

Statistical Analysis of Image Reconstructed Fully-Sampled and Sub-Sampled fMRI Data

Daniel B. Rowe

Program in Computational Sciences

Department of Mathematics, Statistics, and Computer Science

Marquette University



February 1, 2016

NIHR21 NS087450

Outline

Introduction

Varied methods to produce fMRI images with varied properties.

^{3x} Reconstruction

Voxels are not directly measured (k -space). Reconstructed!

Processing

Images are processed for enhancement & artifact reduction.

Implications

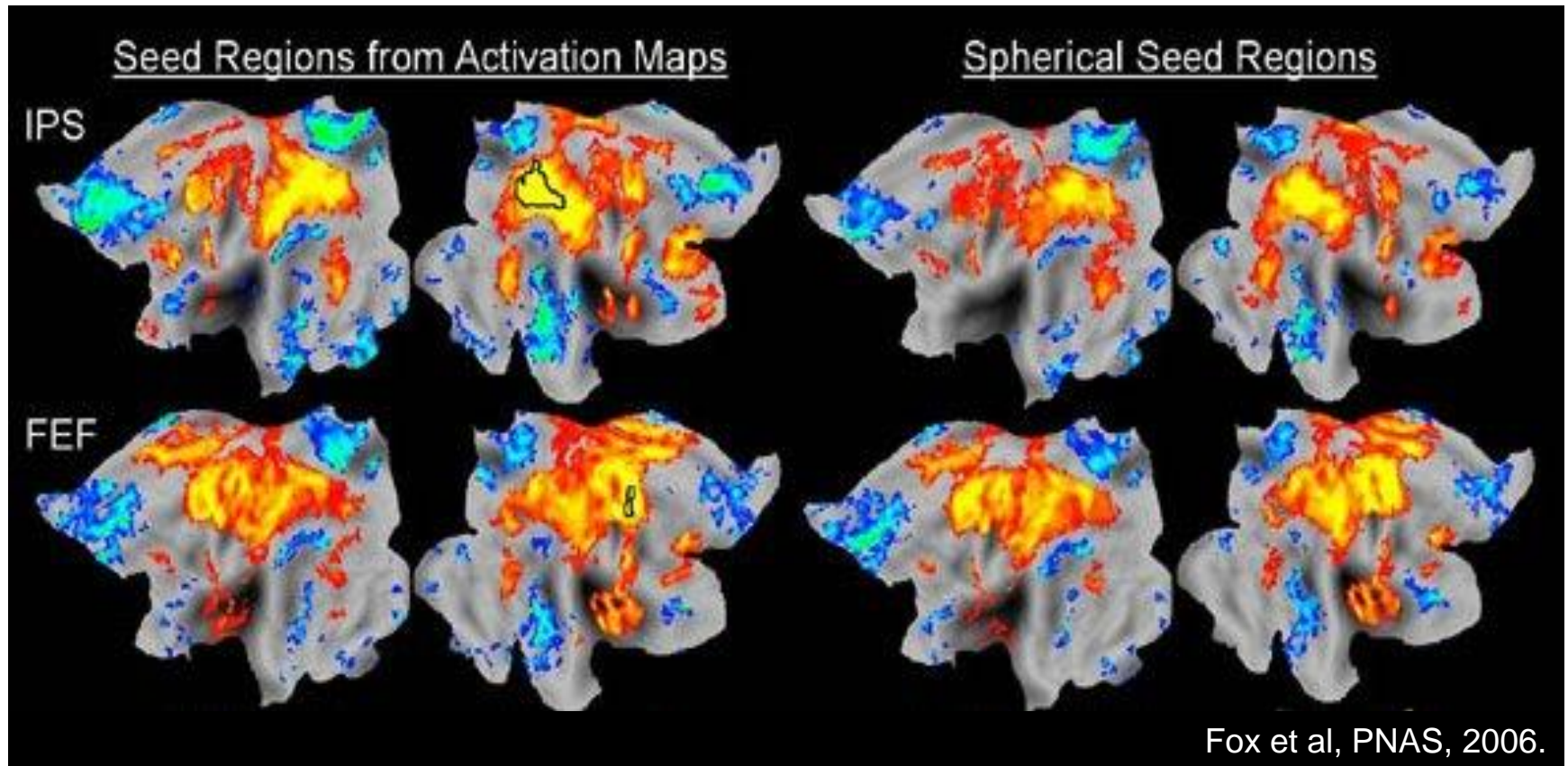
Effects of image reconstruction & processing? Mean, Var, Corr?

Discussion

How was our data was produced and what was done to it?

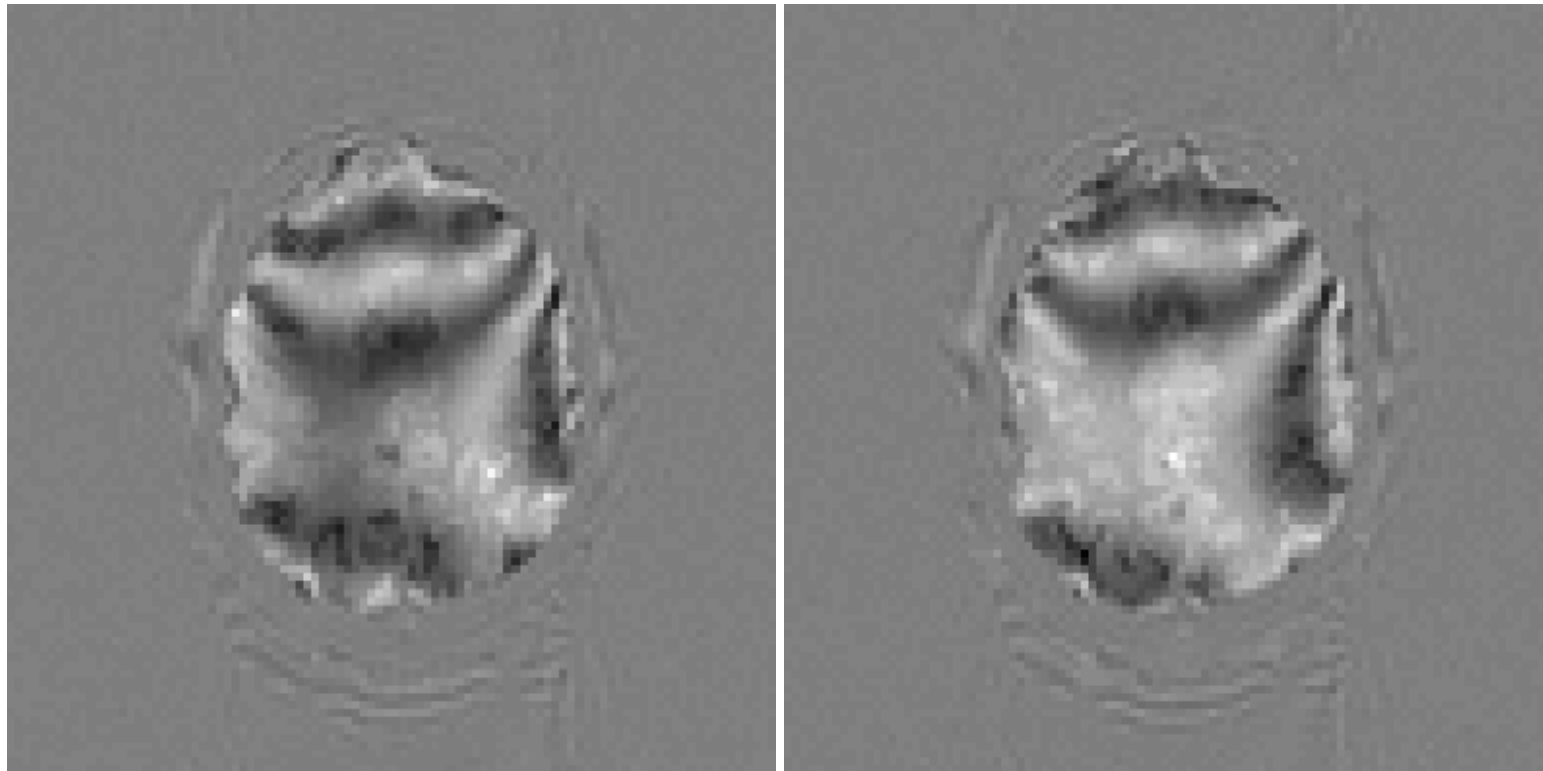
Introduction

In fMRI and fcMRI, there has been an amazing amount of advanced analysis and interpretations presented, but little attention has been paid to what the data truly are.



Introduction

In general, reconstructed GRE EPI images look like below.
How do we get from the below to the previous activation?
And the below isn't even our original measurements.



96×96
240mm FOV
2.5 mm²
In-Plane

Are we ahead of the data with our analyses and interpretations?

Reconstruction

In fMRI and MRI, the measurements taken by the machine are an array of complex-valued spatial frequencies.

This array of complex-valued spatial frequencies need to be reconstructed into an image for us to see, analyze, and interpret.

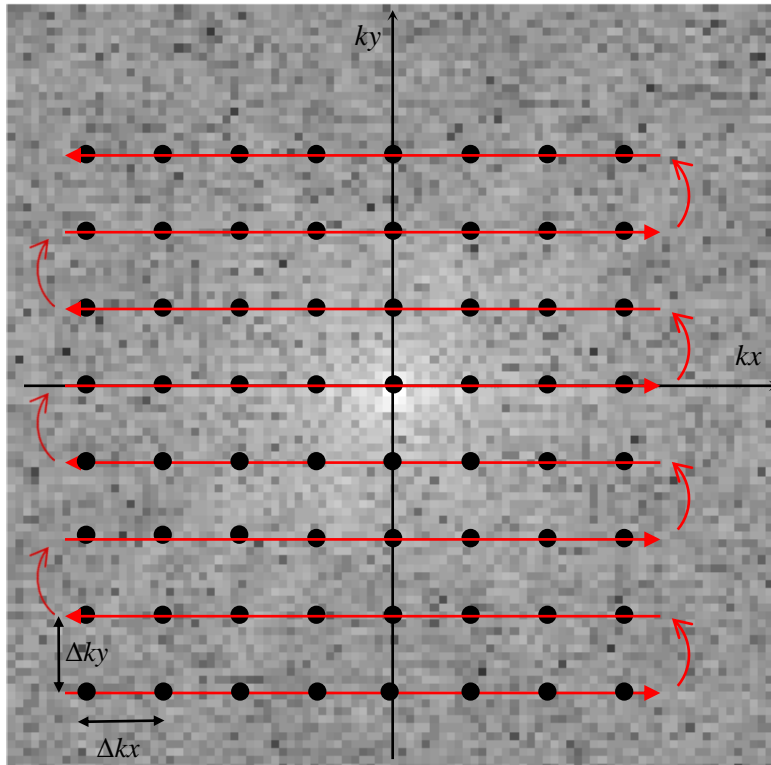
The array of complex-valued spatial frequencies are reconstructed into an image via the inverse Fourier transform.

So lets briefly remind ourselves what the FT and IFT are.

Reconstruction

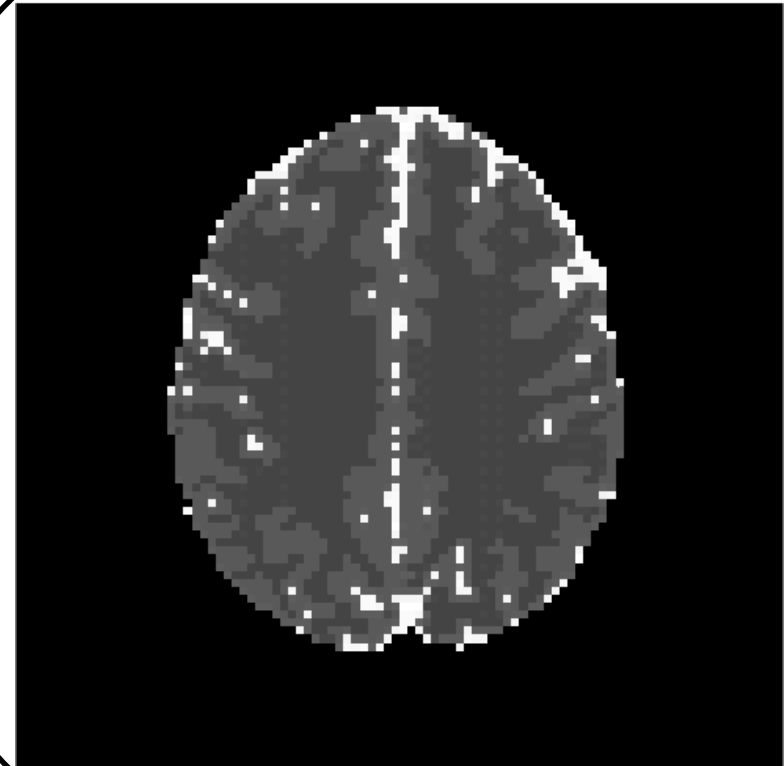
Single Coil Acquisition

Coil measures k -space.



Unaccelerated Acquisition ($A=1$).

Coil

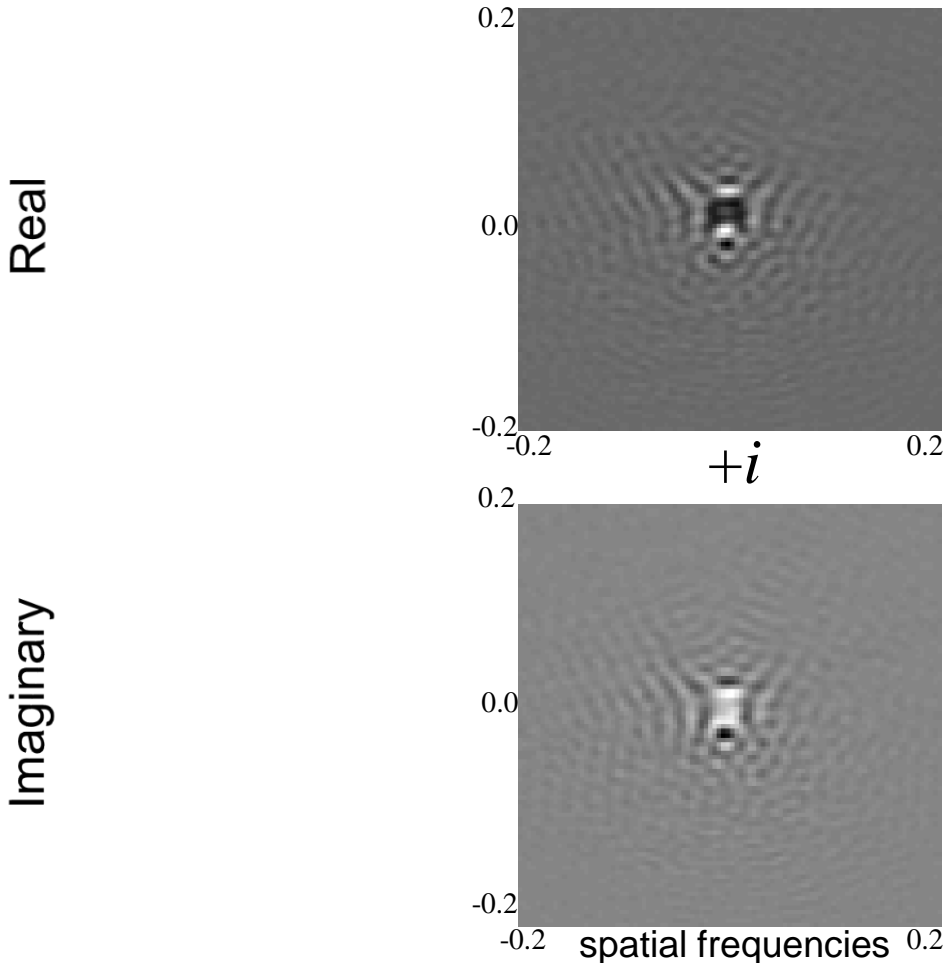


(FOV=240 mm)

 $(n_x=n_y=96, \Delta x=\Delta y=2.5 \text{ mm})$

Reconstruction

We inverse Fourier transform spatial freqs to generate image.

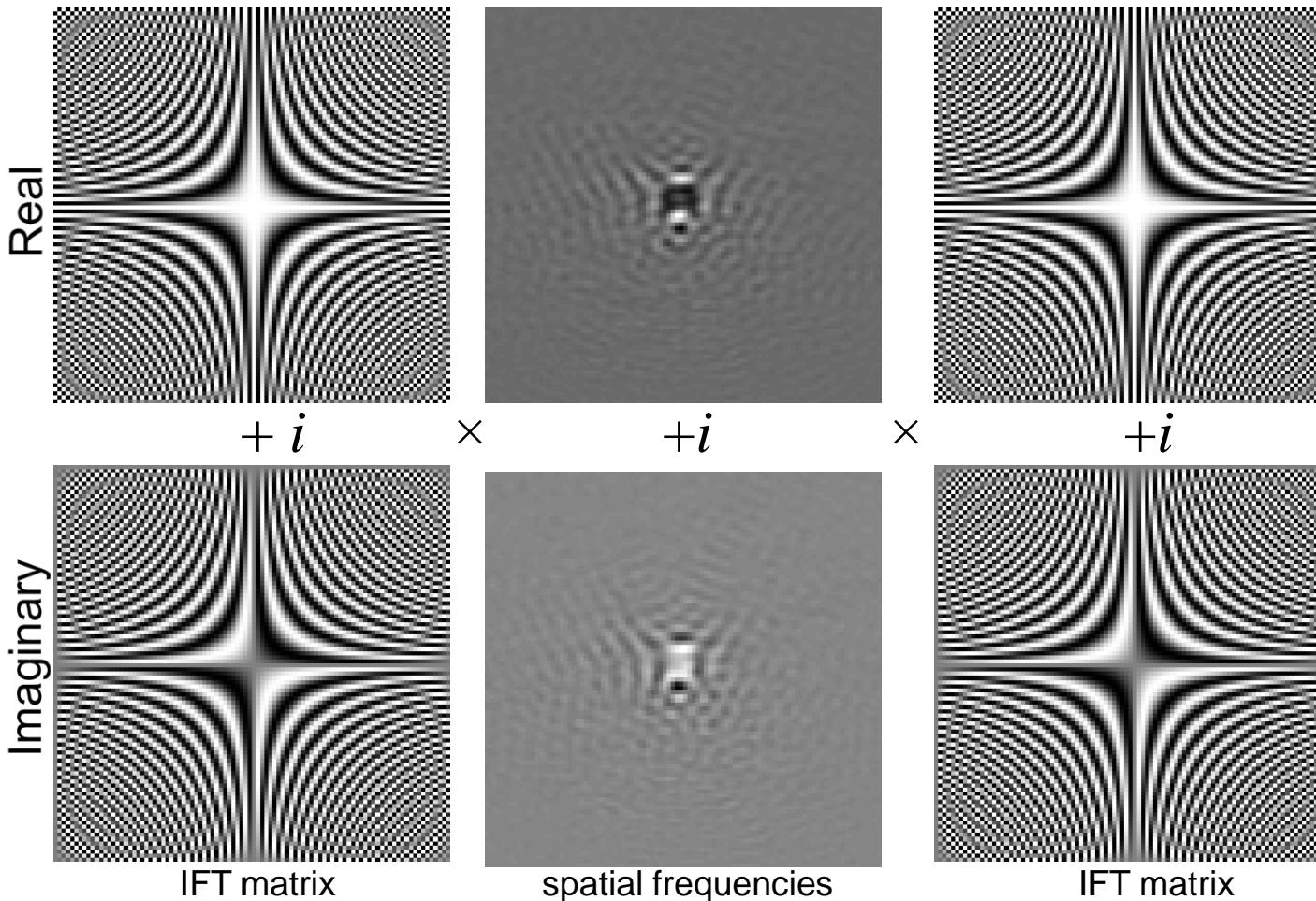


(FOV=240 mm)

 $(n_x=n_y=96, \Delta x=\Delta y=2.5 \text{ mm})$

Reconstruction

We inverse Fourier transform spatial freqs to generate image.

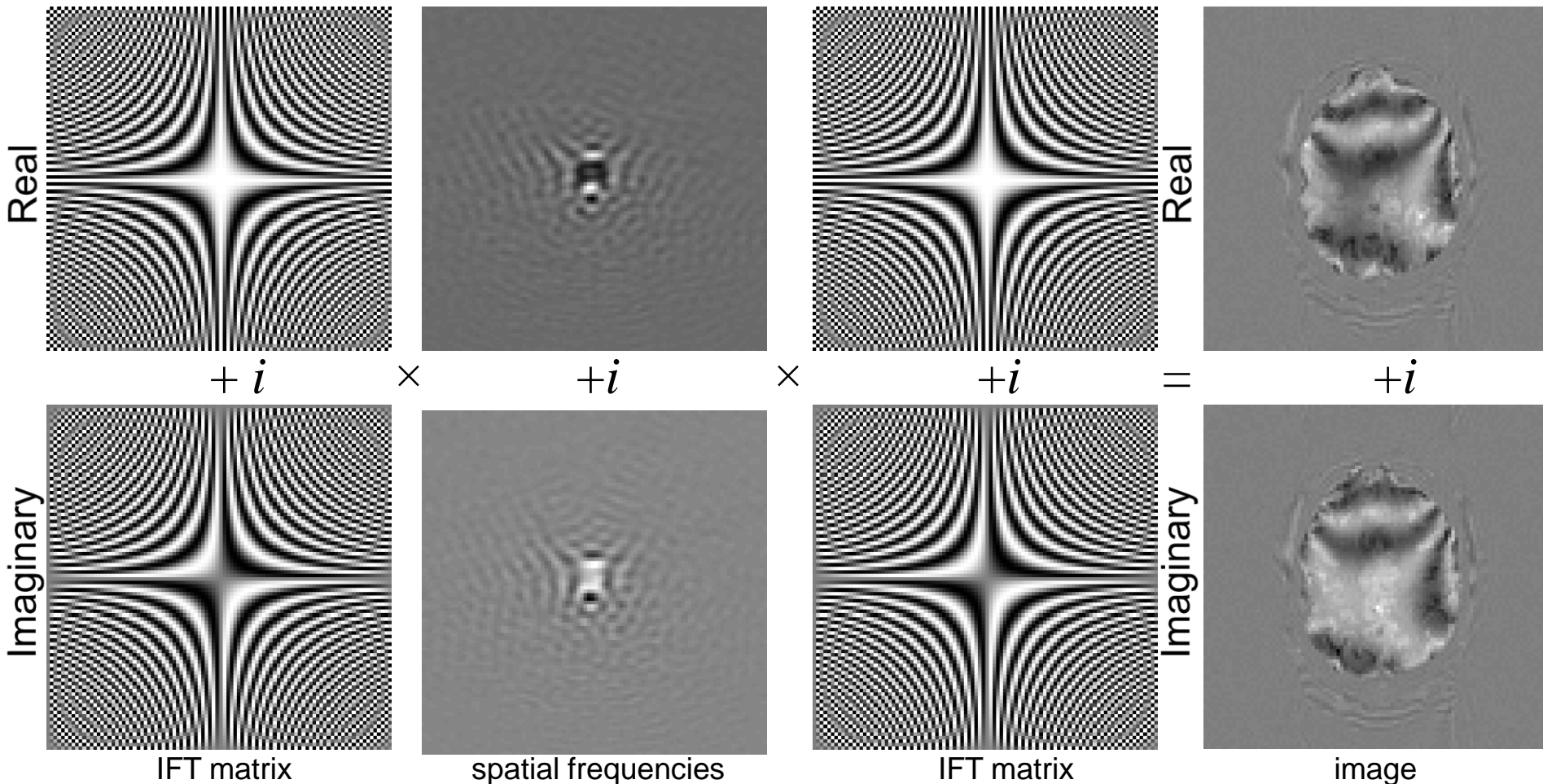


(FOV=240 mm)

 $(n_x=n_y=96, \Delta x=\Delta y=2.5 \text{ mm})$

Reconstruction

We inverse Fourier transform spatial freqs to generate image.

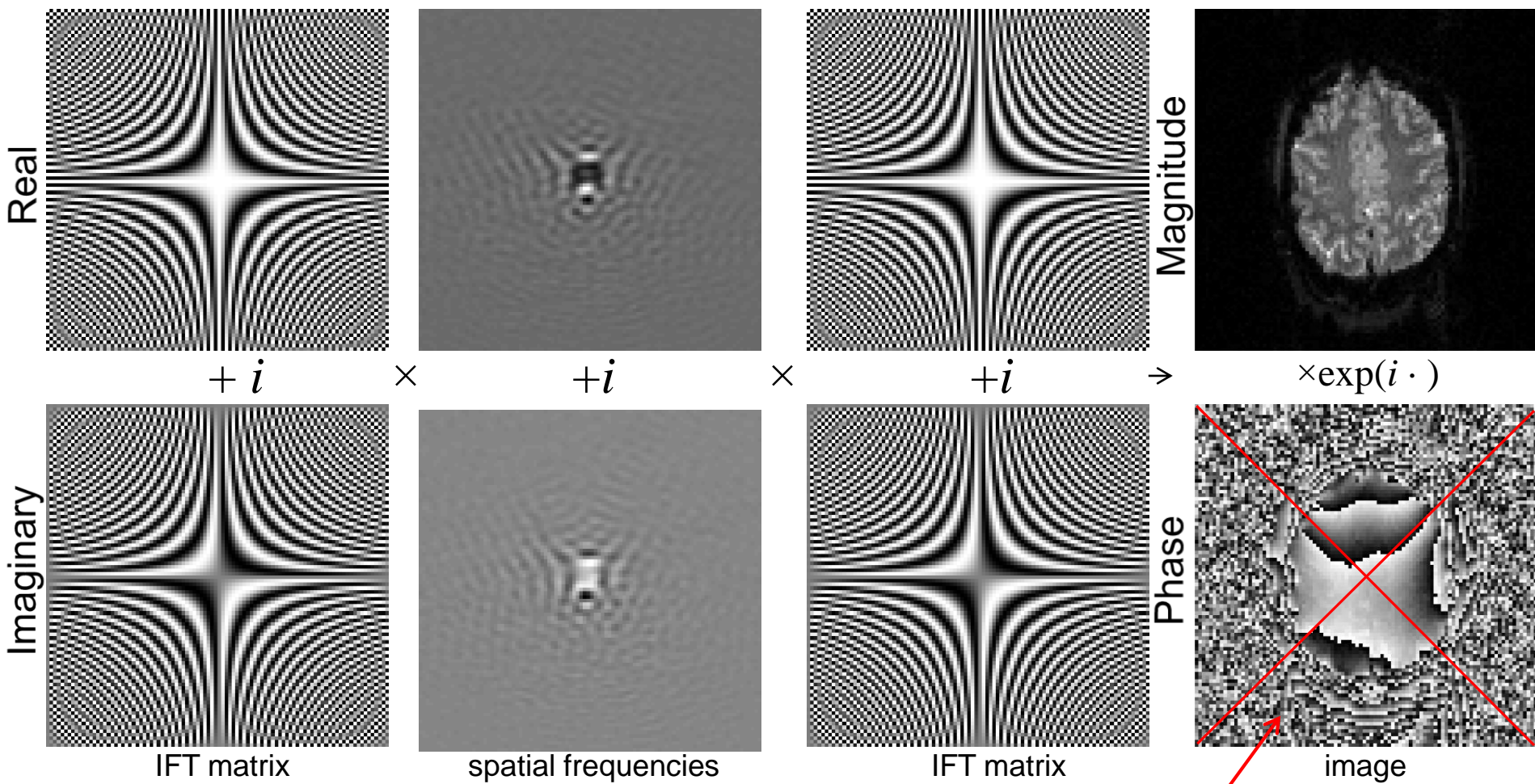


(FOV=240 mm)

($n_x=n_y=96, \Delta x=\Delta y=2.5$ mm)

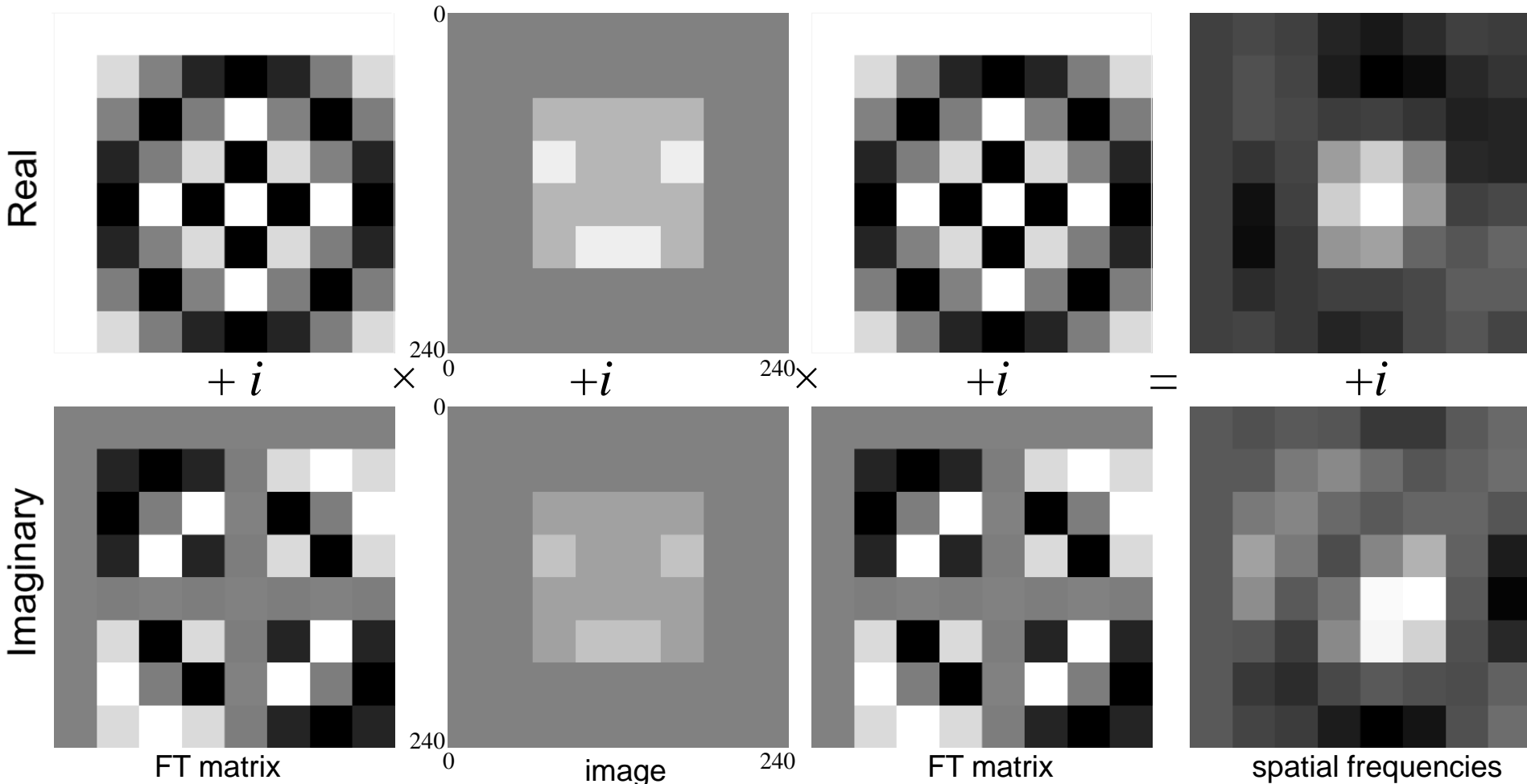
Reconstruction

We inverse Fourier transform spatial freqs to generate image.



Reconstruction

The machine Fourier encodes the image. Measure spatial freq.

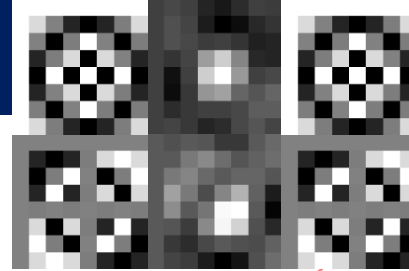




Reconstruction

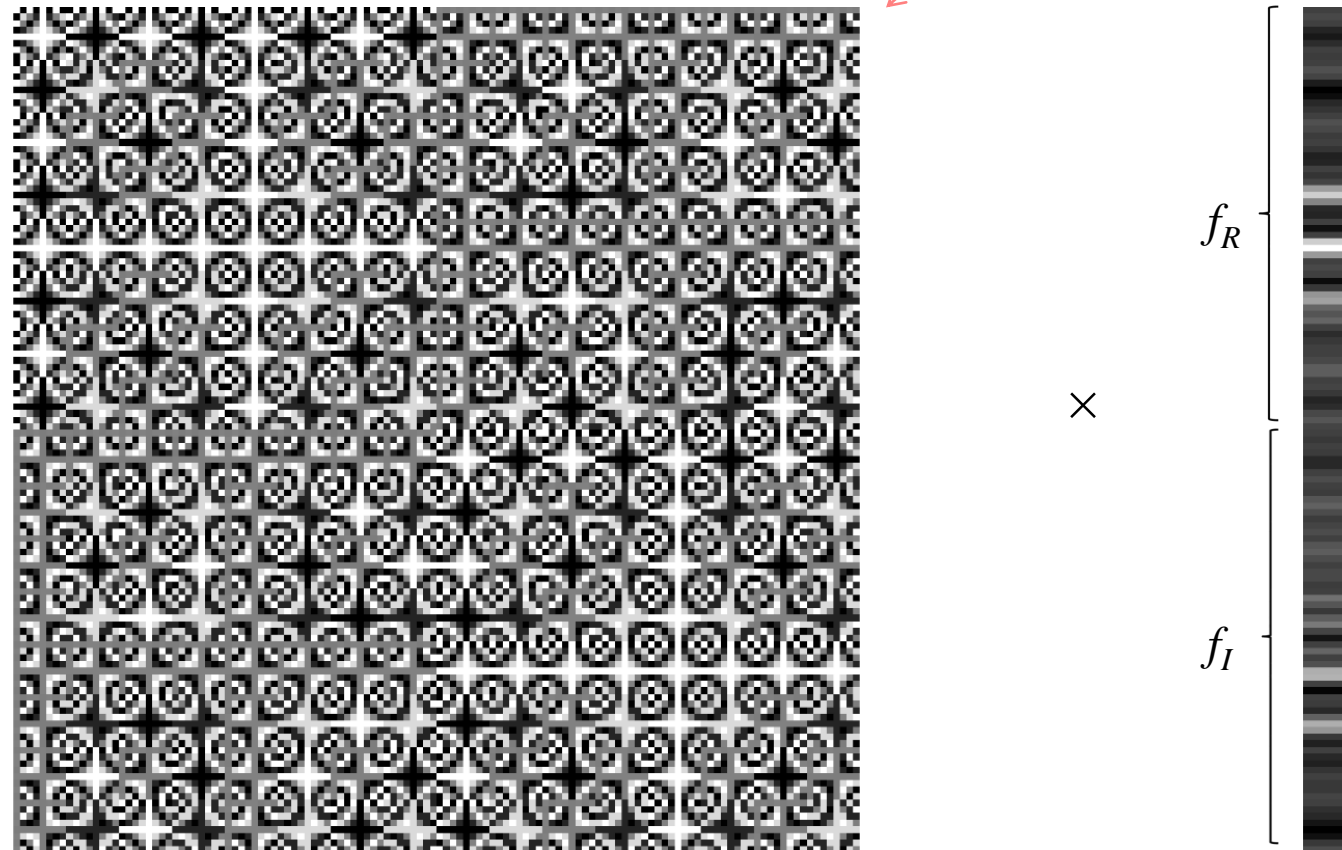
We can stack freq. rows of reals over rows of imaginaries,





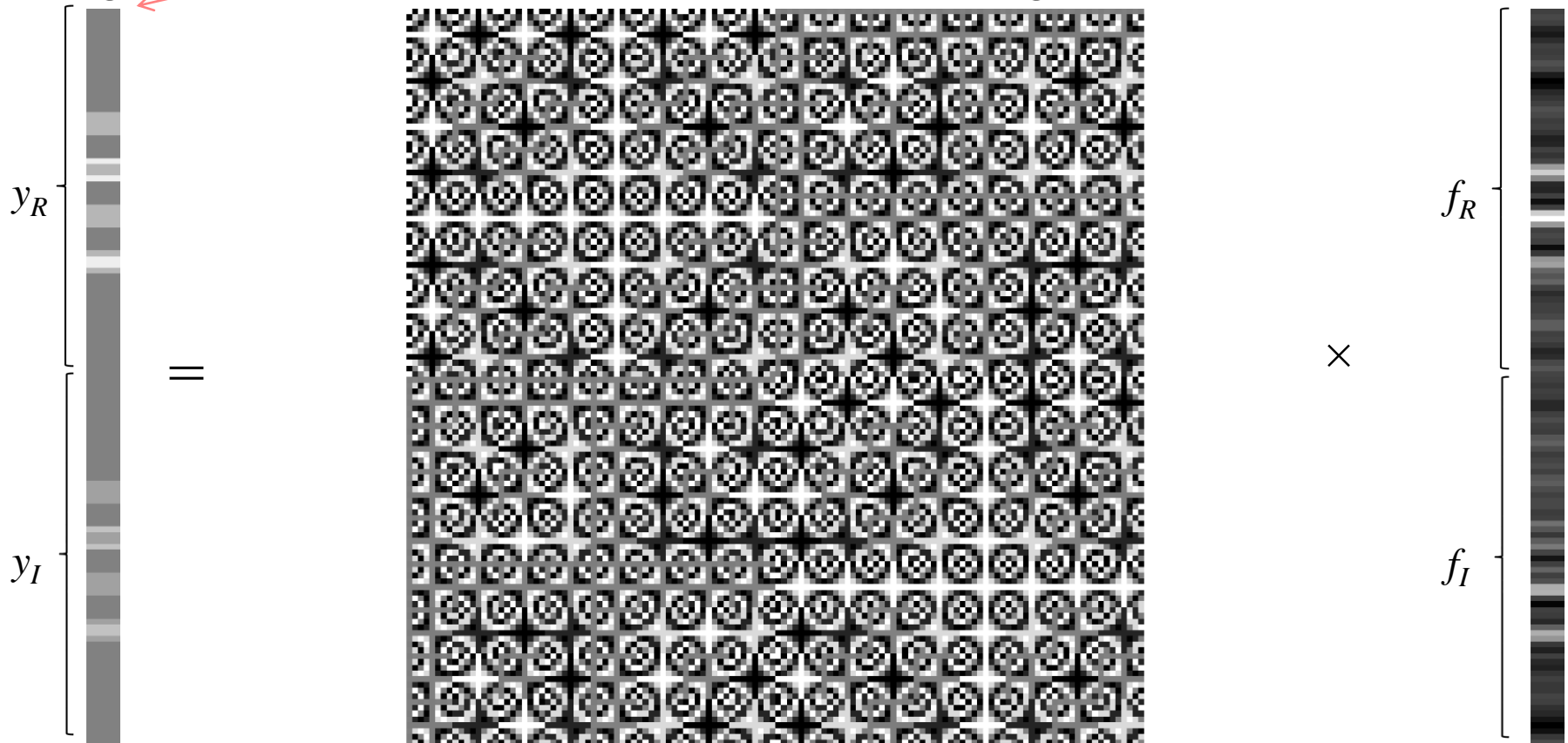
Reconstruction

We can stack freq. rows of reals over rows of imaginaries, make one IFT reconstruction matrix from the two,



Reconstruction

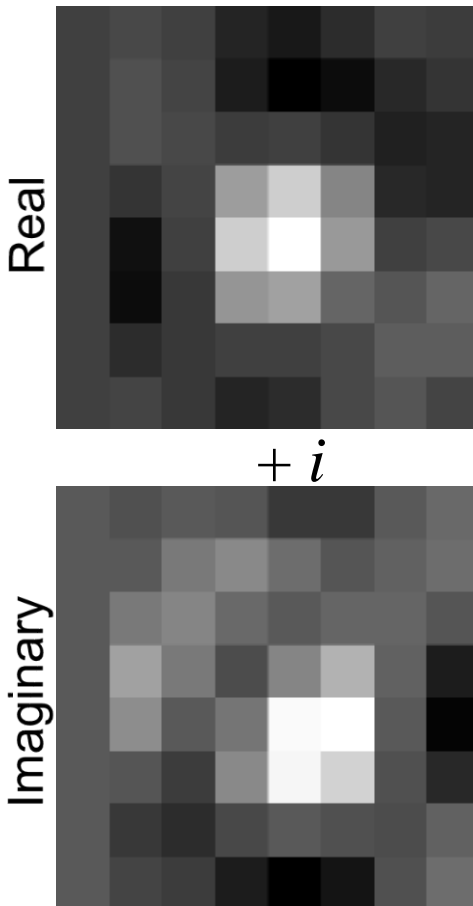
We can stack freq. rows of reals over rows of imaginaries, make one IFT reconstruction matrix from the two, to get the rows of reals over rows of imaginaries.





Processing

Many processing operations are performed by the scanner, by physicists, and by engineers before statistical analysis.



k-space Processing

Nyquist Ghost Correction
 Static B0 Field Correction
 Zero Fill Interpolation
 Non-Cartesian Interpolation
 Ramp Sampling Interpolation
 Homodyne Interpolation
 Apodization
 And many more...

Image Reconstruction

2D inverse Fourier transform
 In-Plane SENSE/GRAPPA
 Through-Plane SENSE

Image Processing

Image Smoothing
 Global Normalization
 Motion Correction
 And many more...

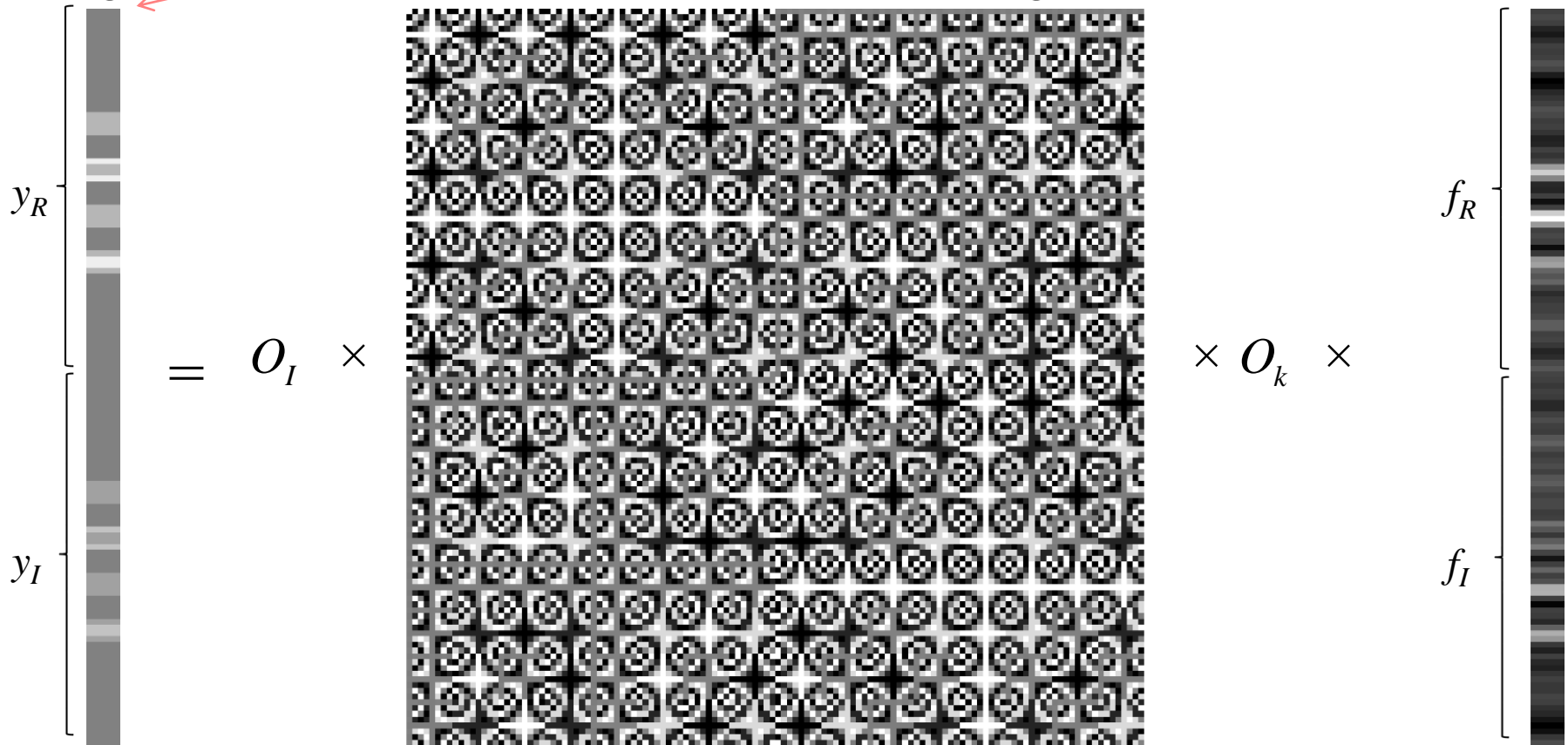
Time Series Processing

Filtering
 Smoothing
 Dynamic B0 Correction
 Slice Timing
 And many more...

Show ones in blue.

Reconstruction

We can stack freq. rows of reals over rows of imaginaries, make one IFT reconstruction matrix from the two, to get the rows of reals over rows of imaginaries.

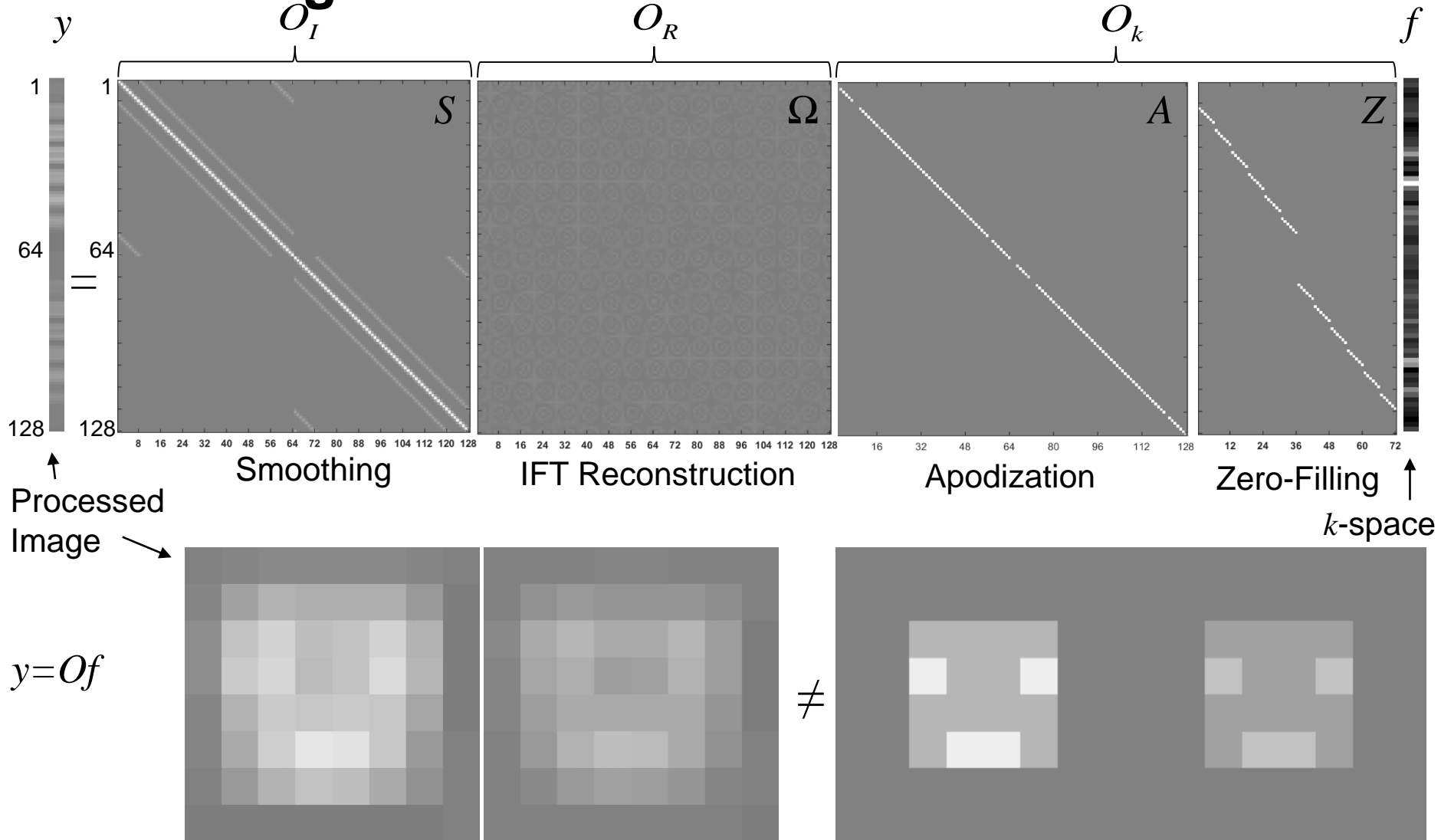


Processing





Processing



Processing

We measure an array of complex-valued numbers, perform complex-valued image reconstruction to this array, to generate complex-valued images in real and imaginary, along the way, there is complex-valued image processing.

What are the implications of what was done to the data?

Implications

In statistics, we know the rule that says:

If a vector f has a mean δ , and a covariance Γ ,

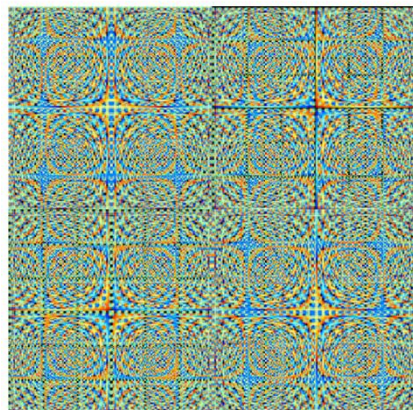
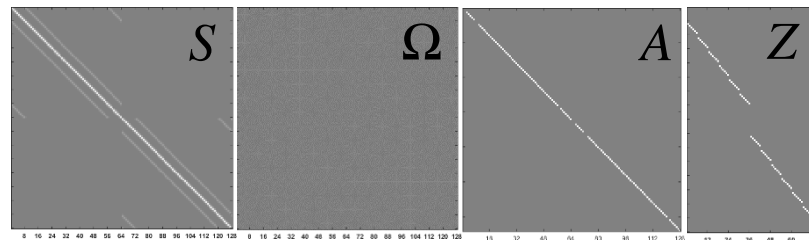
Then $y=Of$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

Then Σ can be converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

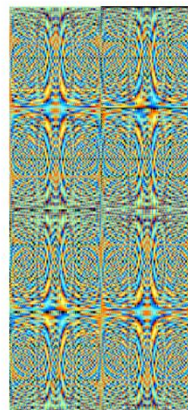
Where $D^{-1/2} = 1 / \sqrt{\text{diag}(\Sigma)}$.

Assume k -space measurements independent so $\Gamma=I$.

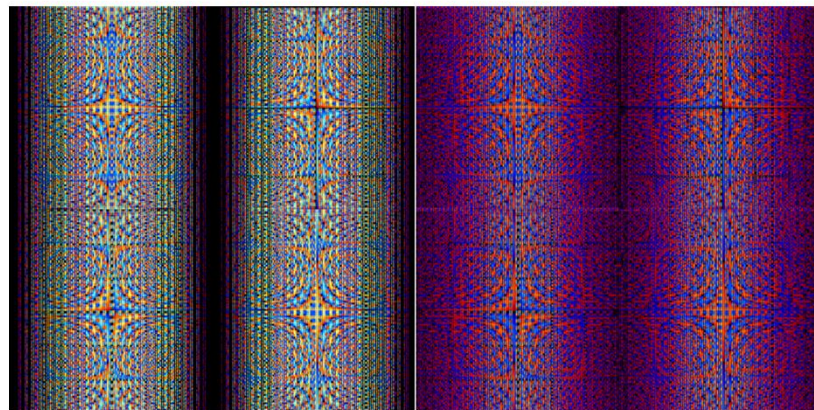
Implications Operators, O .



a) $O = \Omega$



b) $O = \Omega Z$

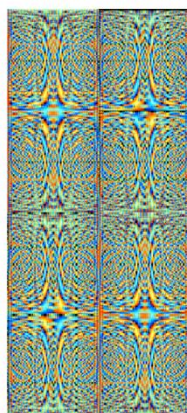


c) $O = \Omega A$

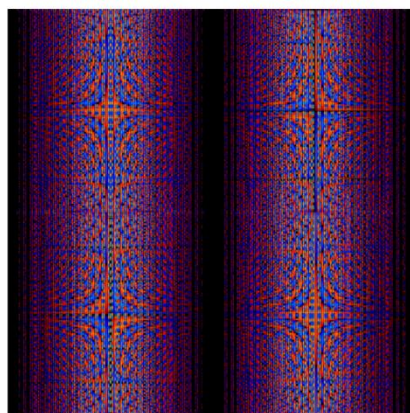
1.1×10^{-4}



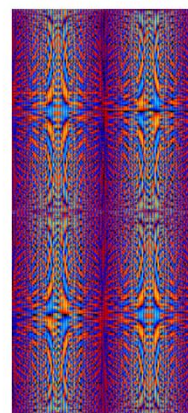
d) $O = S\Omega$



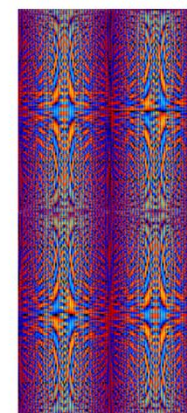
e) $O = \Omega A Z$



f) $O = S\Omega A$



g) $O = S\Omega Z$



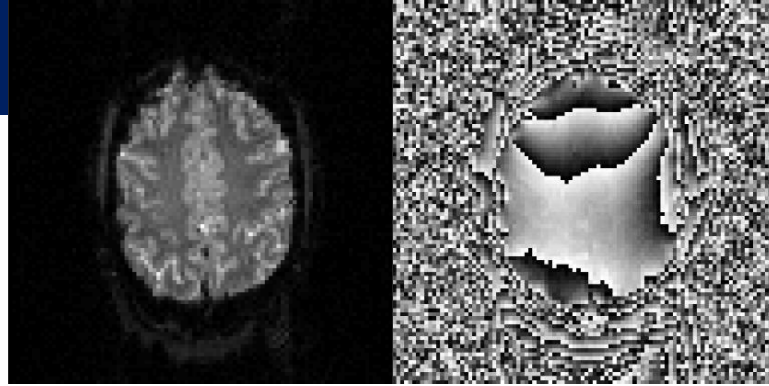
h) $O = S\Omega A Z$

-1.1×10^{-4}

Implications

Mean, $\mu = Of$.

$O = \Omega$



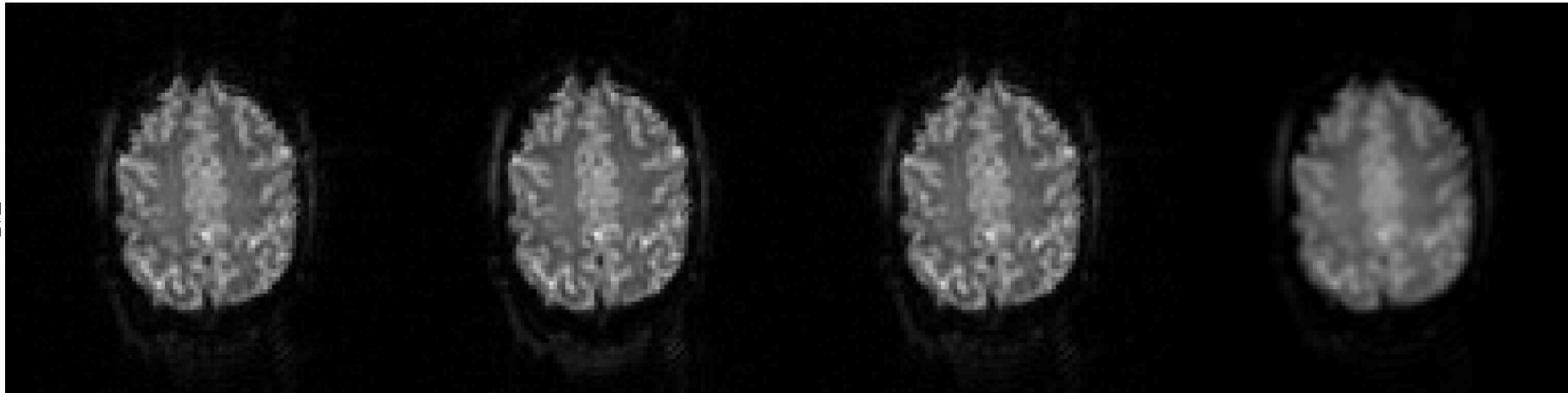
$O = \Omega Z$

$O = \Omega A$

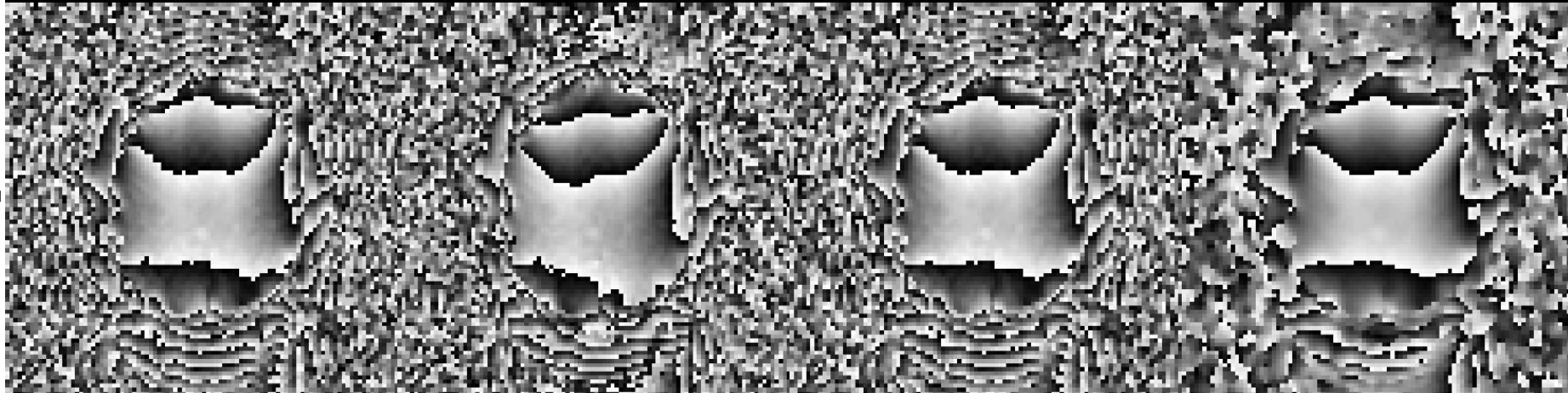
$O = \Omega AZ$

$O = S\Omega AZ$

M



P



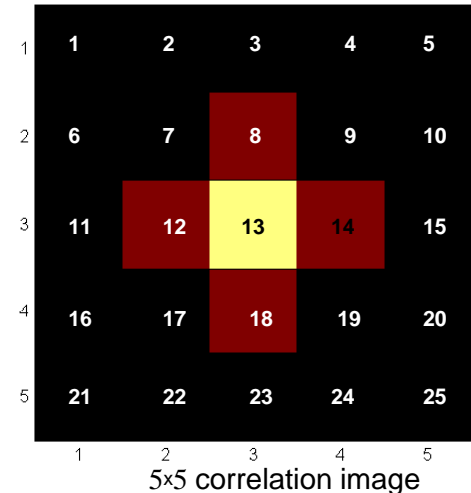
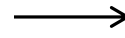
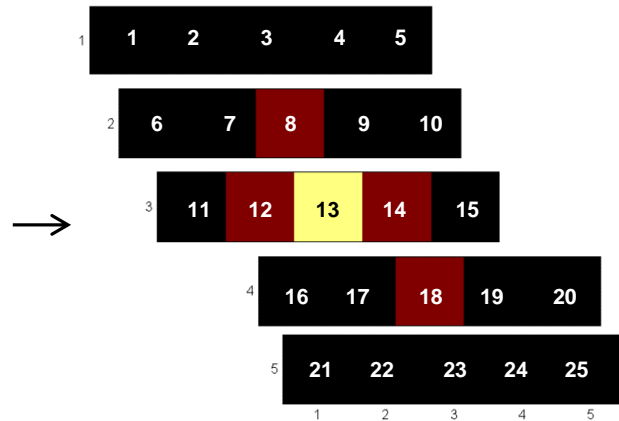
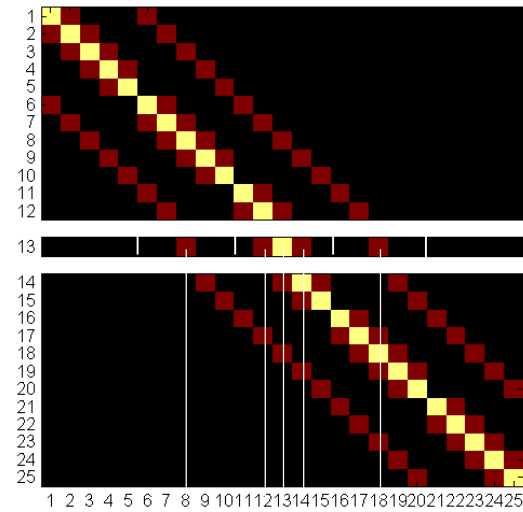
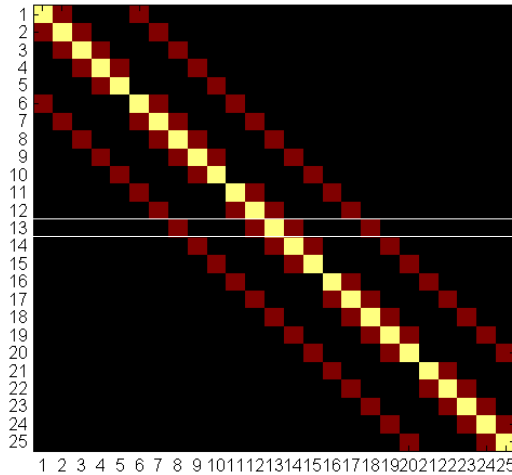
Implications

Correlation matrix and correlation image.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

5x5 image

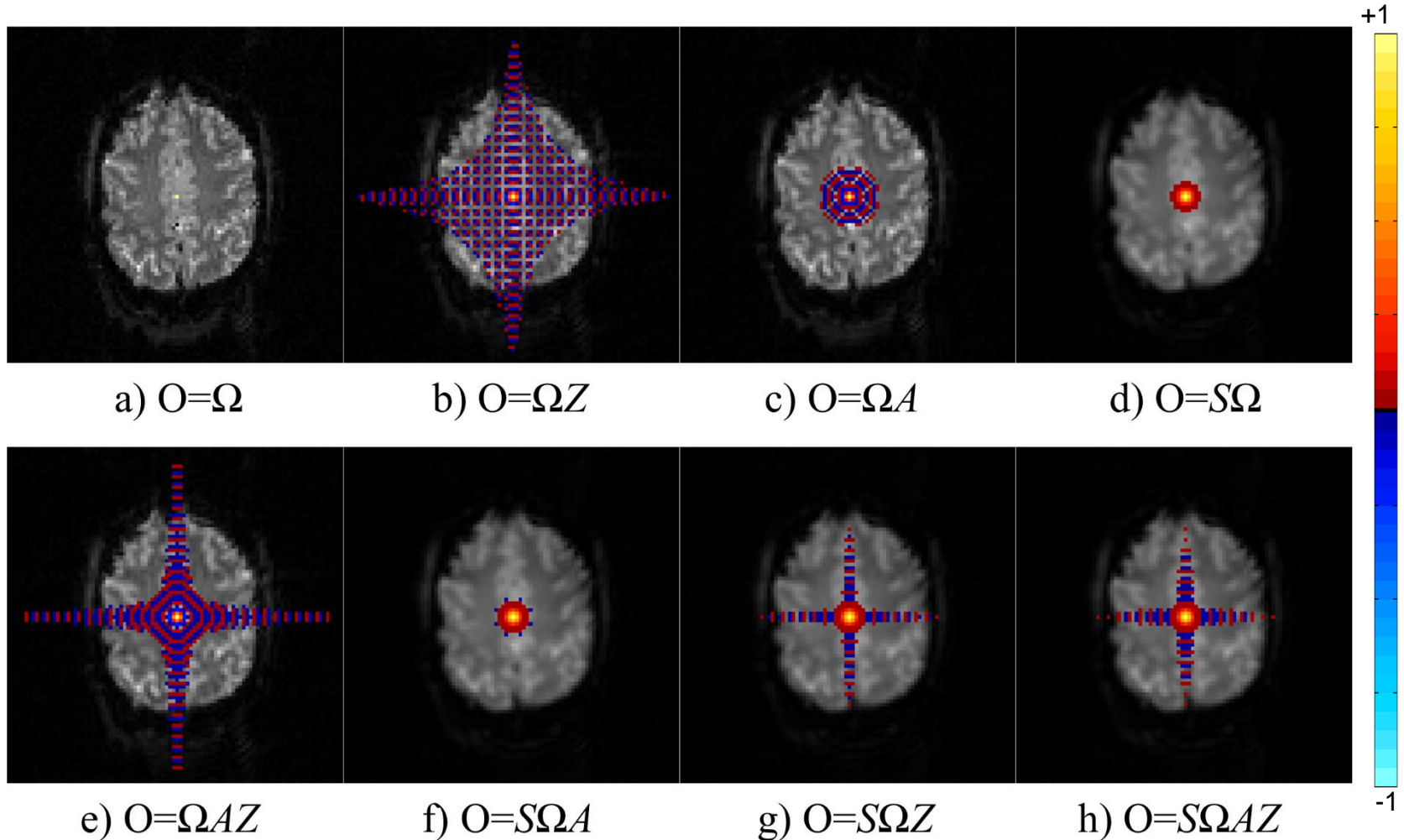
$cor(y) =$
25x25
correlation
matrix



Implications

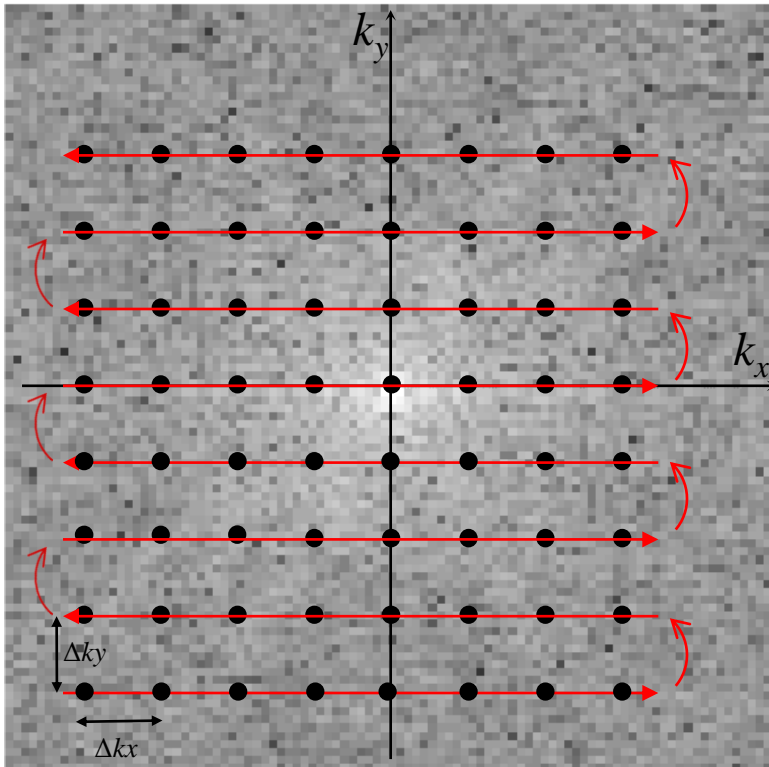
Correlation, $R = D^{-1/2} \Sigma D^{-1/2}$.

$$R = \begin{bmatrix} R_{RR} & R_{RI} \\ R_{IR} & R_{II} \end{bmatrix}$$



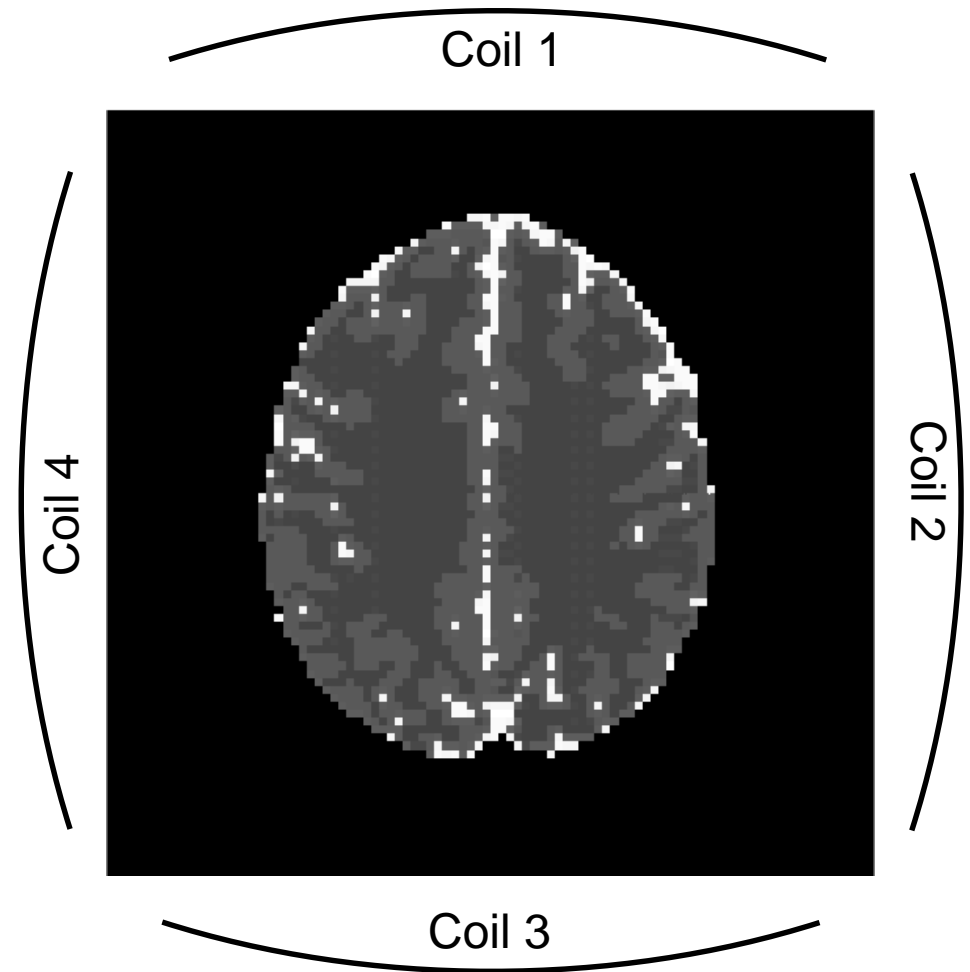
Reconstruction

Multi-Coil Acquisition



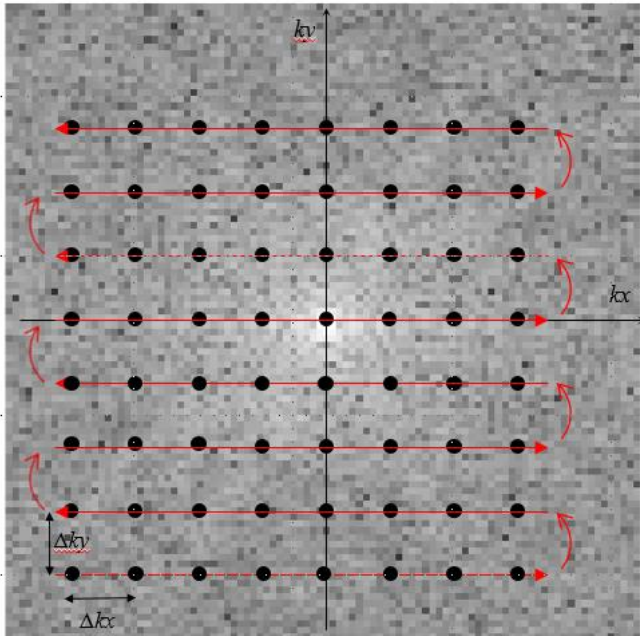
Each coil measures k -space.

$N_C=4, A=1$



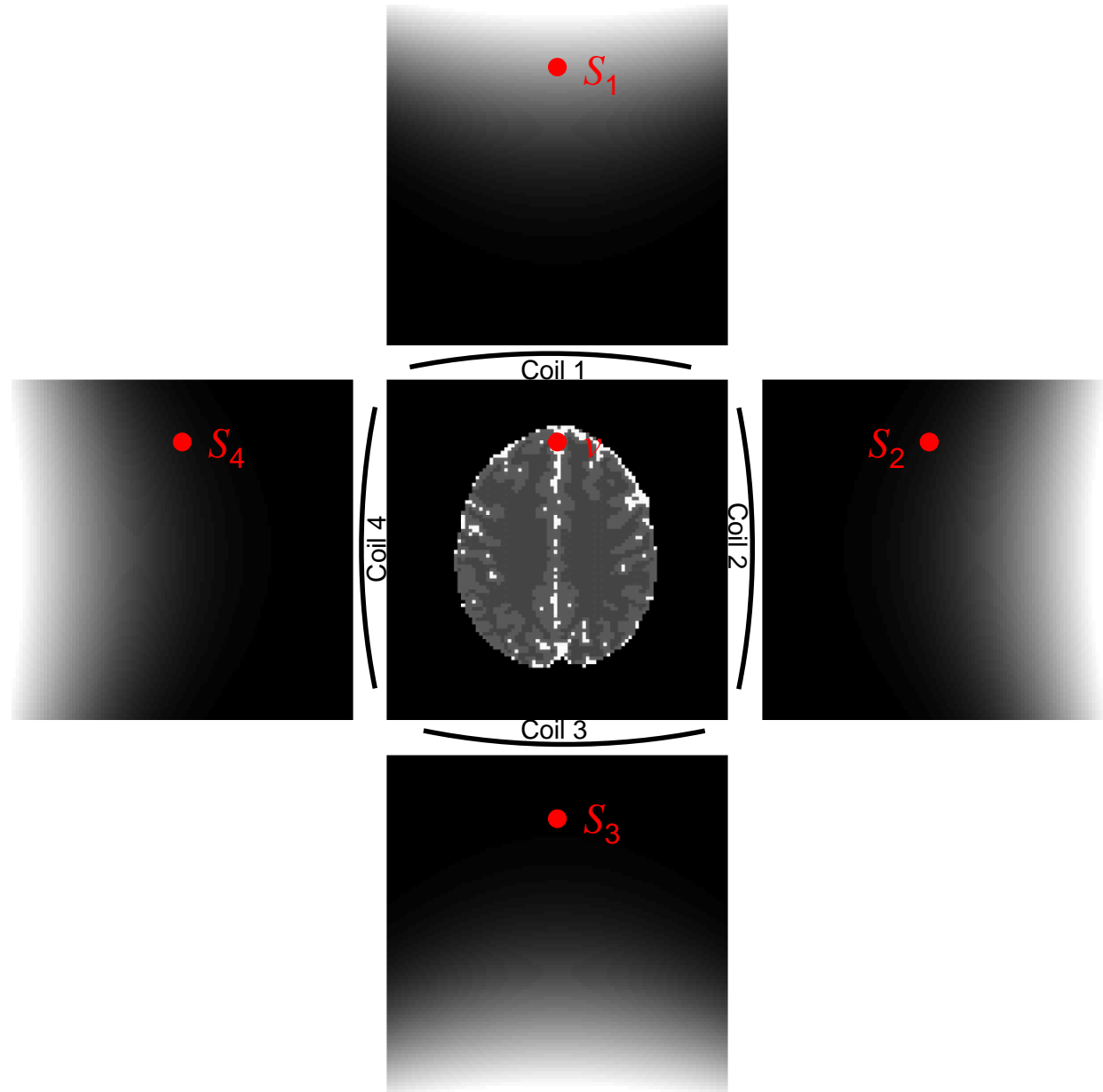
Reconstruction

Multi-Coil Acquisition



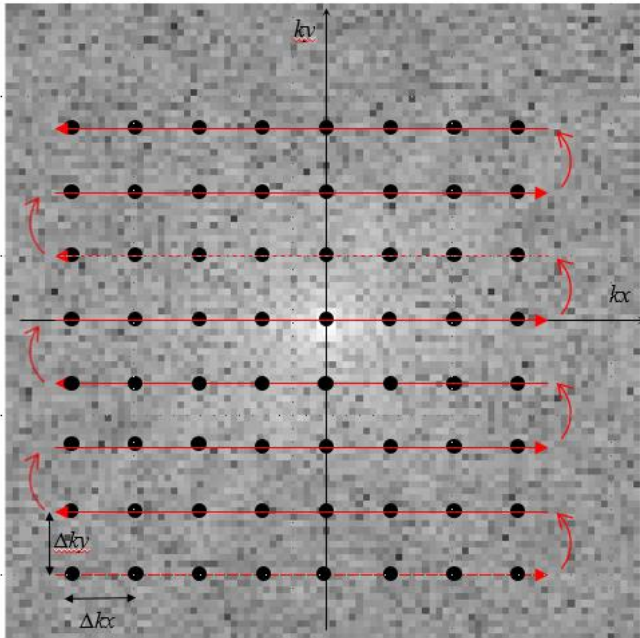
Each coil measures k -space.

$$N_C=4, A=1$$



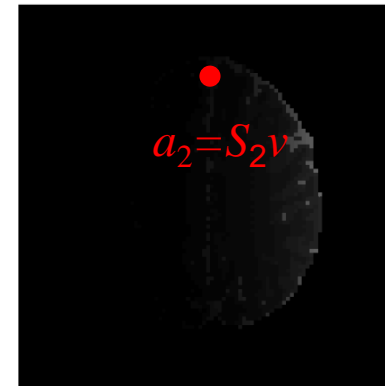
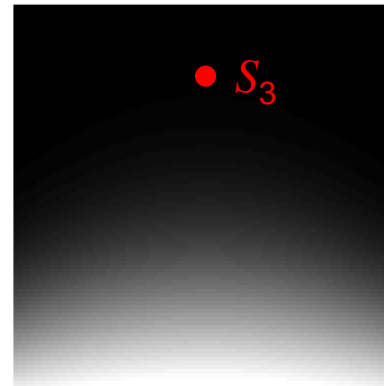
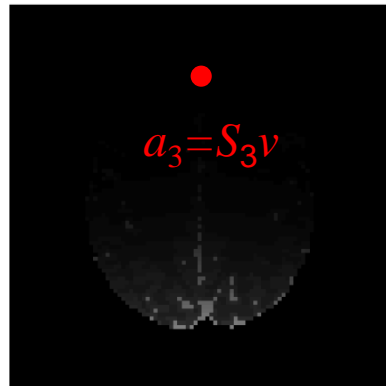
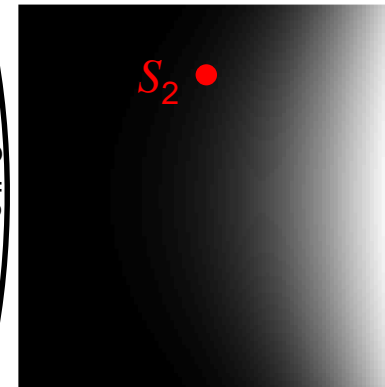
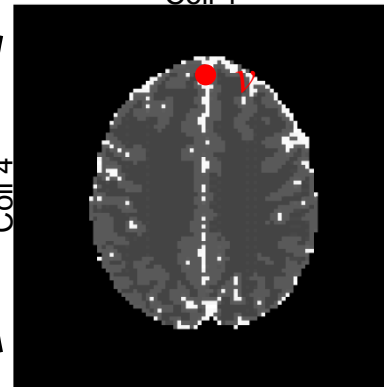
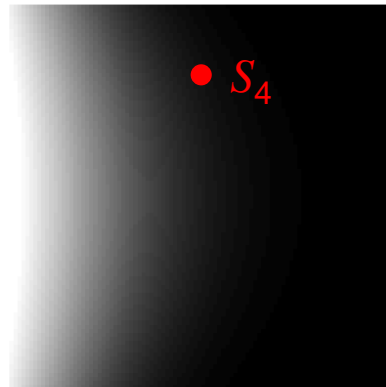
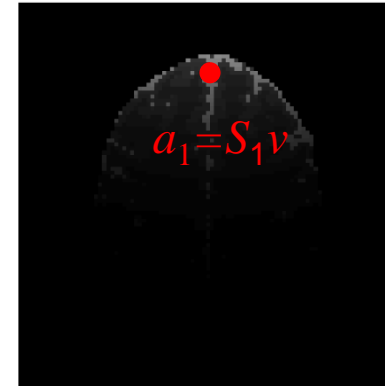
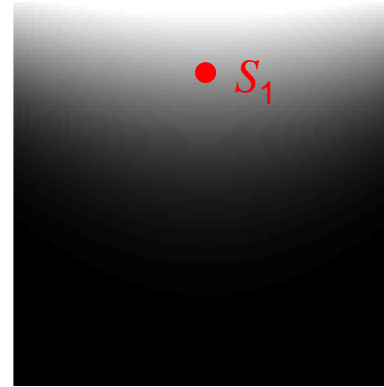
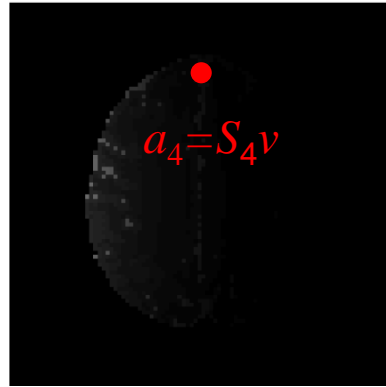
Reconstruction

Multi-Coil Acquisition



Each coil measures k -space.

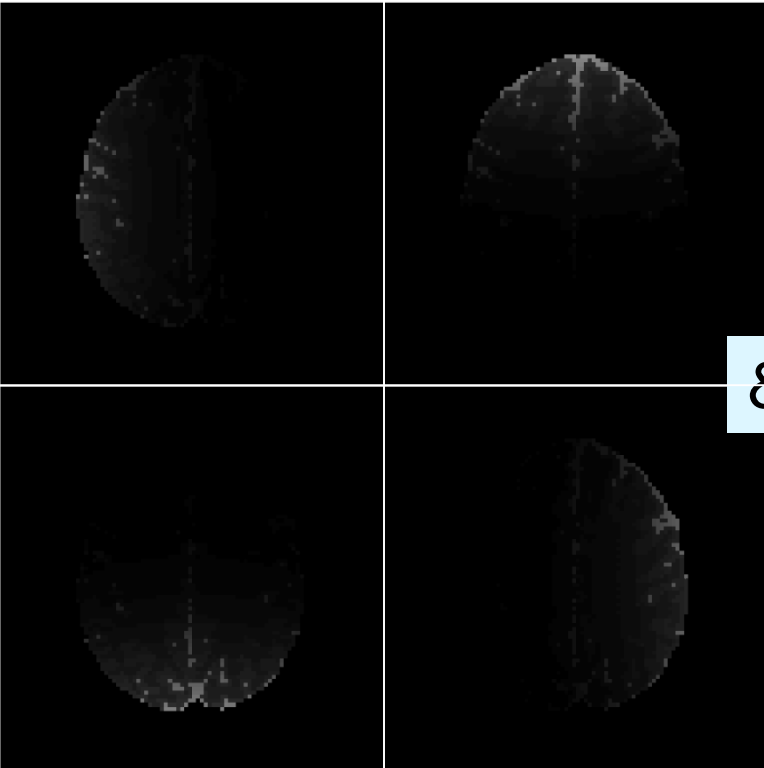
$N_C=4, A=1$



Reconstruction

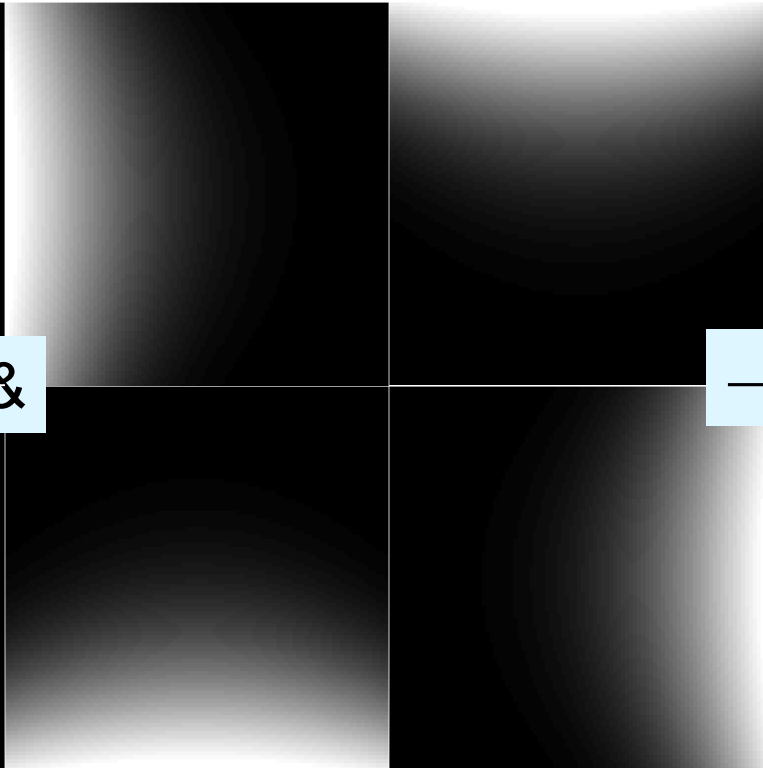
SENSE

Measured Coil Images

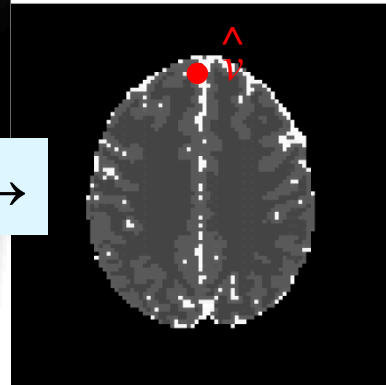


&

Estimated Sensitivities



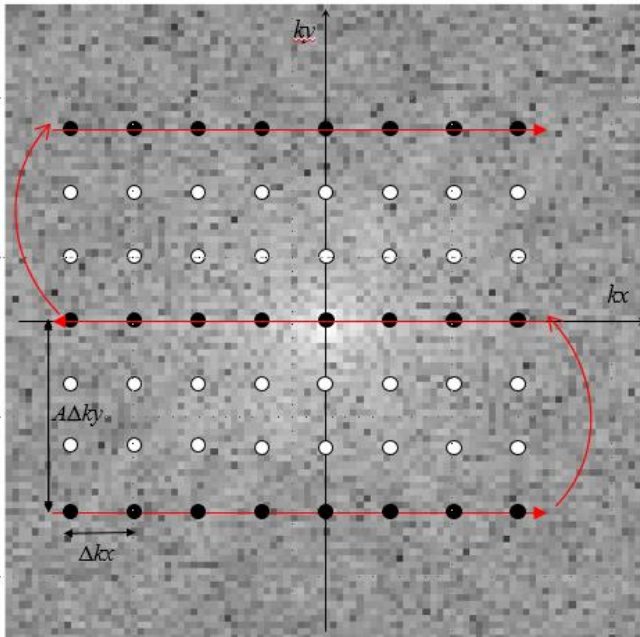
Combined Image



$N_C=4, A=1$

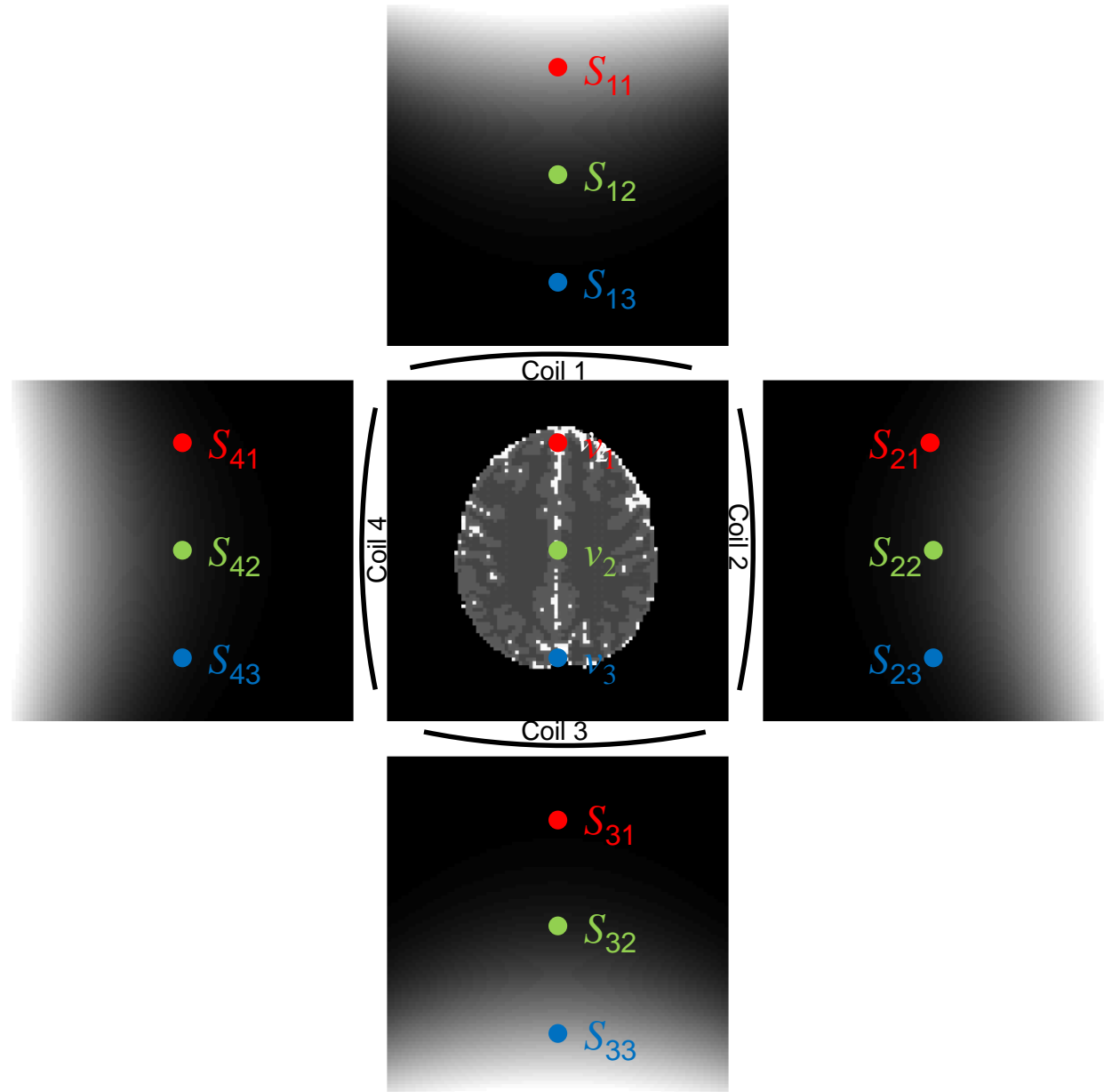
Reconstruction

Multi-Coil Acquisition



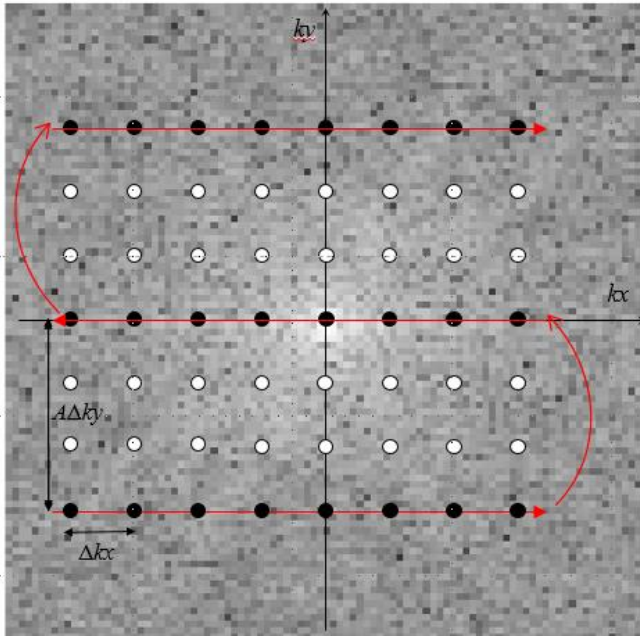
Each coil measures k -space.

$N_C=4, A=3$



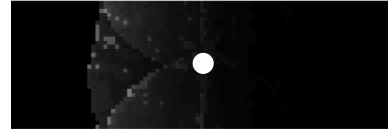
Reconstruction

Multi-Coil Acquisition

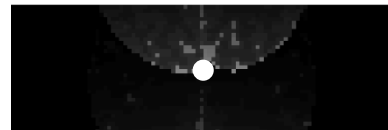
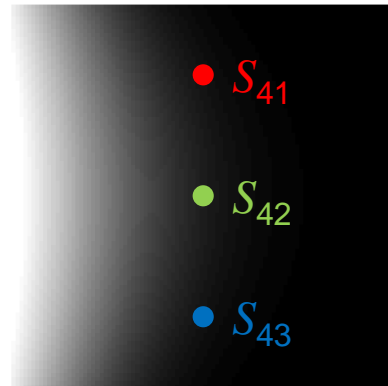


Each coil measures k -space.

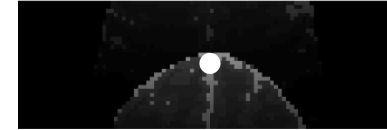
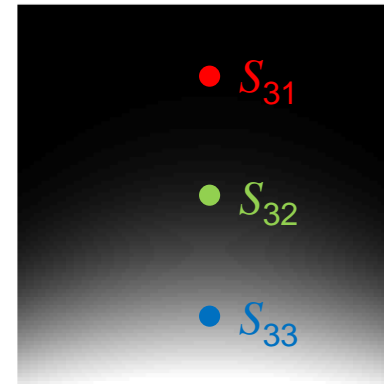
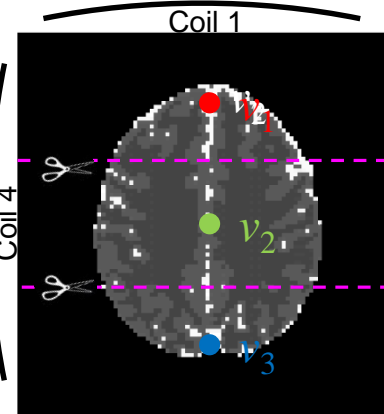
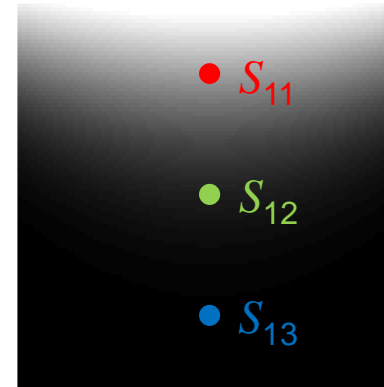
$N_C=4, A=3$



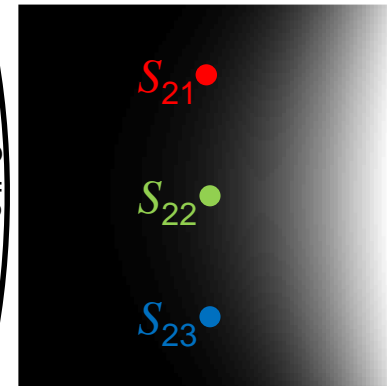
$$a_4 = S_{41}v_1 + S_{42}v_2 + S_{43}v_3$$



$$a_3 = S_{31}v_1 + S_{43}v_2 + S_{33}v_3$$



$$a_1 = S_{11}v_1 + S_{12}v_2 + S_{13}v_3$$

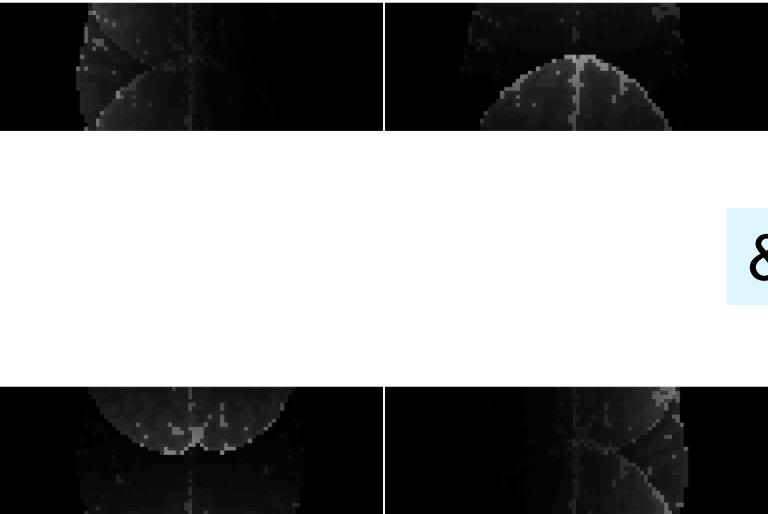


$$a_2 = S_{21}v_1 + S_{22}v_2 + S_{23}v_3$$

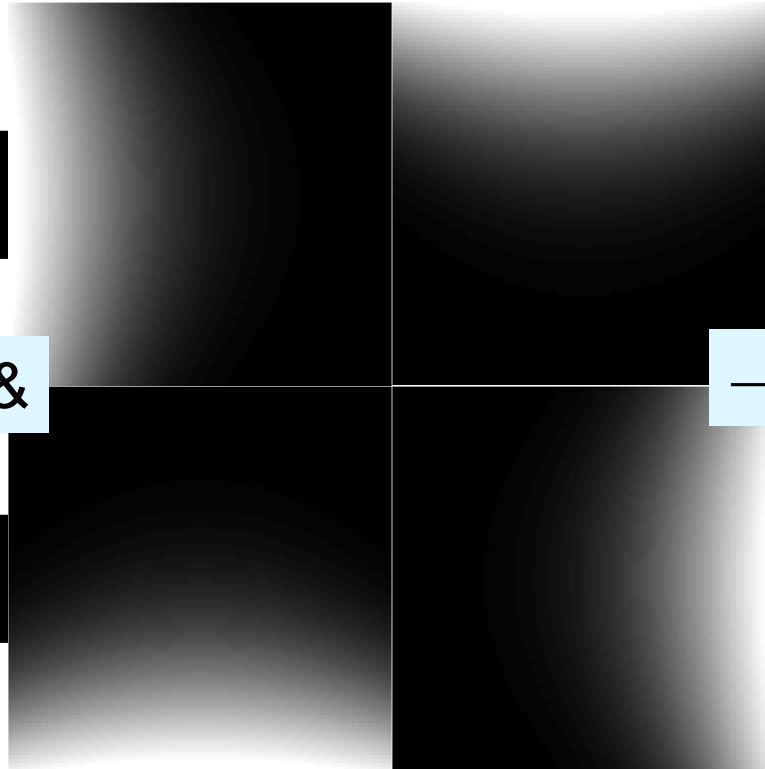
Reconstruction

SENSE

Measured Coil Images



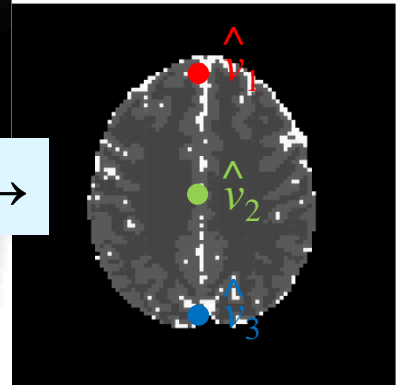
Estimated Sensitivities



&



Separated
Combined
Image

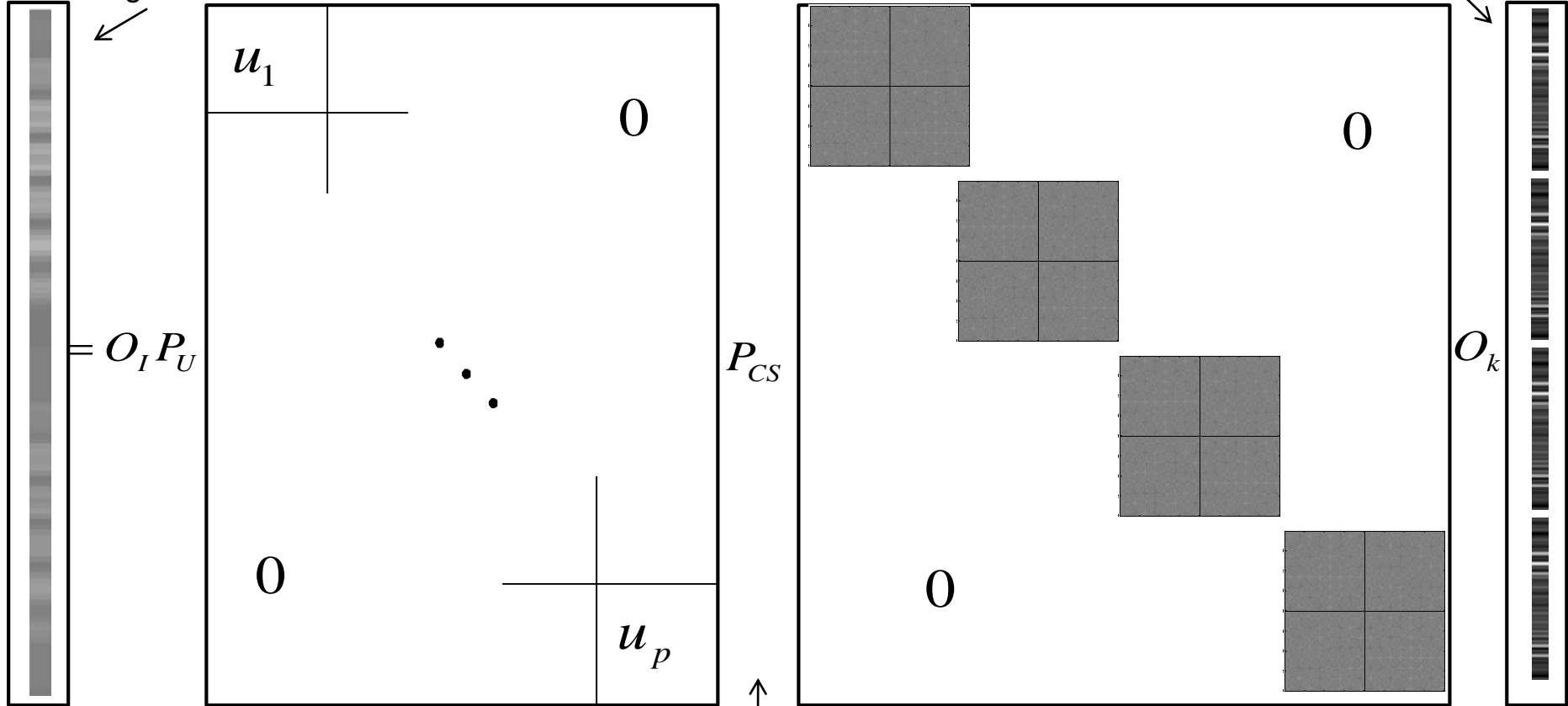
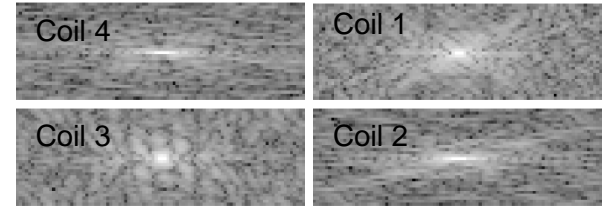


$$N_C=4, A=3$$

Reconstruction/Processing

SENSE

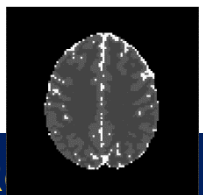
Image vector



permute to by folded voxel

reconstruct $N_c=4$ images

k -space vector of N_c images



Implications

In statistics, we know the rule that says:

If a vector f has a mean δ , and a covariance Γ ,

Then $y=Of$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

Then Σ can be converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

Where $D^{-1/2} = 1 / \sqrt{\text{diag}(\Sigma)}$.

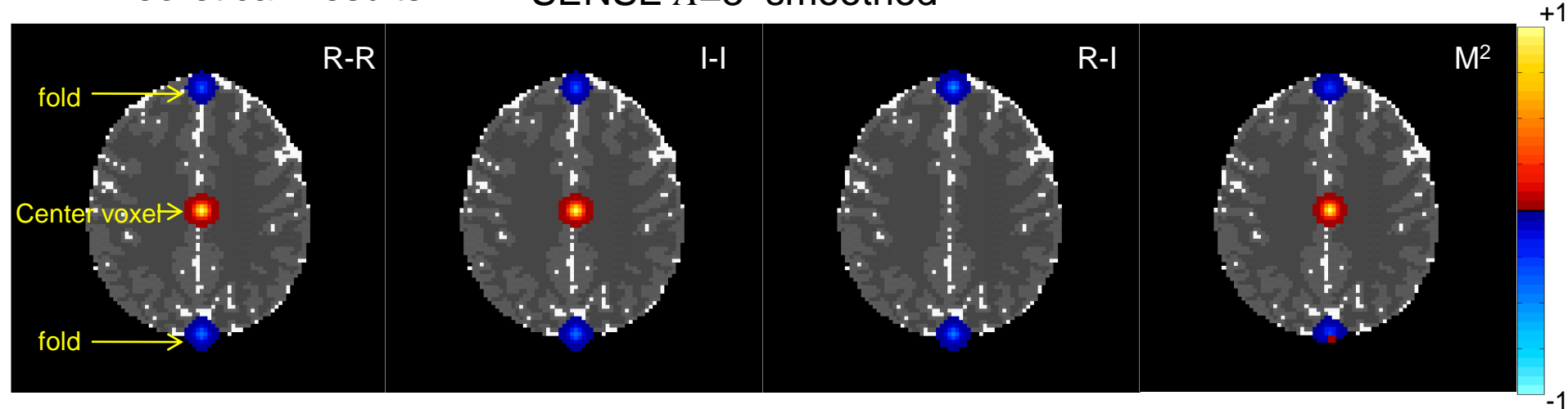
Assume k -space measurements independent so $\Gamma=I$.

Implications

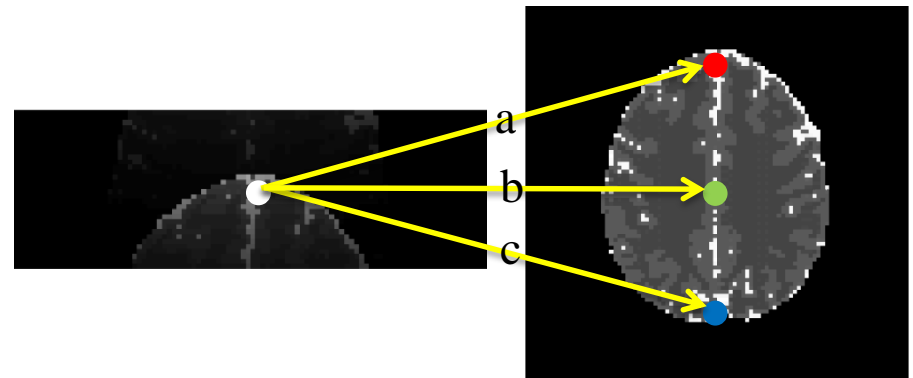
SENSE induces long-range in-plane correlation.

Theoretical Results

SENSE $A=3$ smoothed



Basically multiplying voxel values a_t by same 3 numbers over time t to lead to correlated voxels.



Implications

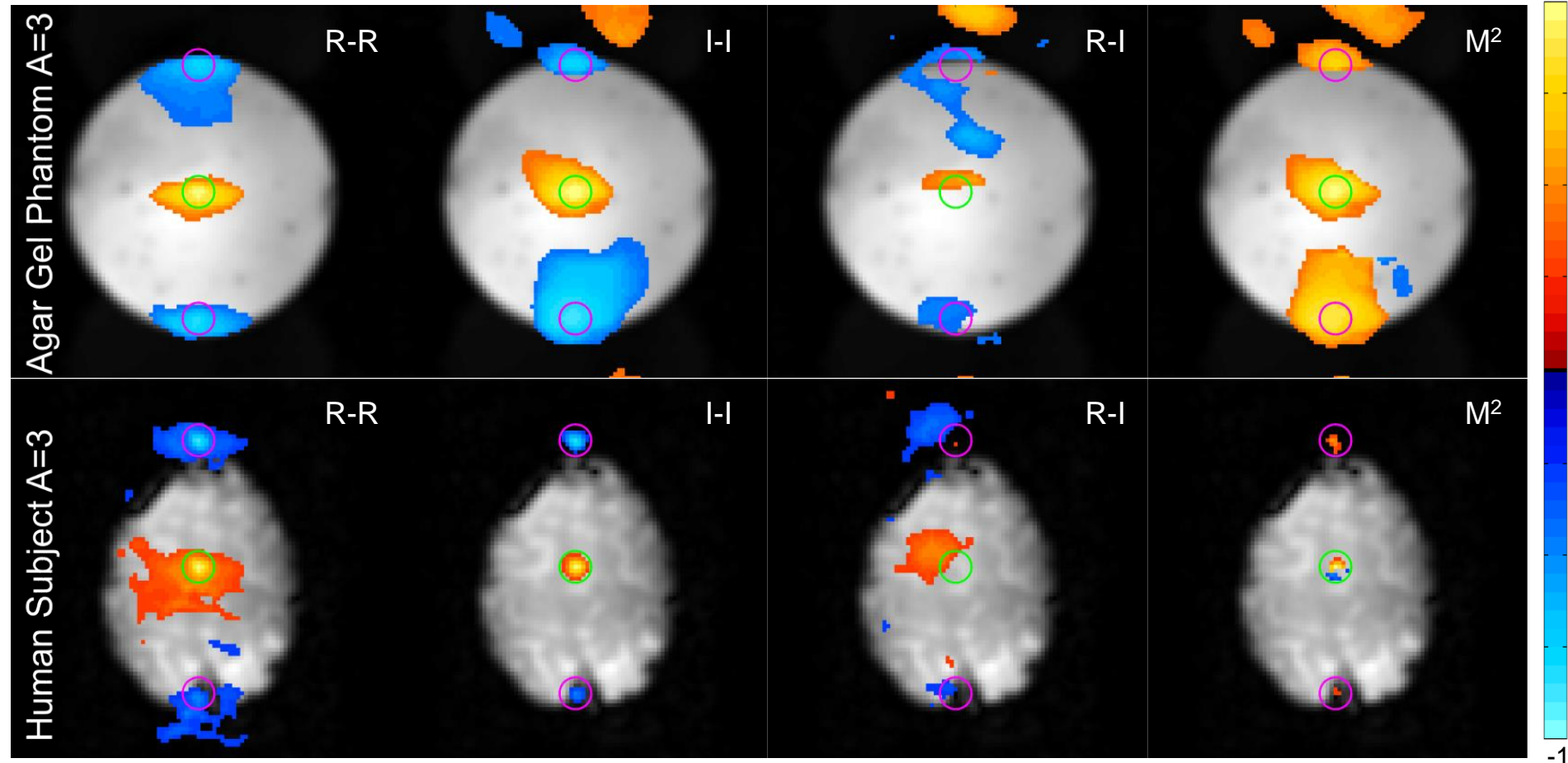
SENSE Reconstruction induces long-range correlation.

Experimental Results

SENSE $A=3$ smoothed

+1

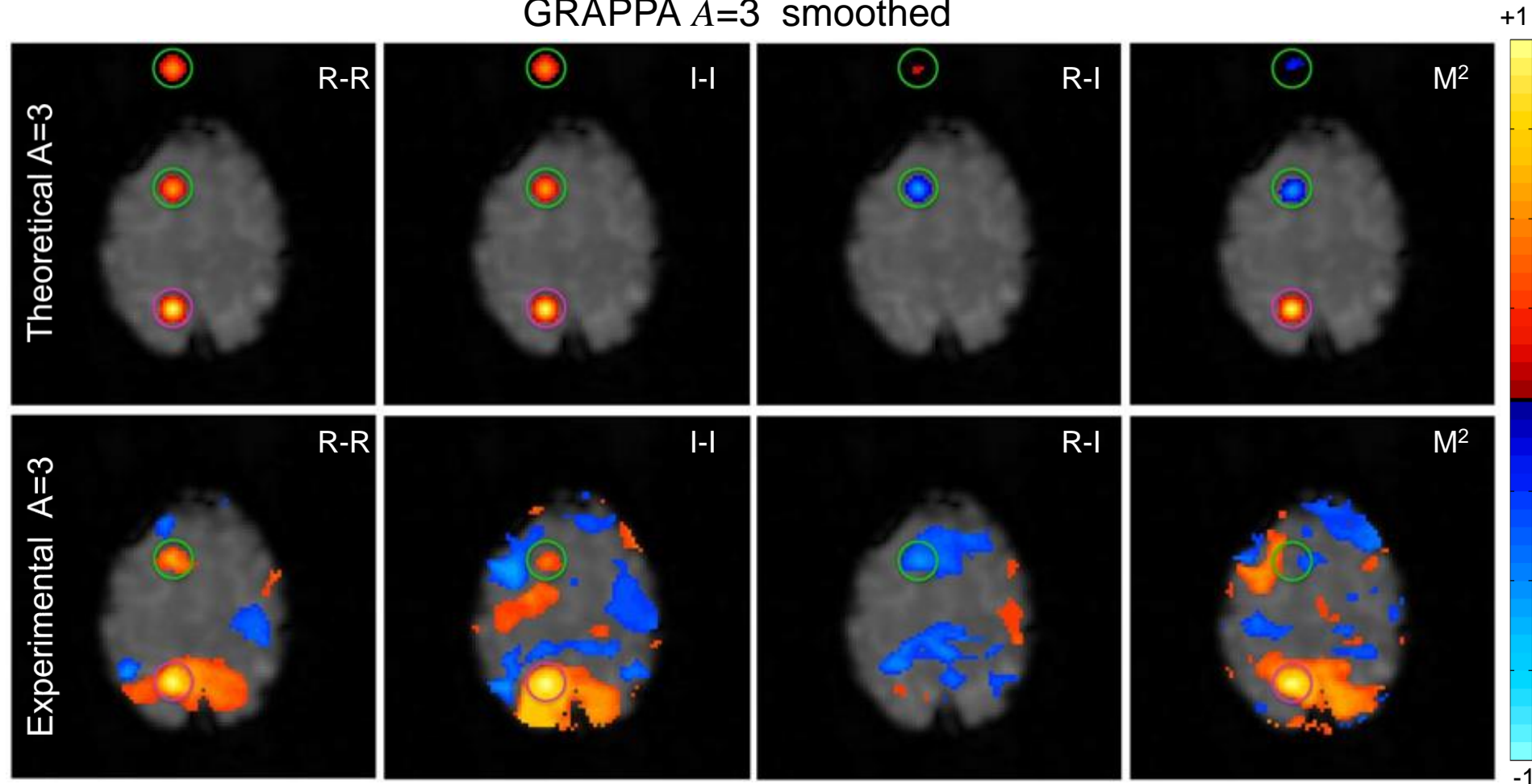
-1



Implications

GRAPPA reconstruction induces long-range correlation.

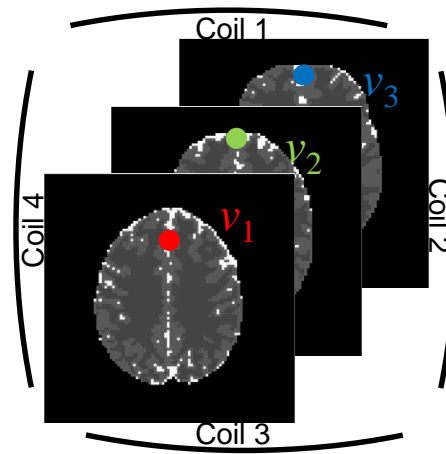
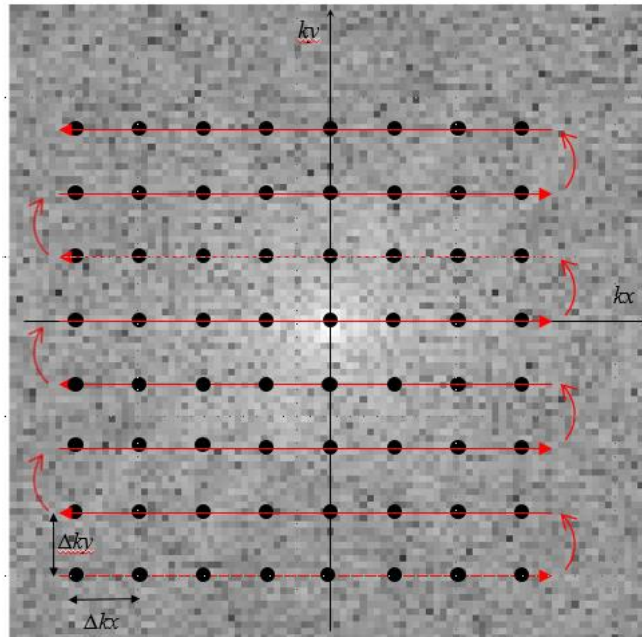
GRAPPA $A=3$ smoothed



Reconstruction

Multi-Coil Acquisition

Simultaneous Multi-Slice (SMS)



Each coil measures k -space.

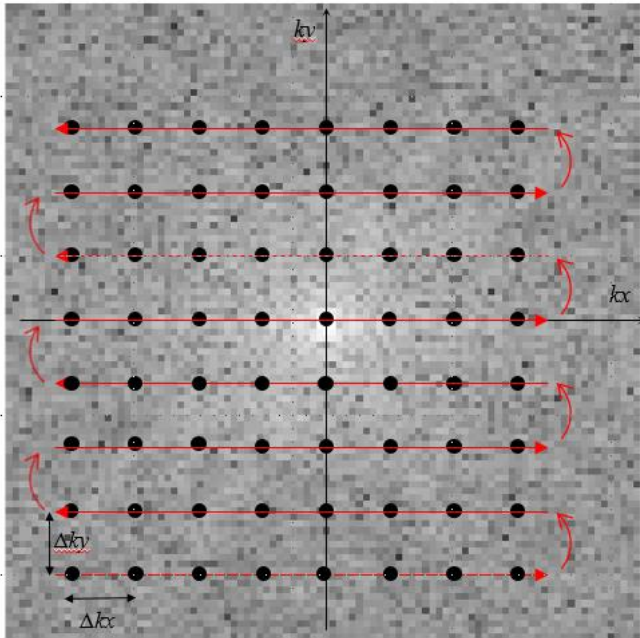
$$v_j$$

$$N_C=4, A=3 \quad j=1:A$$

Reconstruction

Multi-Coil Acquisition

Simultaneous Multi-Slice (SMS)

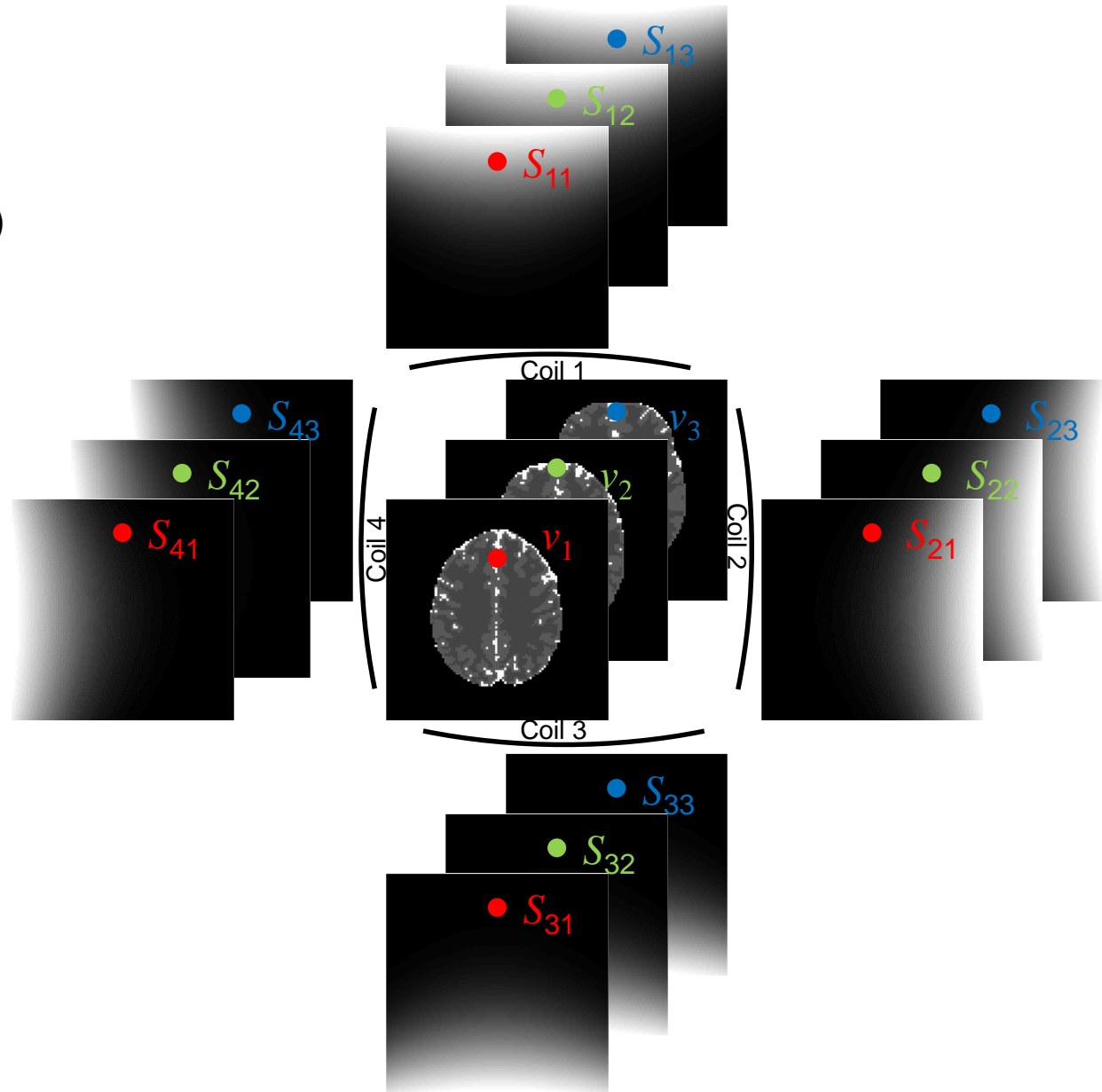


Each coil measures k -space.

$$v_j \cdot S_{kj}$$

$N_C=4, A=3$

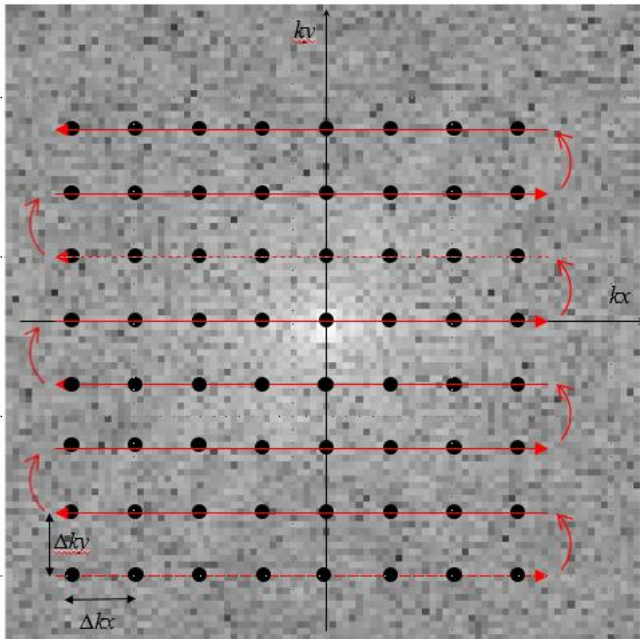
$j=1:A, k=1:N_C$



Reconstruction

Multi-Coil Acquisition

Simultaneous Multi-Slice (SMS)

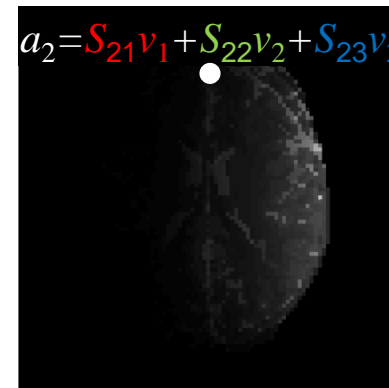
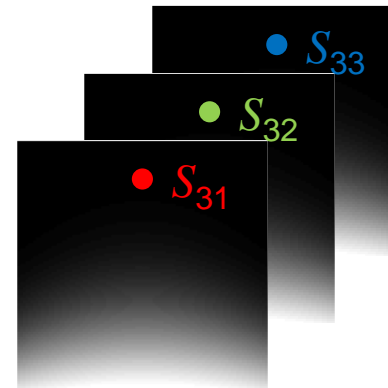
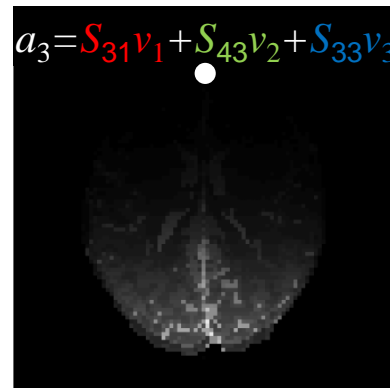
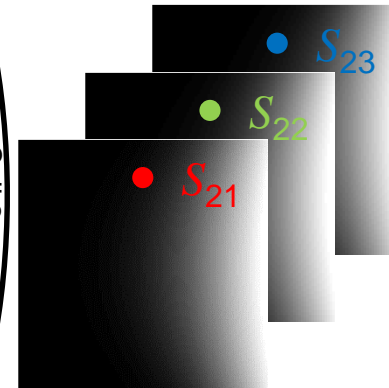
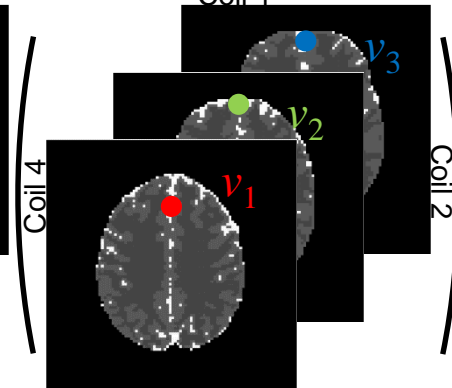
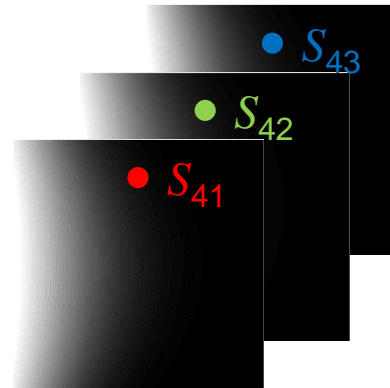
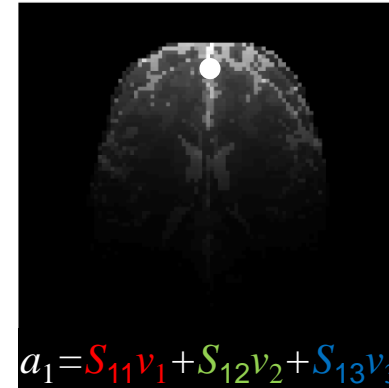
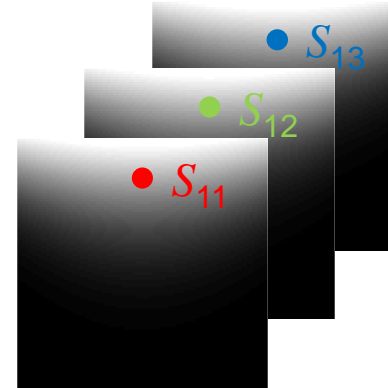
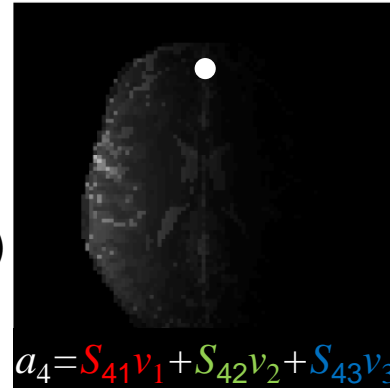


Each coil measures k -space.

$$v_j, S_{kj}, a_k$$

$$N_C=4, A=3$$

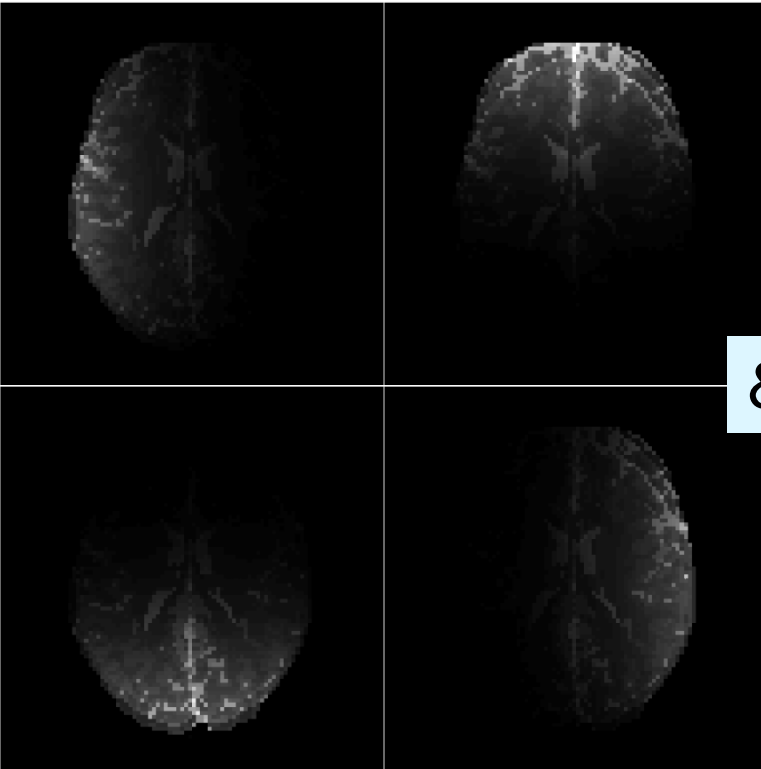
$$j=1:A, k=1:N_C$$



Reconstruction

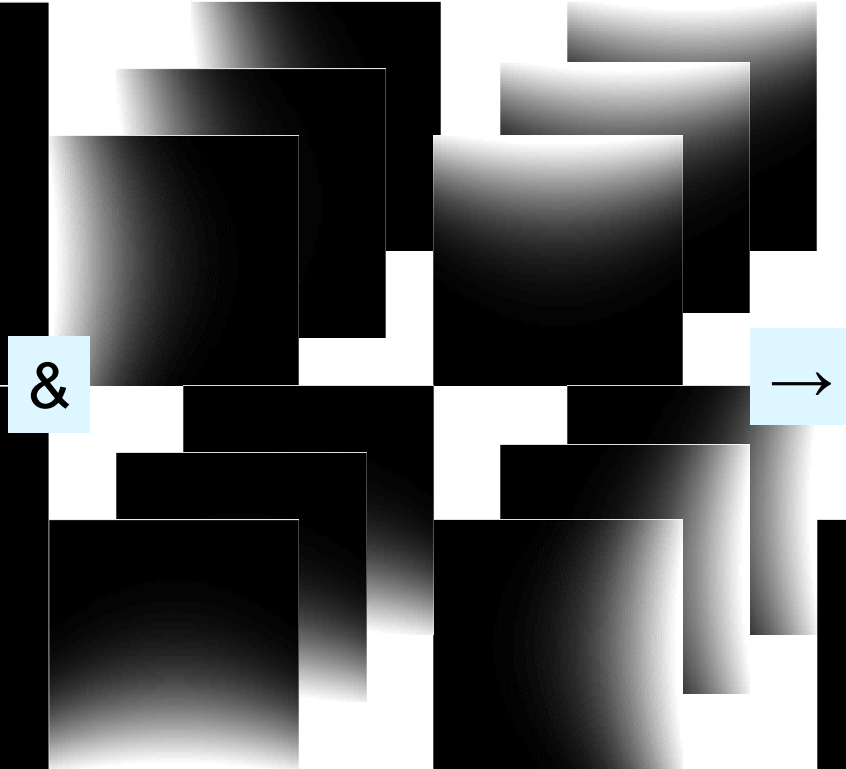
SENSE SMS

Measured Coil Images

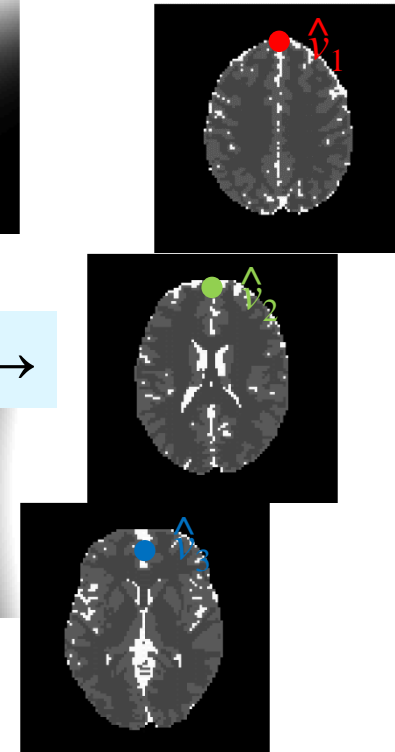


$N_C=4, A=3$

Estimated Sensitivities



Separated Combined Images



Reconstruction/Processing

SENSE SMS

4x

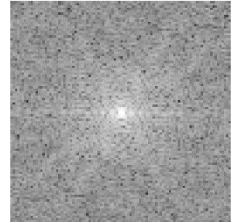
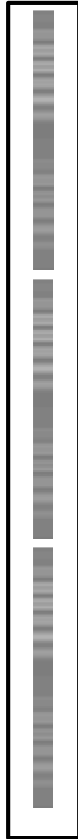
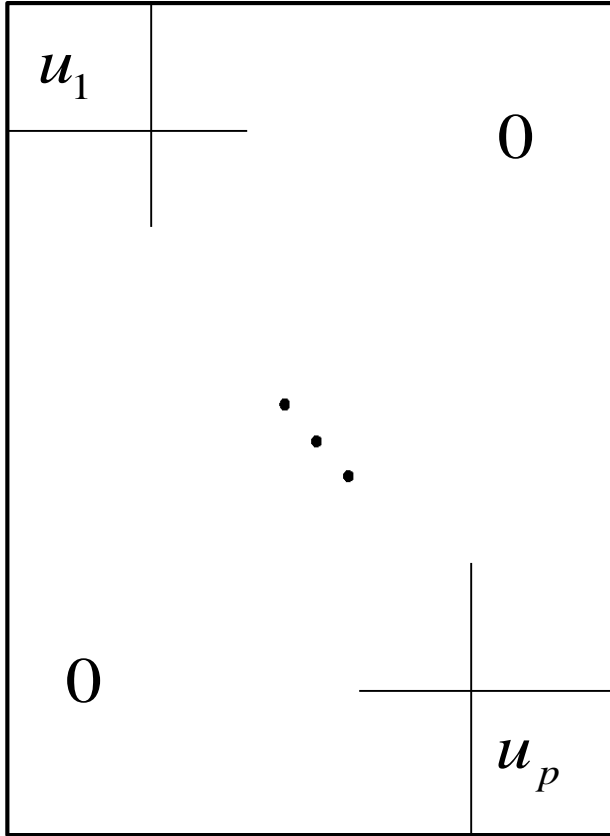


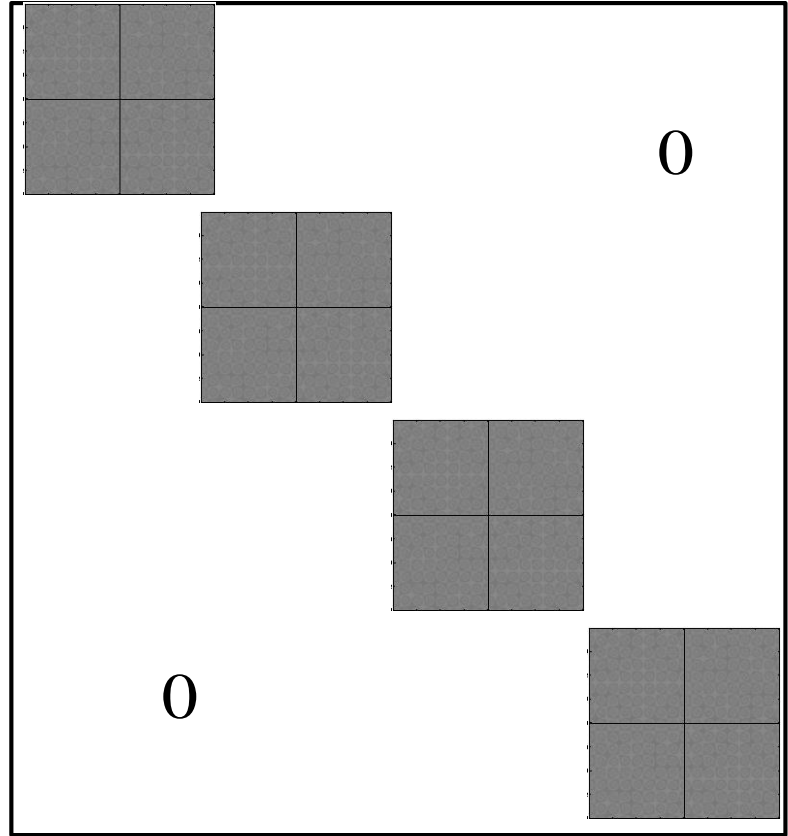
Image vector



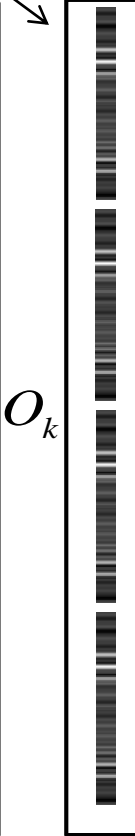
$$= O_I P_U$$



P_{CS}



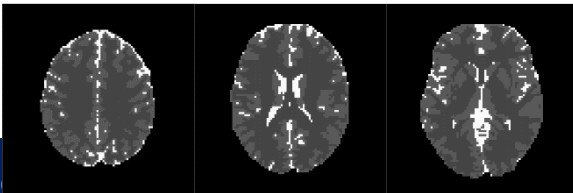
reconstruct $N_c=4$ images



$$O_k$$

k -space vector of N_c images

permute to by folded voxel



Implications

In statistics, we know the rule that says:

If a vector f has a mean δ , and a covariance Γ ,

Then $y=Of$ has a mean $\mu=O\delta$, and a covariance $\Sigma=O\Gamma O^T$.

Then Σ can be converted into a correlation matrix $R=D^{-1/2}\Sigma D^{-1/2}$.

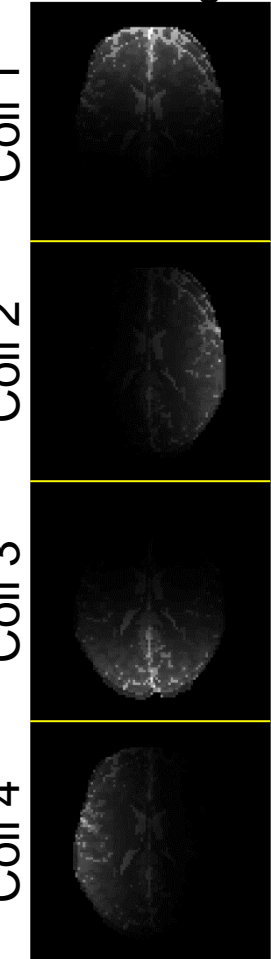
Where $D^{-1/2} = 1 / \sqrt{\text{diag}(\Sigma)}$.

Assume k -space measurements independent so $\Gamma=I$.

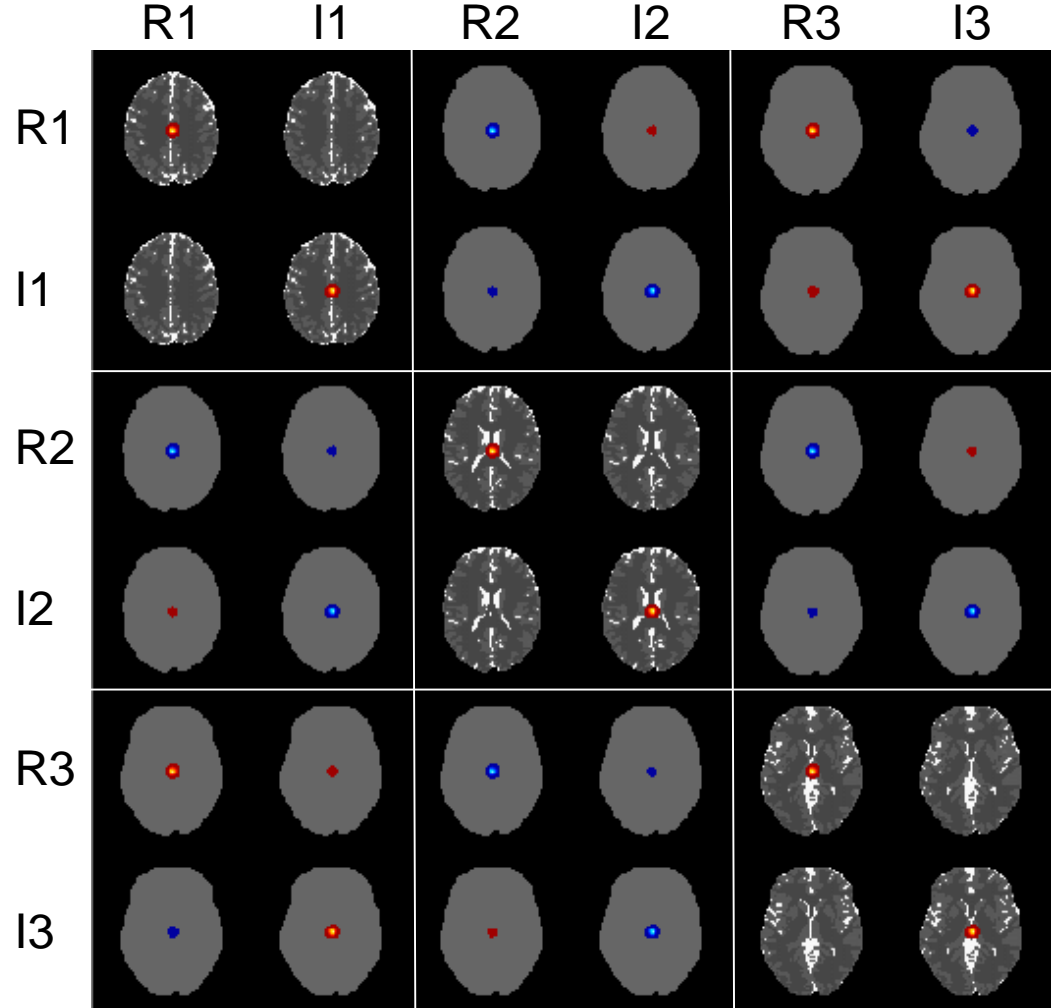
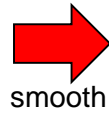
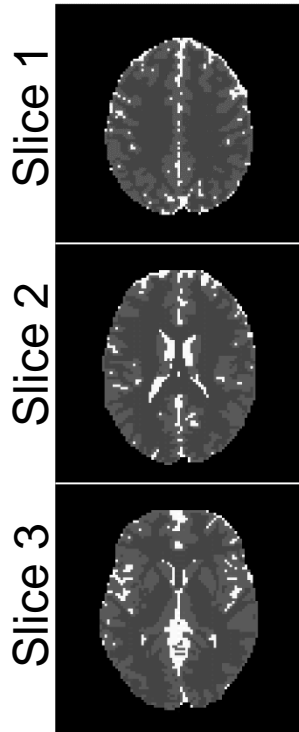
Implications

SENSE induces long-range through-plane correlation.

Coil Images



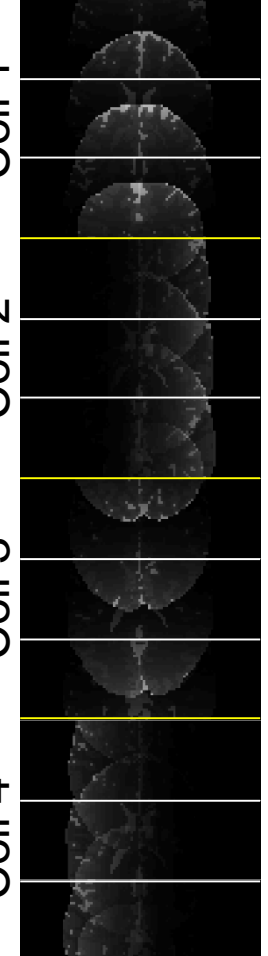
Separated Images



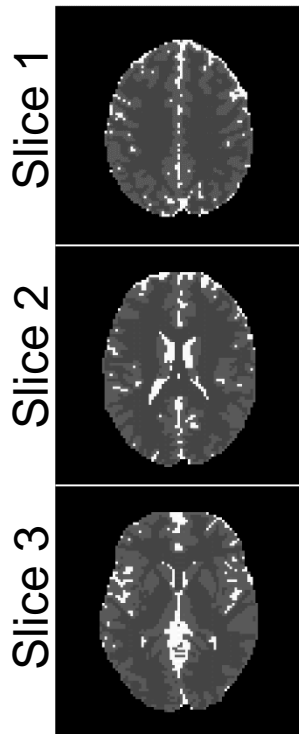
Implications

SENSE induces long-range in-plane correlation.

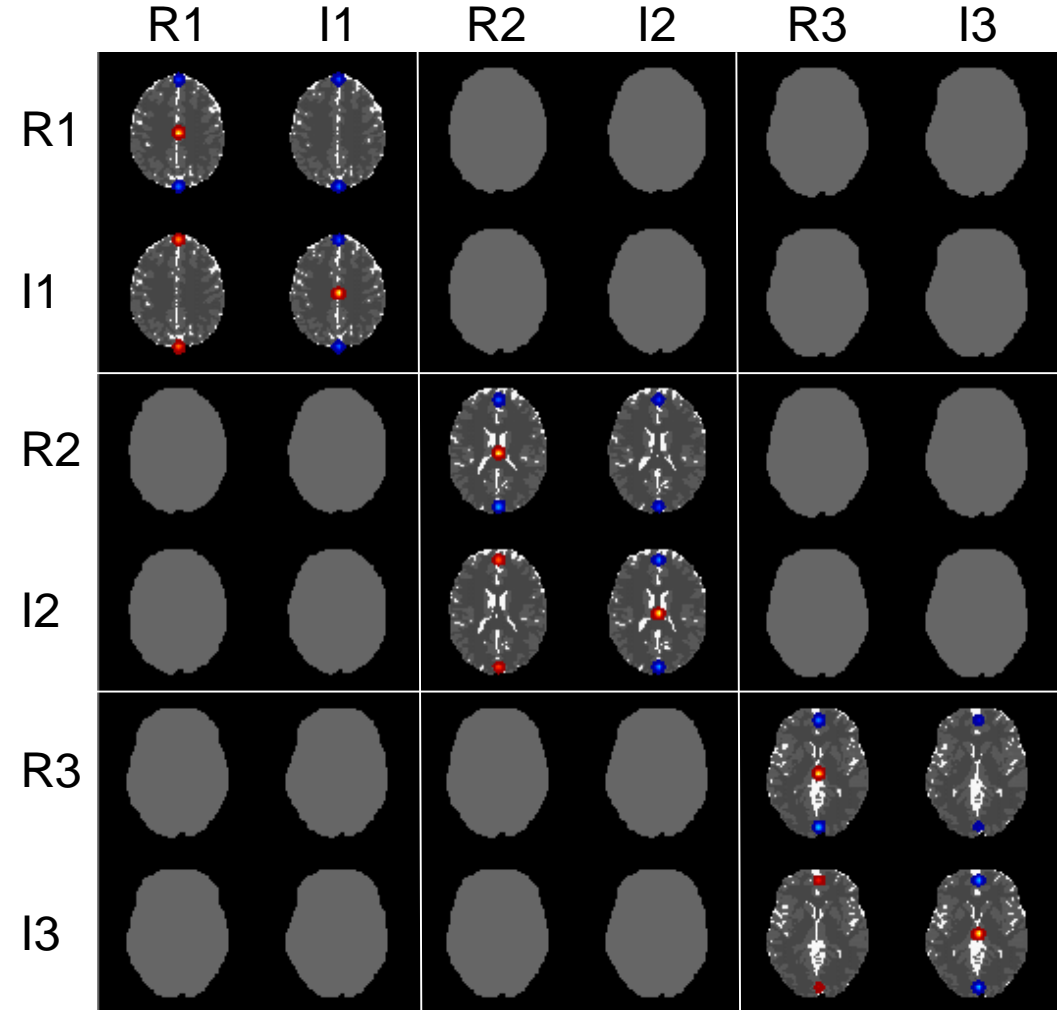
Coil Images



Separated Images



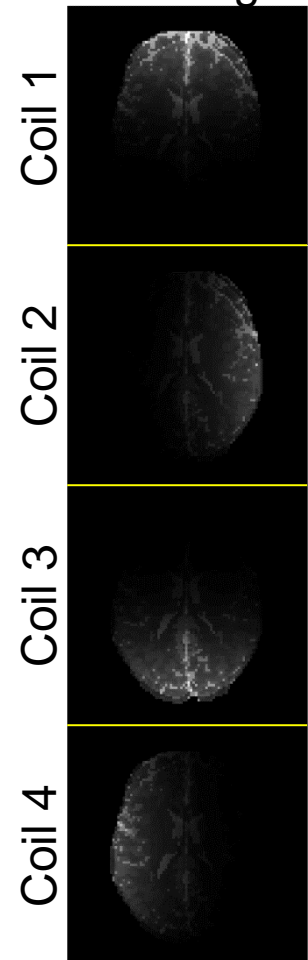
smooth



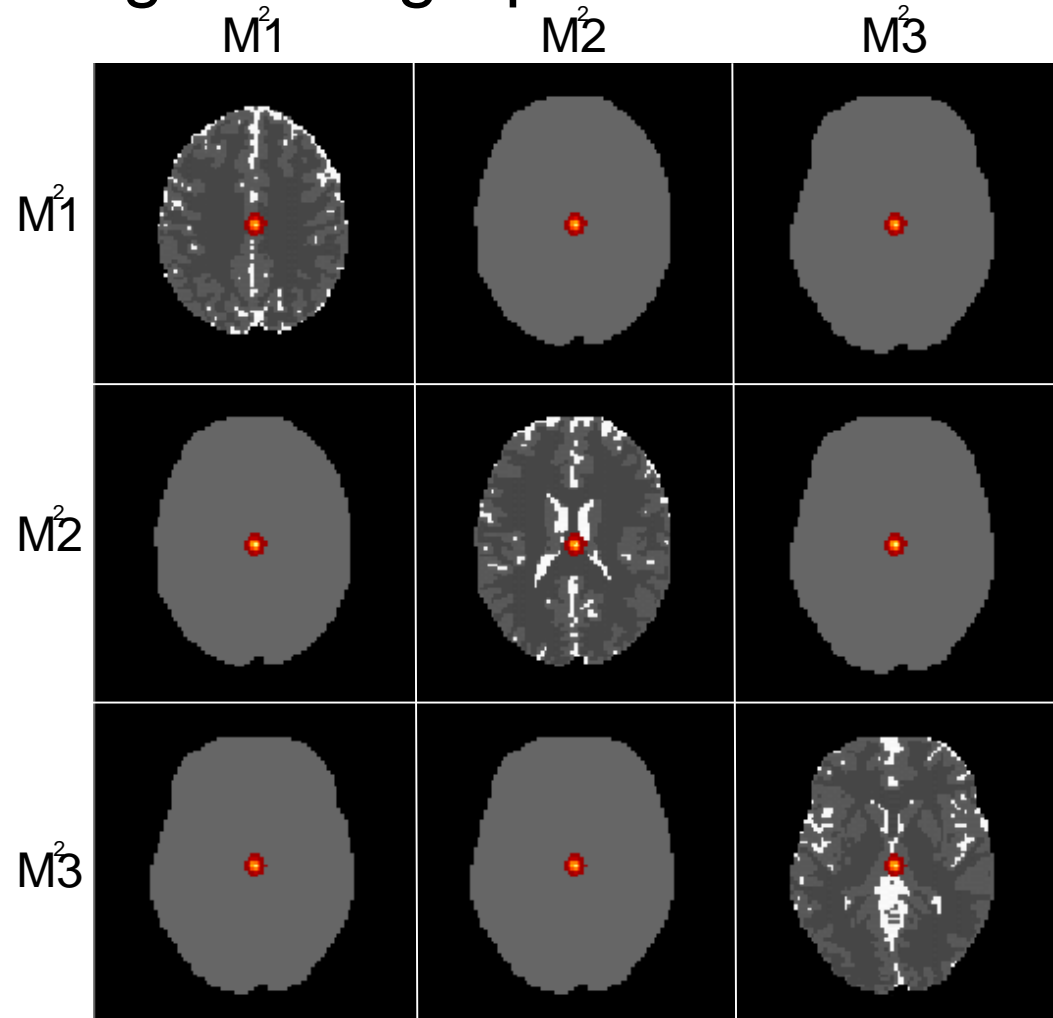
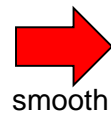
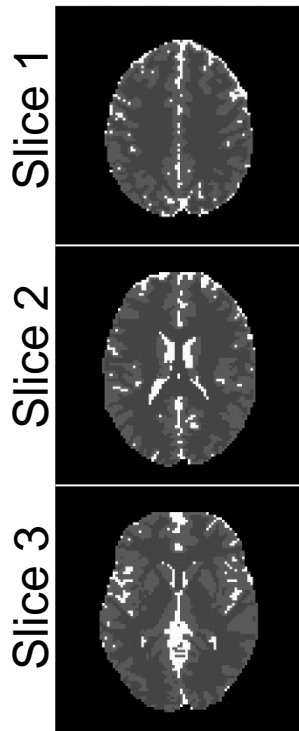
Implications

SENSE induces long-range through-plane correlation.

Coil Images



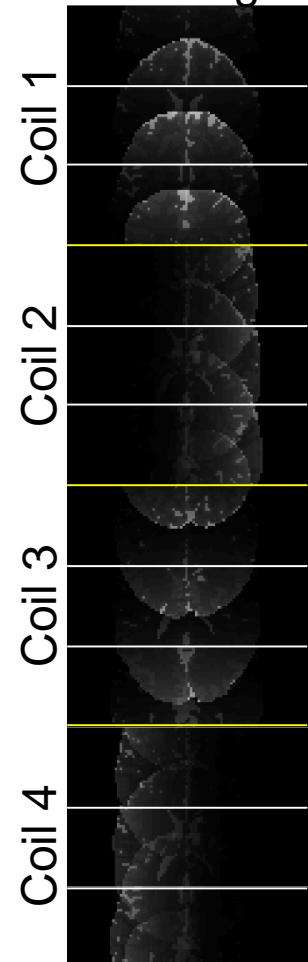
Separated Images



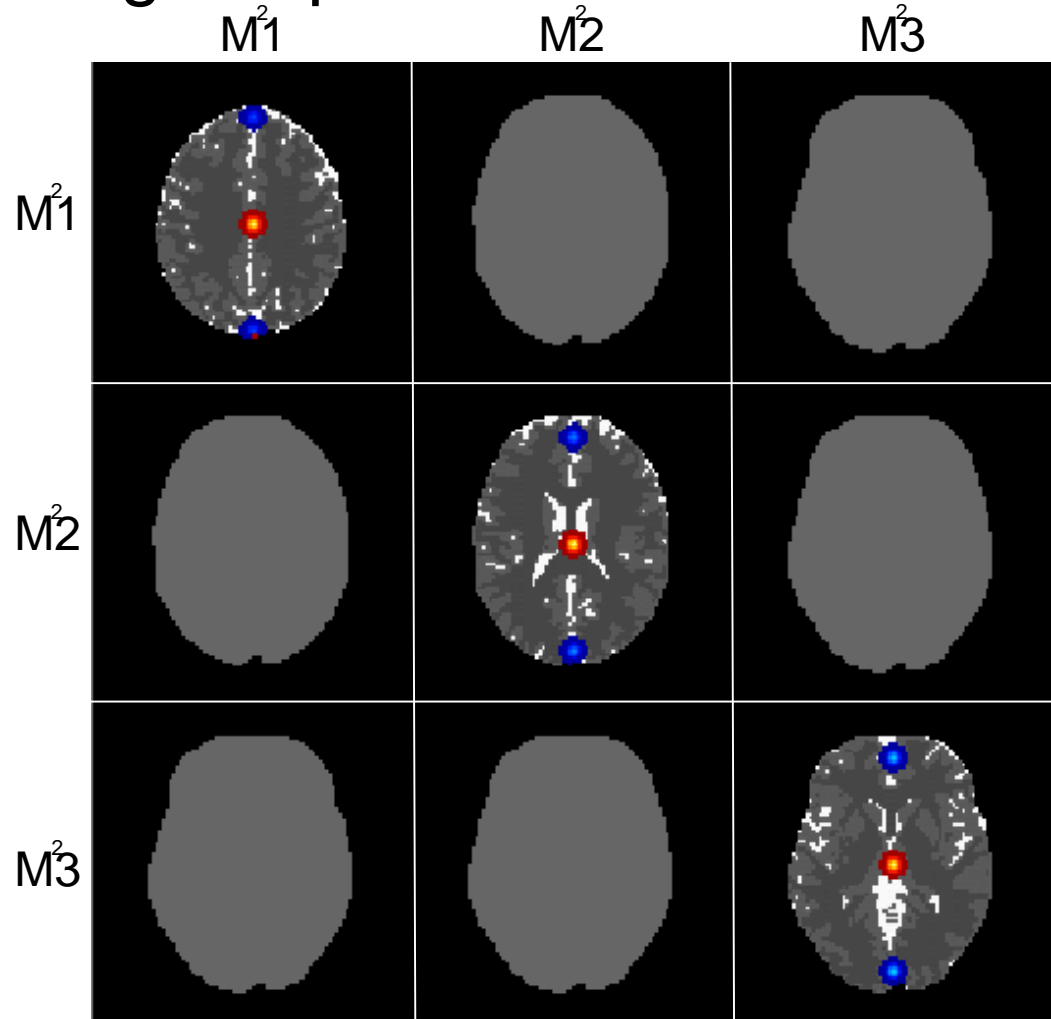
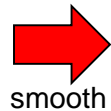
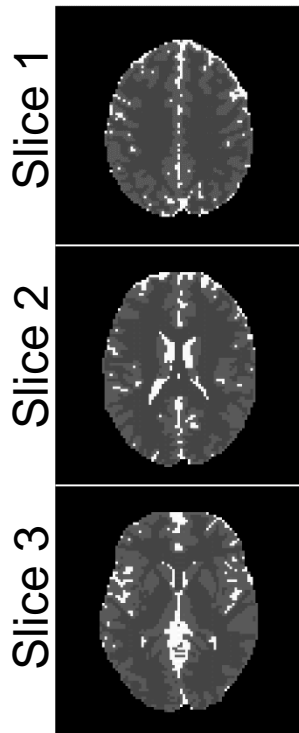
Implications

SENSE induces long-range in-plane correlation.

Coil Images



Separated Images



Discussion

Care needs to be taken when we obtain data.

We should get data in its originally measured form.

We should do any required processing ourselves.

We should develop models that incorporate processing.

We should use all of the data (magnitude and phase).

Thank You!

This work is joint with former & current PhD students:
Dr. Andrew S. Nencka, Medical College of Wisconsin
Dr. Andrew D. Hahn, University of Wisconsin-Madison
Dr. Iain P. Bruce, Duke University
Dr. M. Muge Karaman, University of Illinois-Chicago
Ms. Mary C. Kociuba, Marquette University
Mr. Kevin K. Liu, Marquette University