A Complex data method to Compute fMRI Activation

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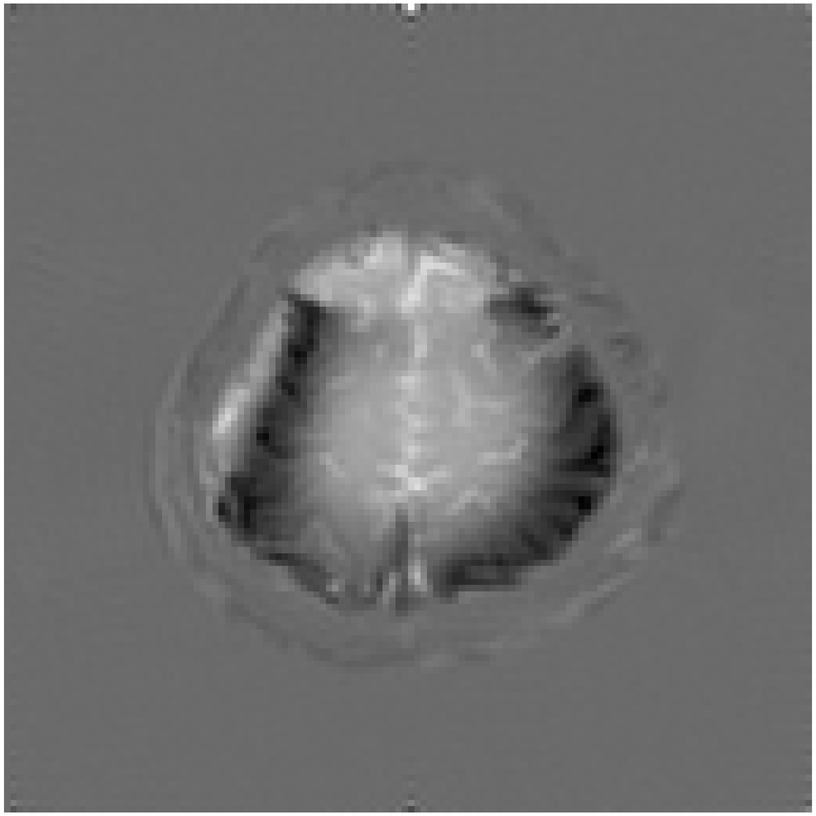
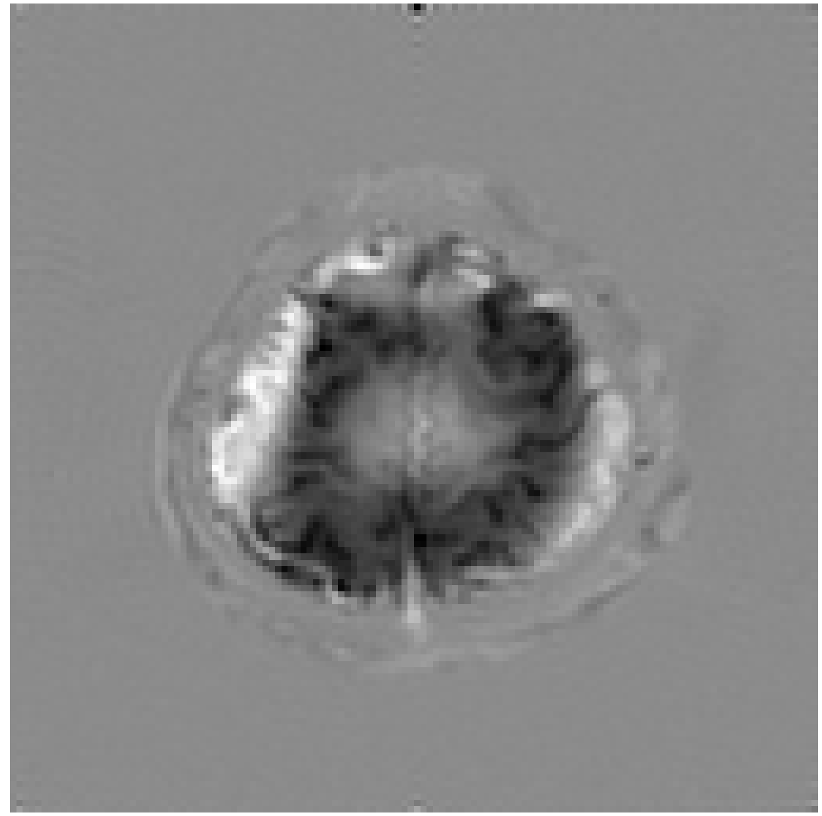
- ☞ **Images**
- ☞ **Complex/Magnitude Time Course Model**
- ☞ **Real fMRI Example**
- ☞ **Simulation Example**
- ☞ **Discussion**

Complex Single Time Images

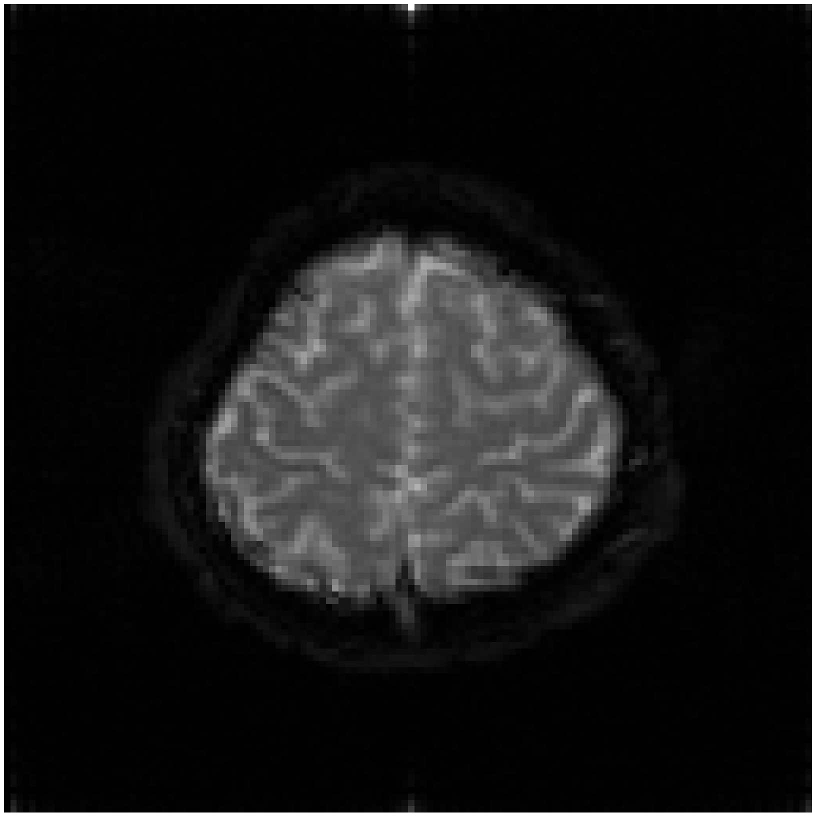


(a) real image

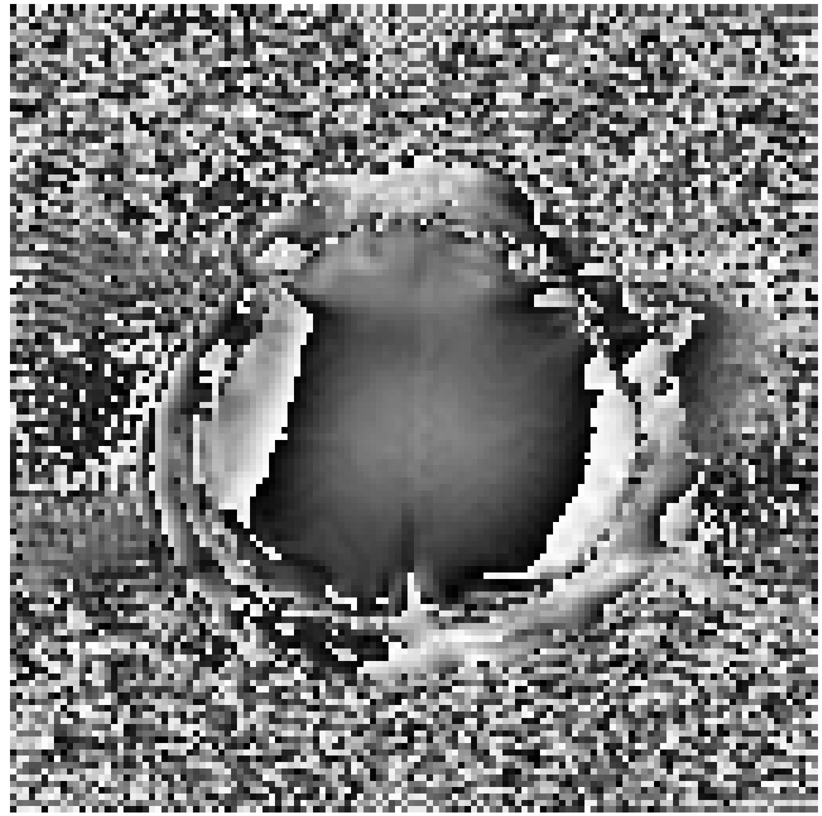


(b) imaginary image

Magnitude/Phase Single Time Images



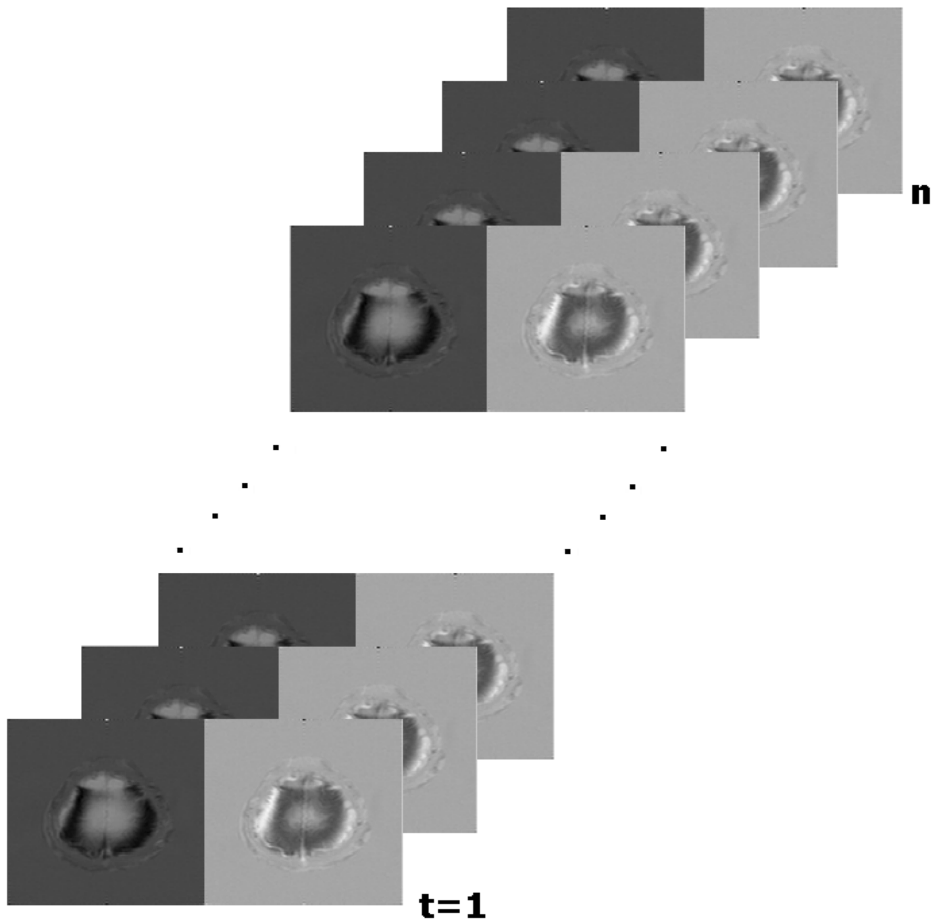
(c) magnitude image



(d) phase image

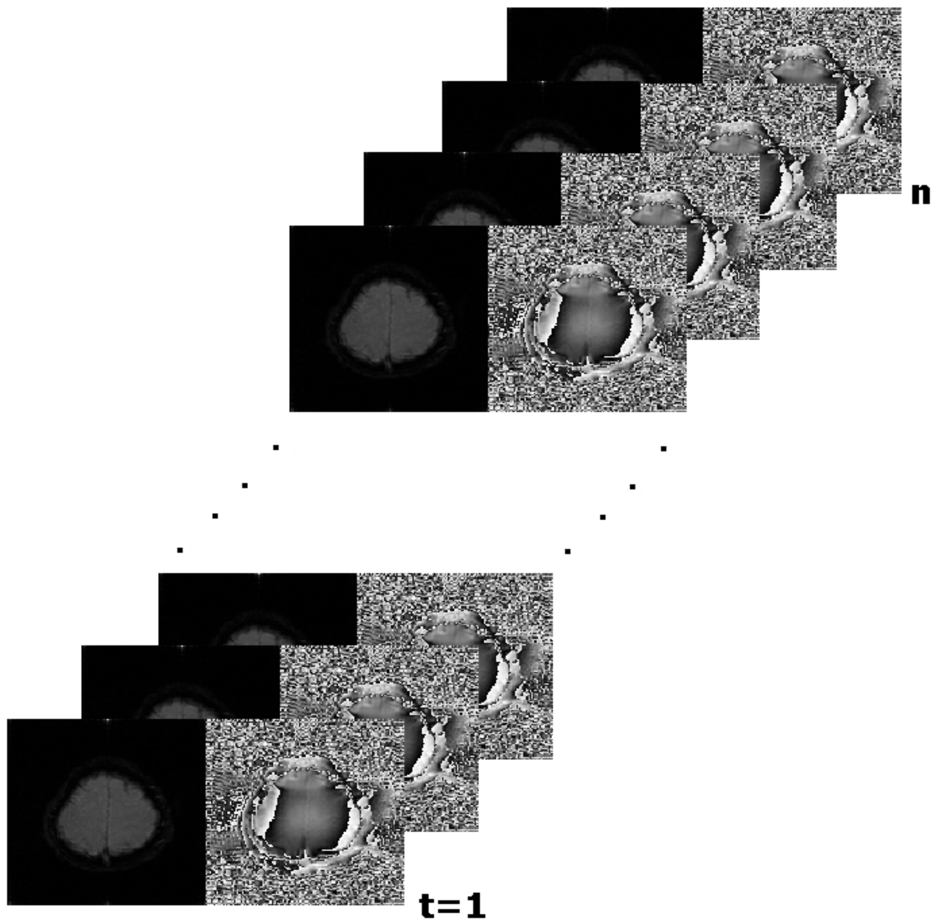
Complex Time Course Images

In fMRI we observe a series of complex images over time.



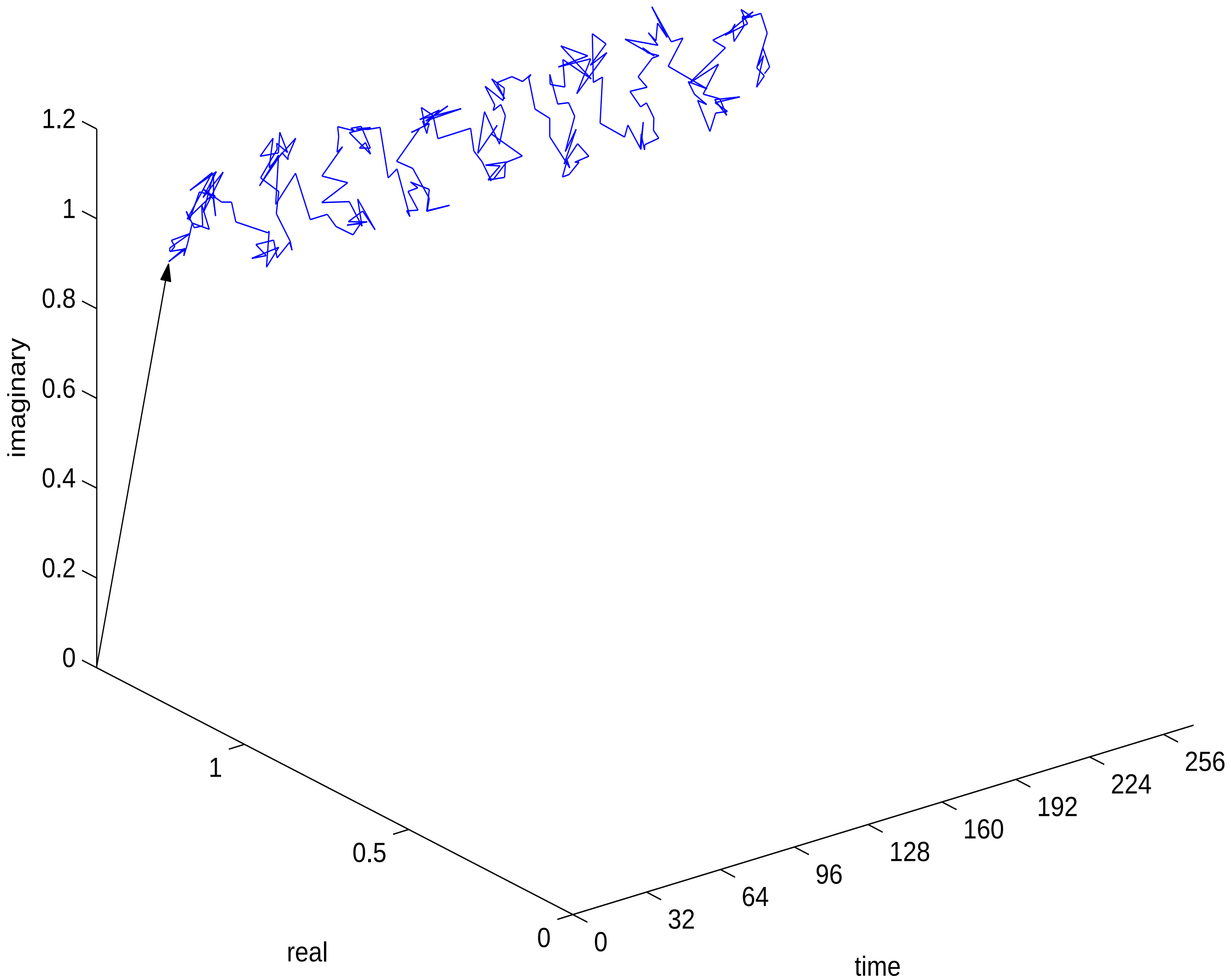
Magnitude Time Course Images

And not a series of real magnitude images. Phase not used.



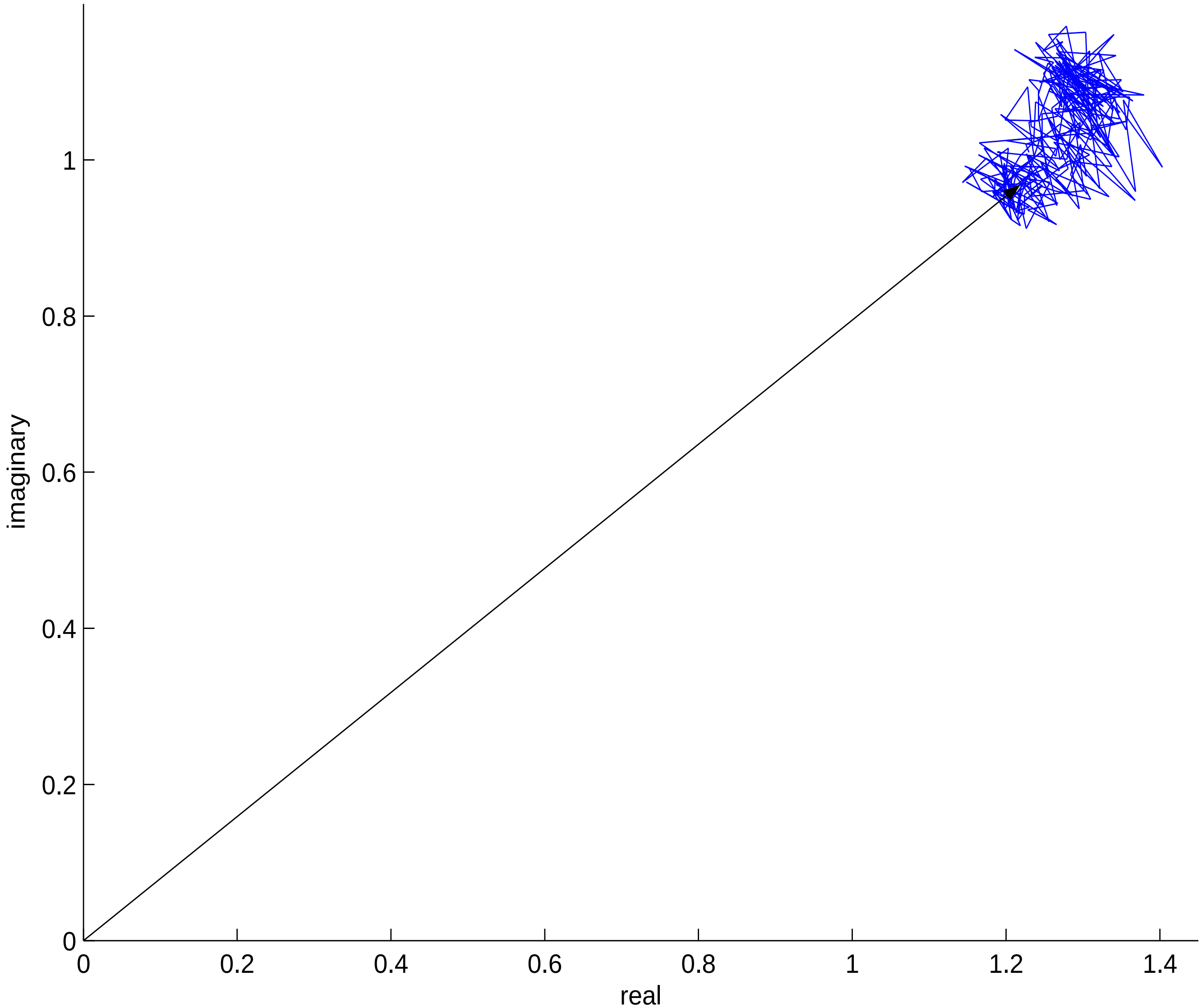
Complex Voxel Time Course

Real/imaginary vector observed over time. Block Design.



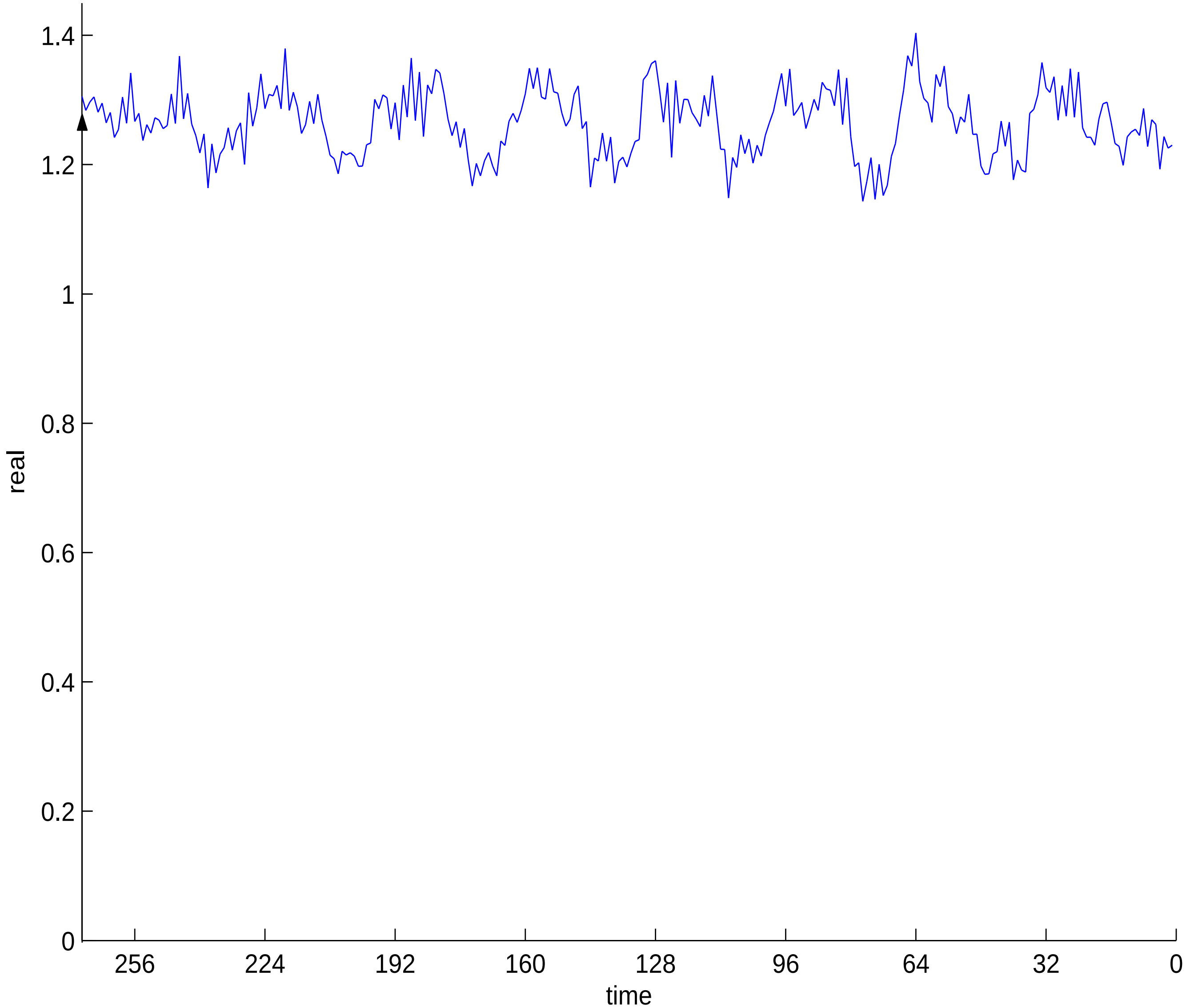
Complex Voxel Time Course

Rotate axis. Real/imaginary scatterplot.



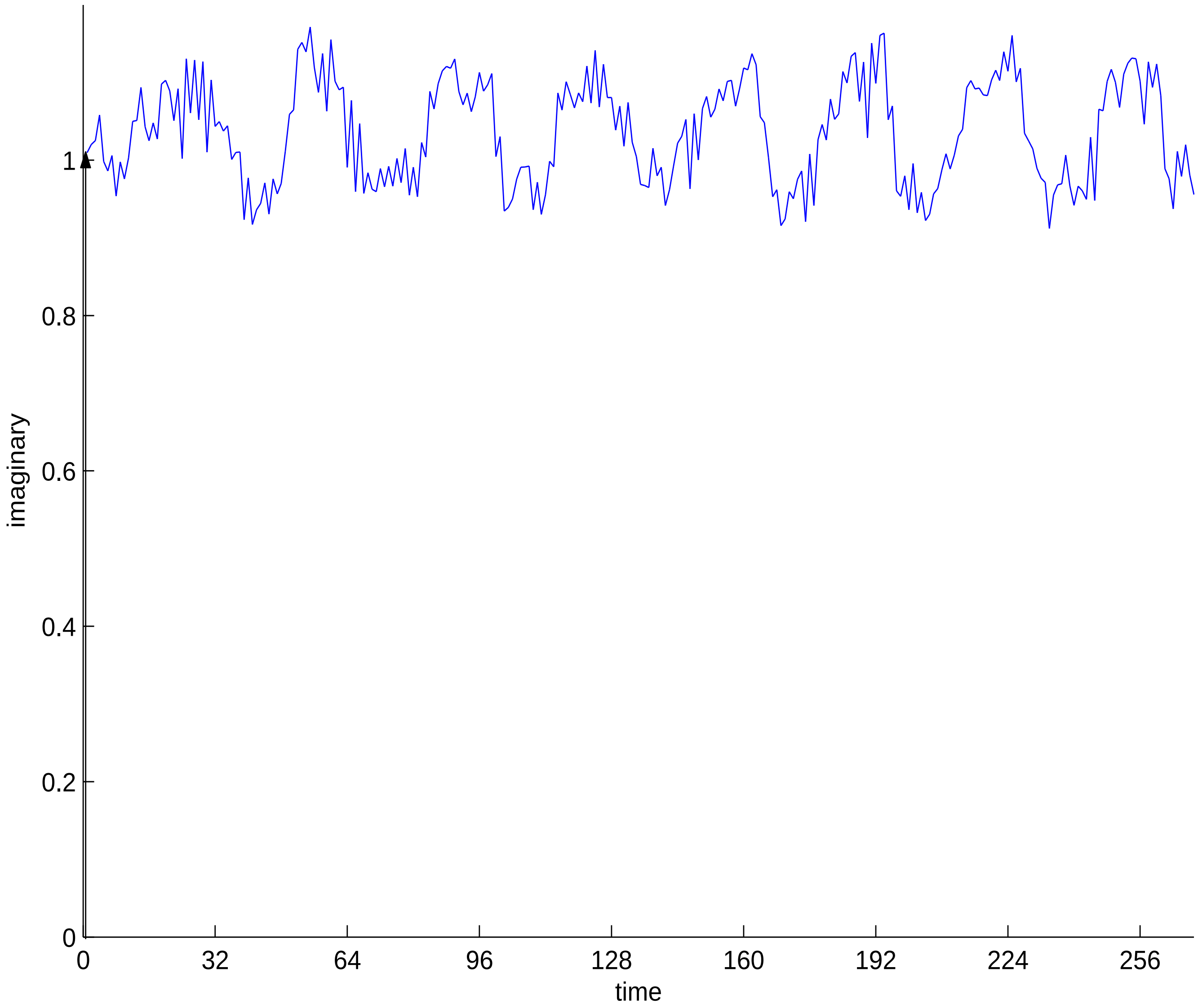
Complex Voxel Time Course

Rotate axis. Real over time plot.



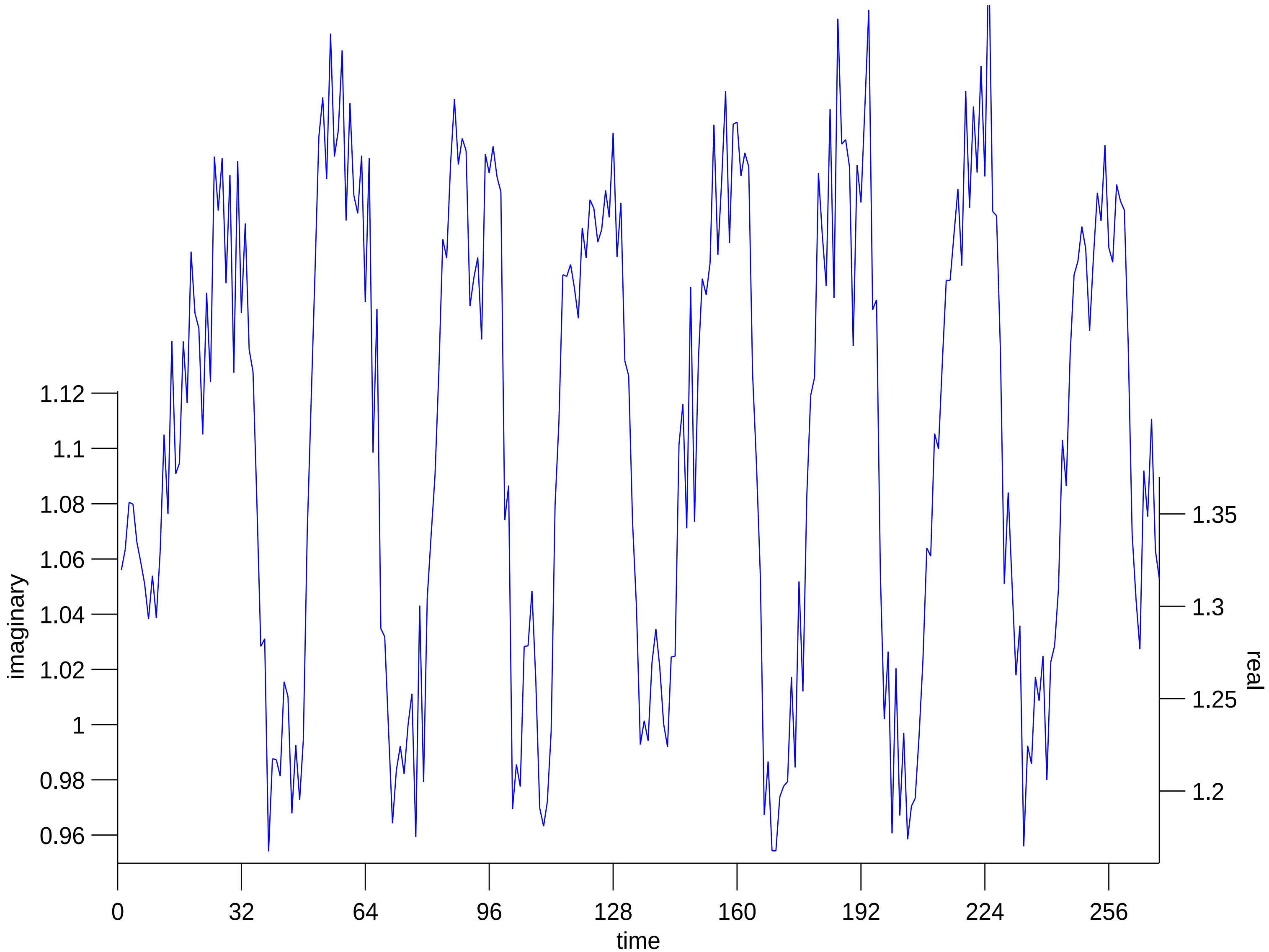
Complex Voxel Time Course

Rotate axis. Imaginary over time plot.



Complex Voxel Time Course

Rotate axis (Avg. Phase). Magnitude over time plot.



Complex Time Course Model

In a voxel, the complex valued quantity measured over time is

$$y_t = (\rho_t \cos\theta + \eta_{Rt}) + i(\rho_t \sin\theta + \eta_{It}), \quad t = 1, ..., n$$

$$\begin{aligned}
y_t &= \text{complex voxel measurement at time } t \\
\rho_t &= \text{true magnitude of voxel measurement at time } t \\
\theta &= \text{true fixed unknown voxel phase} \\
\eta_{Rt} &= \text{noise real part voxel measurement at time } t \\
\eta_{It} &= \text{noise imaginary part voxel measurement at time } t
\end{aligned}$$

$(\eta_{Rt}, \eta_{It})' \sim \mathcal{N}(0, \Sigma)$, $\Sigma = \sigma^2 I_2$.

The distributional specification is on the real and imaginary parts of the image and not on the magnitude.

Magnitude Time Course Model

The magnitude model from the complex phase model

$$r_t = \left[(\rho_t \cos\theta + n_{Rt})^2 + (\rho_t \sin\theta + n_{It})^2\right]^{\frac{1}{2}}, \quad t = 1, ..., n$$

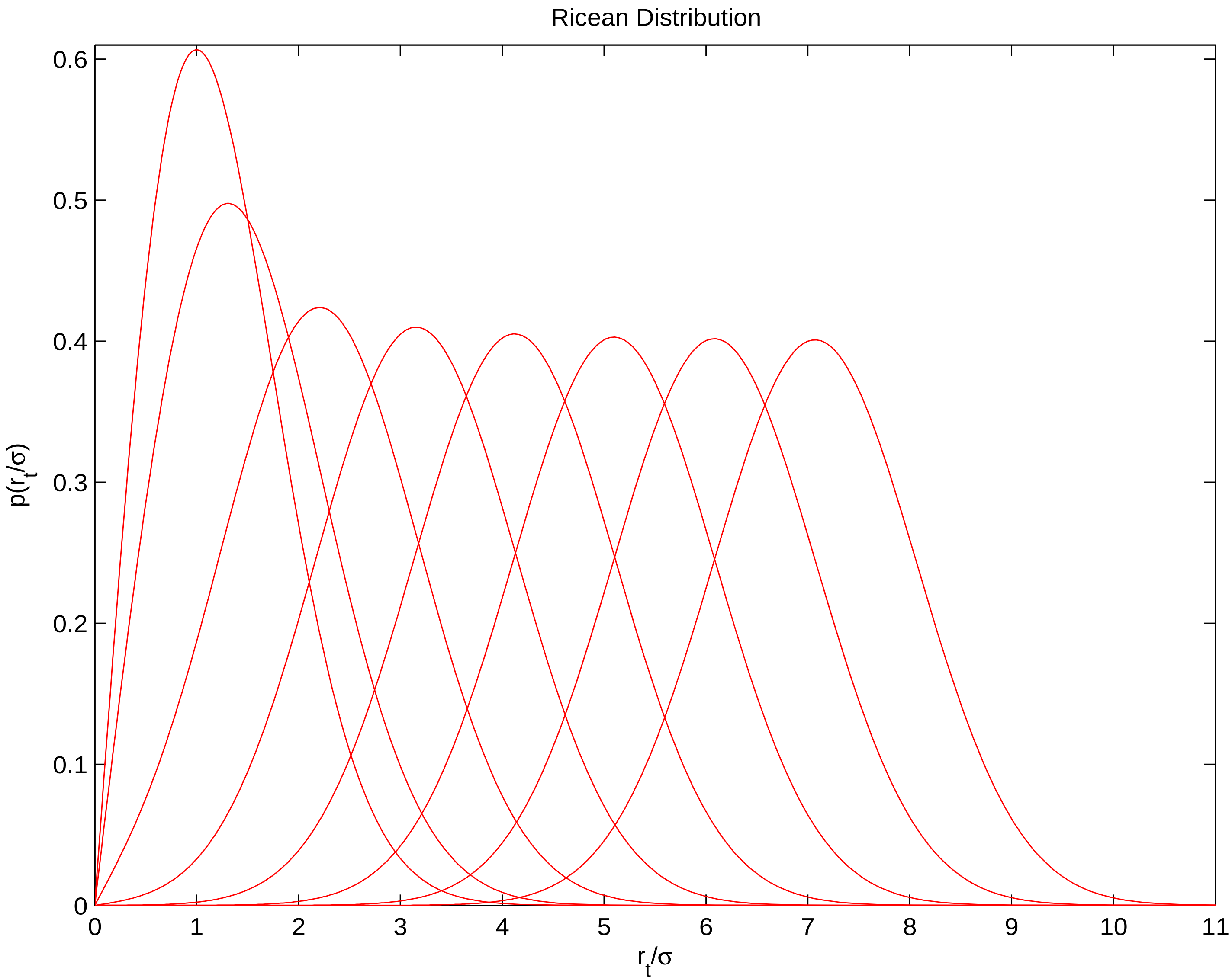
The magnitude, does not have a normal distribution.
The magnitude has a Ricean distribution.

$$p(r_t) = \frac{r_t}{\sigma^2} e^{-\frac{(r_t^2+\rho_t^2)}{2\sigma^2}} I_o\left(\frac{\rho_t \cdot r_t}{\sigma^2}\right), \quad t = 1, ..., n$$

$$I_o\left(\frac{\rho_t \cdot r_t}{\sigma^2}\right) = \int_{\phi_t=-\pi}^{\pi} \frac{1}{2\pi} \exp\left\{\frac{\rho_t r_t}{\sigma^2}\cos(\phi-\theta)\right\} d\phi_t$$

is the zeroth order modified Bessel function of the first kind

Magnitude Ricean Distribution



SNR=ρ_t/σ. Looks normal for deacent SNR. Tails?

Magnitude & Complex Time Course Model

Linear multiple regression model individually for each voxel

$$\rho_t = x_t'\beta = \beta_0 + \beta_1 x_{1t} + \cdots + \beta_q x_{qt}.$$

$$\underset{n\times 1}{r} = \underset{n\times(q+1)}{X} \quad \underset{(q+1)\times 1}{\beta} + \underset{n\times 1}{\epsilon} \qquad \textit{Magnitude}$$

$$\underset{2n\times 1}{y} = \underset{2n\times 2(q+1)}{\begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}} \quad \underset{2(q+1)\times 1}{\begin{pmatrix} \beta\cos\theta \\ \beta\sin\theta \end{pmatrix}} + \underset{2n\times 1}{\eta} \qquad \textit{Complex}$$

where $r = (r_1, \ldots, r_n)'$, $y = (y_R', y_I')'$, and

$\epsilon \sim \mathcal{N}(0, \sigma^2 I_n)$, $\eta = (\eta_{Rt}', \eta_{It}')' \sim \mathcal{N}(0, \sigma^2 I_{2n})$.

Activation Statistics

Both models have normal likelihoods.

We want to see if the observed time course has a component related to the reference function.

$H_0: \ C\beta = \gamma$ vs $H_1: \ C\beta \neq \gamma$

i.e. Is the coefficient for the reference function zero.

$C = (0, ..., 0, 1)$, $\beta' = (\beta_0, \beta_1, \cdots, \beta_q)$, $\gamma = 0$

MLE's from both under null and alternative.

Complex Time Course Model

By maximizing the likelihood under the unconstrained alternative

$$\hat{\theta} = \frac{1}{2}\tan^{-1}\left[\frac{2\hat{\beta}'_R(X'X)\hat{\beta}_I}{(\hat{\beta}'_R(X'X)\hat{\beta}_R - \hat{\beta}'_I(X'X)\hat{\beta}_I)/2}\right]$$

$$\hat{\beta} = \hat{\beta}_R\cos\hat{\theta} + \hat{\beta}_I\sin\hat{\theta}, \quad \Leftarrow Note$$

$$\hat{\sigma}^2 = \frac{1}{2n}\left[y - \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}\begin{pmatrix} \hat{\beta}\cos\hat{\theta} \\ \hat{\beta}\sin\hat{\theta} \end{pmatrix}\right]'\left[y - \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}\begin{pmatrix} \hat{\beta}\cos\hat{\theta} \\ \hat{\beta}\sin\hat{\theta} \end{pmatrix}\right]$$

$$\hat{\beta}_R = (X'X)^{-1}X'y_R, \quad \Leftarrow Note$$
$$\hat{\beta}_I = (X'X)^{-1}X'y_I\ .$$

Complex Time Course Model

By maximizing the likelihood under the constrained null hypotheses

$$\tilde{\theta} = \frac{1}{2}\tan^{-1}\left[\frac{\hat{\beta}_R'\Psi(X'X)\hat{\beta}_I}{(\hat{\beta}_R'\Psi(X'X)\hat{\beta}_R - \hat{\beta}_I'\Psi(X'X)\hat{\beta}_I)/2}\right]$$

$$\tilde{\beta} = \Psi[\hat{\beta}_R\cos\tilde{\theta} + \hat{\beta}_I\sin\tilde{\theta}] \qquad + (X'X)^{-1}C'[C(X'X)^{-1}C']^{-1}\gamma,$$

$$\tilde{\sigma}^2 = \frac{1}{2n}\left[y - \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}\begin{pmatrix} \tilde{\beta}\cos\tilde{\theta} \\ \tilde{\beta}\sin\tilde{\theta} \end{pmatrix}\right]'\left[y - \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}\begin{pmatrix} \tilde{\beta}\cos\tilde{\theta} \\ \tilde{\beta}\sin\tilde{\theta} \end{pmatrix}\right]$$

$$\Psi = I_{q+1} - (X'X)^{-1}C'[C(X'X)^{-1}C']^{-1}C\ .$$

Same Ψ as magnitude model.

Real fMRI Experiment

Imaging Parameters:
1.5T GE Signa
5 axial slices of 128x128
96 acq.-2.0833mm^2
128 recon.-1.5625mm^2
FOV $=$20cm
TR$=$1000ms
TE$=$47ms
FA$=$90°

Task:
Bilateral sequential finger tapping
Block design
16 off $+$ 8$\times$(16on$+$16off);

Time Course Models

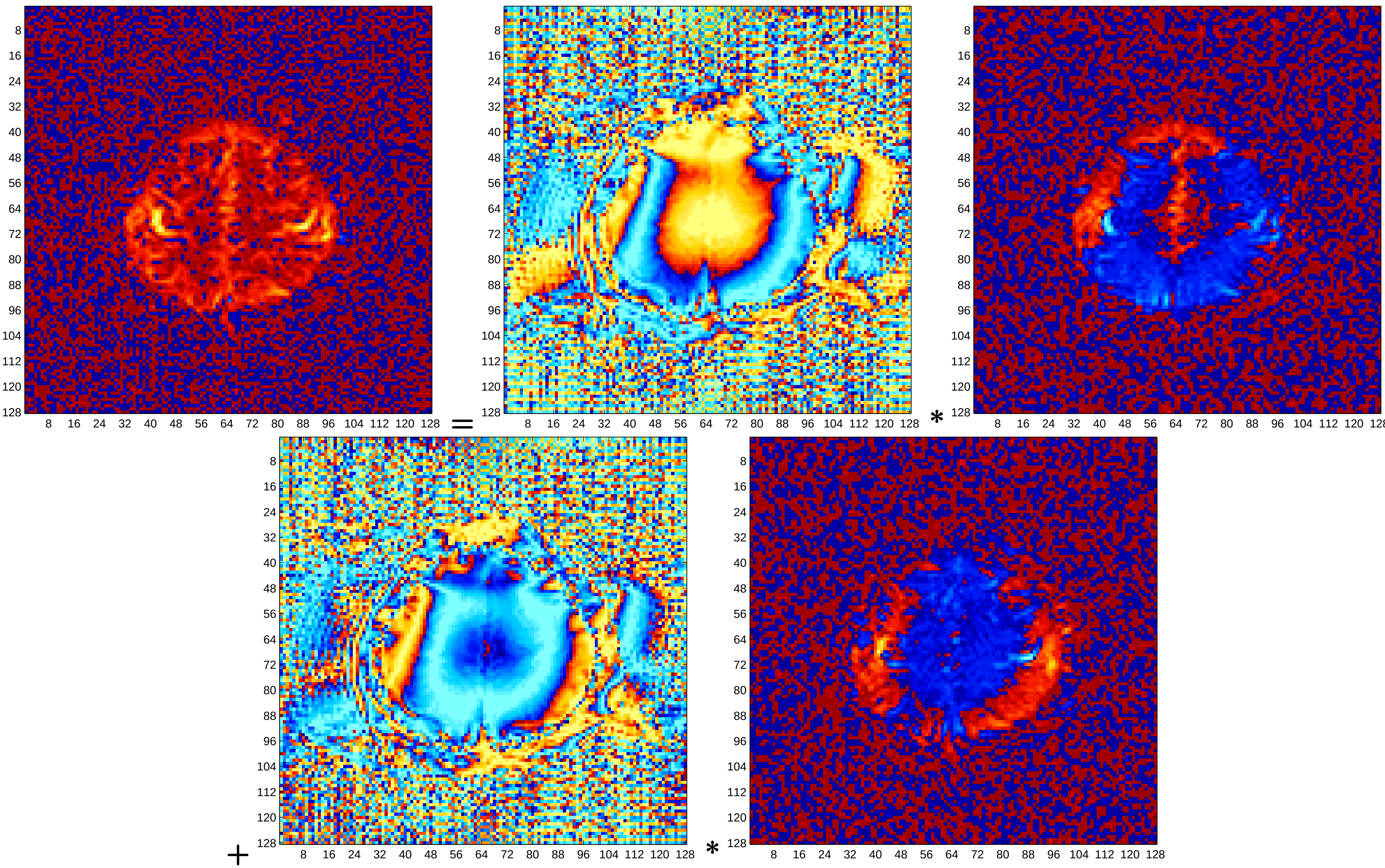
Compare the two models for testing $H_0 : \beta_2 = 0$.
($q = 2$, $X = (e_n, c_n, r_n)$, $C = (0, 0, 1)$, $\gamma = 0$)

$$\chi^2_M \;=\; n \log\left(\tilde{\sigma}^2_M / \hat{\sigma}^2_M\right) \;\dot{\sim}\; \chi^2_1$$

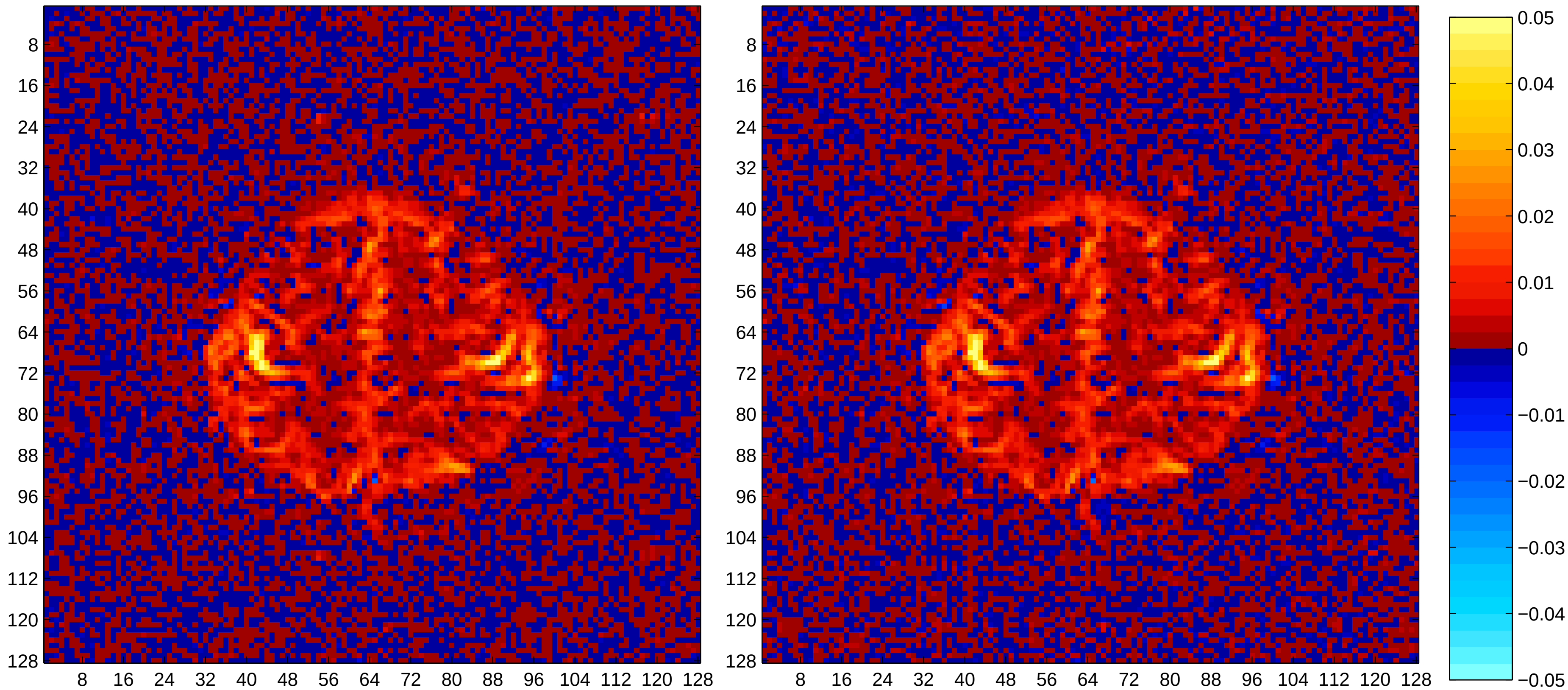
$$\chi^2_C \;=\; 2n \log\left(\tilde{\sigma}^2_C / \hat{\sigma}^2_C\right) \;\dot{\sim}\; \chi^2_1$$

Both χ^2_1 distributed for large samples!

Real fMRI-Complex H1 Estimated

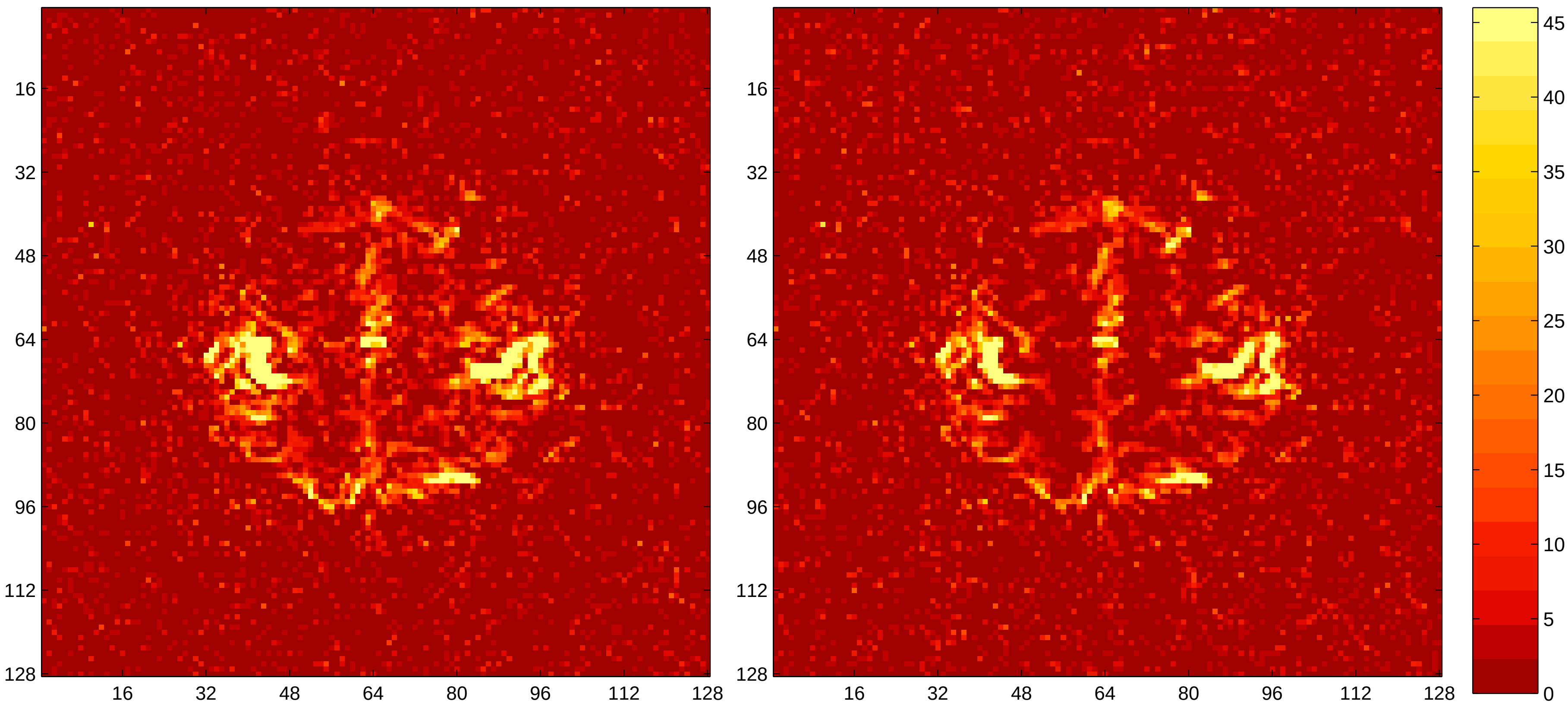


Real fMRI-Magnitude/Complex H1 Estimated $\hat{\beta}_2$



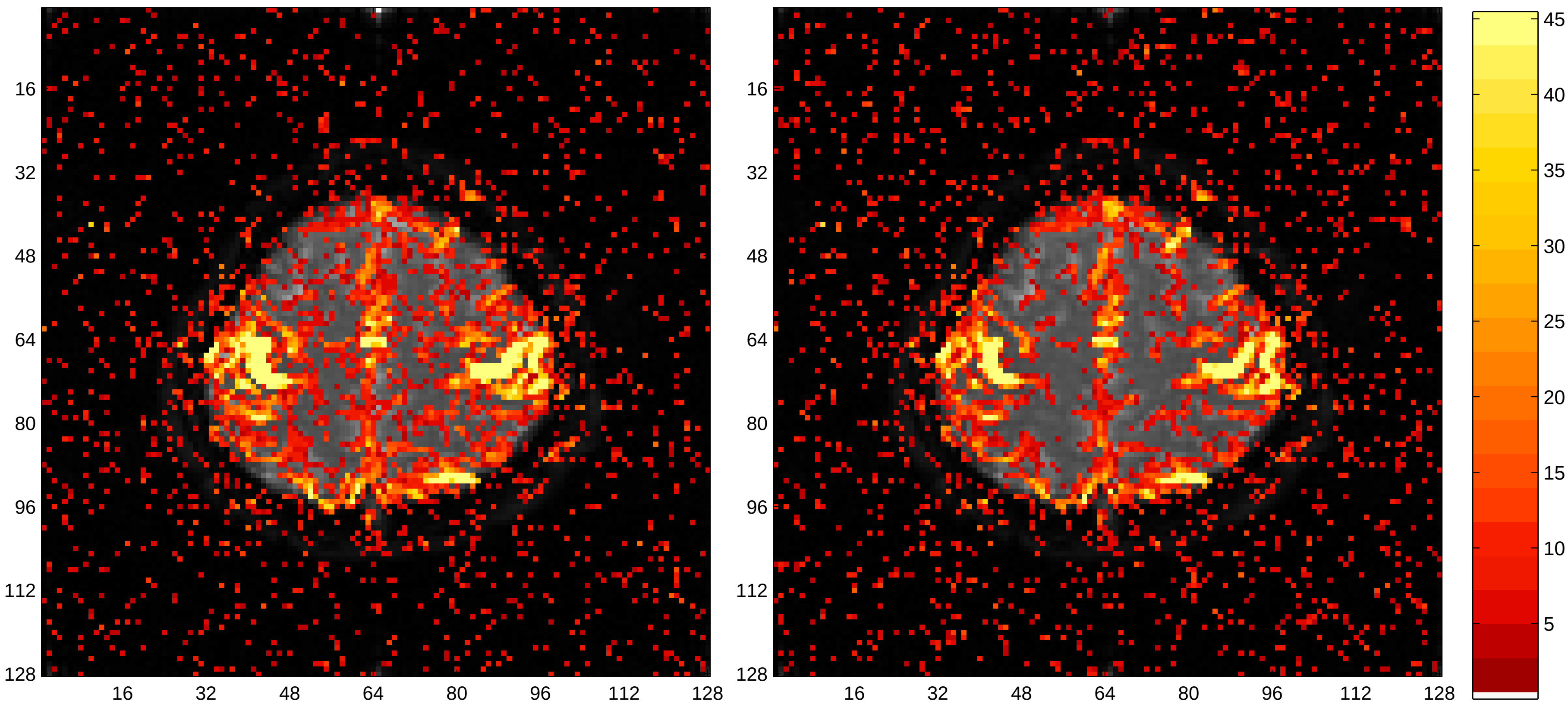
These coefficients are not visually that different but numerically different.

Real fMRI-Magnitude/Complex $-2\log(\lambda)$ Maps



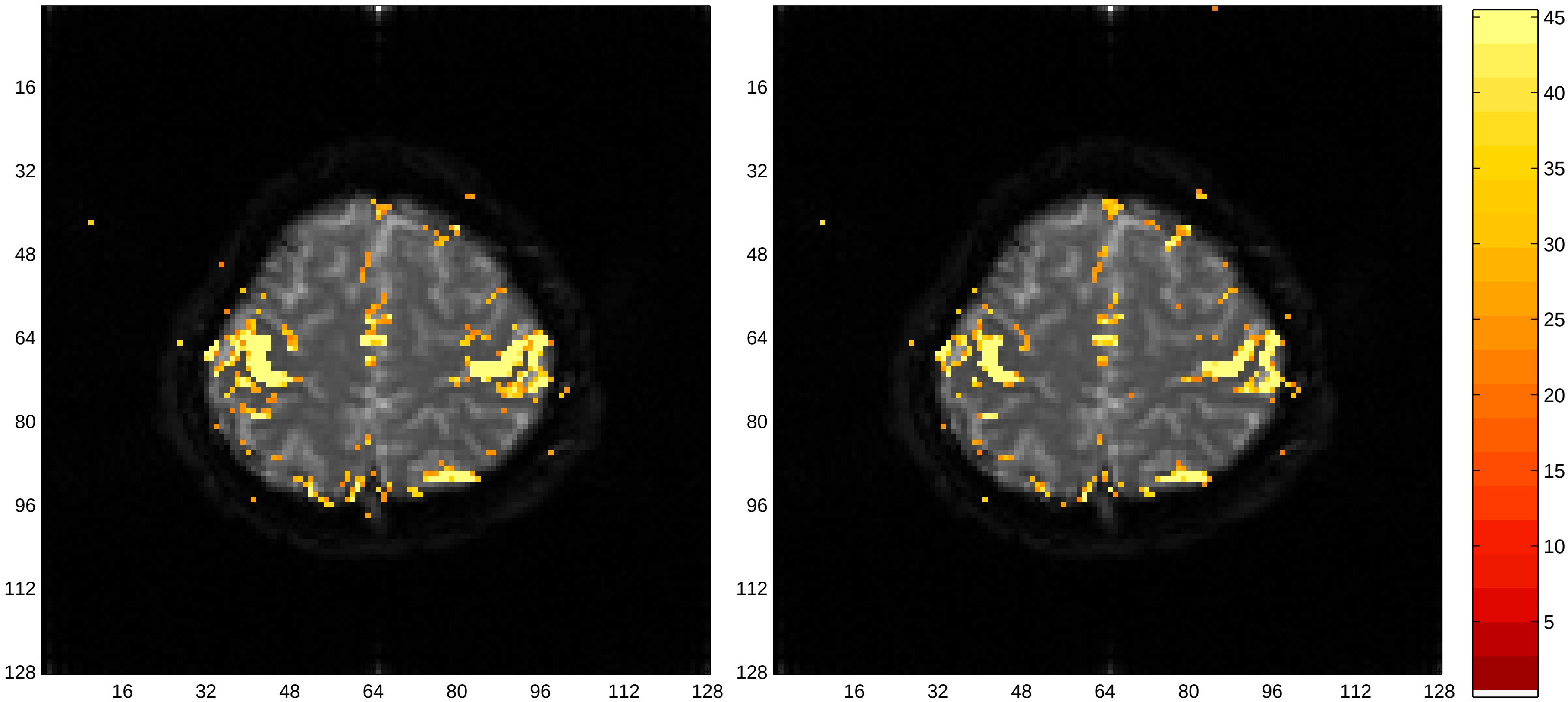
These voxel statistics are $\dot{\sim}\ \chi_1^2$!

Real fMRI-Magnitude/Complex $-2\log(\lambda)$ Maps



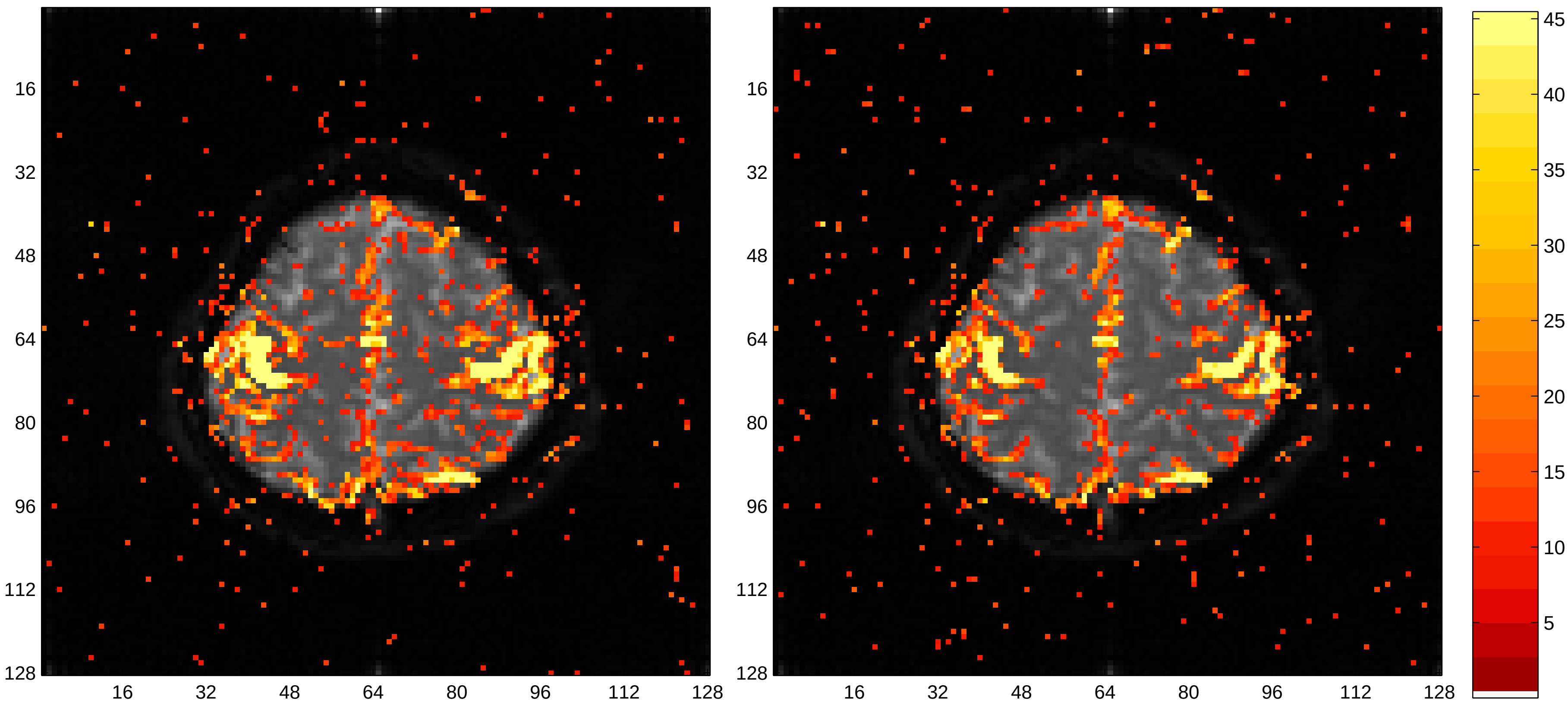
5% Unadjusted Threshold

Real fMRI-Magnitude/Complex $-2\log(\lambda)$ Maps



5% Bonferroni Threshold

Real fMRI-Magnitude/Complex $-2\log(\lambda)$ Maps



5% FDR Threshold

Discussion

A complex data fMRI activation model was presented .

Complex and magnitude models activation compared on real data.

Not shown

Complex and magnitude models power compared on simulated data.

For a given CNR the complex model power constant irrespective of SNR while the magnitude model power decreases.

For smaller SNR's, the complex activation model demonstrated better power

The complex model more useful as SNR decreases with voxel size.

Current/Future Work

Cramer-Rao Lower bounds for parameters (submitted).
-CRLB for SE of variance 1/2 in complex model

$$CRLB_M = \begin{array}{c} \\ \beta \\ \sigma^2 \end{array} \begin{array}{c} \begin{array}{cc} \quad\beta\quad & \quad\sigma^2\quad \end{array} \\ \begin{bmatrix} \sigma^2(X'X)^{-1} & 0 \\ 0 & 2\sigma^4/n \end{bmatrix} \end{array}$$

$$CRLB_C = \begin{array}{c} \\ \beta \\ \sigma^2 \\ \theta \end{array} \begin{array}{c} \begin{array}{ccc} \quad\beta\quad & \quad\sigma^2\quad & \quad\theta\quad \end{array} \\ \begin{bmatrix} \sigma^2(X'X)^{-1} & 0 & 0 \\ 0 & \sigma^4/n & 0 \\ 0 & 0 & \sigma^2/\beta'(X'X)\beta \end{bmatrix} \end{array}$$

Current/Future Work

Phase not constant over time, $\theta_t \neq \theta_0$ (submitted).

$$\underset{2n \times 1}{y} = \underset{2n \times 2n}{\begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}} \underset{2n \times 2(q+1)}{\begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix}} \underset{2(q+1) \times 1}{\begin{pmatrix} \beta \\ \beta \end{pmatrix}} + \underset{2n \times 1}{\eta}$$

A_1 and A_2 hasve $\cos\theta_t$ and $\sin\theta_t$ on diagonals.

θ_t unique at each time point.
-Exactly equivalent to large SNR magnitude-only model.