LETTER TO THE EDITOR

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Magnitude and Phase Signal Detection in Complex-Valued fMRI Data

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In a recent paper on complex-valued functional magnetic resonance imaging (fMRI) detection by Lee et al. (2007), a statistical model for magnitude and phase changes is presented (1). This follows a line of published research on the topic (2–5) motivated by the fact that fMRI phase data contains biological information regarding the vasculature contained within voxels (6,7). The Lee et al. (2007) model is elegant and computationally efficient, but there are four items regarding it that need to be clarified in addition to its relationship to the Rowe (2005) model (5).

The Rowe (2005) model for detecting magnitude and phase changes in complex-valued data is

$$\begin{pmatrix} y_{Rt} \\ y_{It} \end{pmatrix} = \begin{pmatrix} x_t'\beta\cos(u_t'\gamma) \\ x_t'\beta\sin(u_t'\gamma) \end{pmatrix} + \begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} \quad [1]$$

where at time t, $t = 1, \ldots, n$, y_{Rt} and y_{It} are the observed real and imaginary observations. In addition, $x_t'\beta$ is the magnitude signal, x_t' is the tth row of a design matrix X describing temporal magnitude changes, β is a vector of magnitude regression coefficients, $u_t'\gamma$ is the phase signal, u_t' is the tth row of a design matrix U describing temporal phase changes, γ is a vector of phase regression coefficients. Finally, η_{Rt} and η_{It} are the real and imaginary measurement error that are independent and identically distributed $N(0,\sigma^2)$ variables. Several hypothesis pairs are presented with suitable selection from $C\beta = 0$, $C\beta \neq 0$, $D\gamma = 0$, and $D\gamma \neq 0$.

The Lee et al. (2007) model is

$$\begin{pmatrix} y_{Rt} \\ y_{It} \end{pmatrix} = \begin{pmatrix} x_t'\beta_R \\ x_t'\beta_I \end{pmatrix} + \begin{pmatrix} \eta_{Rt} \\ \eta_{It} \end{pmatrix} \quad [2]$$

where β_R and β_I are regression coefficients for the real and imaginary parts of the signal and all other variables are as previously defined. Lee et al. (2007) correctly describe that their model is to be used when the magnitude and phase design matrices are the same ($U = X$) in addition to the same contrast matrices ($\mathbf{v} = C = D$).

The items that need to be clarified are that Lee et al. (2007) state that:

1. A "mathematical proof" is in Appendix B to "show the equivalence" of the Lee et al. model to the Rowe (2005) model. This "proof" is a derivation of their test statistic using a likelihood ratio test. This item is stated without proof.
2. "One can easily incorporate other terms, such as a linear drift, by adding more vectors and parameters into the model (see Appendix B)" and describe that $X = [x_1, x_2, \ldots, x_L]$ "where $x_1, x_2, \ldots, x_L$ are real $n \times 1$ vectors representing such waveforms as a constant, a linear drift, and reference waveforms." This extension of the model to incorporate other terms is not mathematically correct. Simple inspection of Eqs. [1] and [2] reveal that the Lee et al. (2007) model requires $\beta_R = \beta\cos(x_t'\gamma)$ and $\beta_I = \beta\sin(x_t'\gamma)$ for all time t. The Lee et al. (2007) model is only mathematically correct for two regressors, $L = 2$.
3. "One structures the design matrix (X) of the GLM by a constant vector ($\mathbf{1} = [1\ 1, \ldots, 1]^T$, a real $n \times 1$ vector) and a reference waveform vector ($\mathbf{h}$, a real $n \times 1$ vector, the convolution of a stimulus pattern and a hemodynamic response function)." This description of possible reference waveform vectors is not mathematically correct. Consider an example where $L = 2$, $n = 3$, and X has first column $(1,1,1)'$ and second column $(0,1/2,1)'$. Upon equating the means of the Lee et al. (2007) and Rowe (2005) models in Eqs. [2] and [3], the real part is

$$\begin{pmatrix} \beta_{R1} \\ \beta_{R1} + .5\beta_{R2} \\ \beta_{R1} + \beta_{R2} \end{pmatrix} = \begin{pmatrix} \beta_0\cos(\gamma_0) \\ (\beta_0 + .5\beta_1)\cos(\gamma_0 + .5\gamma_1) \\ (\beta_0 + \beta_1)\cos(\gamma_0 + \gamma_1) \end{pmatrix}. \quad [3]$$

Upon inserting $\beta = (10,1)'$ and $\gamma = (\pi/4, \pi/9)'$ into the right side of Eq. [3] one obtains $\beta_{R1} = 7.0711$, $\beta_{R1} + .5\beta_{R2} = 6.0226$, and $\beta_{R1} + \beta_{R2} = 4.6488$. Using $\hat{\beta}_R = (X'X)^{-1} X' y_R$ from Lee et al. (2007) one obtains from these three noiseless observations $(\hat{\beta}_{R1}, \hat{\beta}_{R2})' = (7.1253, -2.4223)'$. It can be seen that $\hat{\beta}_{R1} \neq \beta_{R1}$, $\hat{\beta}_{R1} + .5\hat{\beta}_{R2} \neq \beta_{R1} + .5\beta_{R2}$, and $\hat{\beta}_{R1} + \hat{\beta}_{R2} \neq \beta_{R1} + \beta_{R2}$. The Lee et al. (2007) model is only mathematically correct with a constant baseline and an on/off (0/1 or −1/+1) reference vector. With a 0/1 reference vector, the observation means when the reference vector value is 0 are β_{R1} and $\beta_0\cos(\gamma_0)$ for the Lee et al. (2007) and Rowe (2005) models, while the means when the reference vector

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value is 1 are $\beta_{R1} + \beta_{R2}$ and $(\beta_0+\beta_1)\cos(\gamma_0+\gamma_0)$ in the Lee et al. (2007) and Rowe (2005) models. Additionally, $\beta_{R2} = (\beta_0+\beta_1)\cos(\gamma_0+\gamma_1) - \beta_0\cos(\gamma_0)$ and $\beta_{I2} = (\beta_0+\beta_1)\sin(\gamma_0+\gamma_1) - \beta_0\sin(\gamma_0)$ are real and imaginary parts of the differential effect. The Lee et al. (2007) null hypothesis, $\mathbf{v}[\beta_R,\beta_I] = 0$ implies that $(\beta_0+\beta_1)\cos(\gamma_0+\gamma_1) = \beta_0\cos(\gamma_0)$ and $(\beta_0 + \beta_1)\sin(\gamma_0+\gamma_1) = \beta_0\sin(\gamma_0)$, while indirectly implying that $\beta_1 = 0$ and $\gamma_1 = 0$.

4. "For a given significance level α, the null hypothesis is rejected when"

$$F = \frac{n-m}{m(n-1)}T^2 > F_{m,n-m}(\alpha) \qquad [4]$$

where $m = 2$. This test statistic and critical value equation is not mathematically correct. The likelihood ratio statistic λ when $m = 2$ and $L = 2$ can be rewritten as

$$F = \frac{(2n-4)}{2}(1-\lambda^{-1/n}) = \frac{X_1/2}{X_2/(2n-4)}. \qquad [5]$$

One can show that $X_1 = (\hat{\beta}_R - \tilde{\beta}_R)'(X'X)(\hat{\beta}_R - \tilde{\beta}_R) + (\hat{\beta}_I - \tilde{\beta}_I)'(X'X)(\hat{\beta}_I - \tilde{\beta}_I)]/\sigma^2$ is $\chi^2(2)$ and $X_2 = [(y_R - X\hat{\beta}_R)'(y_R - X\hat{\beta}_R) + (y_I - X\hat{\beta}_I)'(y_I - X\hat{\beta}_I)]/\sigma^2$ is $\chi^2(2n-4)$, then the ratio in Eq. [5] is F distributed with 2 and $2n-4$ degrees of freedom. The proper Lee et al. (2007) test statistic and critical value that it should be compared to are

$$F = \frac{2}{(2n-4)}T^2 > F_{2,(2n-4)}(\alpha). \qquad [6]$$

Despite these inaccuracies, the Lee et al. (2007) model is elegant and is recommended when the magnitude and phase design matrices are identical with a column of ones for a constant baseline and a column with on/off (0/1 or −1/+1) elements for the reference waveform vector.

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