with Constant Phase Activation

➢ Contrast hypotheses $C\beta = 0$ vs $C\beta \neq 0$, $C = (0,0,1)$, with MLEs:

∘ $\beta_2 \neq 0$

$$\hat{\beta} = \hat{\beta}_R \cos\hat{\theta} + \hat{\beta}_I \sin\hat{\theta}$$

$$\hat{\sigma}^2 = \frac{1}{2n}\begin{pmatrix} y_R - X\hat{\beta}\cos\hat{\theta} \\ y_I - X\hat{\beta}\sin\hat{\theta}\end{pmatrix}'\begin{pmatrix} y_R - X\hat{\beta}\cos\hat{\theta} \\ y_I - X\hat{\beta}\sin\hat{\theta}\end{pmatrix}$$

$$\hat{\theta} = \frac{1}{2}\text{atan}\left[\frac{\hat{\beta}_I'(X'X)\hat{\beta}_R}{\left(\hat{\beta}_R'(X'X)\hat{\beta}_R - \hat{\beta}_I'(X'X)\hat{\beta}_I\right)/2}\right]$$

∘ $\beta_2 = 0$

$$\tilde{\beta} = \Psi\left[\hat{\beta}_R \cos\tilde{\theta} + \hat{\beta}_I \sin\tilde{\theta}\right]$$

$$\tilde{\sigma}^2 = \frac{1}{2n}\begin{pmatrix} y_R - X\tilde{\beta}\cos\tilde{\theta} \\ y_I - X\tilde{\beta}\sin\tilde{\theta}\end{pmatrix}'\begin{pmatrix} y_R - X\tilde{\beta}\cos\tilde{\theta} \\ y_I - X\tilde{\beta}\sin\tilde{\theta}\end{pmatrix}$$

$$\tilde{\theta} = \frac{1}{2}\text{atan}\left[\frac{\hat{\beta}_R'\Psi(X'X)\hat{\beta}_I}{\left(\hat{\beta}_R'\Psi(X'X)\hat{\beta}_R - \hat{\beta}_I'\Psi(X'X)\hat{\beta}_I\right)/2}\right]$$

$\hat{\beta}_R = (X'X)^{-1}X_R'y_R$ $\quad$ $\hat{\beta}_I = (X'X)^{-1}X_I'y_I$ $\quad$ Same $\Psi$ as MO

## Complex Magnitude with Constant Phase Activation

- Like other methods, the significance of the hypotheses can be evaluated using a LRT which results in a large sample $\chi^2$ statistic:

$$-2\log\lambda_{CP} = n\log\frac{\widehat{\sigma}^2}{\widetilde{\sigma}^2}$$

- Like other methods, this can be changed into an (large sample) F or t statistic with some algebra
- Again the map of activation statistics can be thresholded to leave "active" voxels above threshold

## Complex Magnitude and/or Phase Activation

Recent work has suggests phase time courses may exhibit TRPCs (Hoogenrad et al., 1998; Borduka et al.,1999; Menon,2002).

Considered magnitude linearly changing w/ constant & unrestricted phase.

Both magnitude and phase change linearly over time.

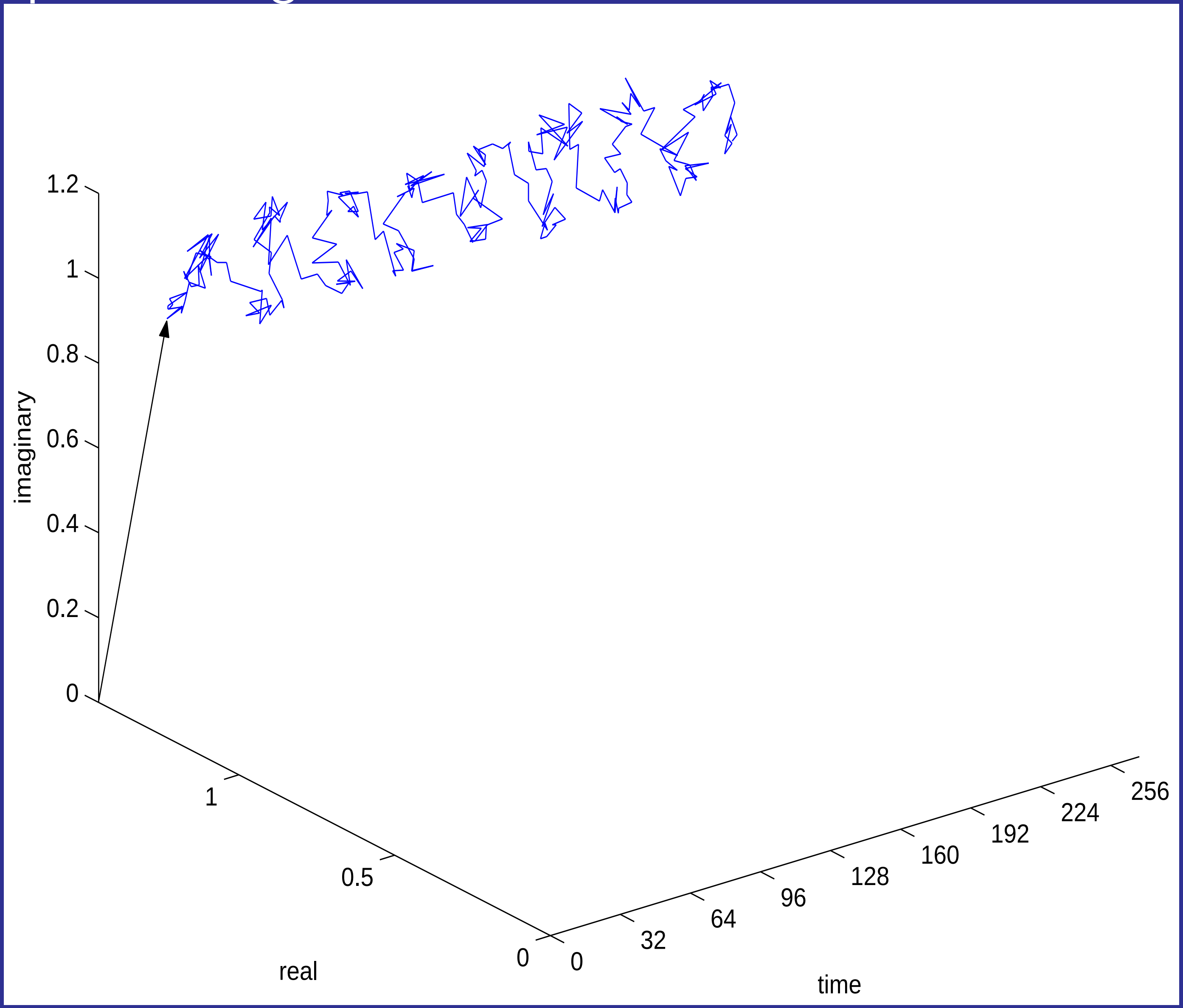
$$\begin{aligned} y_t &= [\rho_t \cos\theta_t + \eta_{Rt}] + i[\rho_t \sin\theta_t + \eta_{It}] \\ \rho_t &= x_t'\beta = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_{q_1} x_{q_1 t} \\ \theta_t &= u_t'\gamma = \gamma_0 + \gamma_1 u_{1t} + \cdots + \gamma_{q_2} u_{q_2 t}, \qquad t = 1, \ldots, n \end{aligned}$$

$x_t'$ is the $t^{th}$ row of a design matrix $X$ for the magnitude and
$u_t'$ is the $t^{th}$ row of a design matrix $U$ for the phase.

Last Col of $X$ and $U$ are (different) task related reference functions.

## Complex Magnitude and/or Phase Activation

Magnitude and phase changes with task in this voxel!



## Complex Magnitude and/or Phase Activation

We want to see if there is anything in either the magnitude or phase of the observed complex-valued time course has a component related to the reference function.

i.e.
$C = (0, ..., 0, 1),\ \beta = (\beta_0, \beta_1, \cdots, \beta_{q_1})'$
$D = (0, ..., 0, 1),\ \gamma = (\gamma_0, \gamma_1, \cdots, \gamma_{q_2})'$

MLE's from both under null and alternative hypotheses.

Form GLR test statistic, $\lambda$ and $-2\log\lambda$.

## Complex Magnitude and/or Phase Activation

Four readily visible hypotheses for testing.

$$H_a : C\beta \neq 0,\ \ D\gamma \neq 0$$
$$H_b : C\beta = 0,\ \ D\gamma \neq 0$$
$$H_c : C\beta \neq 0,\ \ D\gamma = 0$$
$$H_d : C\beta = 0,\ \ D\gamma = 0$$

We can combine these four hypotheses in different ways to form specific meaningful hypothesis pairs.

Other models and hypotheses are supported within this framework!

Can write down and maximize the log likelihood under each hypothesis using various Lagrange constraints.

Other Hypotheses: One sided or Interval

## Complex Magnitude and/or Phase Activation

$$
\begin{aligned}
LL &= -n\log(2\pi) - \sum_{t=1}^{n}\log r_t - n\log\sigma^2 \\
&\quad - \frac{1}{2\sigma^2}\sum_{t=1}^{n}\left[r_t^2 + (x_t'\beta)^2 - 2(x_t'\beta)\underbrace{r_t\cos(\phi_t - u_t'\gamma)}_{r_{*t}}\right] \\
&= -n\log(2\pi) - \sum_{t=1}^{n}\log r_t - n\log\sigma^2 \\
&\quad - \frac{1}{2\sigma^2}\left[(r - X\beta)'(r - X\beta) + 2(r - r_*)'X\beta\right] \\
&\quad + \psi'(C\beta - 0) + \delta'(D\gamma - 0)
\end{aligned}
$$

$H_0 : (\tilde{\beta}, \tilde{\gamma}, \tilde{\sigma}^2)$ and $H_1 : (\hat{\beta}, \hat{\gamma}, \hat{\sigma}^2)$

$\cos(\alpha) \doteq 1 - \alpha^2/2.$

$\alpha = \pi/12$ radians or $15$ degrees,

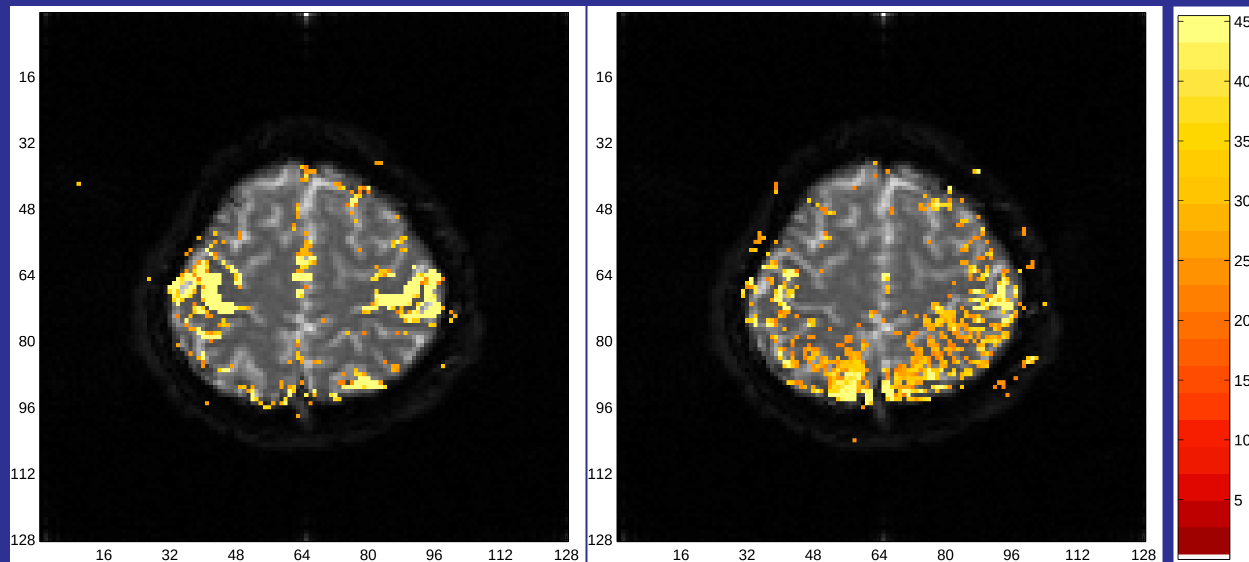
$\cos(\alpha) = 0.9659$ while $1 - \alpha^2/2 = 0.9657$

## Activations in Human Experimental Data

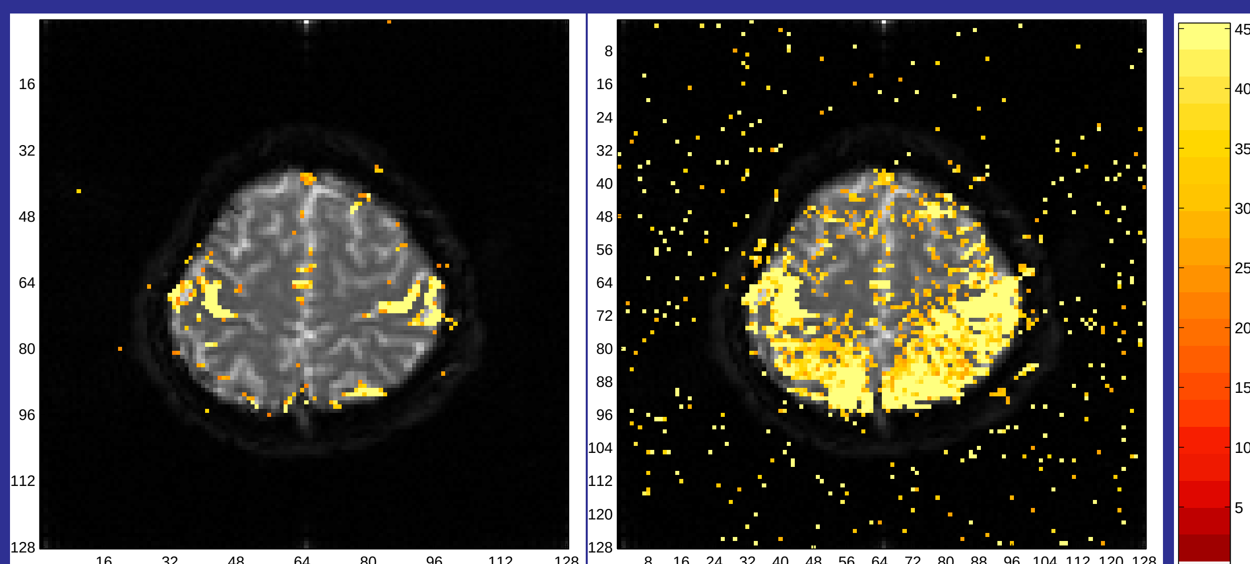
Imaging Parameters:
1.5T GE Signa
5 axial slices of 128x128
96 acq.-2.0833mm$^2$
128 recon.-1.5625mm$^2$
FOV $=$20cm
TR$=$1000ms
TE$=$47ms
FA$=90^\circ$

Task:
Bilateral sequential finger tapping light triggered
Block design
16 off $+$ 8$\times$(16on$+$16off);

# $\chi^2$ Maps: 5% Bonferroni Threshold

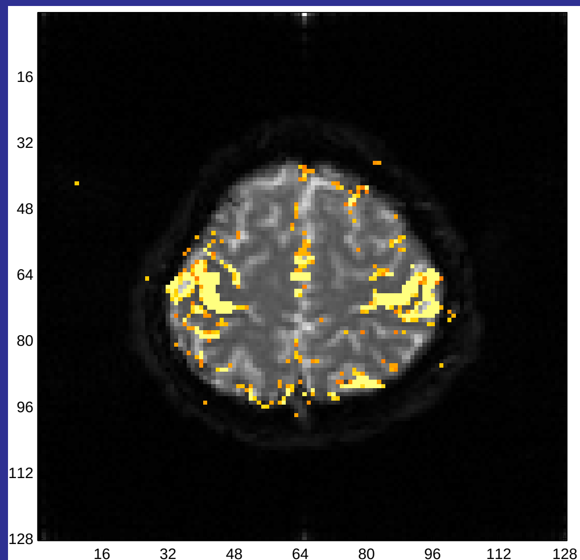



(a) UP/MO (b) PO

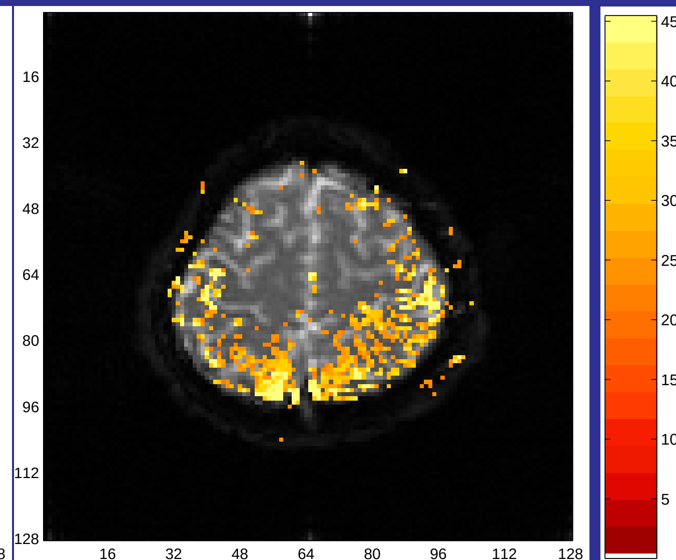



(c) CP (d) $C\beta = 0, D\gamma = 0; C\beta \neq 0, D\gamma \neq 0$

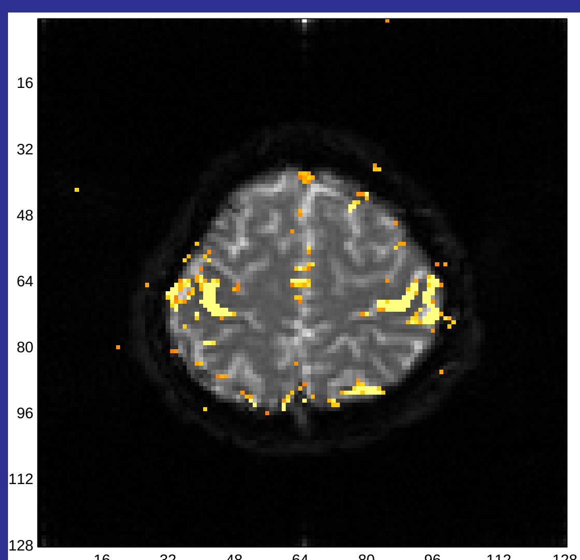
# $\chi^2$ Maps: 5% Bonferroni Threshold



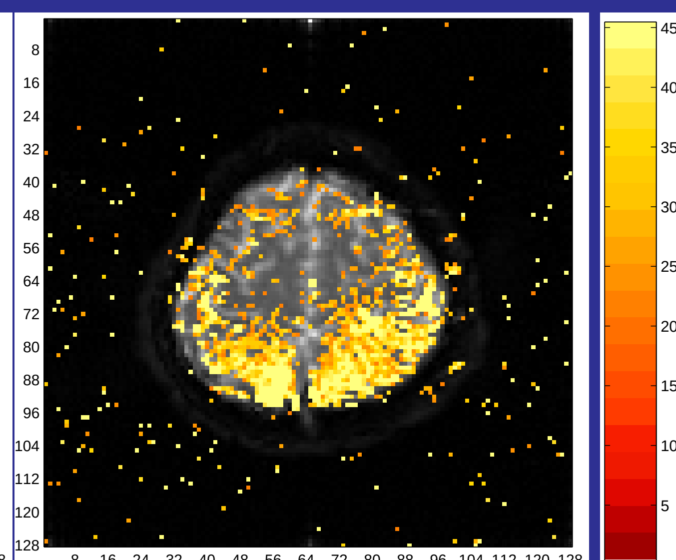

(a) UP/MO




(b) PO

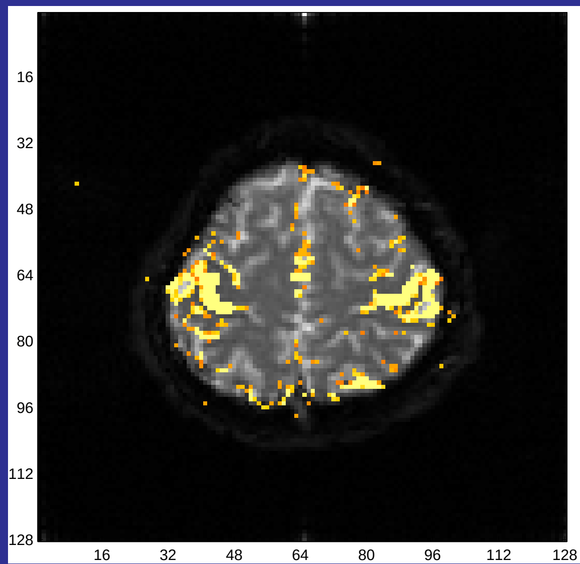



(c) CP

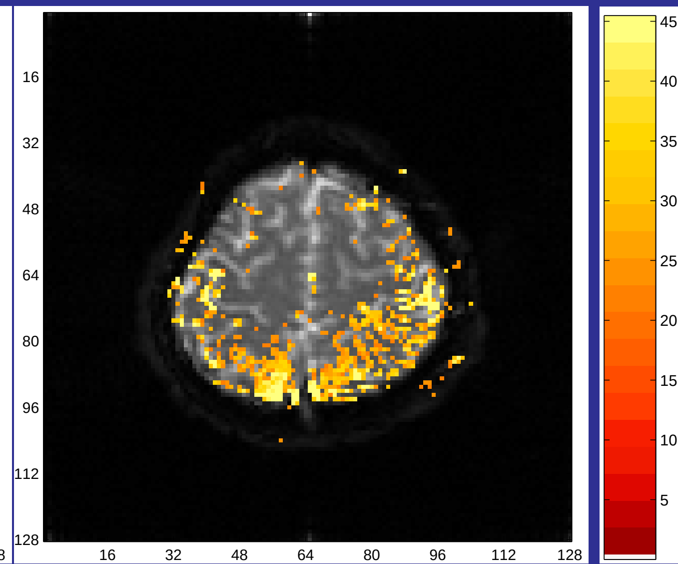



(d) $C\beta = 0, D\gamma = 0; C\beta = 0, D\gamma \neq 0$

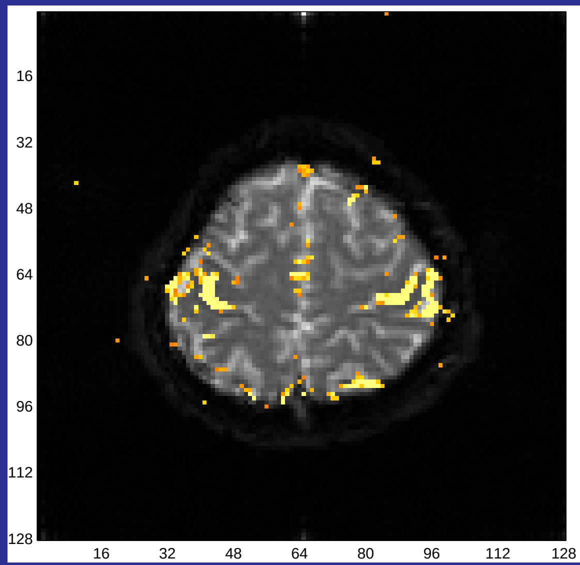
# $\chi^2$ Maps: 5% Bonferroni Threshold



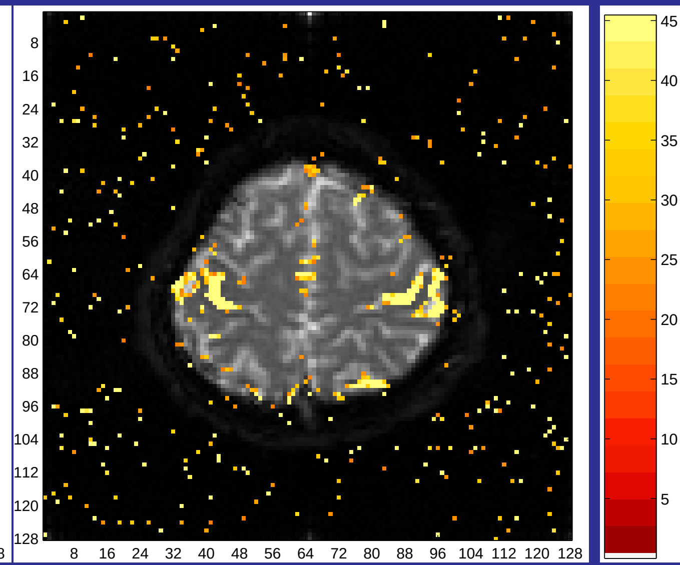

(a) UP/MO




(b) PO

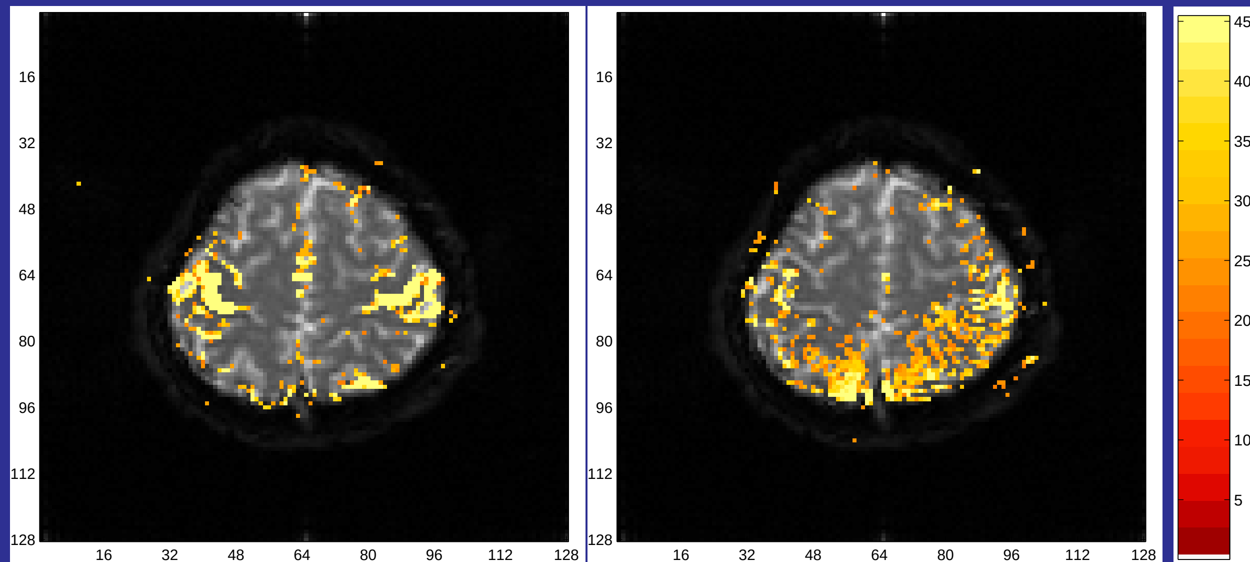



(c) CP

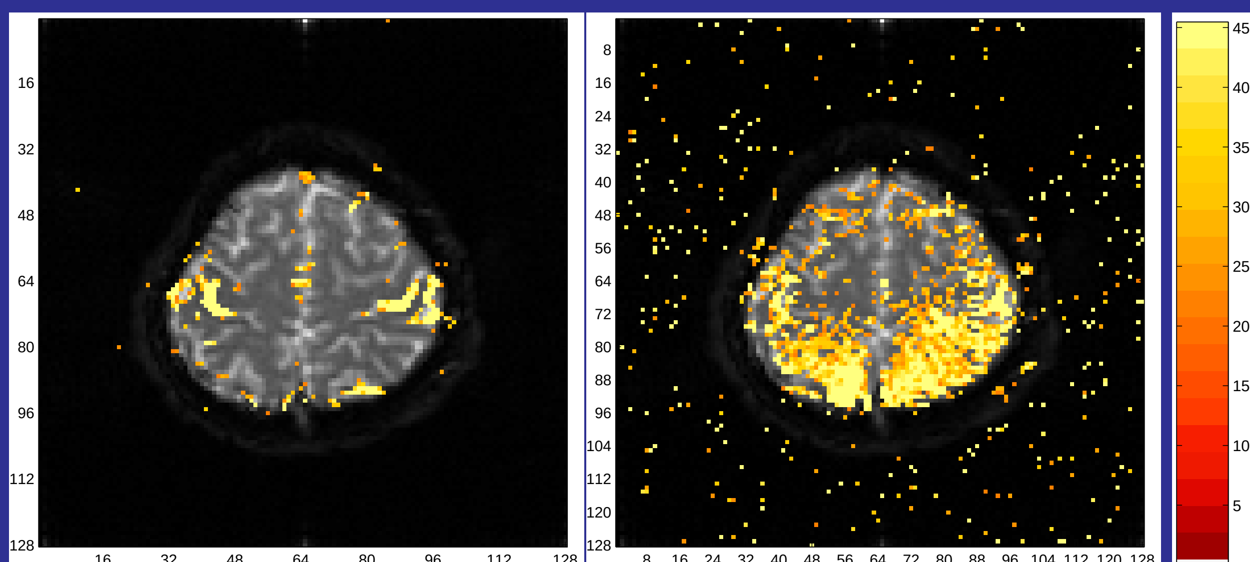



(d) $C\beta = 0, D\gamma = 0; C\beta \neq 0, D\gamma = 0$

# $\chi^2$ Maps: 5% Bonferroni Threshold

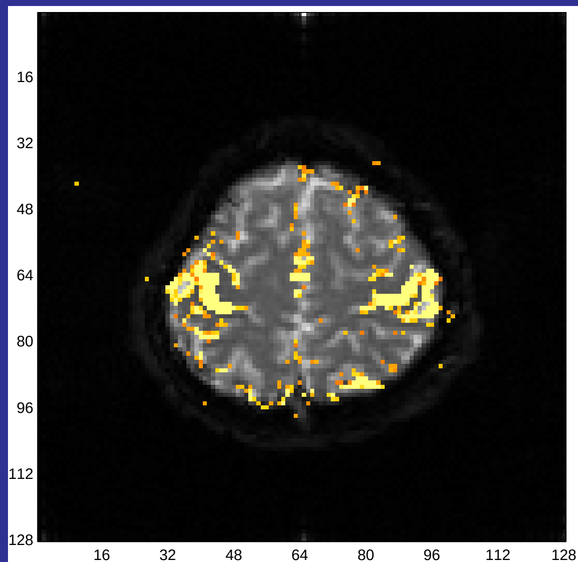



(a) UP/MO (b) PO

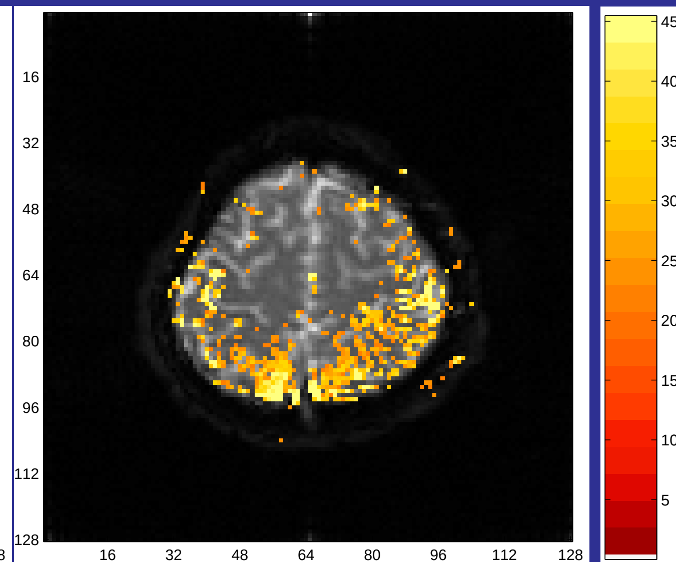



(c) CP (d) $C\beta \neq 0, D\gamma = 0; C\beta \neq 0, D\gamma \neq 0$

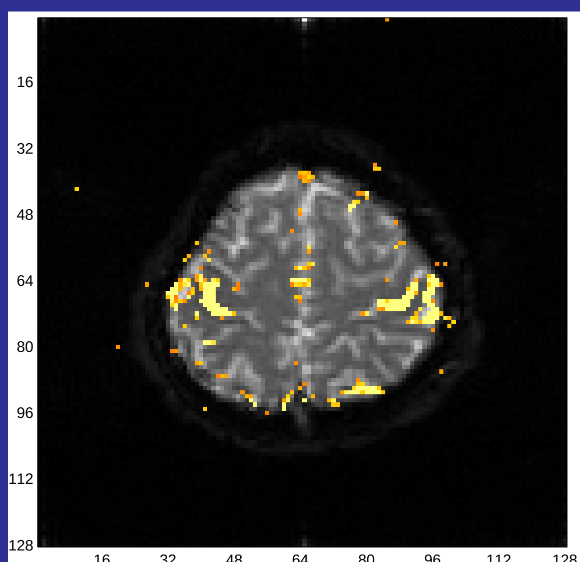
# $\chi^2$ Maps: 5% Bonferroni Threshold



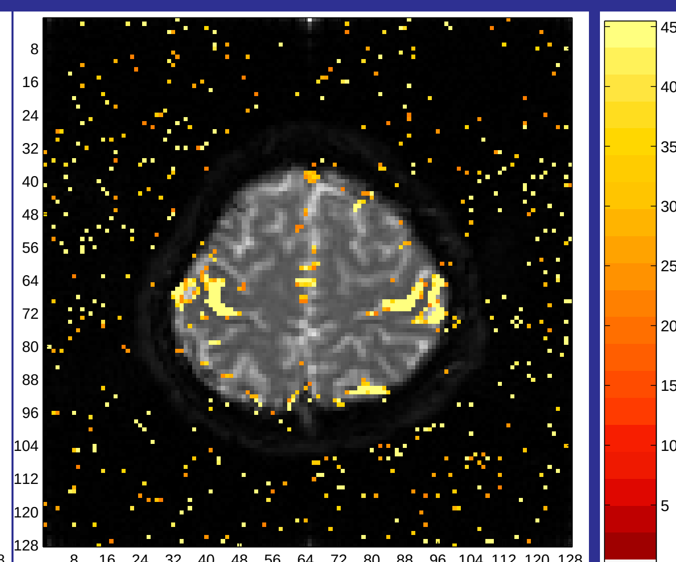

(a) UP/MO




(b) PO




(c) CP




(d) $C\beta = 0, D\gamma \neq 0; C\beta \neq 0, D\gamma \neq 0$

## Discussion

New methodological development.

Applied to real data.

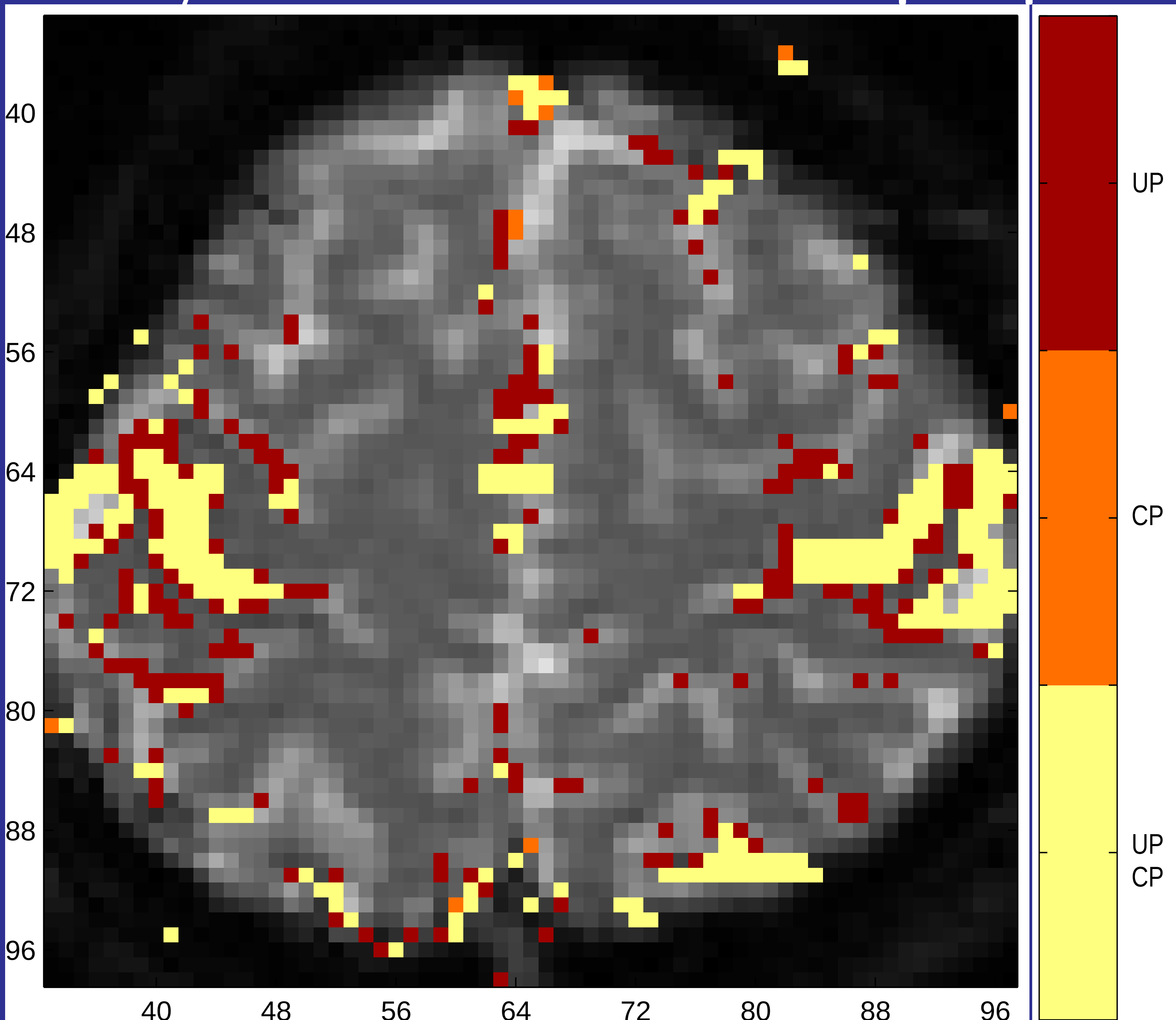
Applied to simulated data (not shown).

Further research is needed.

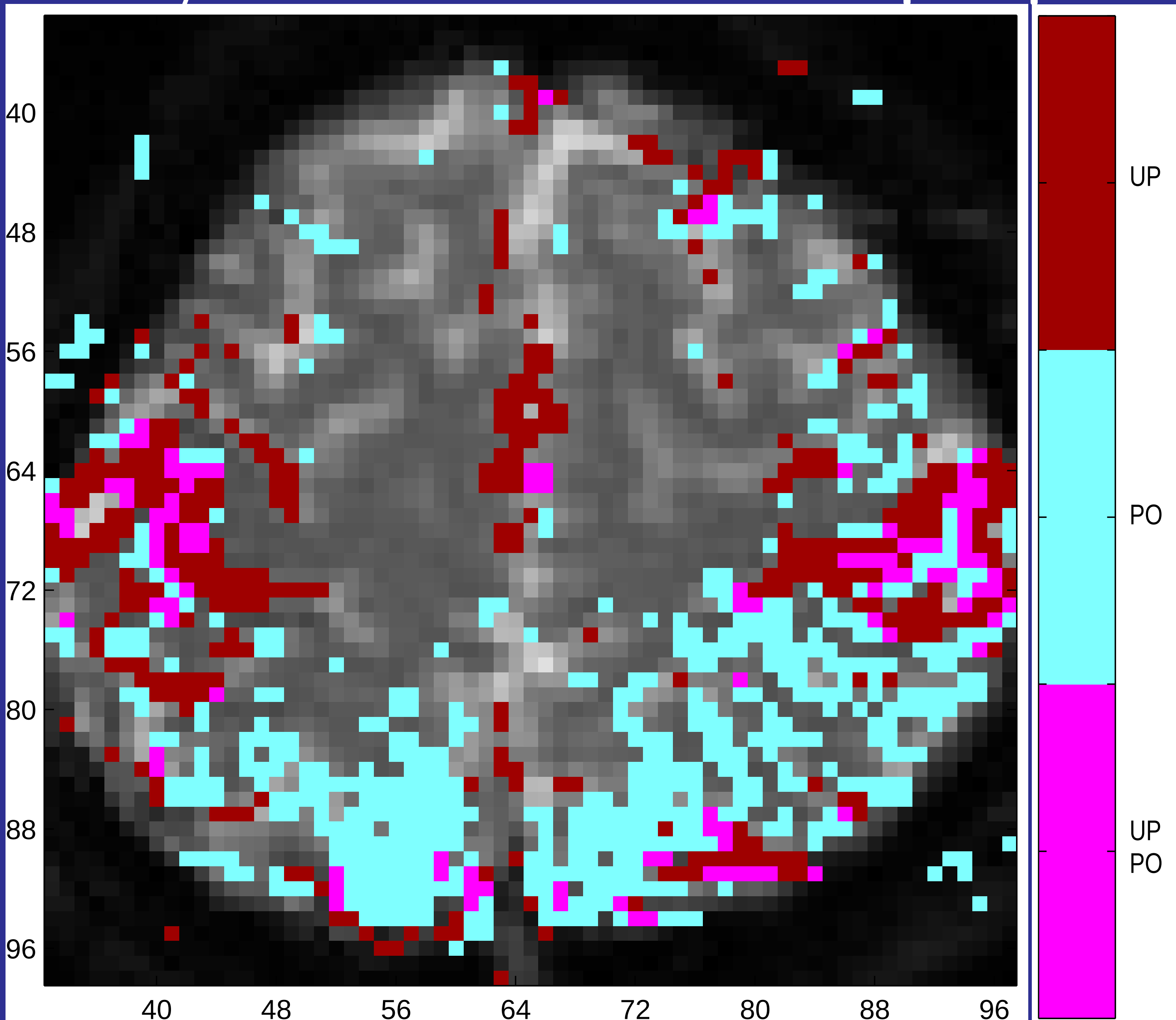
To be continued ...

Thank You

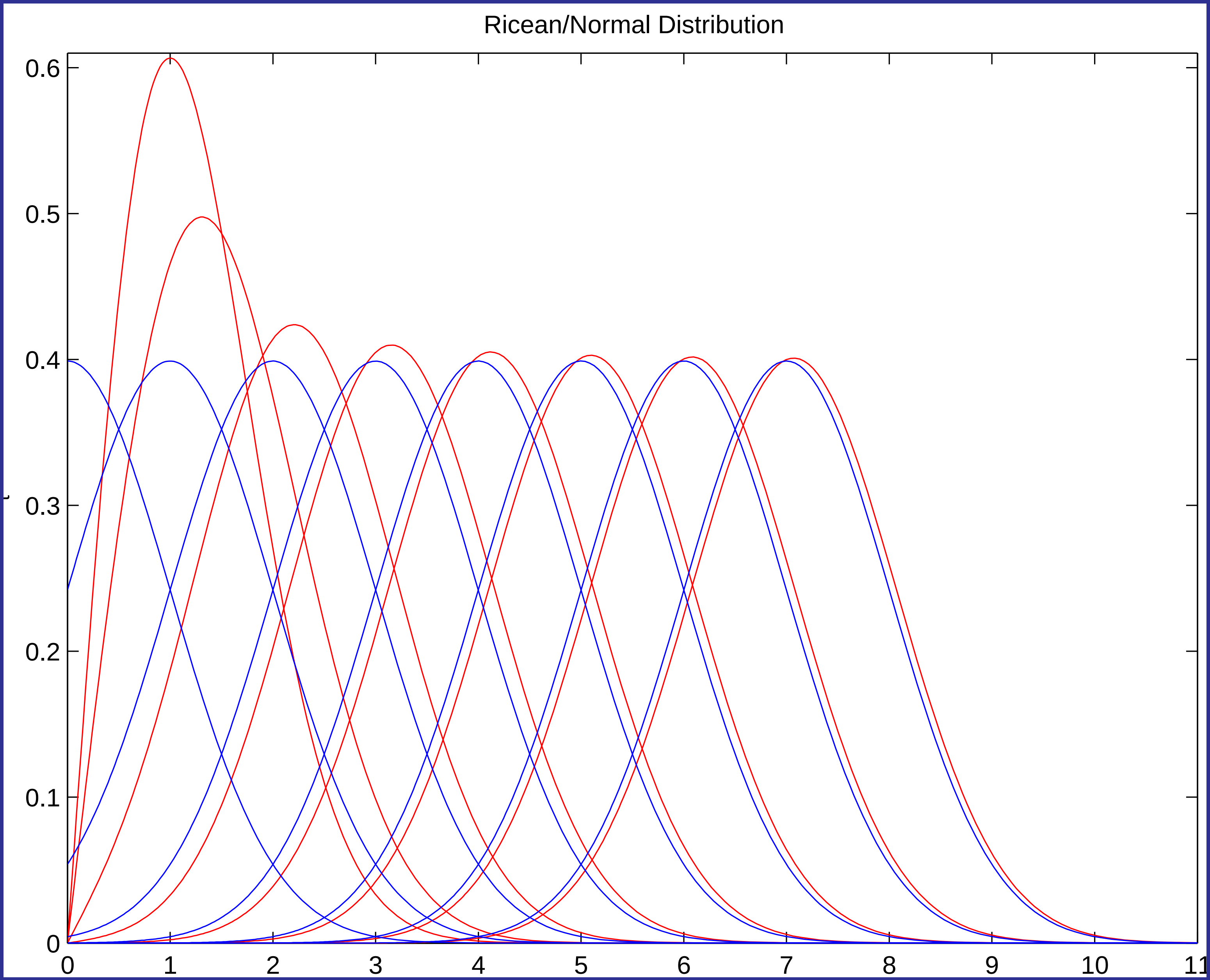
# Real fMRI-UP/MO & CP Bonferroni Overlap Maps



# Real fMRI-UP/MO & PO Bonferroni Overlap Maps



# Magnitude-Only




SNR=$m_t/\sigma$. Looks normal for decent SNR. Tails?