FMRI Activation in Image Space from k-space Data

Daniel B. Rowe[1,2]
[1]Department of Biophysics, Medical College of Wisconsin
[2]Division of Biostatistics, Medical College of Wisconsin

Introduction: In fMRI, image reconstruction and statistical activation are treated separately. The relationship between complex-valued encoded *k*-space measurements and complex-valued image measurements from reconstructed images has recently been described [1]. Here, voxel time courses are written in terms of spatio-temporal *k*-space measurements and voxel fMRI activation [2,3] is determined in image space from the original *k*-space measurements. Thus activation and reconstruction are all in a single step. This is one move closer to the actual acquired data and opens the potential for statistical modeling in *k*-space.

Model: Denote a complex-valued *k*-space matrix by $S_C=(S_{0R}+iS_{0I})+(E_R+iE_I)$ [1]. Traditionally each S_C is reconstructed by pre-multiplying by the reconstruction matrix $\Omega_C=\Omega_R+i\Omega_I$ and post-multiplying by its transpose to produce the complex-valued image $R_C=(R_{0R}+iR_{0I})+(N_R+iN_I)$. This linear reconstruction process can be equivalently described as the pre-multiplication of the complex-valued spatial frequencies in the form of a real-valued vector $s=\text{vec}(S_R,S_I)^T$ by a real-valued matrix representation $\Omega=[\Omega_1,-\Omega_2;\Omega_2,\Omega_1]$ of Ω_C where $\Omega_1=\Omega_R\otimes\Omega_R-\Omega_I\otimes\Omega_I$ and $\Omega_2=\Omega_R\otimes\Omega_I+\Omega_I\otimes\Omega_R$ with $\otimes$ being the Kronecker product. This produces a real-valued vector $r=\text{vec}(R_R,R_I)^T$ containing real and imaginary parts of the complex-valued image. If the mean and covariance of s are s_0 and Γ, then they are Ωs_0 and $\Omega\Gamma\Omega^T$ for r.

In fMRI a series of slices are acquired. Let the random complex-valued spatial frequency matrix at time t be S_{Ct} and $s_t=\text{vec}(S_{Rt},S_{It})^T$, for $t=1,\dots,n$. Define a matrix $S=(s_1,\dots,s_n)$ and $R=\Omega S$ where the t^{th} column is the real *k*-space measurements stacked upon the imaginary *k*-space measurements for time t. All measurements can be stacked as newly defined $s=\text{vec}(S)$ and $r=\text{vec}(R)$. If the mean and covariance of s are s_0 and Δ, then the mean and covariance of the reconstructed voxel measurements r are $(I_n\otimes\Omega)s_0$ and $(I_n\otimes\Omega)\Delta(I_n\otimes\Omega^T)$. Thus, the fMRI voxel measurements are a linear function of the *k*-space measurements. We can reorder the voxel measurements to be reals then imaginaries for each successive voxel in a single column vector by the linear transformation $y=Pr=P(I_n\otimes\Omega)s$ where P is a permutation matrix. The mean and covariance of y are $\mu=P(I_n\otimes\Omega)s_0$ and $\Lambda=P(I_n\otimes\Omega)\Delta(I_n\otimes\Omega^T)P^T$. The data can now be modeled as in Fig. 1

$$\begin{bmatrix}\begin{pmatrix}y_{R1}\\ y_{I1}\end{pmatrix}\\ \vdots \\ \begin{pmatrix}y_{Rp}\\ y_{Ip}\end{pmatrix}\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}C_1X & 0\\ 0 & S_1X\end{pmatrix} & & 0\\ & \ddots & \\ 0 & & \begin{pmatrix}C_pX & 0\\ 0 & S_pX\end{pmatrix}\end{bmatrix}\begin{bmatrix}\begin{pmatrix}\beta_1\\ \beta_1\end{pmatrix}\\ \vdots \\ \begin{pmatrix}\beta_p\\ \beta_p\end{pmatrix}\end{bmatrix}+\begin{bmatrix}\begin{pmatrix}\eta_{R1}\\ \eta_{I1}\end{pmatrix}\\ \vdots \\ \begin{pmatrix}\eta_{Rp}\\ \eta_{Ip}\end{pmatrix}\end{bmatrix}$$

where C_j and S_j are diagonal matrices with cosine and sine terms respectively. Different complex fMRI activation models [2,3] are found by different choices of C and S. Since Ω and P are known a priori, we can write $s=(I_n\otimes\Omega^{-1})P^{-1}y$ then the optimization for the regression coefficients (β) and phases (θ) could be performed in *k*-space to yield the same parameter estimates. Activations can then be computed from one of the complex activation models and thresholded [4].

Conclusion: Complex-valued voxel measurements have been written in terms of the original complex-valued *k*-space measurements. This allows the computation of statistically significant fMRI brain activation directly from the original *k*-space measurements but in image space. The correlation between voxel measurements can also be written in terms of the correlation between *k*-space measurements.

References: 1. DB Rowe et al., JNEUMETH, 159:361-369, 2007. 2. DB Rowe & BR Logan, NIMG 23:1078-1092, 2004. 3. DB Rowe, NIMG 25:1310-1324, 2005. 4. BR Logan & DB Rowe, NIMG 22:95-108, 2004.

Category: *Modeling and Analysis*
Sub-Category: *Multivariate modeling, PCA & ICA*