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"Discovery, Innovation & Application – Advancing MR for Improved Health"

Declaration of Relevant Financial Interests or Relationships

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I have no relevant financial interest or relationship to disclose with regard to the subject matter of this presentation.

Separation of Two Simultaneously Encoded Slices with a Single Coil

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Outline:

1. Single Coil Two Slice Encoding

2. Image Separation
   Magnitude-Only & Complex-Valued

3. Statistical Properties

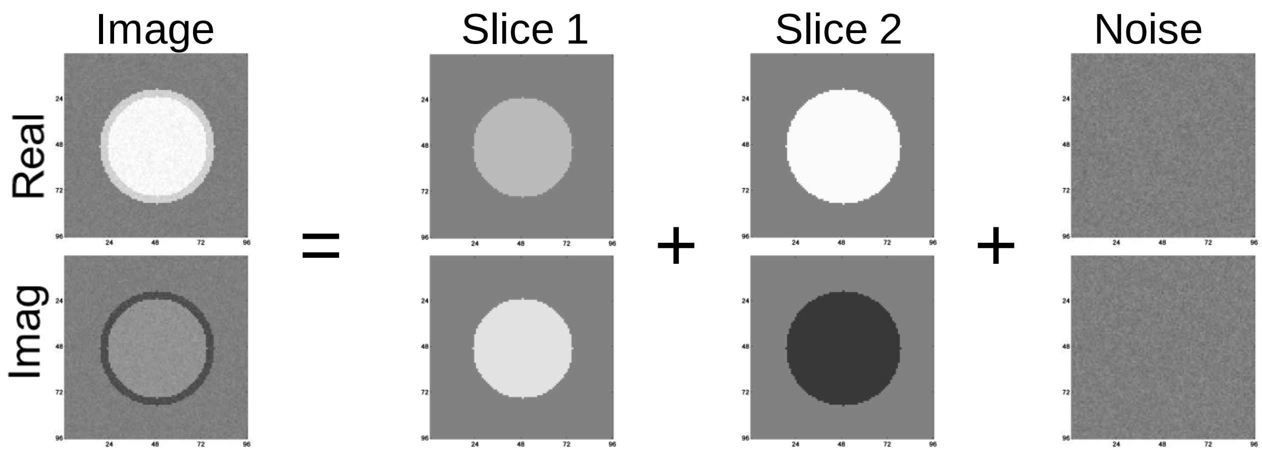
4. Experimental Results

5. Discussion

1. Single Coil Two Slice Encoding

In each voxel:

	Image		Slice 1		Slice 2		Noise
Real	y_R	$=$	$\rho_1 \cos\theta_1$	$+$	$\rho_2 \cos\theta_2$	$+$	ε_R
	$+$		$+$		$+$		$+$
Imag	$i y_I$		$i\rho_1 \sin\theta_1$		$i\rho_2 \sin\theta_2$		$i\varepsilon_I$



1. Single Coil Two Slice Encoding

In each voxel:

$$
\begin{aligned}
(y_R + i y_I) &= (\rho_1 \cos\theta_1 + i \rho_1 \sin\theta_1) \\
&+ (\rho_2 \cos\theta_2 + i \rho_2 \sin\theta_2) \\
&+ (\varepsilon_R + i \varepsilon_I)
\end{aligned}
$$

$$
\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}
$$

Aliased Image ↑ — Aliasing Matrix ↑ — True Unaliased Images ↑ — Measurement Error ↑

$$
y \quad = \quad X \quad \beta \quad + \quad \varepsilon
$$

(2 linear equations and 4 unknowns)

1. Single Coil Two Slice Encoding

The goal is to estimate (separate) the two images

$$\hat{\beta} = (X'X)^{-1} X'y \; .$$

However, we have 2 equations and 4 unknowns and $X'X$ is not square or invertible or of full rank.

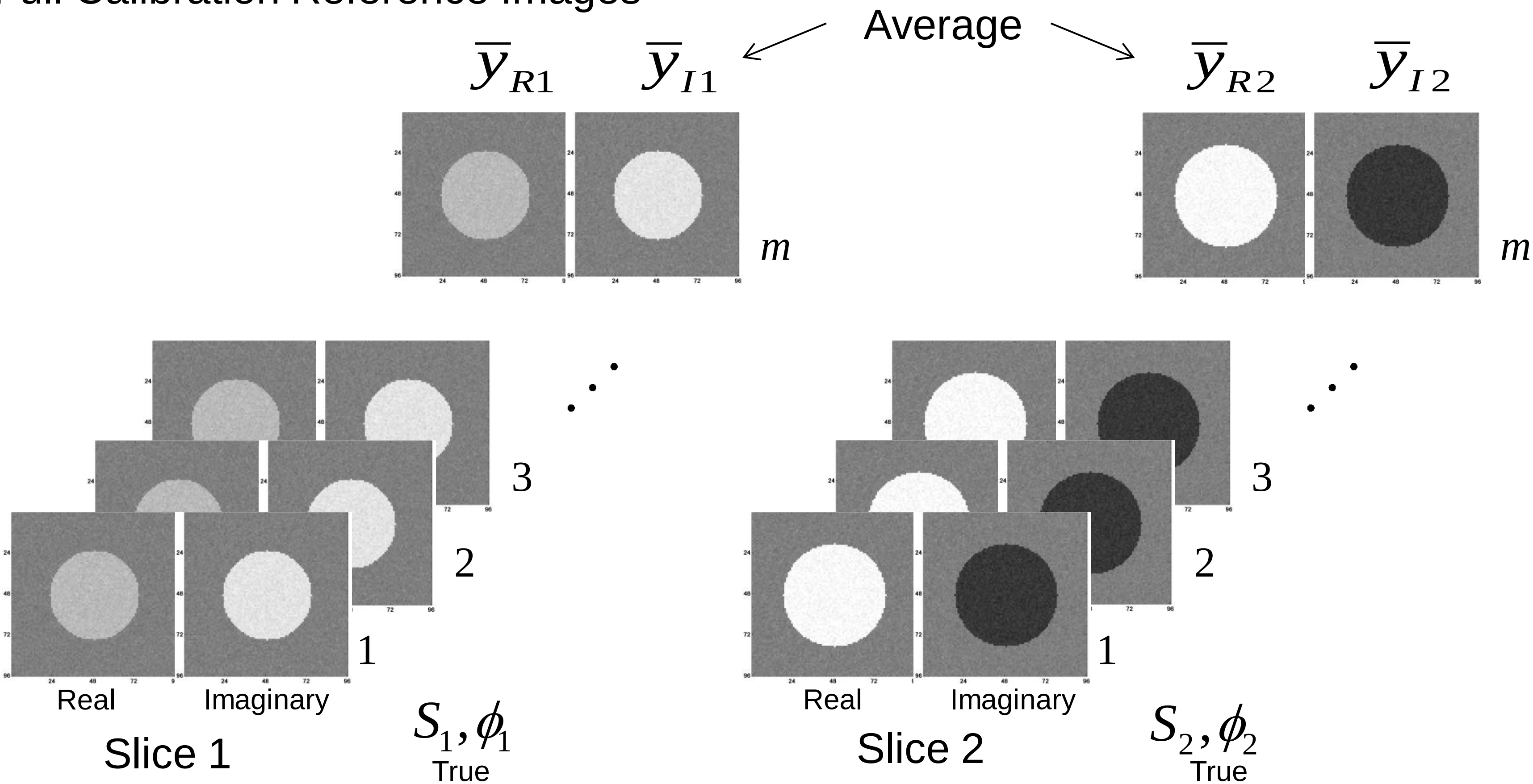
Approach:

Acquire full calibration reference images.

Magnitude-Only & Complex-Valued separation.

2. Image Separation

Full Calibration Reference Images



2. Image Separation, Magnitude-Only

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \cos\theta_2 \\ \sin\theta_1 & \sin\theta_2 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}$$

$$y = X\beta + \varepsilon$$

reduce dimension of the problem

$$\left(\bar{y}_{R1}, \bar{y}_{I1}, \bar{y}_{R2}, \bar{y}_{I2}\right) \longrightarrow \left(\bar{r}_1, \bar{\phi}_1, \bar{r}_2, \bar{\phi}_2\right)$$

$$\left(\theta_1 = \bar{\phi}_1, \theta_2 = \bar{\phi}_2\right)$$

$$\begin{pmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{pmatrix} = \frac{1}{\sin(\bar{\phi}_1 - \bar{\phi}_2)} \begin{pmatrix} -\sin\bar{\phi}_2 & \cos\bar{\phi}_2 \\ \sin\bar{\phi}_1 & -\cos\bar{\phi}_1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \end{pmatrix}$$

Invert X

$$\hat{\beta} = X^{-1} y$$

provided $\bar{\phi}_1 - \bar{\phi}_2 \neq k\pi, \; k = 0, \pm 1, \ldots$

Jesmanowicz, Li, Hyde: ISMRM,2009.
Islam, Glover: ISMRM, 2012.

2. Image Separation, Complex-Valued

$$\begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \rho_1 \cos\theta_1 \\ \rho_1 \sin\theta_1 \\ \rho_2 \cos\theta_2 \\ \rho_2 \sin\theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \\ 0 \\ 0 \end{pmatrix}$$

$$y = X\beta + \varepsilon$$

← added two linear constraints

Observed Aliased
Reference Aliased

$$\begin{pmatrix} v_R \\ v_I \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{y}_{R1} \\ \bar{y}_{I1} \\ \bar{y}_{R2} \\ \bar{y}_{I2} \end{pmatrix}$$

Invert X

$$\hat{\beta} = X^{-1} y$$

Rowe, Nencka, Jesmanowicz,Hyde: ISMRM,2013.

2. Image Separation, Complex-Valued

$$\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}}_{\text{rank=4}}{}^{-1} \begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix}$$

$$\hat{\beta} = X^{-1} y$$

X is the same for each voxel and its inverse can be precomputed.

$$\underbrace{\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix}}_{\text{separated}} = \frac{1}{2} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} y_R \\ y_I \\ v_R \\ v_I \end{pmatrix}$$

y_R, y_I: Observed Aliased

v_R, v_I: Reference Aliased

3. Statistical Properties, Magnitude-Only

True Image For Unaliasing

$$S_1,\phi_1 \qquad S_2,\phi_2$$

True Image That Is Aliased

$$\rho_1,\theta_1 \qquad \rho_2,\theta_2$$

$$E\begin{pmatrix}\hat{\rho}_1\\ \hat{\rho}_2\end{pmatrix}=\frac{1}{\sin(\bar{\phi}_1-\bar{\phi}_2)}\begin{pmatrix}\sin(\theta_1-\bar{\phi}_2) & \sin(\theta_2-\bar{\phi}_2)\\ \sin(\bar{\phi}_1-\theta_1) & \sin(\bar{\phi}_1-\theta_2)\end{pmatrix}\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix}$$

mean, variance and correlation phase dependent

$$\mathrm{cov}\begin{pmatrix}\hat{\rho}_1\\ \hat{\rho}_2\end{pmatrix}=\frac{\sigma^2}{\sin^2(\bar{\phi}_1-\bar{\phi}_2)}\begin{pmatrix}1 & -\cos(\bar{\phi}_1-\bar{\phi}_2)\\ -\cos(\bar{\phi}_1-\bar{\phi}_2) & 1\end{pmatrix}\begin{pmatrix}\rho_1\\ \rho_2\end{pmatrix}$$

Voxels are correlated with their counterpart in the other slice.
No Free Lunch!

3. Statistical Properties, Complex-Valued

$$E\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \begin{bmatrix} \frac{1}{2}(\rho_1\cos\theta_1 + S_1\cos\phi_1) + \frac{1}{2}(\rho_2\cos\theta_2 - S_2\cos\phi_2) \\ \frac{1}{2}(\rho_1\sin\theta_1 + S_1\sin\phi_1) + \frac{1}{2}(\rho_2\sin\theta_2 - S_2\sin\phi_2) \\ \frac{1}{2}(\rho_2\cos\theta_2 + S_2\cos\phi_2) + \frac{1}{2}(\rho_1\cos\theta_1 - S_1\cos\phi_1) \\ \frac{1}{2}(\rho_2\sin\theta_2 + S_2\sin\phi_2) + \underbrace{\frac{1}{2}(\rho_1\sin\theta_1 - S_1\sin\phi_1)}_{\text{small}} \end{bmatrix}$$

$$\operatorname{cov}\begin{pmatrix} \hat{\rho}_1 \cos\hat{\theta}_1 \\ \hat{\rho}_1 \sin\hat{\theta}_1 \\ \hat{\rho}_2 \cos\hat{\theta}_2 \\ \hat{\rho}_2 \sin\hat{\theta}_2 \end{pmatrix} = \frac{\sigma^2}{4}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

Voxels are correlated with their counterpart in the other slice.

No Free Lunch!

4. Experimental Results

Data: Spherical Agar phantom

10 full reference slices and 5 aliased slices (1&6, 2&7,...)
TRs=720, TE=42.5 ms, TR=1 s, FA=45°, BW=166 kHz, FOV=24 cm, SLTH=4 mm, matrix size 96×96

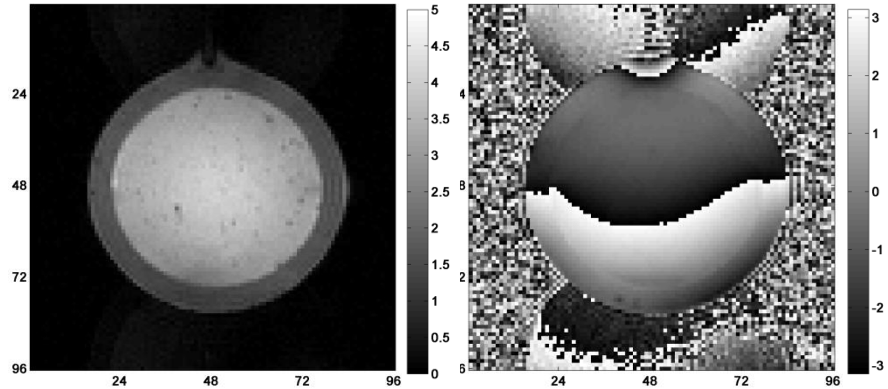
1st aliased and the 1st and 6th fully acquired slices analyzed

Each scan's first 5 TRs deleted, next 2 reference images averaged for separation of 715 aliased images.
Plane fit to phase of aliased images and corrected over time.

4. Results

Data:

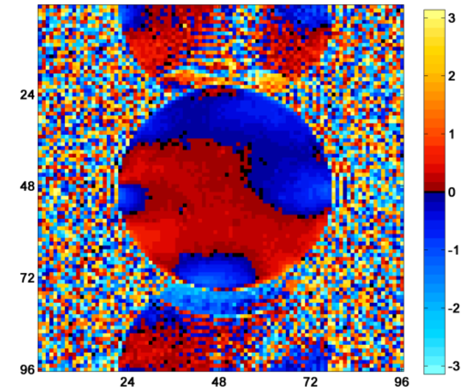
First Aliased Image



Magnitude

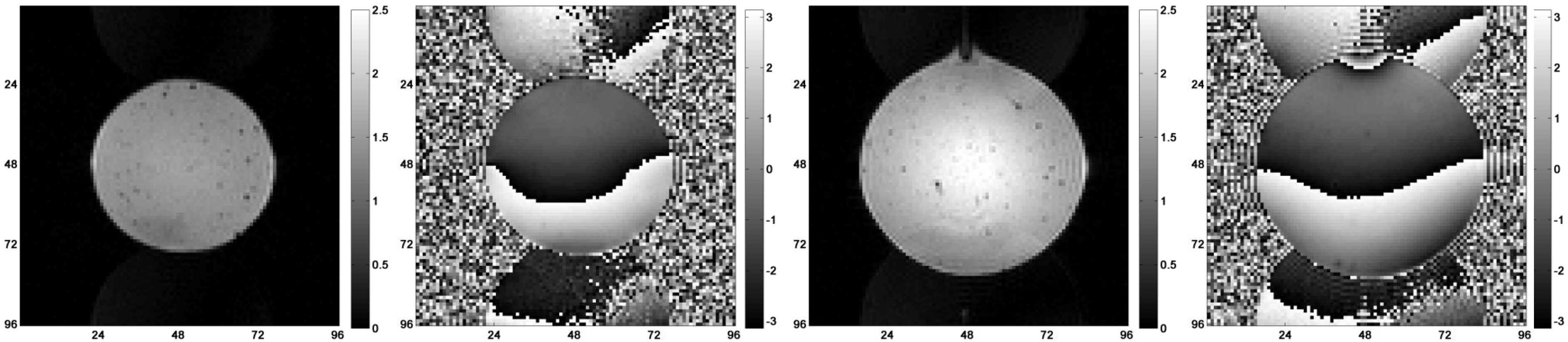
Phase

Phase Difference



$$\bar{\phi}_1 - \bar{\phi}_2$$

Reference Images

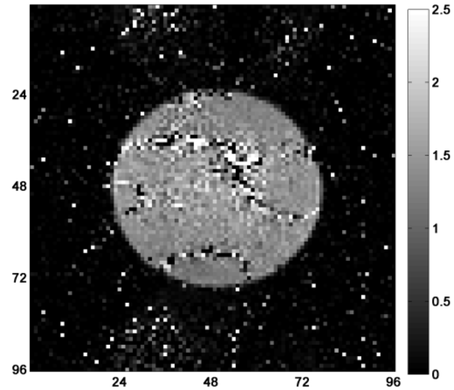


Magnitude 1 Phase 1 Magnitude 2 Phase 2

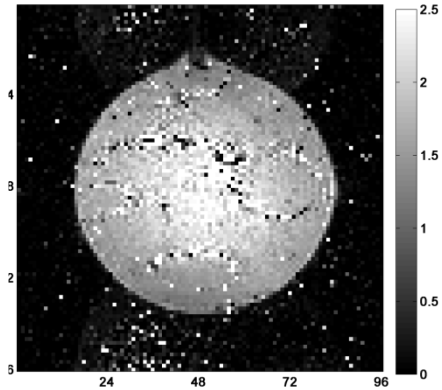
4. Results

Data:

MO Separated First Image

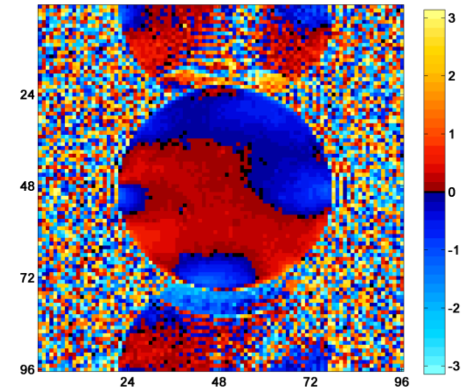


Magnitude



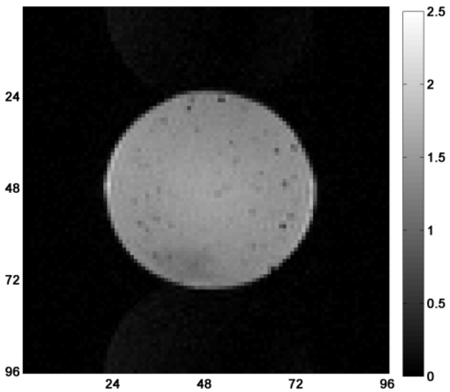
Phase

Phase Difference

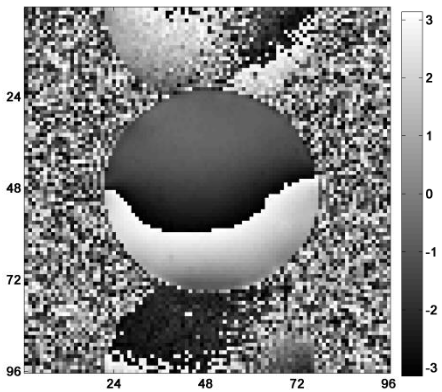


$\bar{\phi}_1 - \bar{\phi}_2$

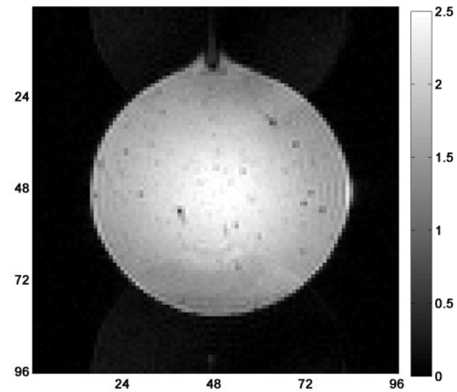
CV Separated First Image



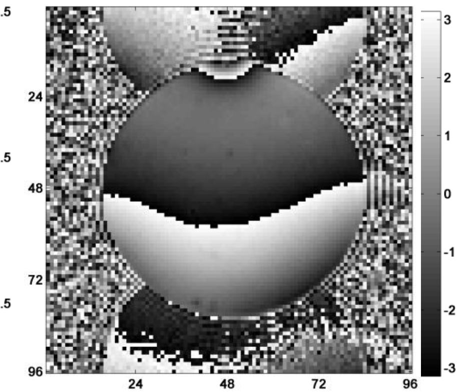
Magnitude 1



Phase 1



Magnitude 2



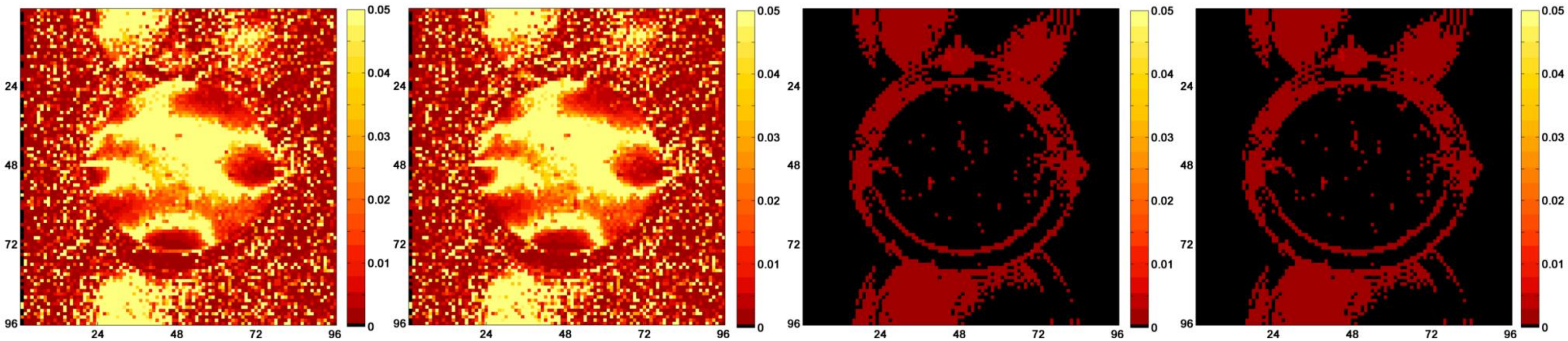
Phase 2

4. Results

Data:

Variances Over Series

MO Separated Images **CV Separated Images**

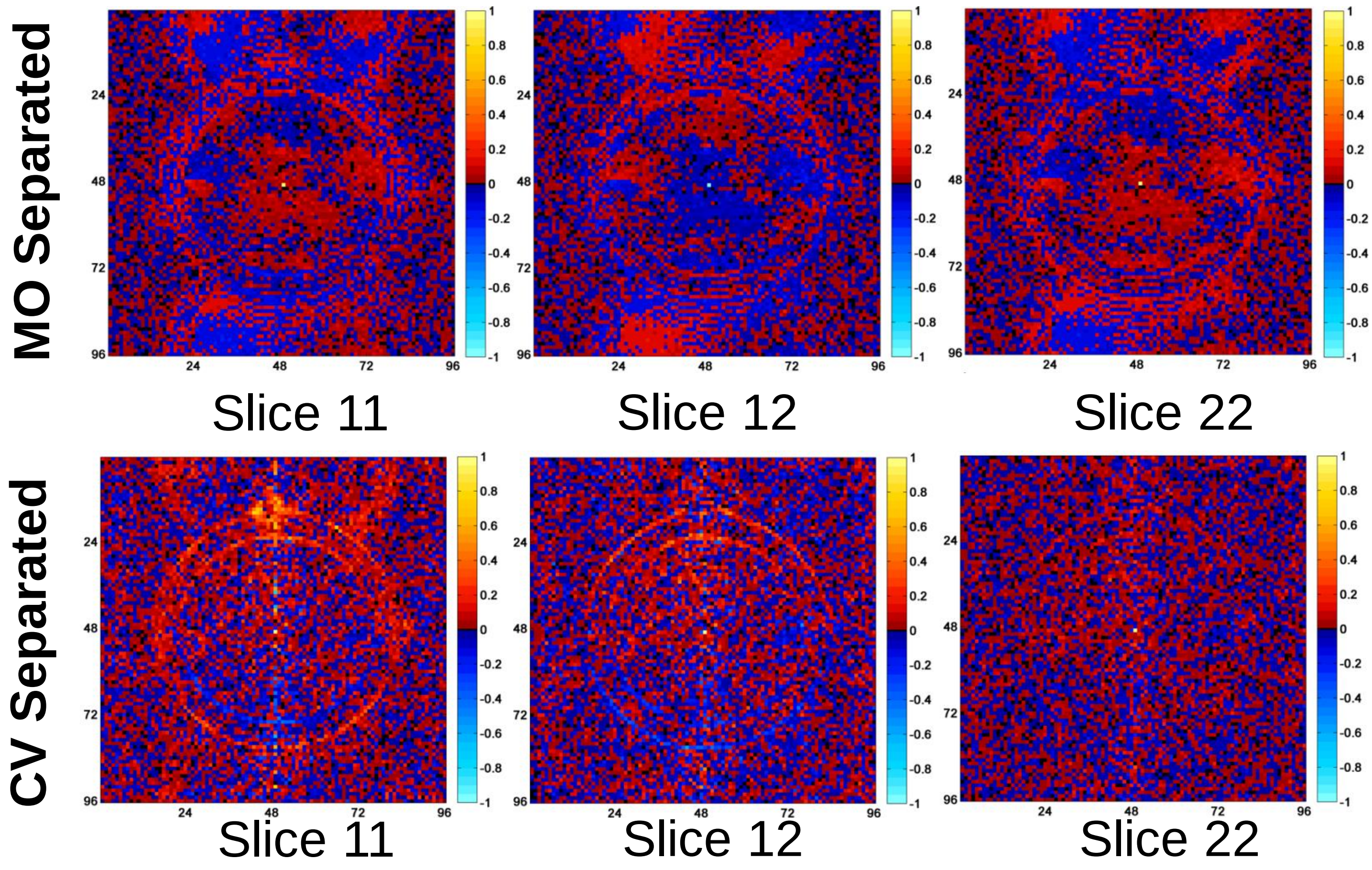


Slice 1 Slice 2 Slice 1 Slice 2

4. Results

Correlations Over Series

Data:



5. Discussion

Description of the 2 slice 1 coil aliasing process.

Description of new complex-valued constrained separation.

Statistical properties of the MO and CVC separation.

Results on experimental phantom data.

As usual, any subsampling yields correlated voxels.

Can be used with single channel animal scanners.