The Fourier Transform in MRI/fMRI

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Outline

- Introduction
- One Dimensional FT
  - Time series constituents and Fourier spectrum.
- Two Dimensional FT
  - An image constituents and Fourier spectrum.
- Two Dimensional MR Image Formation.
  - k-space and MR Image Reconstruction.
- Two Dimensional MR Image Processing
  - Smoothing/Edge detetection.
- One Dimensional fMRI Time Series
  - fMRI time series Fourier spectrum and filtering.
- FMRI Time Series Statistics

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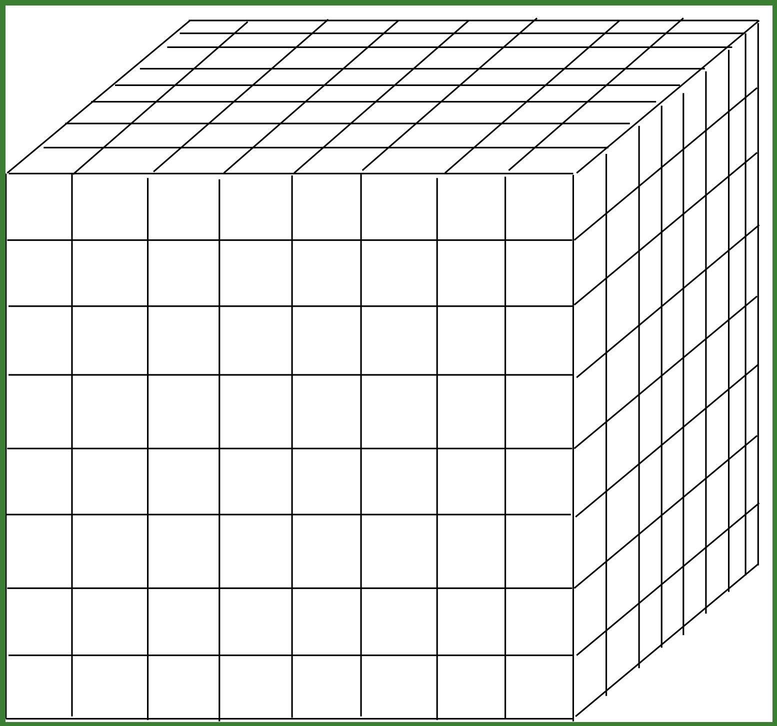
Introduction

Place volunteer/patient into MRI scanner.

MCW GE 3T Long Bore

Introduction

In MRI we image a real-valued 3D object, $R(x, y, z)$.
Lattice of volume elements, voxels.



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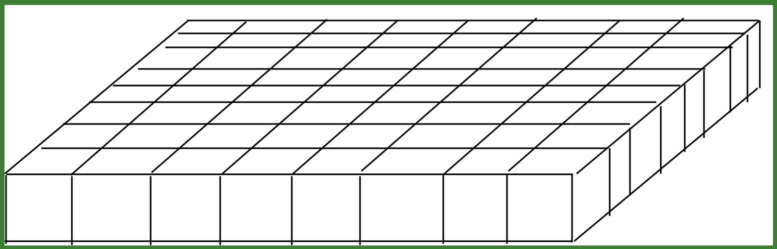
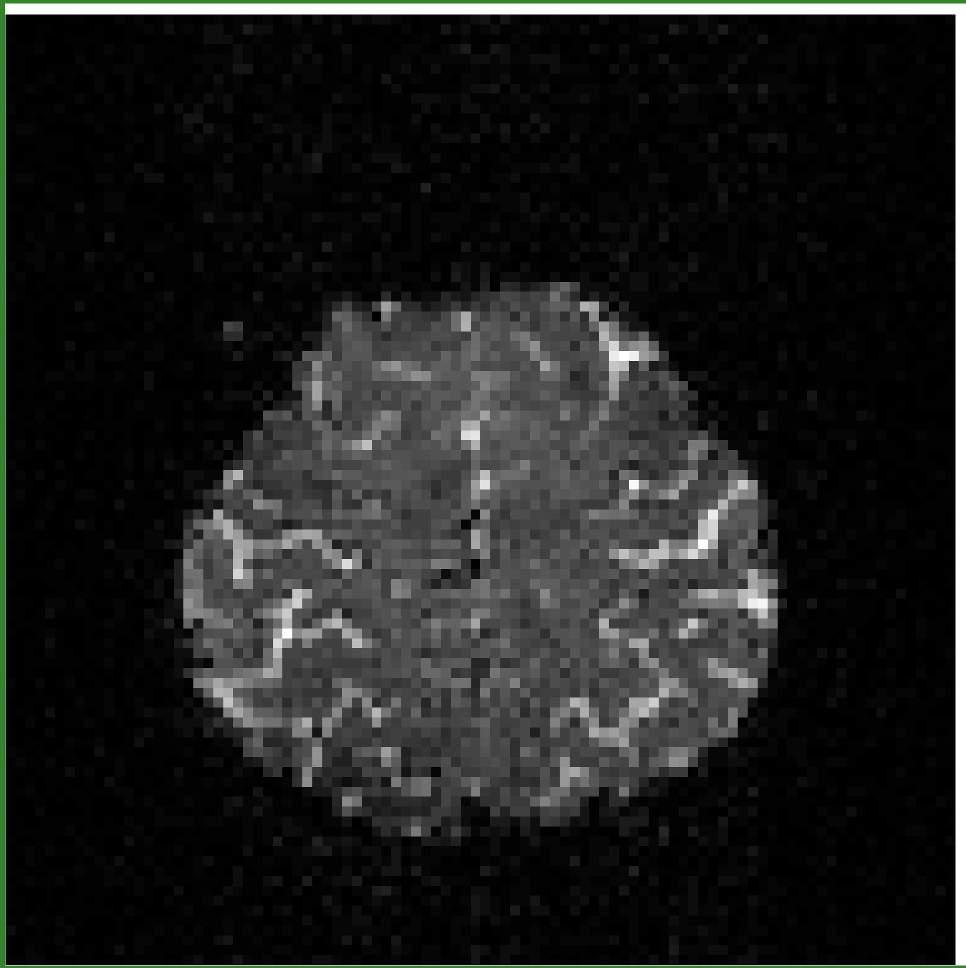


Volume Slice

Consider a single slice $R(x, y)$.

Introduction

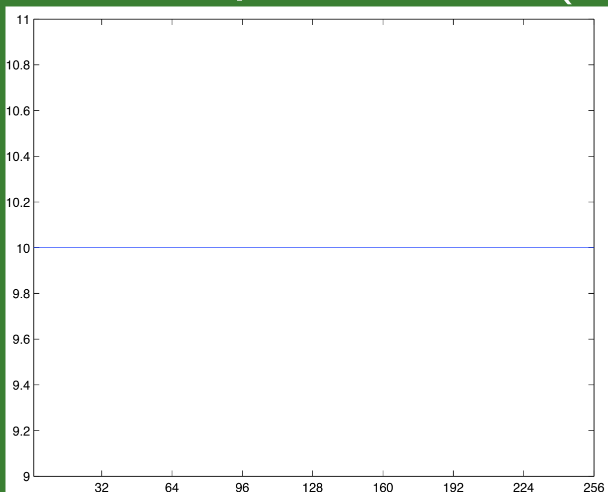
In MRI we aim to image a real-valued object, $R(x, y)$.
Different tissues have different magnetic properties yielding contrast.



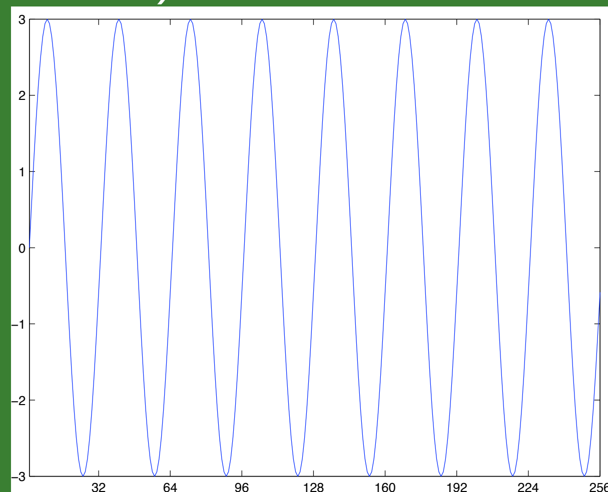
T_2^* weighted image

One Dimensional FT

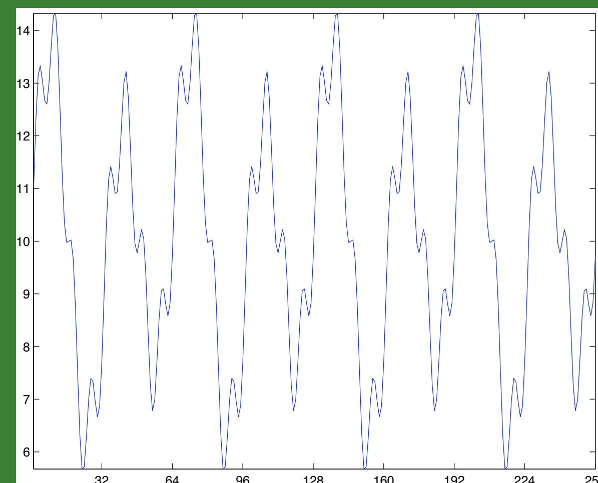
The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)



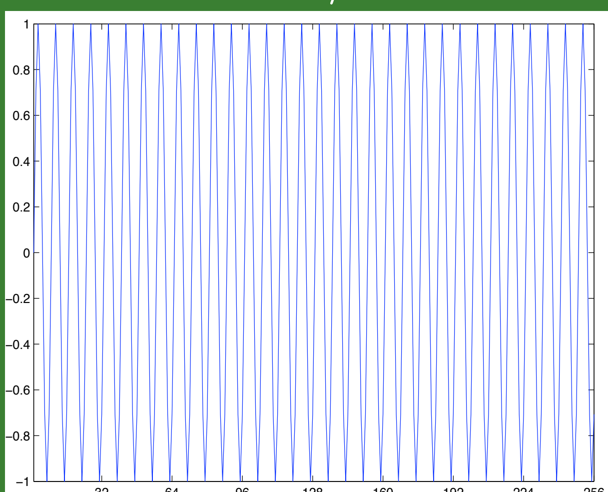
$10 * \cos 0/512$ Hz



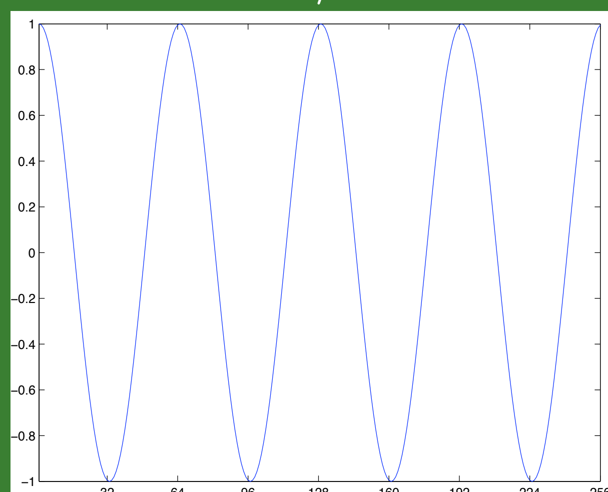
$3 * \sin 8/512$ Hz



Sum



$\sin 32/512$ Hz



$\cos 4/512$ Hz

No noise added!

One Dimensional FT-Continuous

The FT of a continuous function $f(x)$ is

$$F(k) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi kx} \, dx$$

also denoted as $\mathcal{F}\{f(x)\}$ and its inverse to be

$$f(x) = \int_{-\infty}^{+\infty} F(k) e^{+i2\pi kx} \, dk$$

also denoted as $\mathcal{F}^{-1}\{F(k)\}$.

Don't forget that

$$e^{i\alpha} = \cos(\alpha) + i \sin(\alpha) \ .$$

One Dimensional FT-Continuous

$$
\begin{aligned}
F(k) &= \int_{-\infty}^{\infty} f(x)[\cos(2\pi kx) - i\sin(2\pi kx)]\, dx \\
&= \int_{-\infty}^{\infty} f(x)\cos(2\pi kx)\, dx - i\int_{-\infty}^{\infty} f(x)\sin(2\pi kx)\, dx \\
&= F_C(k) - iF_S(k) \\
F_C(k) &= \int_{-\infty}^{\infty} \left[\sum_j A_j \cos(2\pi\nu_j x) + \sum_j B_j \sin(2\pi\nu_j x)\right] \cos(2\pi kx)\, dx \\
F_S(k) &= \int_{-\infty}^{\infty} \left[\sum_j A_j \cos(2\pi\nu_j x) + \sum_j B_j \sin(2\pi\nu_j x)\right] \sin(2\pi kx)\, dx
\end{aligned}
$$

The $\cos()\sin()$ and $\sin()\cos()$ cross terms are zero.

Nonzero values at constituent frequencies where A_j and B_j nonzero.

One Dimensional FT-Continuous

Fourier Transform properties.

Property	Function	Transform
Linearity	$af(x) + bg(x)$	$aF(k) + bG(k)$
Similarity	$f(ax)$	$\frac{1}{\lvert a \rvert} F(\frac{k}{a})$
Shifting	$f(x-a)$	$e^{-i2\pi ka} F(k)$
Derivative	$\frac{d^\ell f(x)}{dx^\ell}$	$(i2\pi k)^\ell F(k)$

One Dimensional FT-Continuous

Convolution of functions $f(x)$ and $g(x)$ is defined as

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(\alpha)\ g(x-\alpha)\ d\alpha\ .$$

Further

$$\mathcal{F}\ \{f(x) \cdot g(x)\} = F(k) * G(k)\ ,$$

and

$$\mathcal{F}\ \{f(x) * g(x)\} = F(k) \cdot G(k)\ .$$

One Dimensional FT-Continuous

Convolution properties.

$$f(x) * g(x) = g(x) * f(x) \quad \text{commutative}$$

$$f(x) * [g(x) * h(x)] = [f(x) * g(x)] * h(x) \quad \text{associative}$$

$$f(x) * [g_1(x) + g_2(x)] = f(x) * g_1(x) + f(x) * g_2(x) \quad \text{distributive}$$

$$\frac{d\ \ f(x)*g(x)}{dx} = \frac{d\ \ f(x)}{dx} * g(x) = f(x) * \frac{d\ \ g(x)}{dx} \quad \text{derivative}$$

$$h(x - x_0) = f(x - x_0) * g(x) = f(x) * g(x - x_0) \quad \text{shift}$$

if $h(x) = f(x) * g(x)$

One Dimensional FT-Discrete

The (finite) Discrete FT can be derived from the continuous FT (with assumptions).

$$\underbrace{F(p\Delta k)}_{Complex} = \sum_{q=-n}^{n-1} \underbrace{f(q\Delta x)}_{Complex} e^{-\frac{i2\pi pq}{2n}}$$
$$\text{for} \quad p = -n, \ldots, n-1$$

$$\underbrace{f(q\Delta x)}_{Complex} = \frac{1}{2n} \sum_{p=-n}^{n-1} \underbrace{F(p\Delta k)}_{Complex} e^{\frac{i2\pi pq}{2n}}$$
$$\text{for} \quad q = -n, \ldots, n-1 \ .$$

There are some assumptions here.
The constituent frequencies do not change in time.
The constituent frequencies are $\leq 1/(2\Delta x)$.

One Dimensional FT-Discrete

The DFT can be represented as

$$\begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} = \bar{\Omega} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

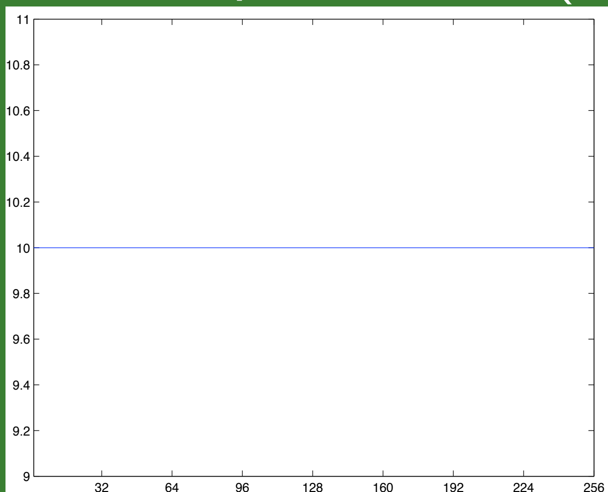
$$n \times 1 \qquad n \times n \qquad n \times 1$$

$$\text{Complex} \qquad \text{Complex} \qquad \text{Complex (Real)}$$

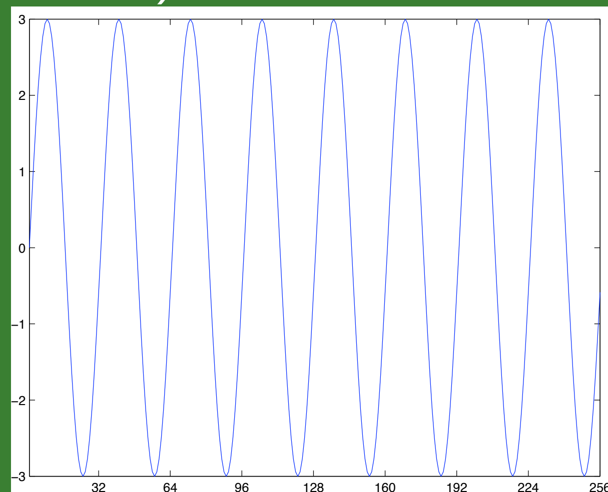
$$(f_R + i f_I) = \left(\bar{\Omega}_R + i\ \bar{\Omega}_I\right)(y_R + i y_I)$$

One Dimensional FT-Discrete

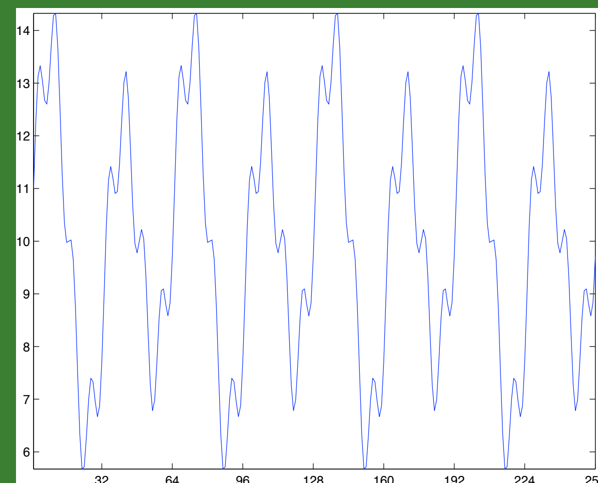
The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)



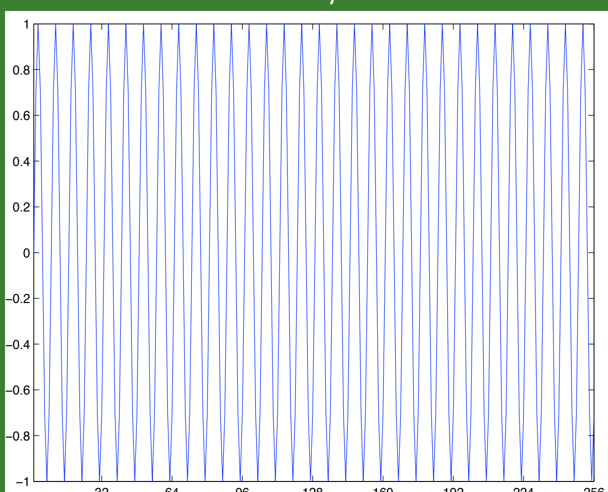
$10 * \cos 0/512$ Hz



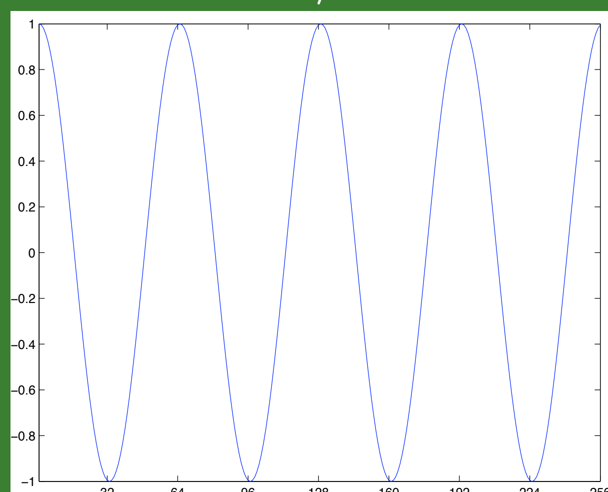
$3 * \sin 8/512$ Hz



Sum



$\sin 32/512$ Hz



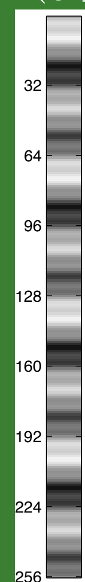
$\cos 4/512$ Hz

No noise added!

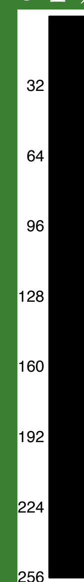
One Dimensional FT-Discrete

The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)

$$(\bar{\Omega}_R \quad +i \quad \bar{\Omega}_I) * (y_R \; +i \quad y_I) \; = \; (f_R \quad f_I)$$



$+ \; i$

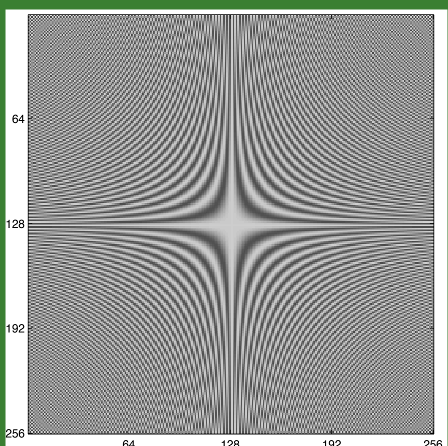


Represent complex-valued time series as an image.

One Dimensional FT-Discrete

The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)

$$(\bar{\Omega}_R \quad +i \quad \bar{\Omega}_I) \, * (y_R \; +i \; y_I) \; = \; (f_R \; +i \; f_I)$$



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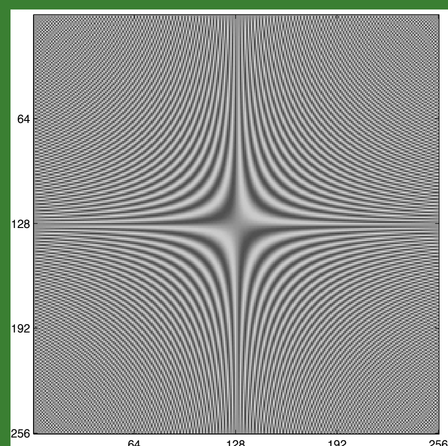

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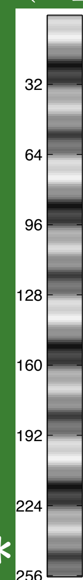
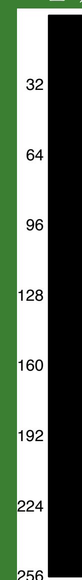


$+ \; i$

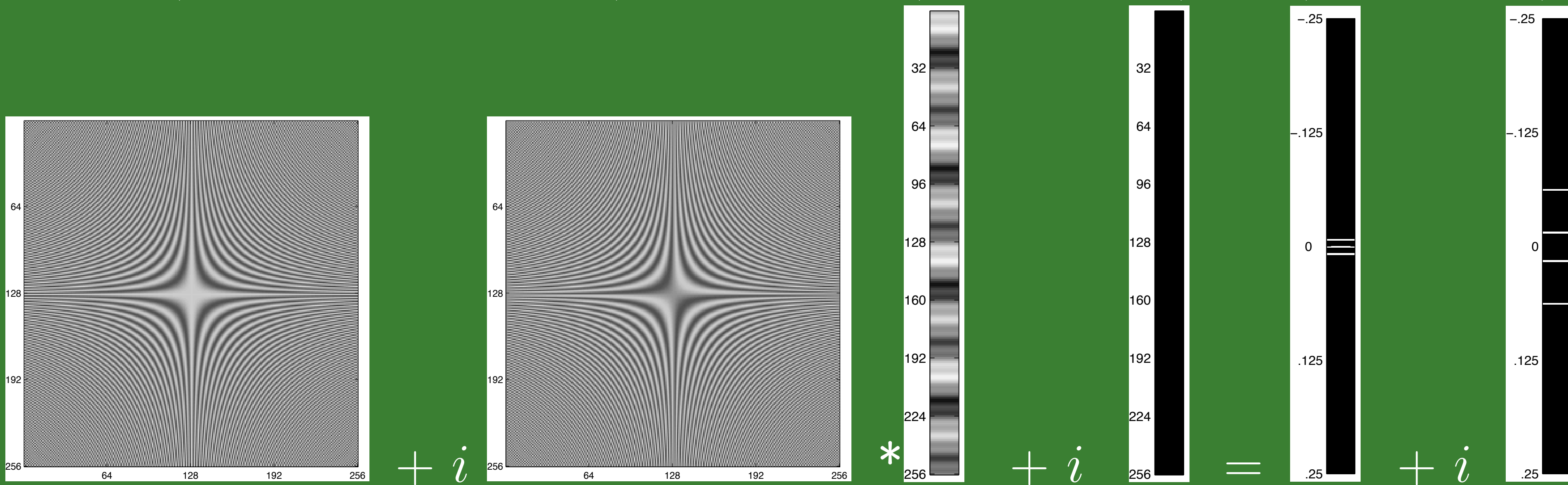


Pre-multiply by complex-valued forward Fourier Matrix as an image.

One Dimensional FT-Discrete

The Complex-Valued (Discrete) Fourier Transform (n=256, TR=2s)

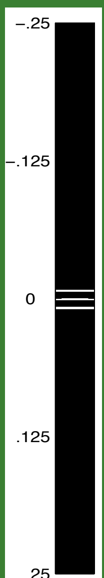
$$(\bar{\Omega}_R \quad +i \quad \bar{\Omega}_I) \quad *(y_R \quad +i \quad y_I) \quad = \quad (f_R \quad +i \quad f_I)$$

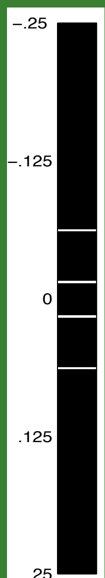


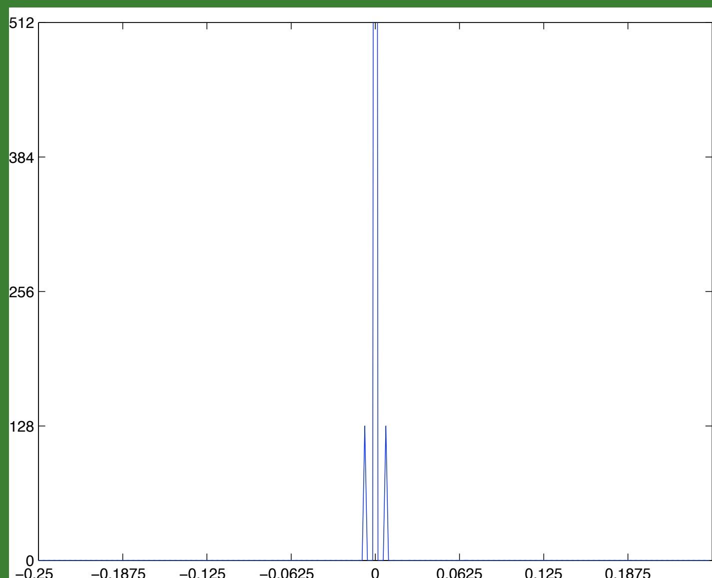
There are lines at the frequency locations.
Real part (image) represents constituent cosine frequencies.
Imaginary part (image) represents constituent sine frequencies.
The intensity of the lines represents amplitude of that frequency.

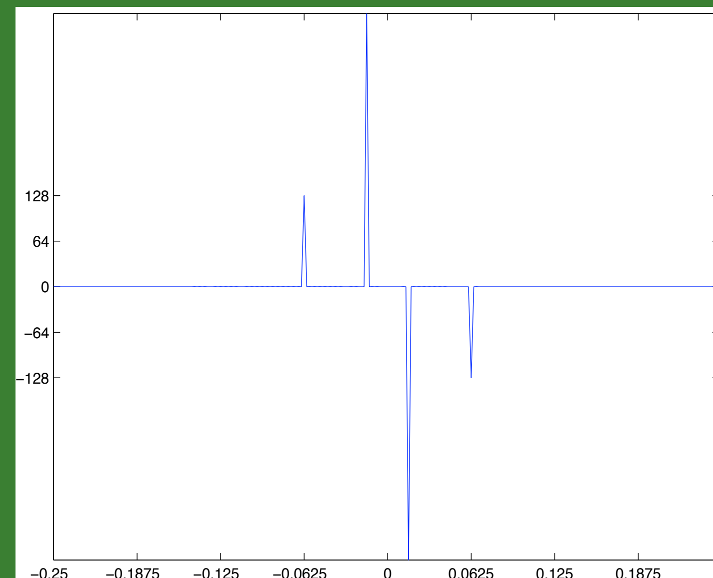
One Dimensional FT-Discrete

The Complex-Valued (Discrete) Fourier Transform (TR=2s)

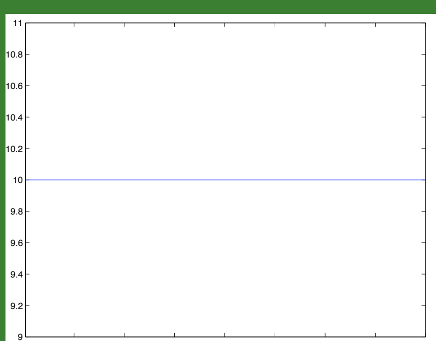








Cosines Sines Cosines: 0Hz, .0078Hz Sines: .0156Hz, .0625Hz



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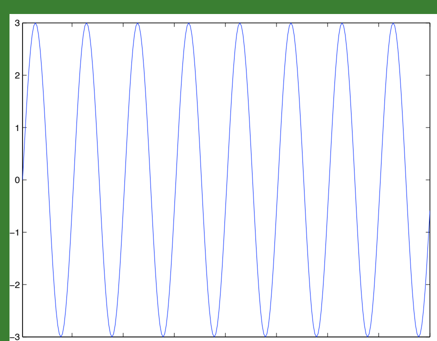

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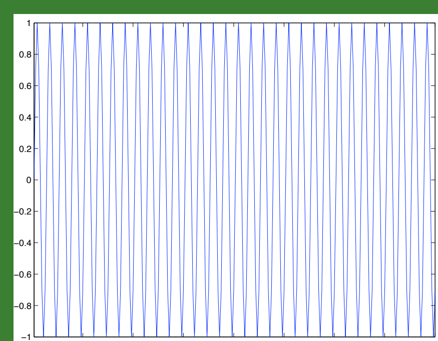
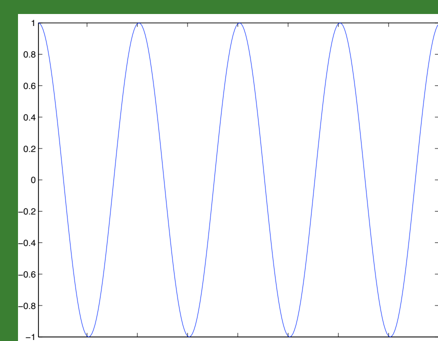
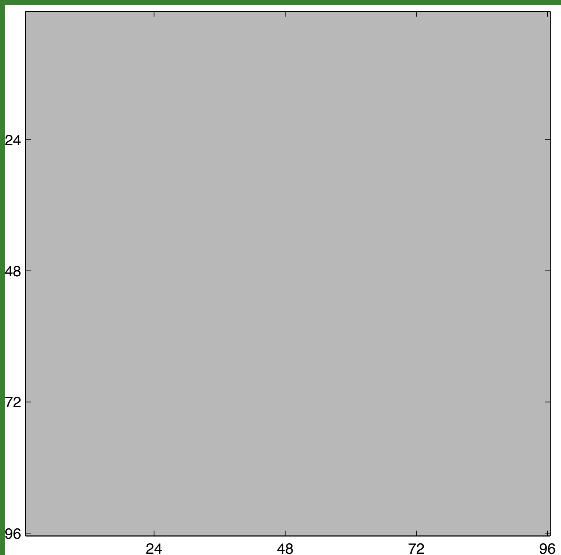


$\sin 32/512$ Hz

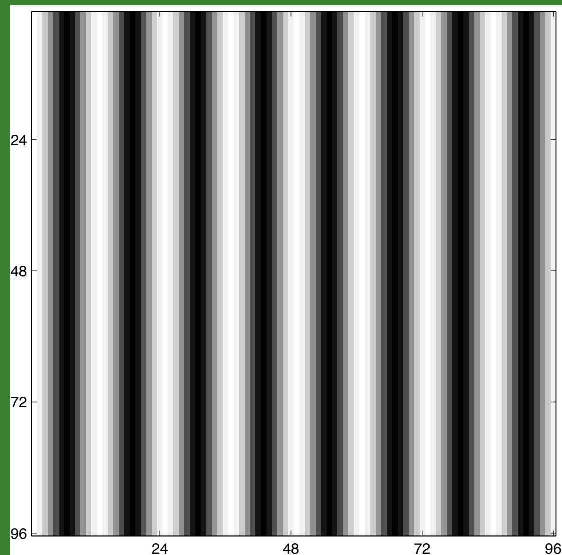


$\cos 4/512$ Hz

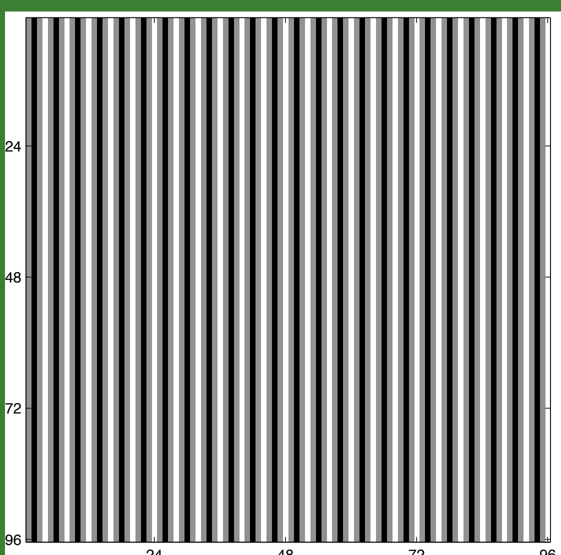
Two Dimensional FT-Discrete



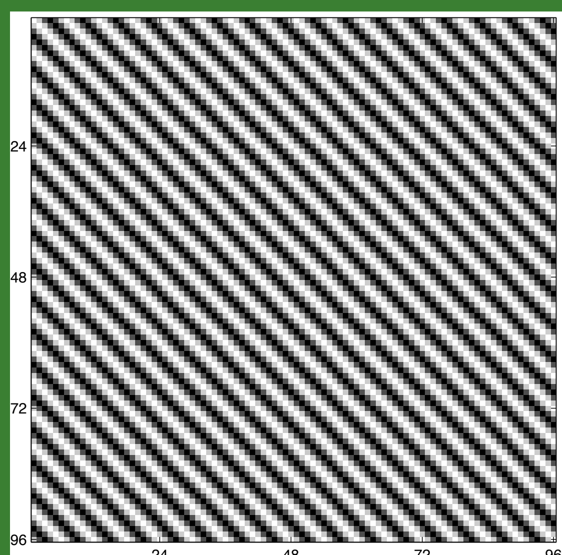
Cos



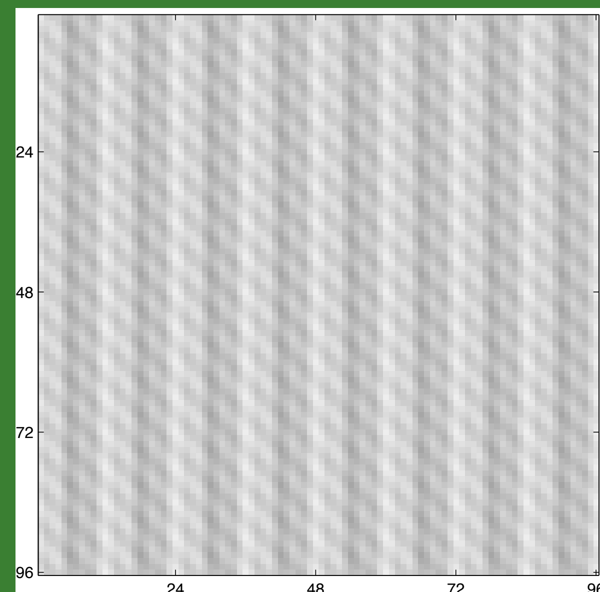
Cos



Sin



Cos



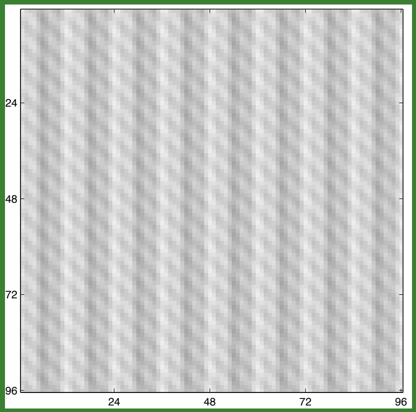
$$\sum = \cos + \cos + \sin + \cos$$

FOV=192 mm, mat=96×96, vox=2 mm^3

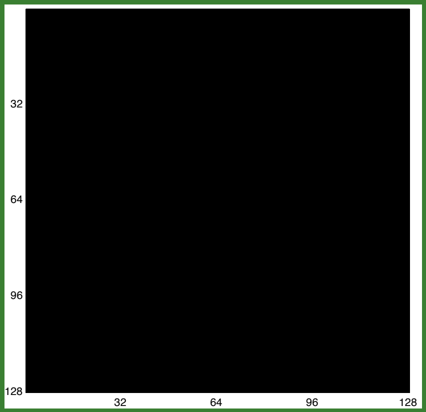
Two Dimensional FT-Discrete

The Complex-Valued 2D (Discrete) Fourier Transform

$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) \quad * \quad (Y_R + iY_I) \quad * \quad (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T \; = \; (F_R + iF_I)$$



$+\, i$



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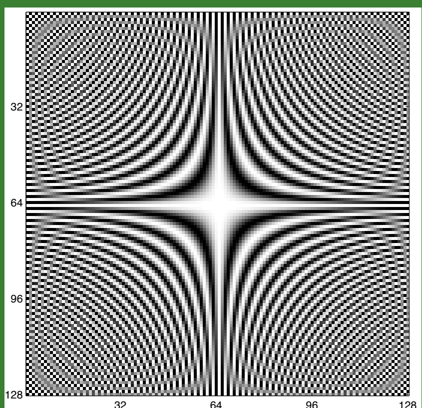
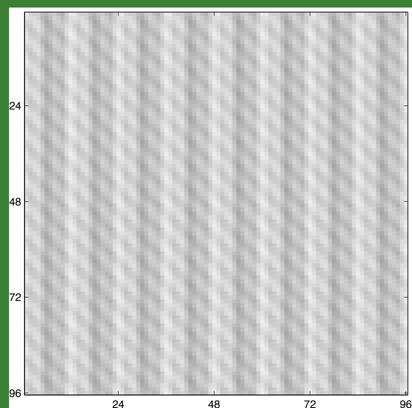


$*$



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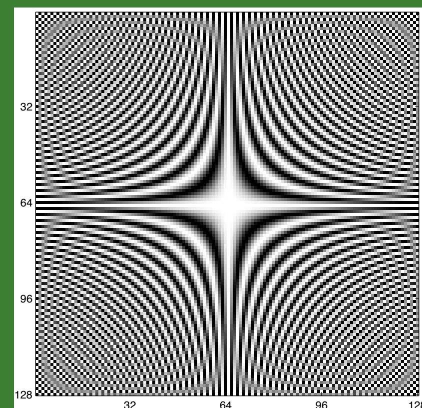
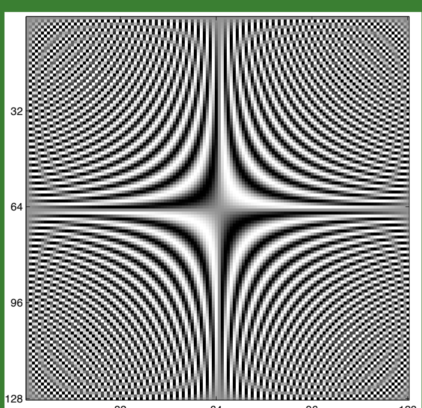
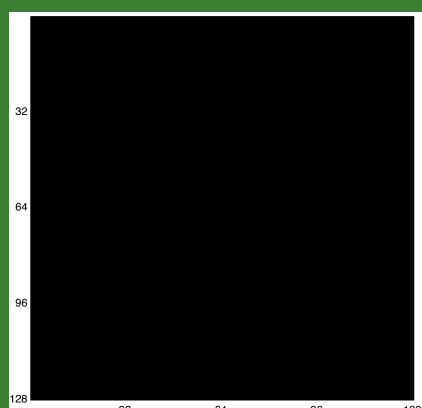
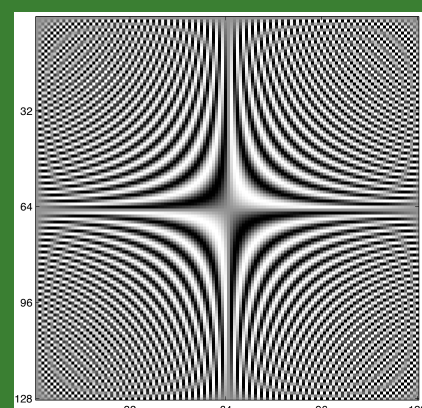


$$+\, i \quad * \quad +\, i \quad * \quad +\, i \quad =$$







FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional FT-Discrete

The Complex-Valued 2D (Discrete) Fourier Transform

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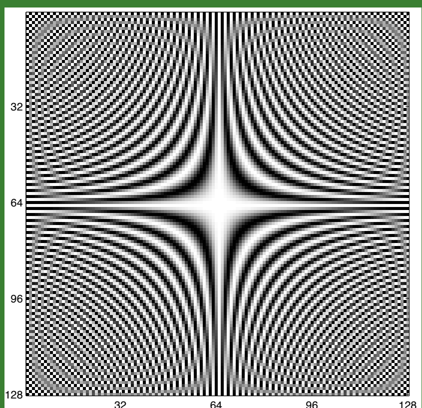
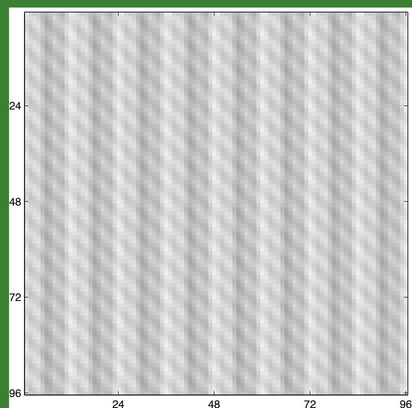
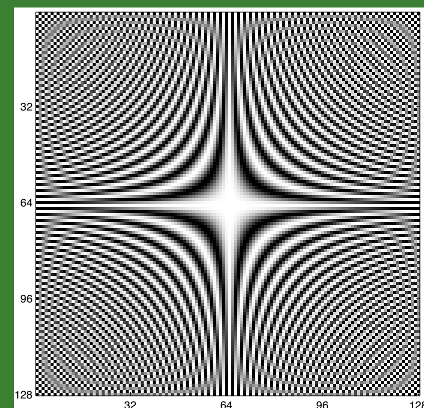
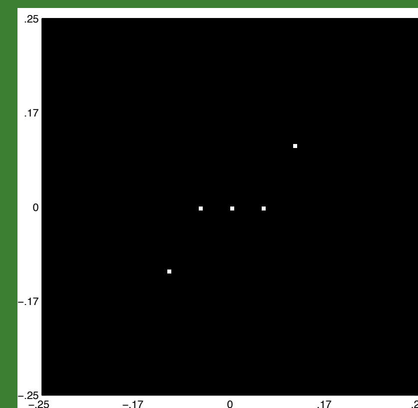
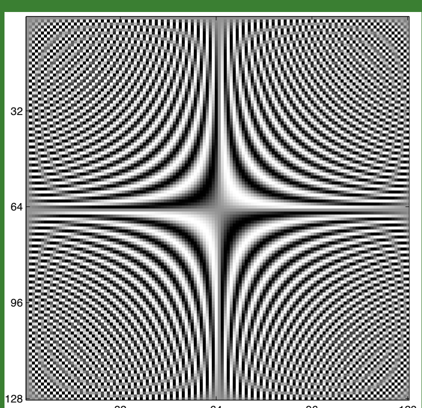


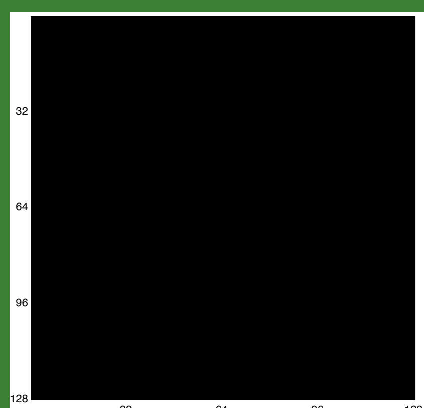


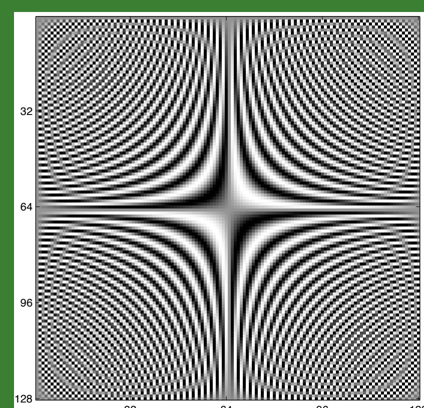


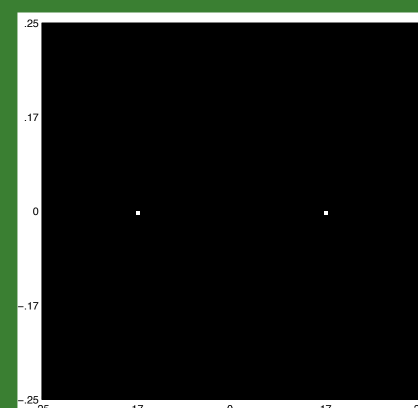


$+i$ $\quad * \quad$ $+i$ $\quad * \quad$ $+i$ $\quad = \quad$ $+i$



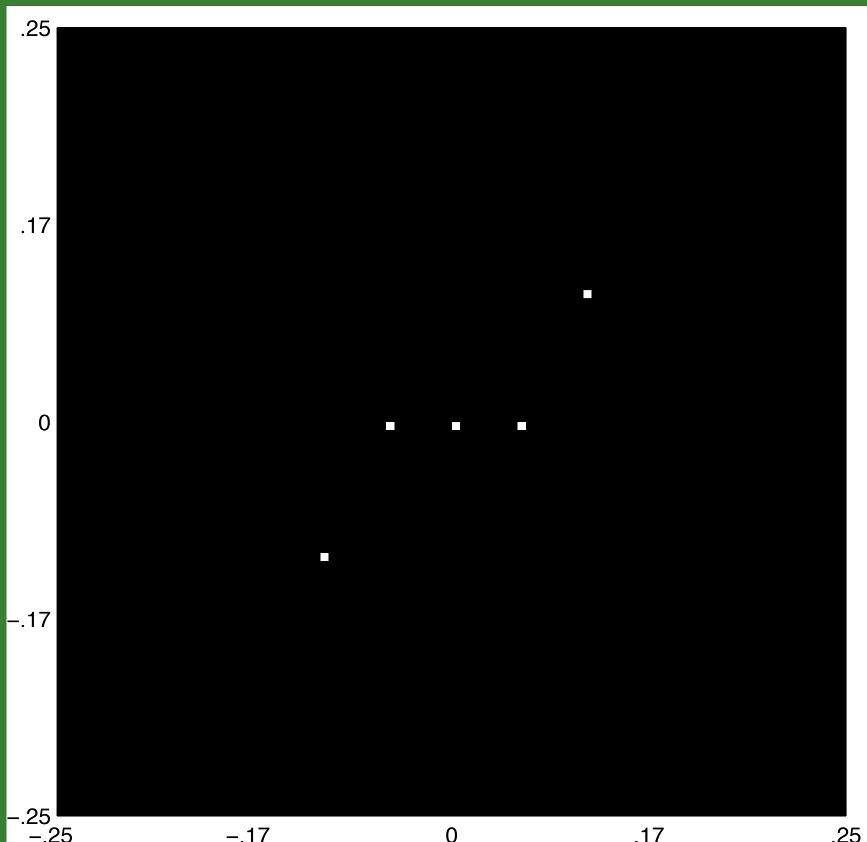


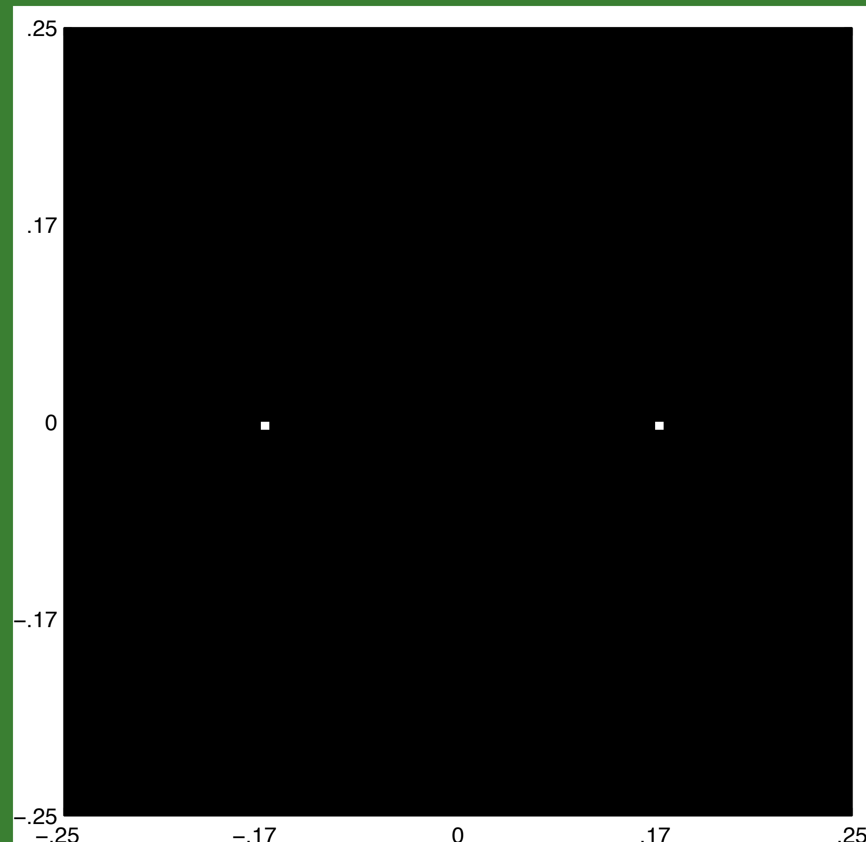




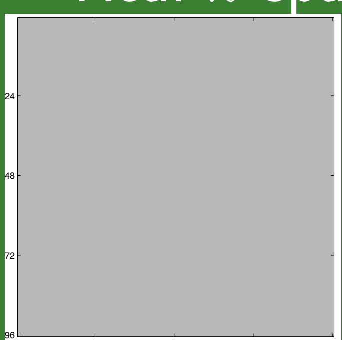
FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional FT-Discrete

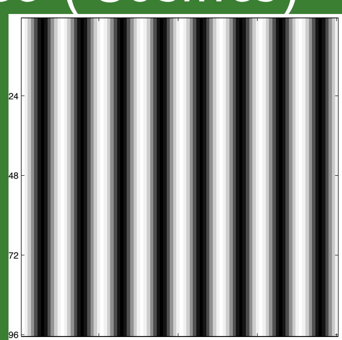




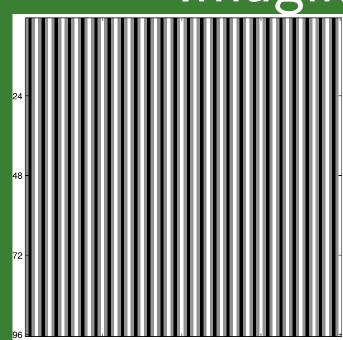
Real k-space (Cosines) Imaginary k-space (Sines)



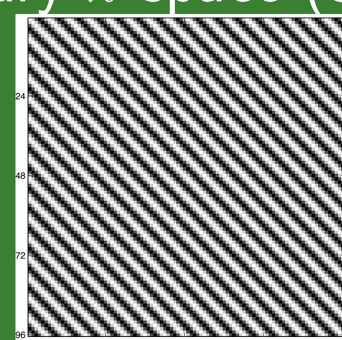
cos



cos

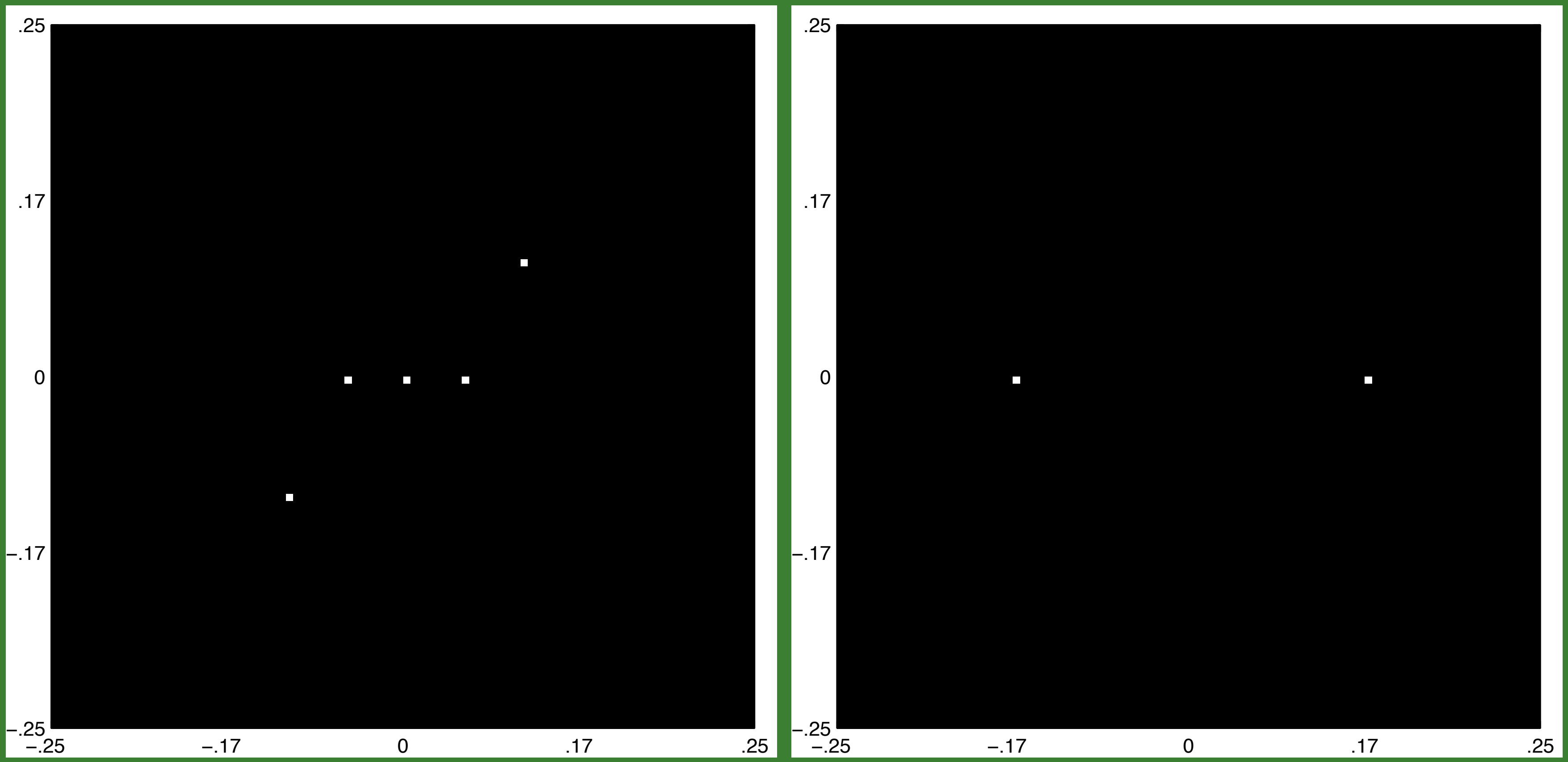


sin



cos

Two Dimensional FT-Discrete



Real k-space (Cosines) Imaginary k-space (Sines)

Note: Rotate bottom half (complex conjugate) up to get top!
Hermetian symmetry (property).

Two Dimensional MR Image Formation

So why do we need Fourier Transforms?

Two Dimensional MR Image Formation

So why do we need Fourier Transforms?

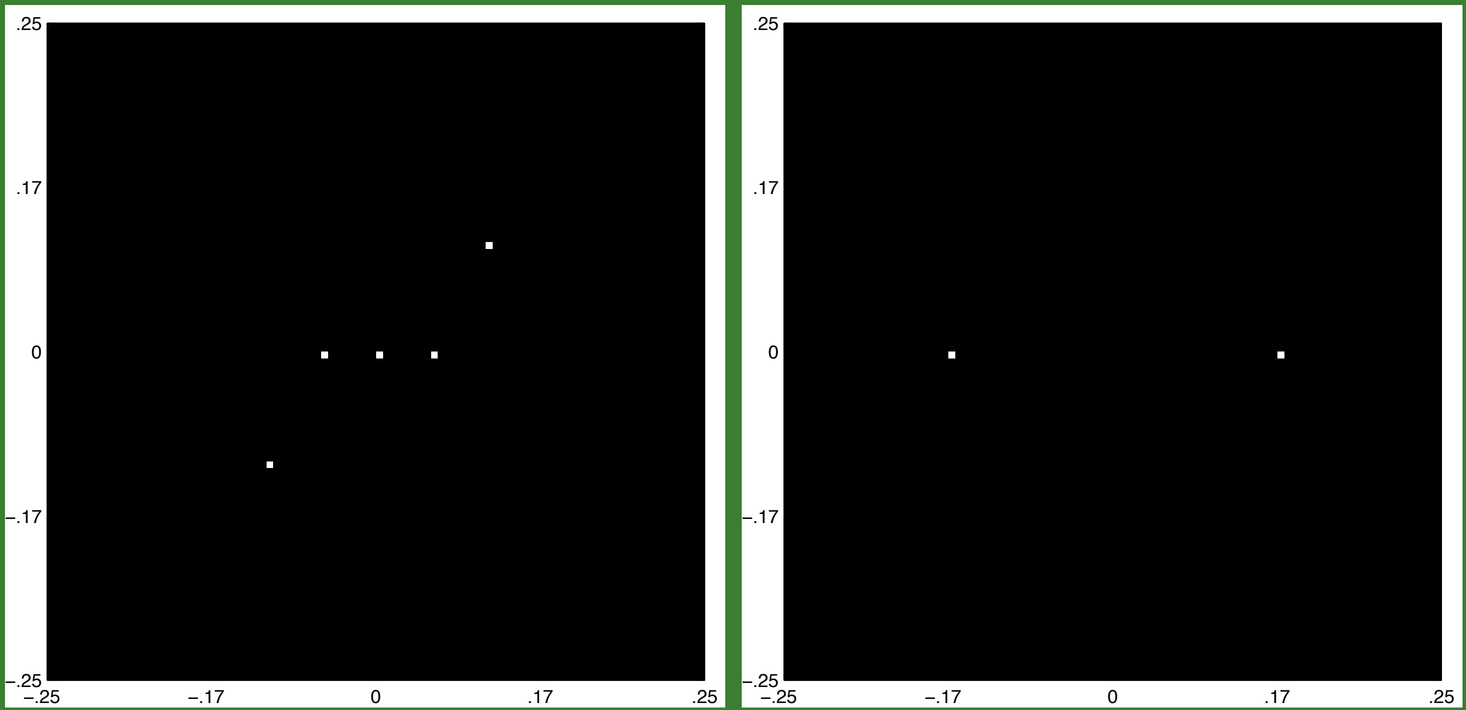
In MRI/fMRI our measurements are not voxel values!

Two Dimensional MR Image Formation

So why do we need Fourier Transforms?

In MRI/fMRI our measurements are not voxel values!

Our measurements are spatial frequencies!



Real k-space (Cosines) Imaginary k-space (Sines)

Two Dimensional MR Image Formation

How do we get spatial frequencies?

Two Dimensional MR Image Formation

How do we get spatial frequencies?

We apply G_x & G_y magnetic field gradients to encode then we measure the complex-valued DFT of the object.

Two Dimensional MR Image Formation

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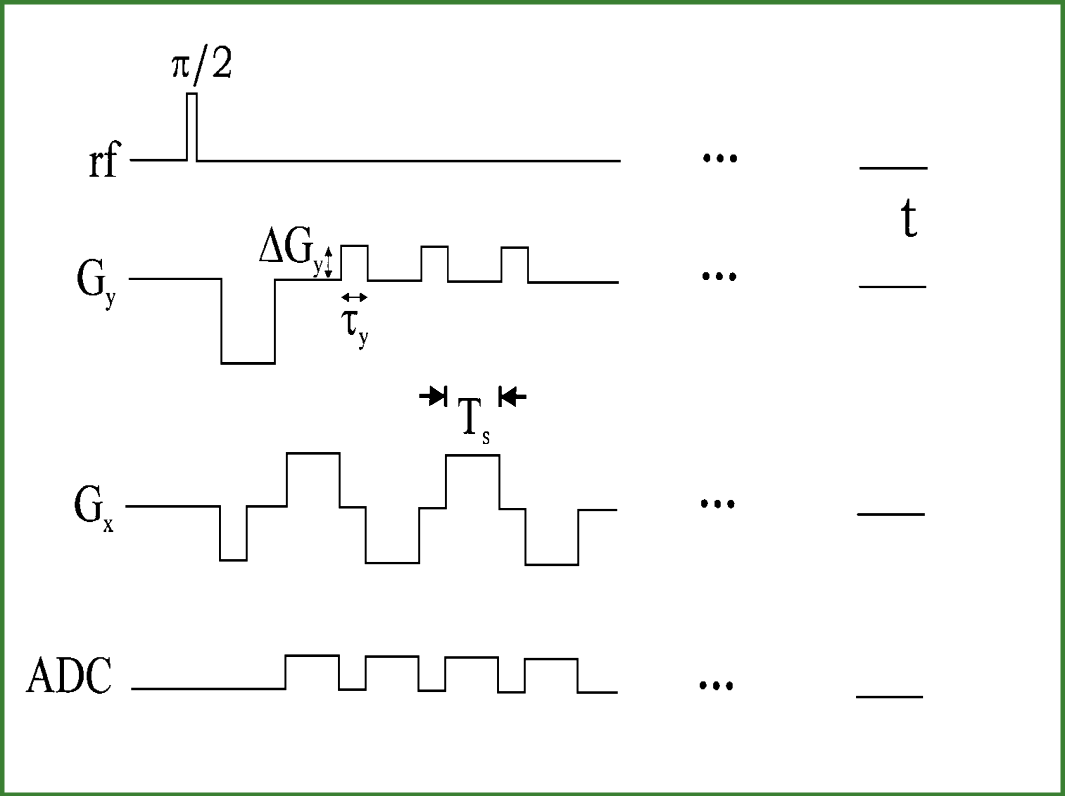
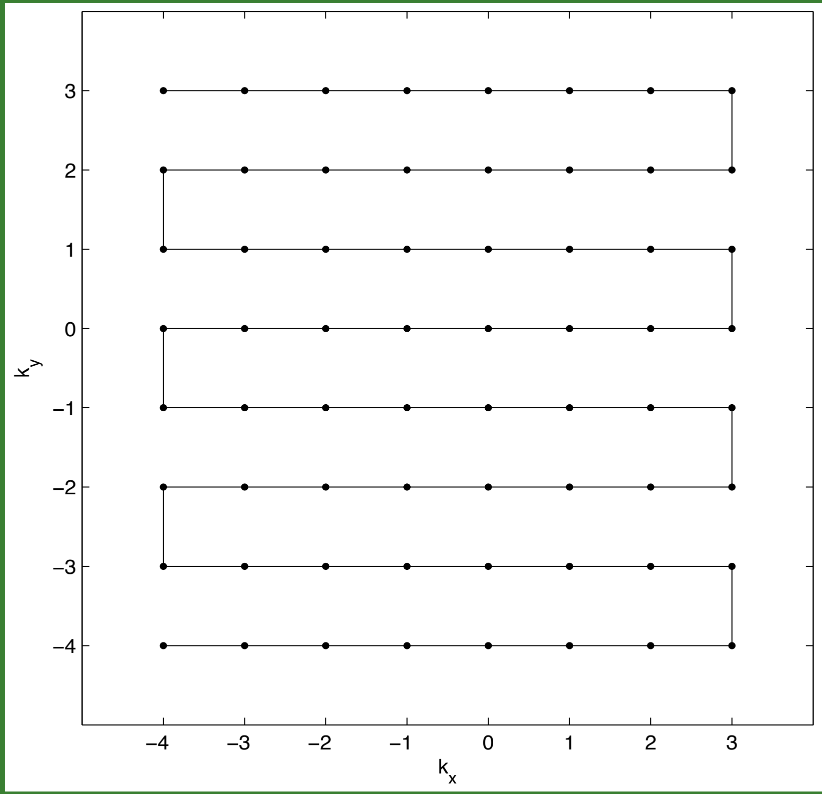
We apply G_x & G_y magnetic field gradients to encode then we measure the complex-valued DFT of the object.

Images are formed (Reconstructed) by a 2D IFT

Two Dimensional MR Image Formation



(a) Gradient Echo-EPI Pulse Sequence



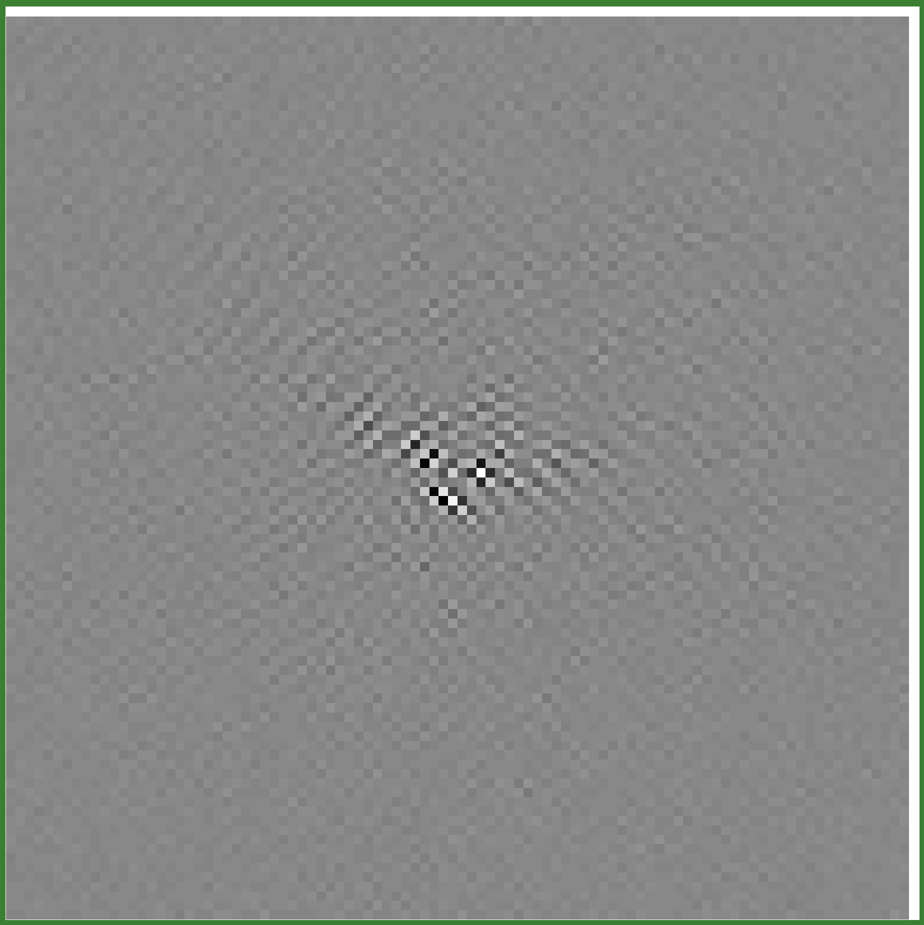
(b) k-Space Trajectory

Kumar, Welti and Ernst: NMR Fourier Zeugmatography, J. Magn. Reson. 1975

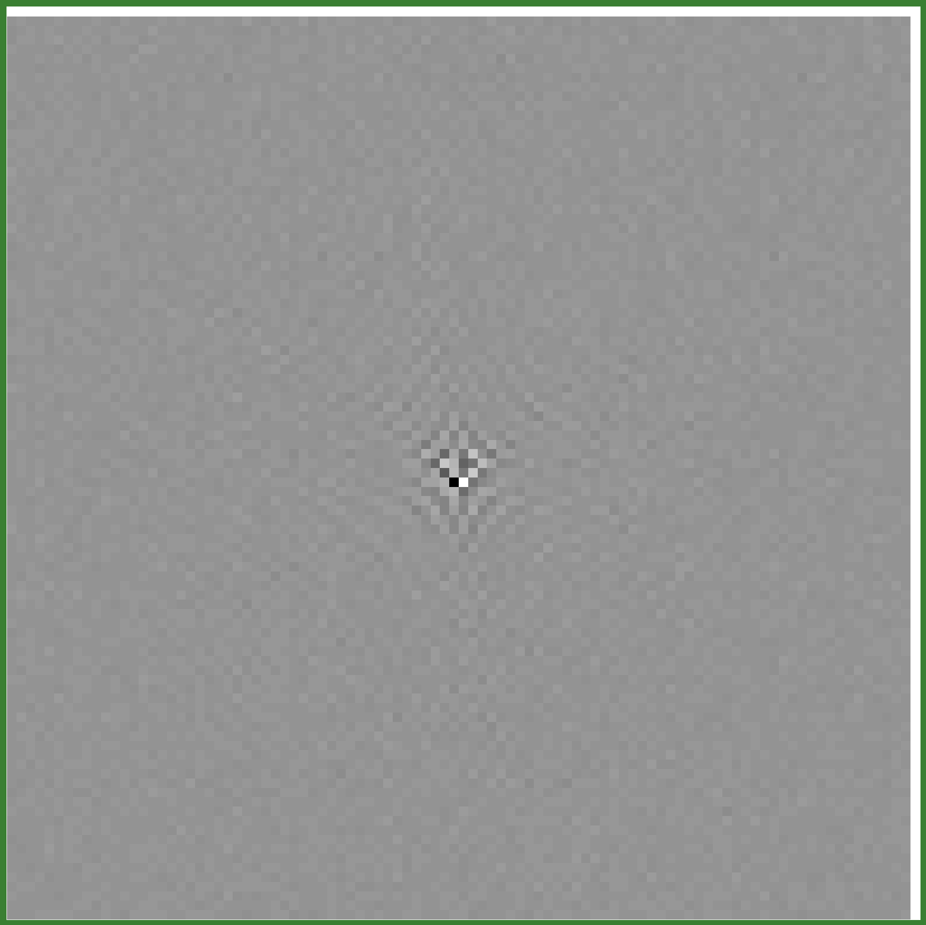
Haacke et al.: *Magnetic Resonance Imaging: Physical Principles and Sequence Design*, 1999.

Two Dimensional MR Image Formation

$F(k_x, k_y) = F_R(k_x, k_y) + iF_I(k_x, k_y)$, the complex-valued DFT of object



(a) real: 96×96



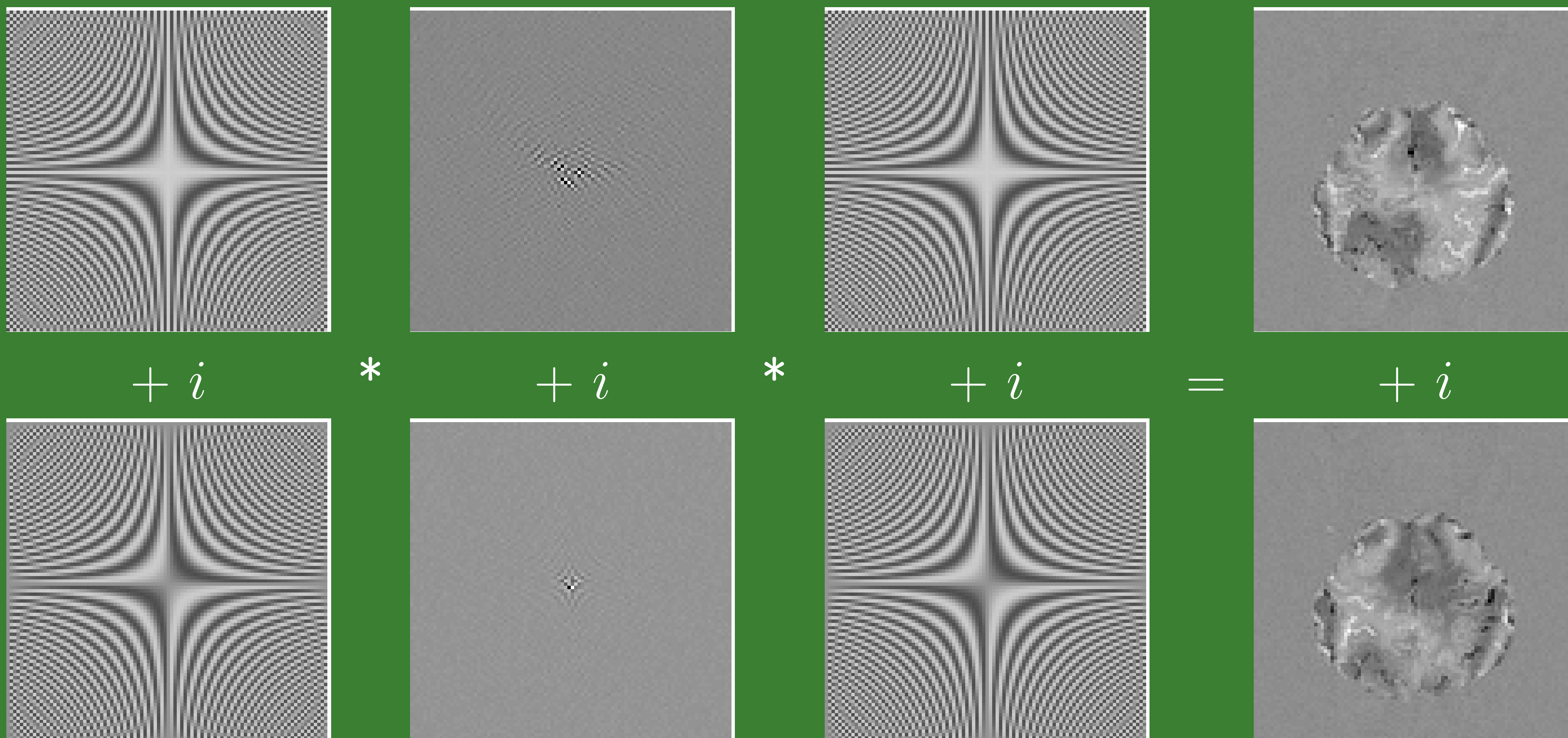
(b) imaginary: 96×96

FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional MR Image Formation

complex-valued 2D IFT

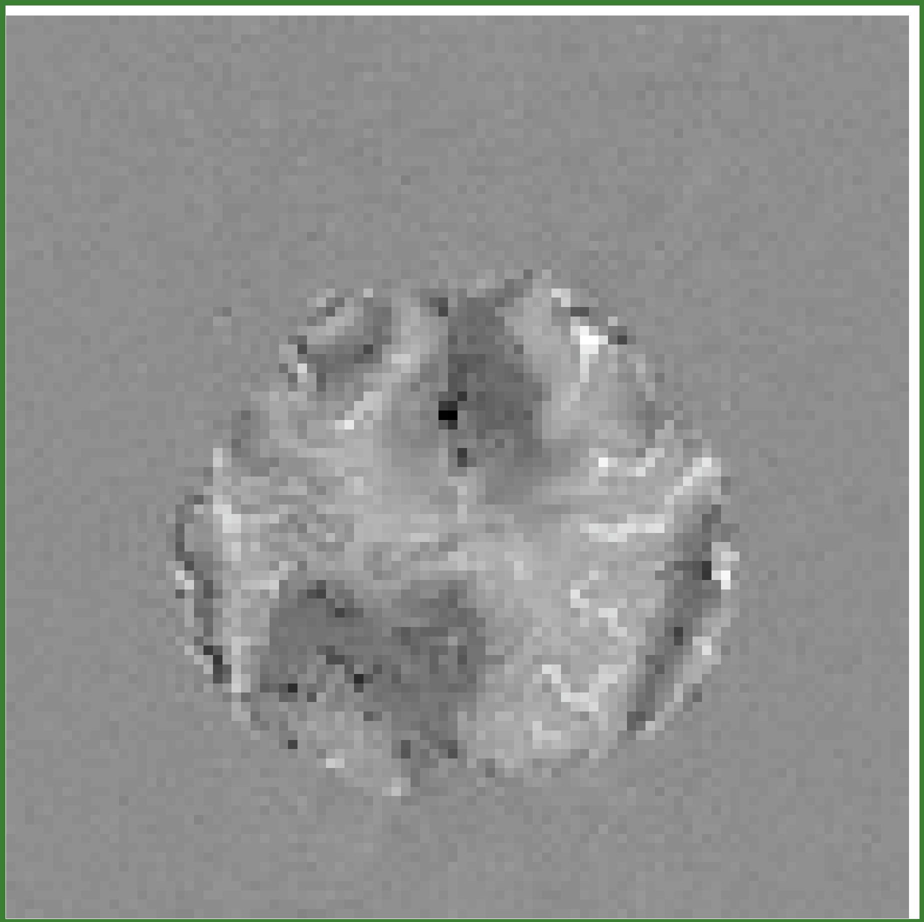
$$(\Omega_{yR} + i\Omega_{yI}) \ * \ (F_R + iF_I) \ * \ (\Omega_{xR} + i\Omega_{xI})^T = (Y_R + iY_I)$$



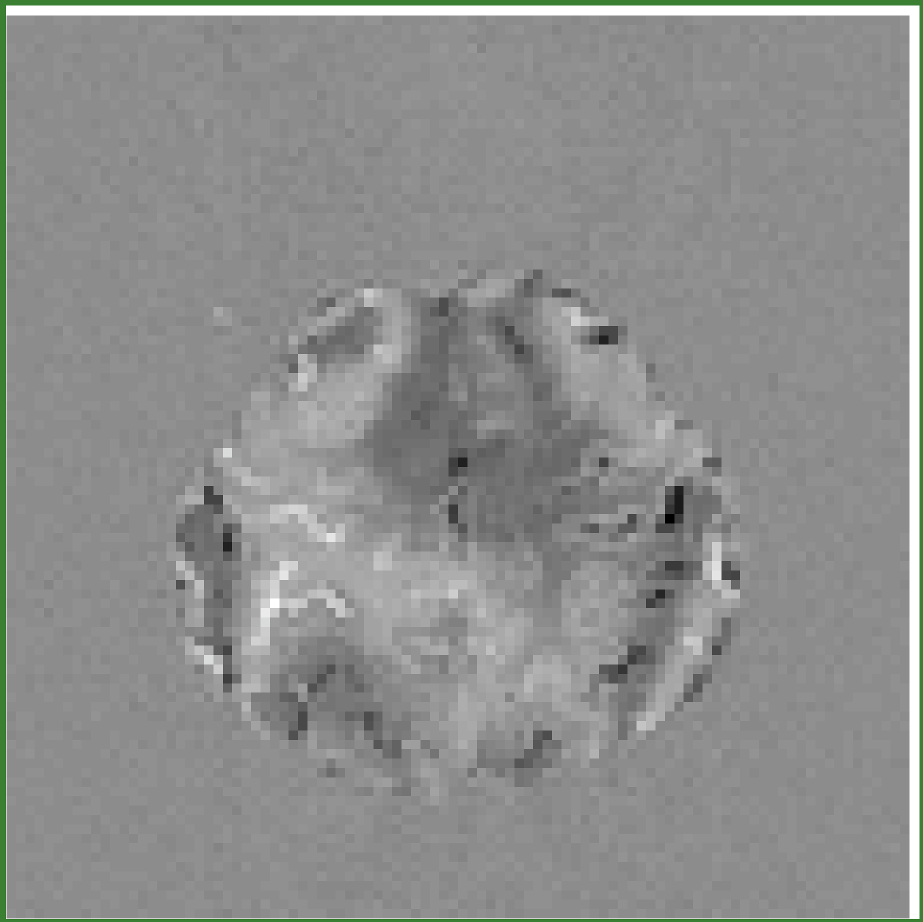
FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional MR Image Formation

Due to the imperfect Fourier encoding, the IFT reconstructed object is complex-valued, $Y(x,y) = Y_R(x,y) + iY_I(x,y)$.



(a) Real image, y_R

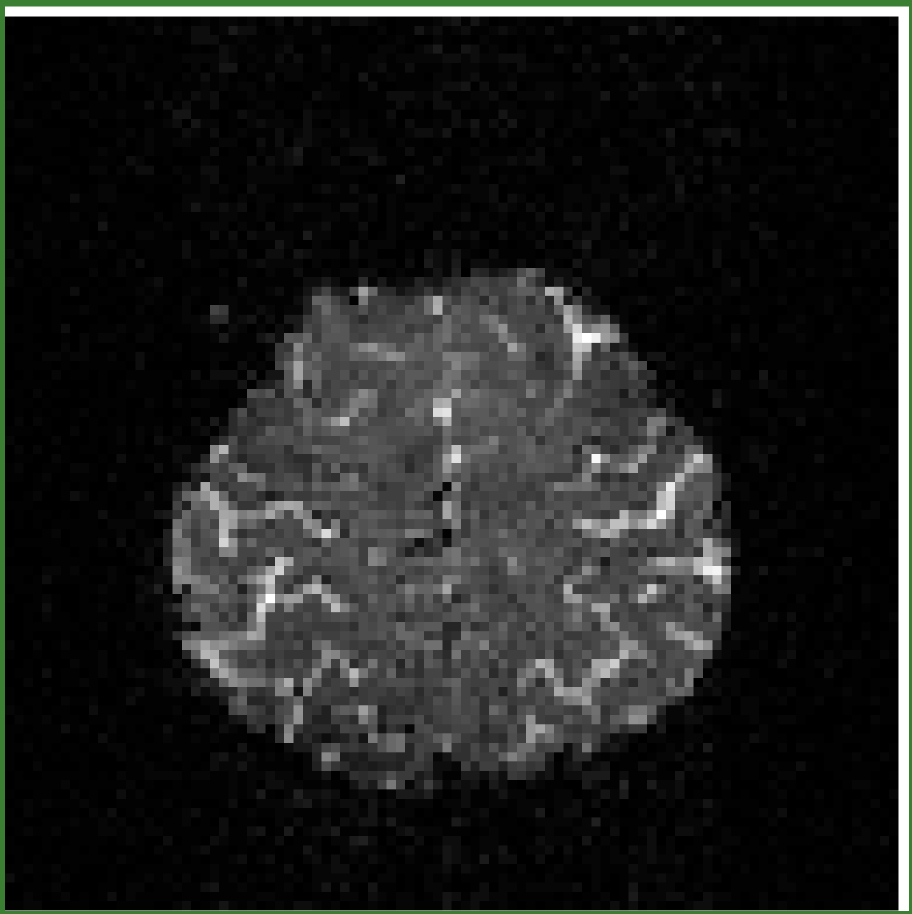


(b) Imaginary image, y_I

FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional MR Image Formation

Most fMRI studies transform from real-imaginary rectangular coordinates to magnitude-phase polar coordinates, $\rho(x,y) = m(x,y)e^{i\phi(x,y)}$.



(a) Magnitude, $m = \sqrt{y_R^2 + y_I^2}$

(b) Phase, $\phi = \text{atan}_4(y_I/y_R)$

FOV=192 mm, mat=96×96, vox=2 mm^3

Two Dimensional MR Image Formation

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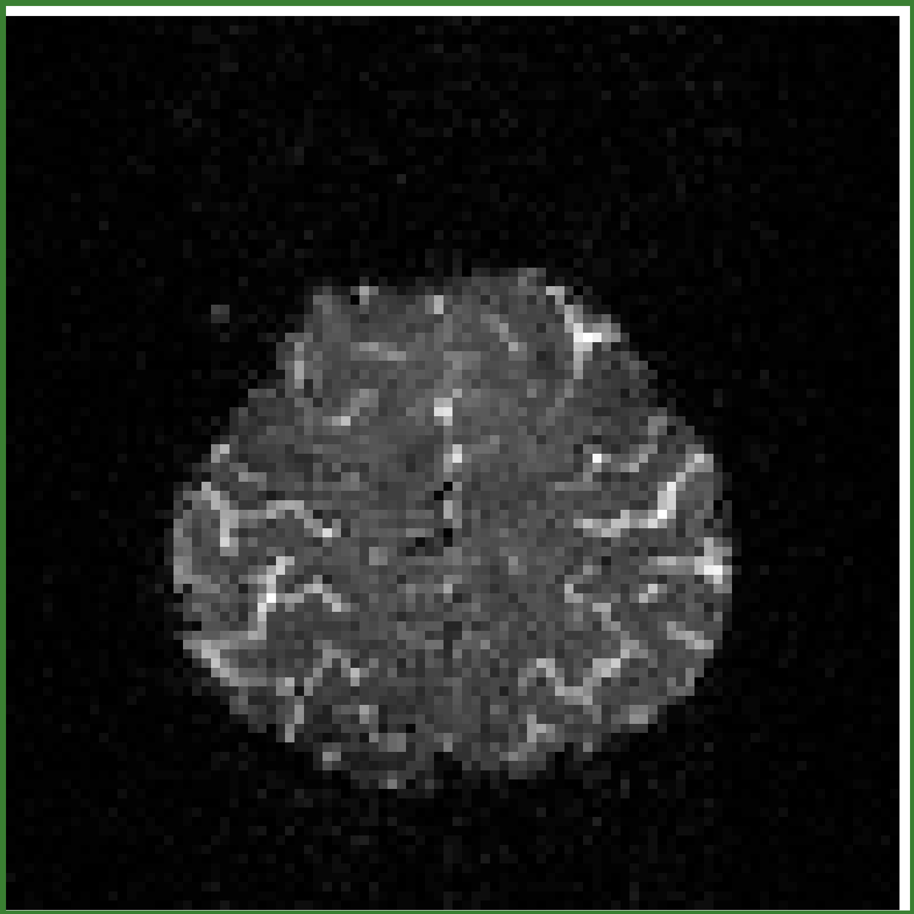
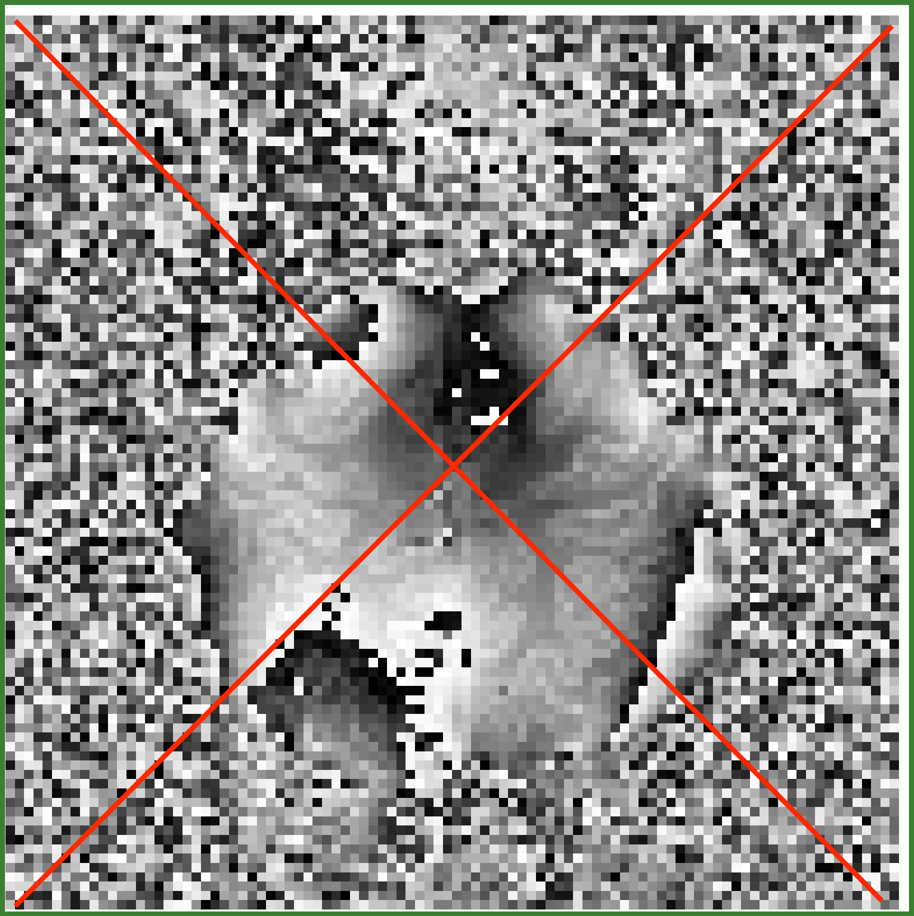


(a) Magnitude, $m = \sqrt{y_R^2 + y_I^2}$



(b) Phase, $\phi = \text{atan}_4(y_I/y_R)$

Two Dimensional MR Image Processing

There are two basic image (filtering) processing categories.

1) Image smoothing

2) Image sharpening

Both can be performed with the DFT.

Two Dimensional MR Image Processing

For image processing, we first define a kernel (AKA mask).
A 3×3 kernel with weights denoted by w's.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

We take this kernel and move it around the image.

A new image is made by summing the product of the kernel weights with the pixel intensity values under the kernel.

The kernel weights typically sum to unity.

Two Dimensional MR Image Processing

$u(x,y)$

$1/16$	$1/8$	$1/16$
$1/8$	$1/4$	$1/8$
$1/16$	$1/8$	$1/16$

$Y(x,y)$

·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·
·	·	·	202	198	207	·	·	·
q_y	·	·	195	186	201	·	·	·
·	·	·	211	189	208	·	·	·
·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·
·	·	·	·	q_x	·	·	·	·

Make new image Z with value at (q_x, q_y) that is

$$Z(q_x,q_y) = 1/16 * 202 + 1/8 * 198 + ... + 1/8 * 189 + 1/16 * 208$$

Two Dimensional MR Image Processing

The previous procedure of moving the kernel around and making new voxel values is the definition of convolution!

$$Z(q_x, q_y) = \sum_{s=-m}^{m} \sum_{r=-n}^{n} Y(q_x - r, q_y - s) u(r, s)$$

where n and m are the x and y dimensions of Y.
u is zero padded to be of the same dimension as Y.

Image filtering (Smoothing) is computationally faster in freq space.

Two Dimensional MR Image Processing-Smoothing

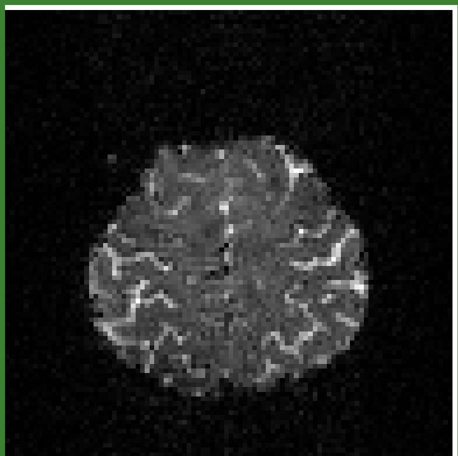
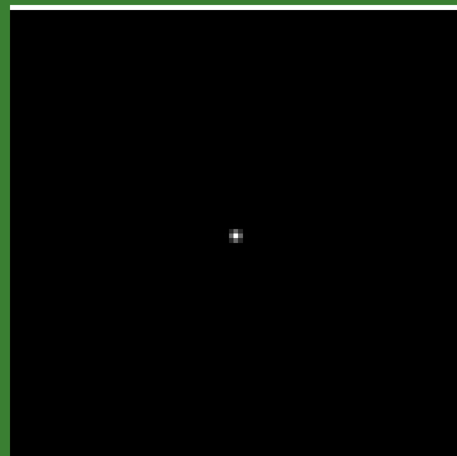


Image Real



Kernel Real

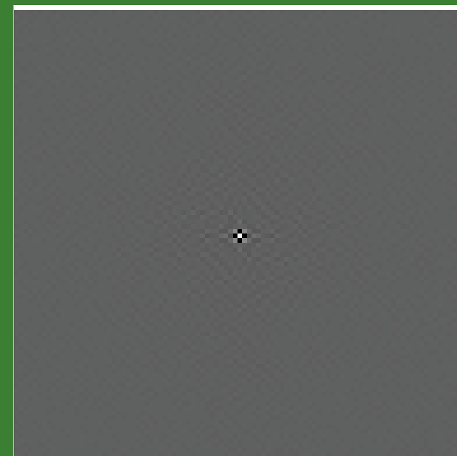
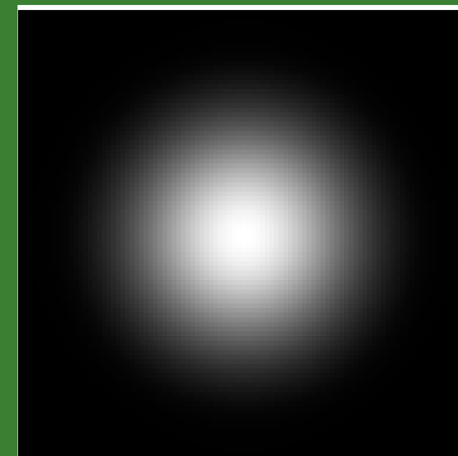


Image FT Real



Kernel FT Real

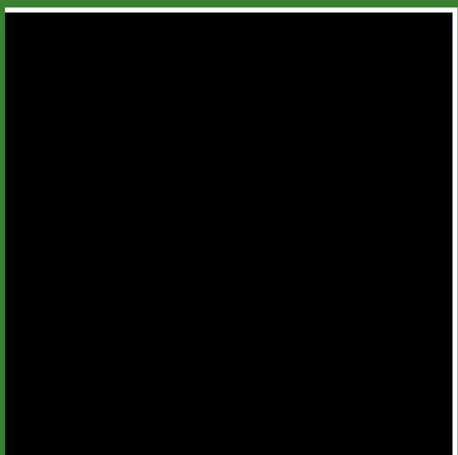
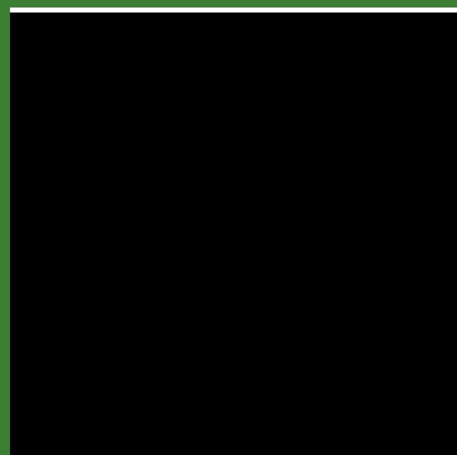
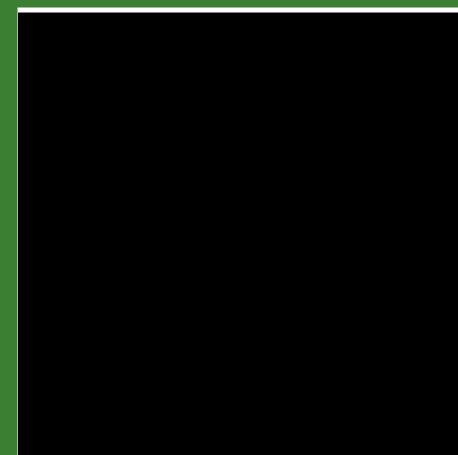


Image Imag.



Kernel Imag.

Image FT Imag.



Kernel FT Imag.

kernel=

.0000	.0000	.0000	.0001	.0001	.0001	.0000	.0000	.0000
.0000	.0000	.0004	.0017	.0026	.0017	.0004	.0000	.0000
.0000	.0004	.0041	.0154	.0239	.0154	.0041	.0004	.0000
.0001	.0017	.0154	.0581	.0906	.0581	.0154	.0017	.0001
.0001	.0026	.0239	.0906	.1412	.0906	.0239	.0026	.0001
.0001	.0017	.0154	.0581	.0906	.0581	.0154	.0017	.0001
.0000	.0004	.0041	.0154	.0239	.0154	.0041	.0004	.0000
.0000	.0000	.0004	.0017	.0026	.0017	.0004	.0000	.0000
.0000	.0000	.0000	.0001	.0001	.0001	.0000	.0000	.0000

Two Dimensional MR Image Processing-Smoothing

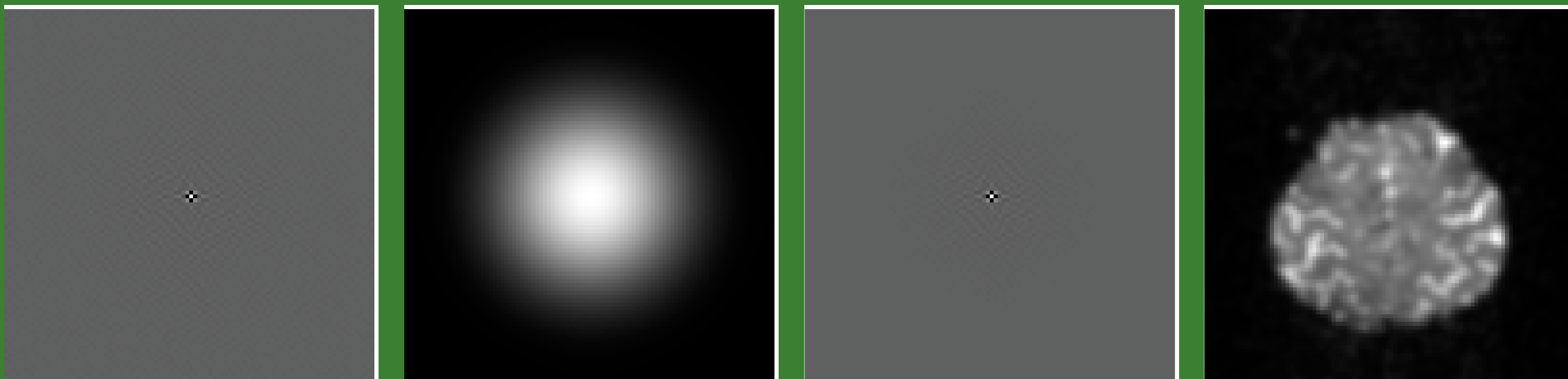


Image FT Real   Kernel FT Real   Prod. FT Real   IFT Prod. Real

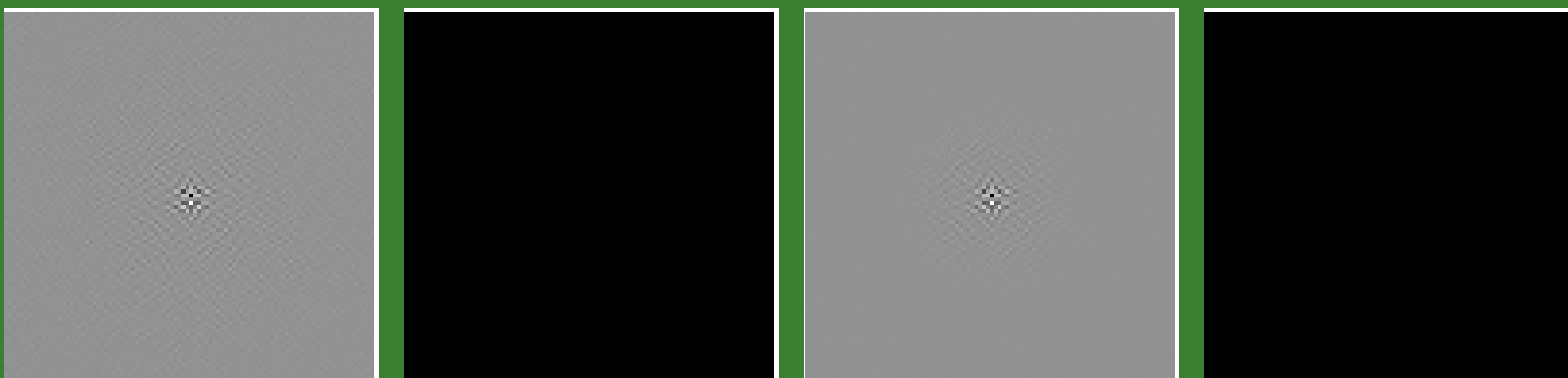
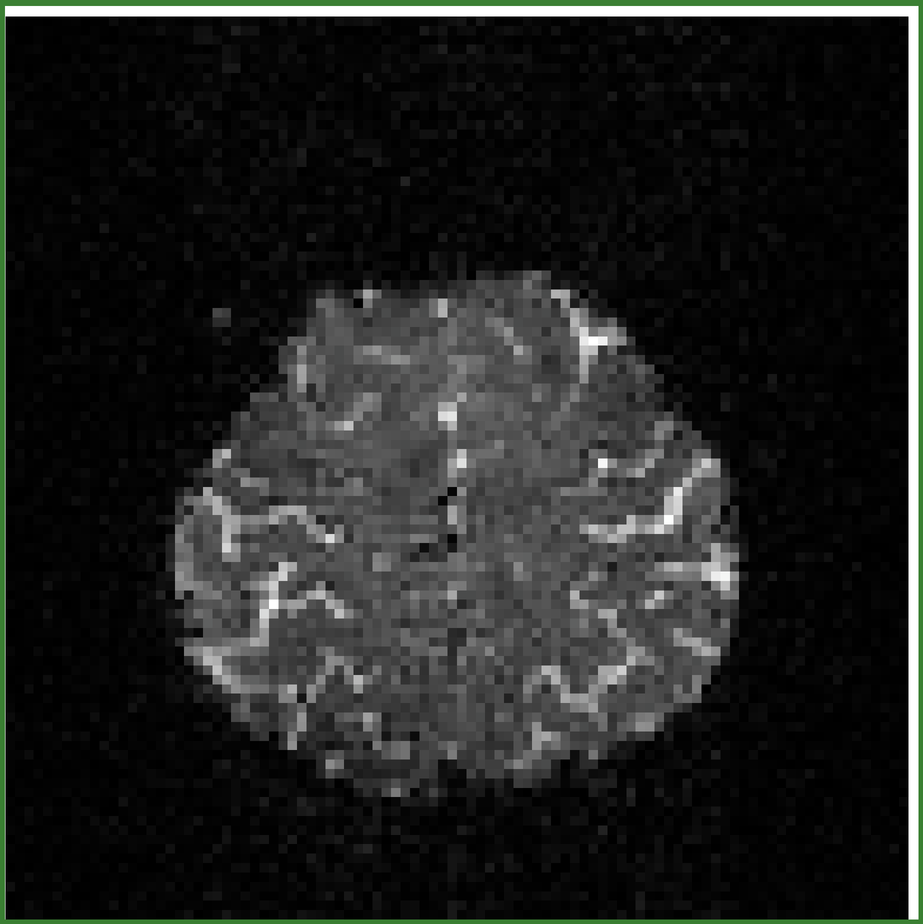
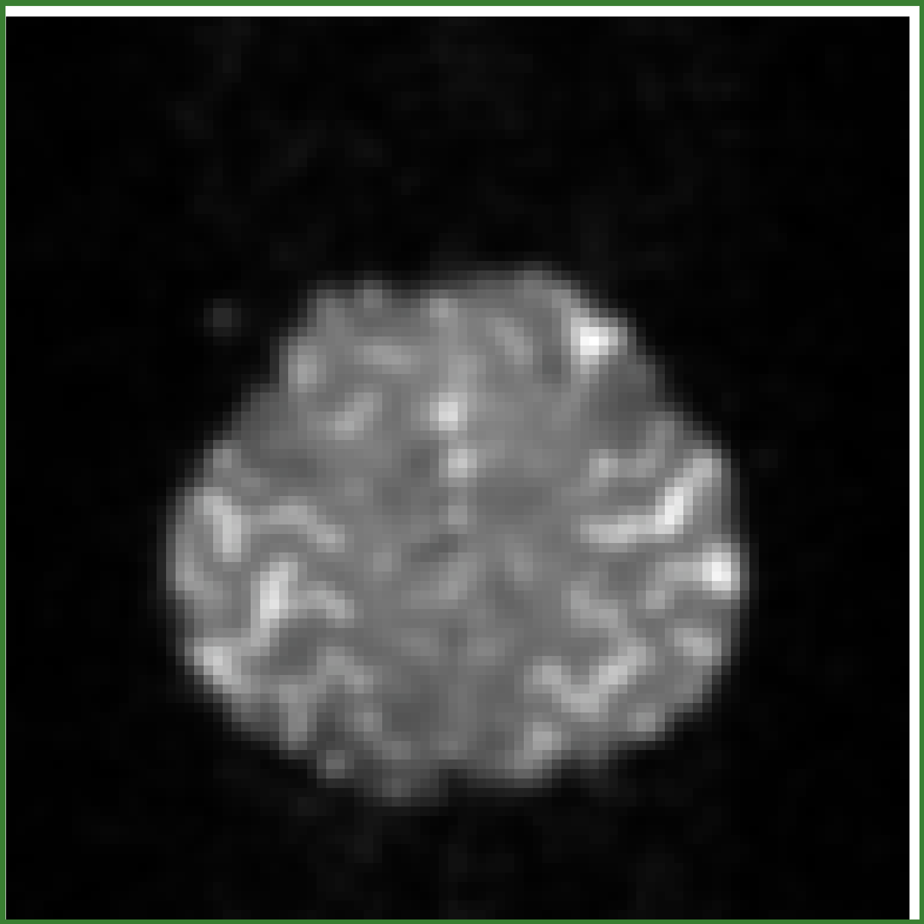


Image FT Imag.   Kernel FT Imag.   Prod. FT Imag.   IFT Prod. Imag.

Two Dimensional MR Image Processing-Smoothing



Original Image



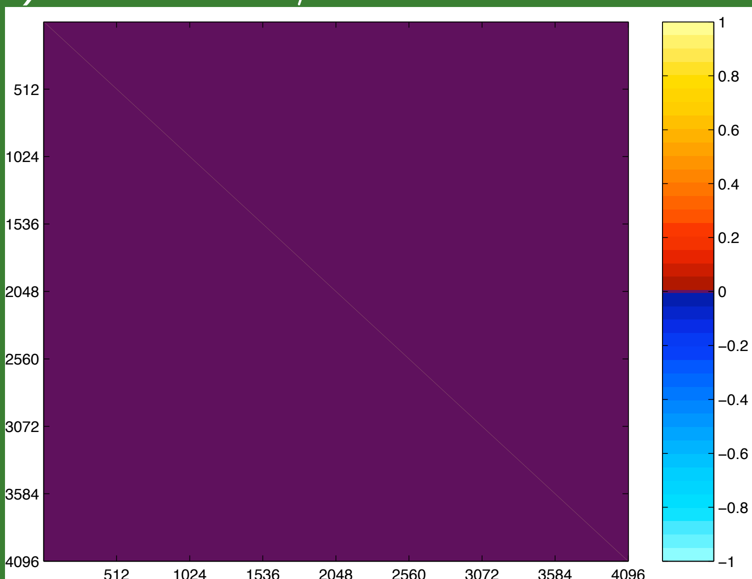
Smoothed Image

FOV=192 mm, mat=96×96, vox=2 mm^3

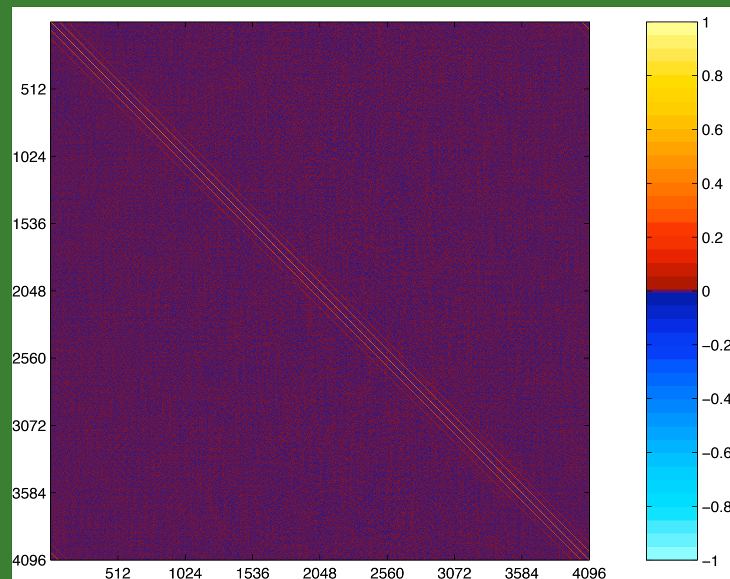
Two Dimensional MR Image Processing-Smoothing

Smothing Caveats:

1) Increases/Induces local voxel correlation



Original Image Corr



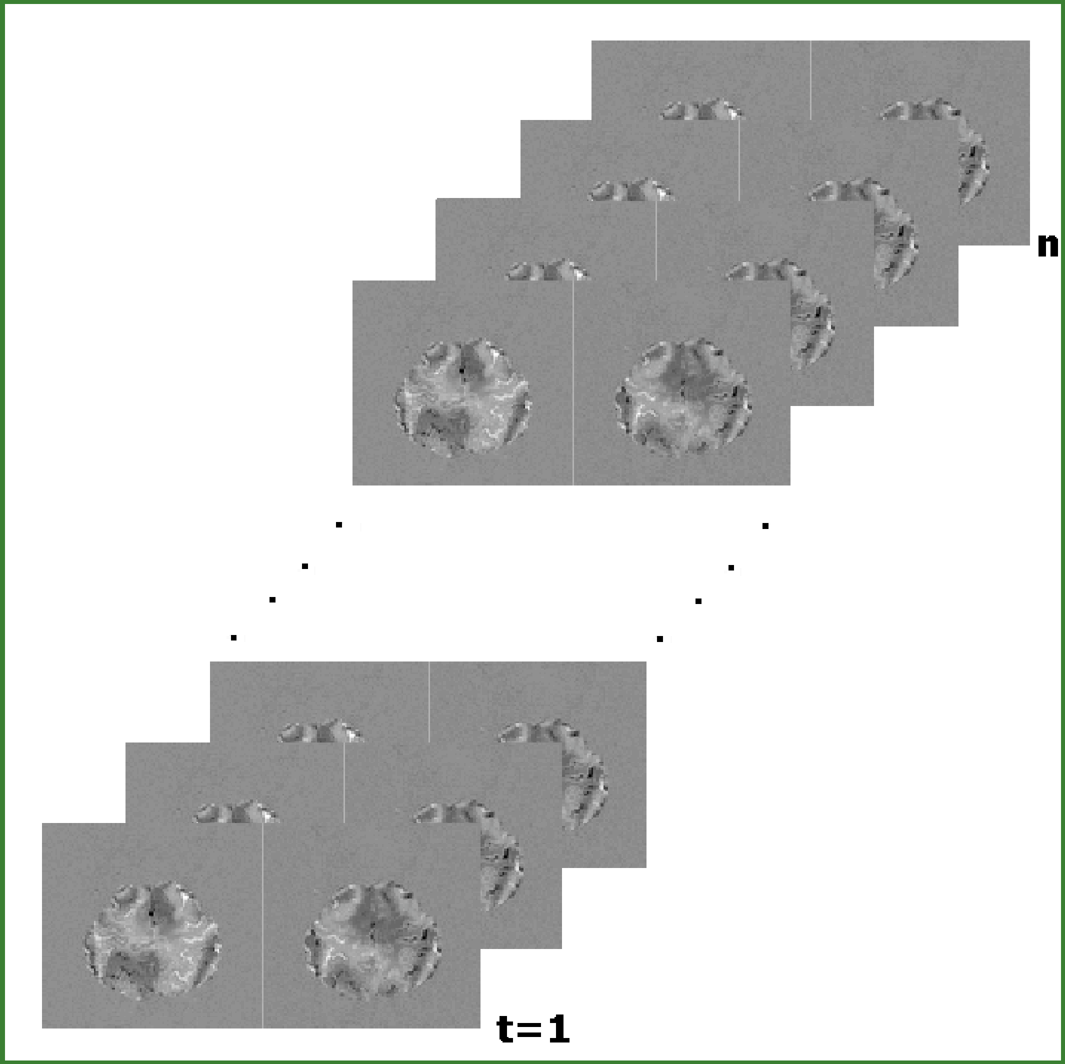
Smoothed Image Corr

2) t-statistics need to be renormalized

$K = \sqrt{\sum w_j}$ under independence

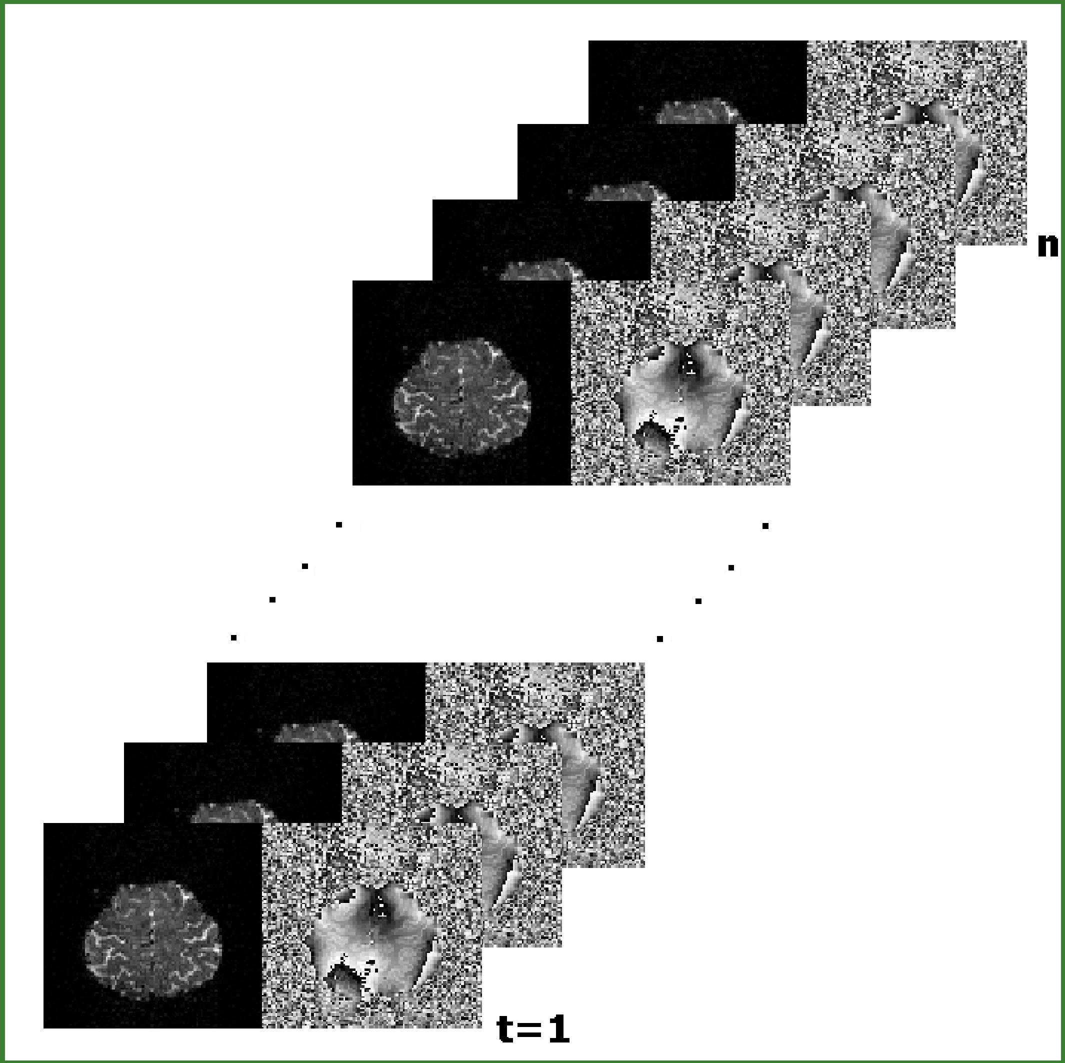
One Dimensional fMRI Time Series

In fMRI we get complex-valued images over time and voxel time course observations, $y_t = y_{Rt} + iy_{It}$.



One Dimensional fMRI Time Series

Collect a sequence of these reconstructed images over time.
Form voxel time courses, $y_t = r_t e^{i\phi_t}$.



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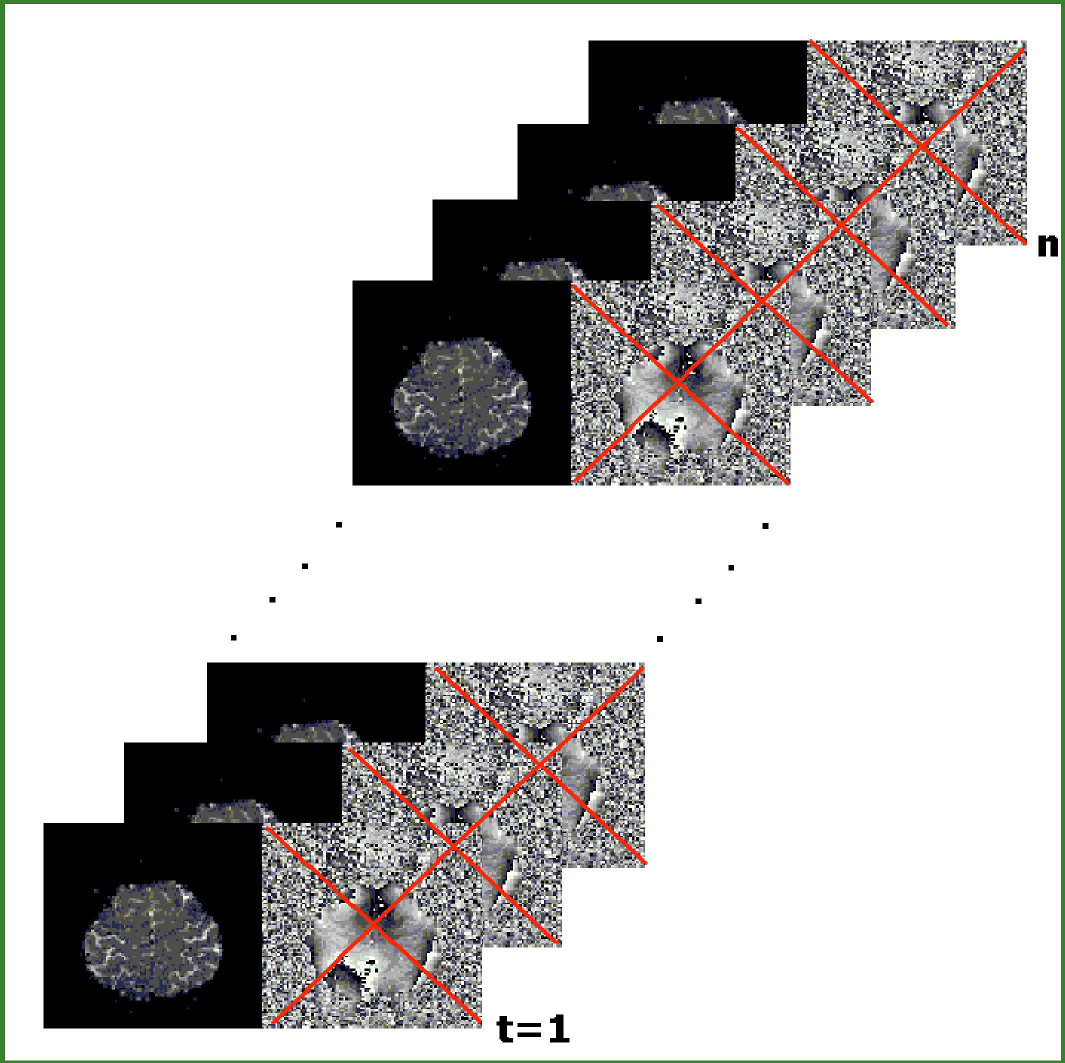
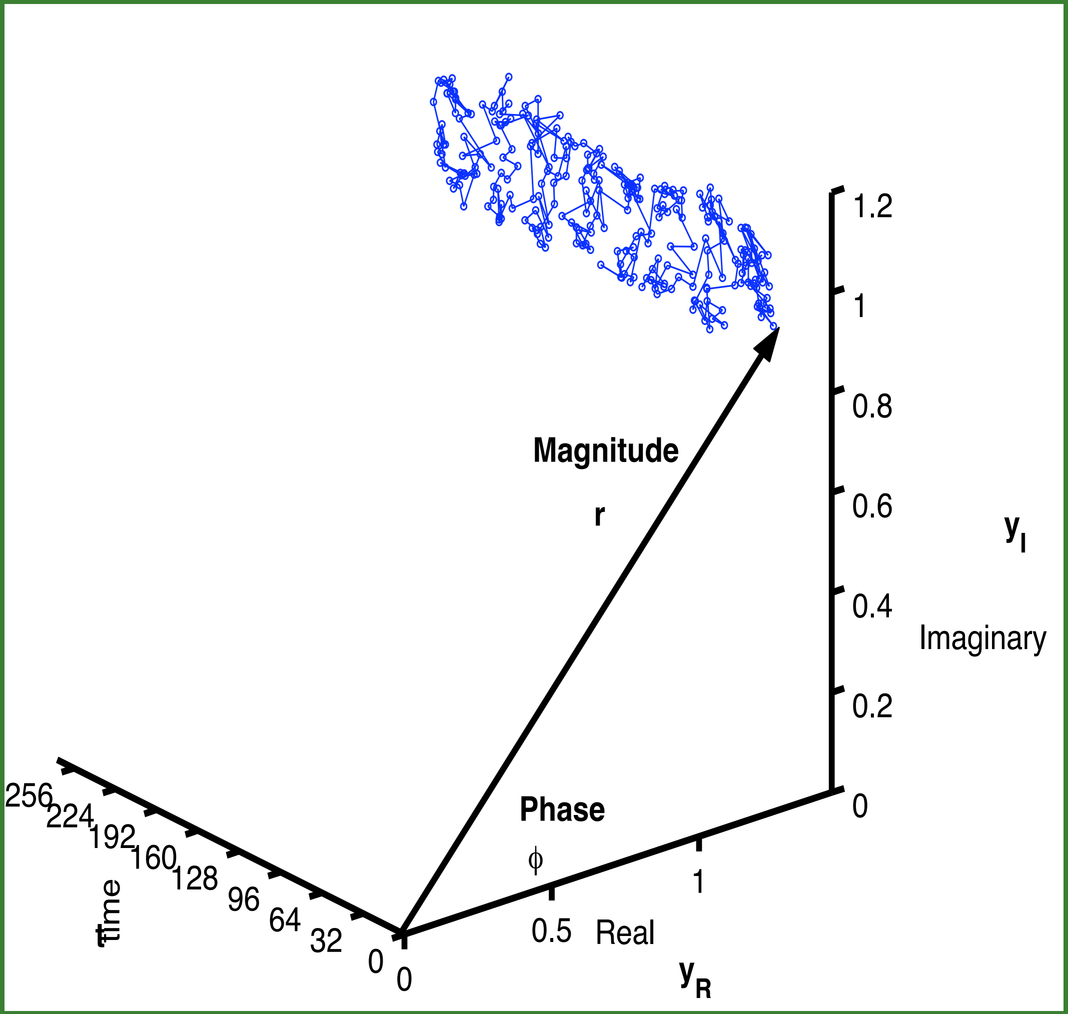
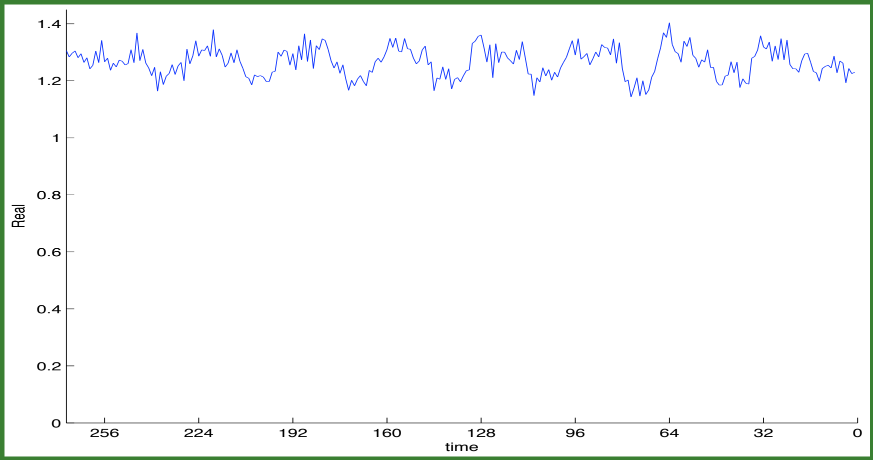


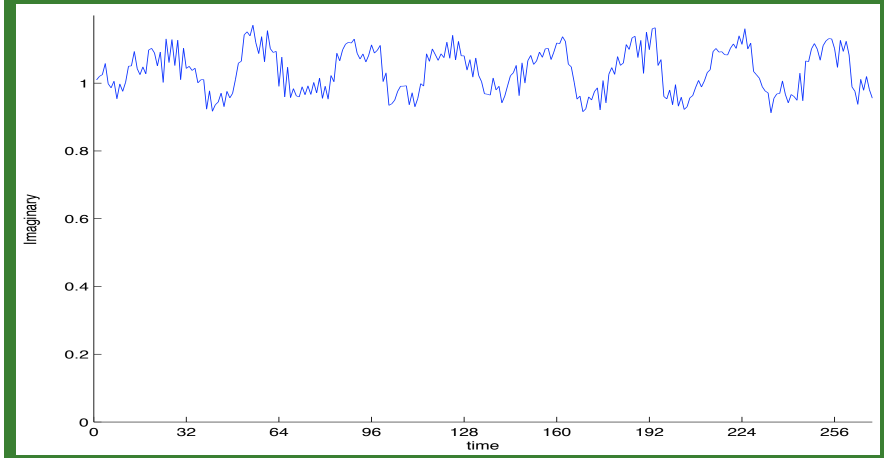
One Dimensional fMRI Time Series

Time series are complex-valued or bivariate with phase coupled means.





Real: Task related changes!



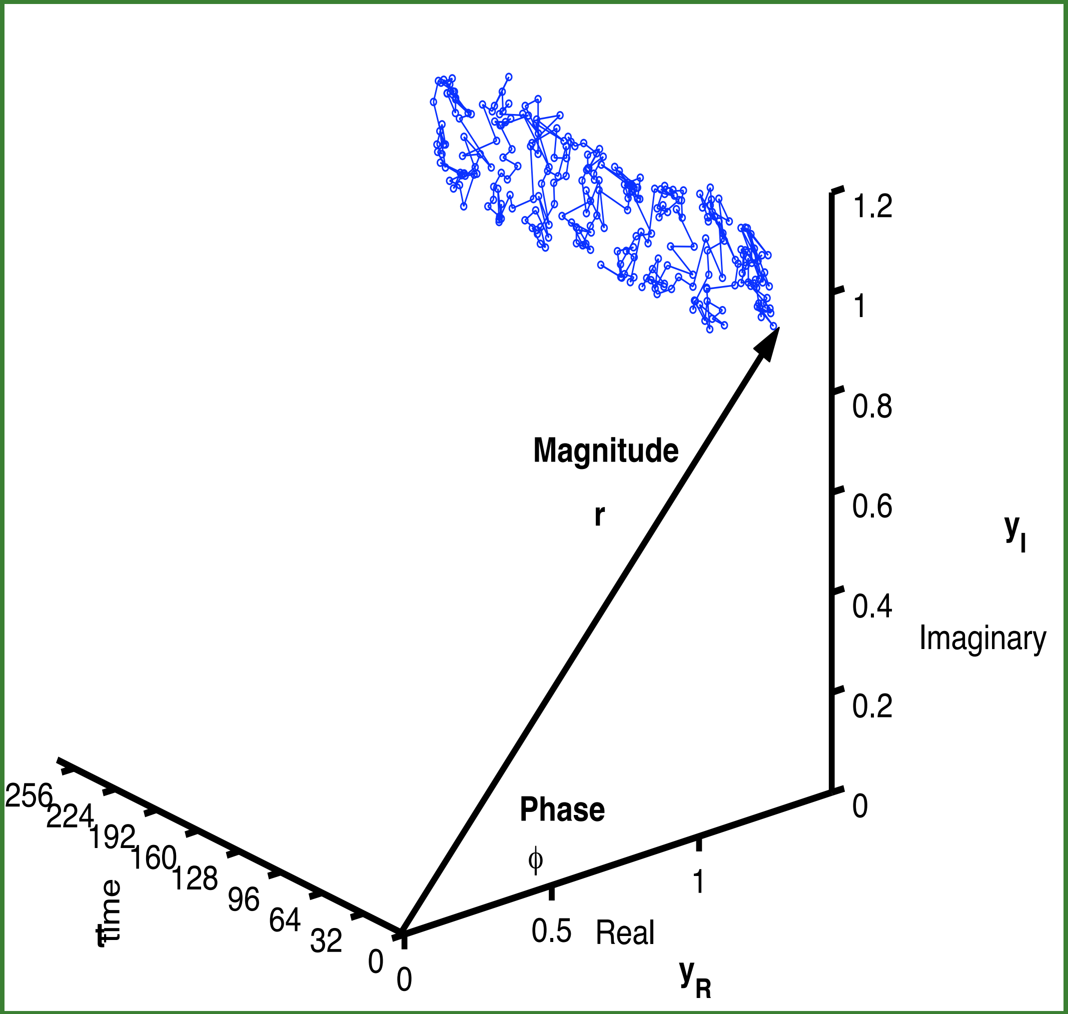
Imaginary: Task related changes!

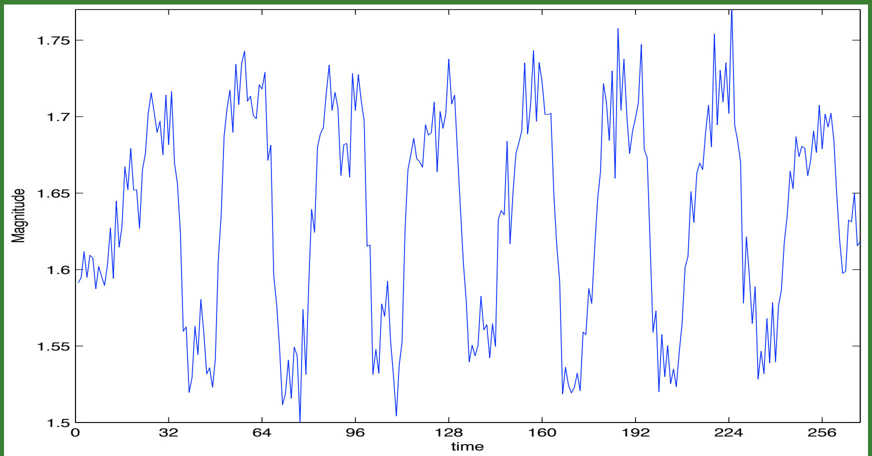
The y_R and y_I time courses have related vector length info!

This is a time series from a actual human experimental data!

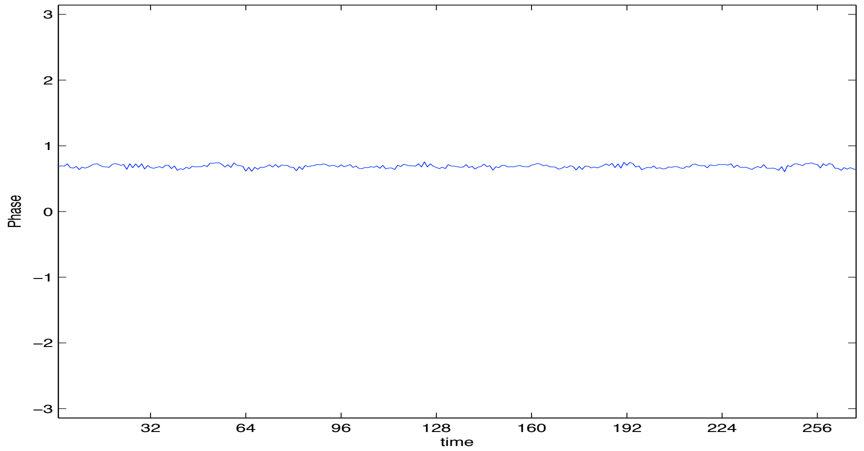
One Dimensional fMRI Time Series

Time series are complex-valued or bivariate with phase coupled means.





Magnitude: Task related magnitude changes!



Phase: Often relatively constant temporally.

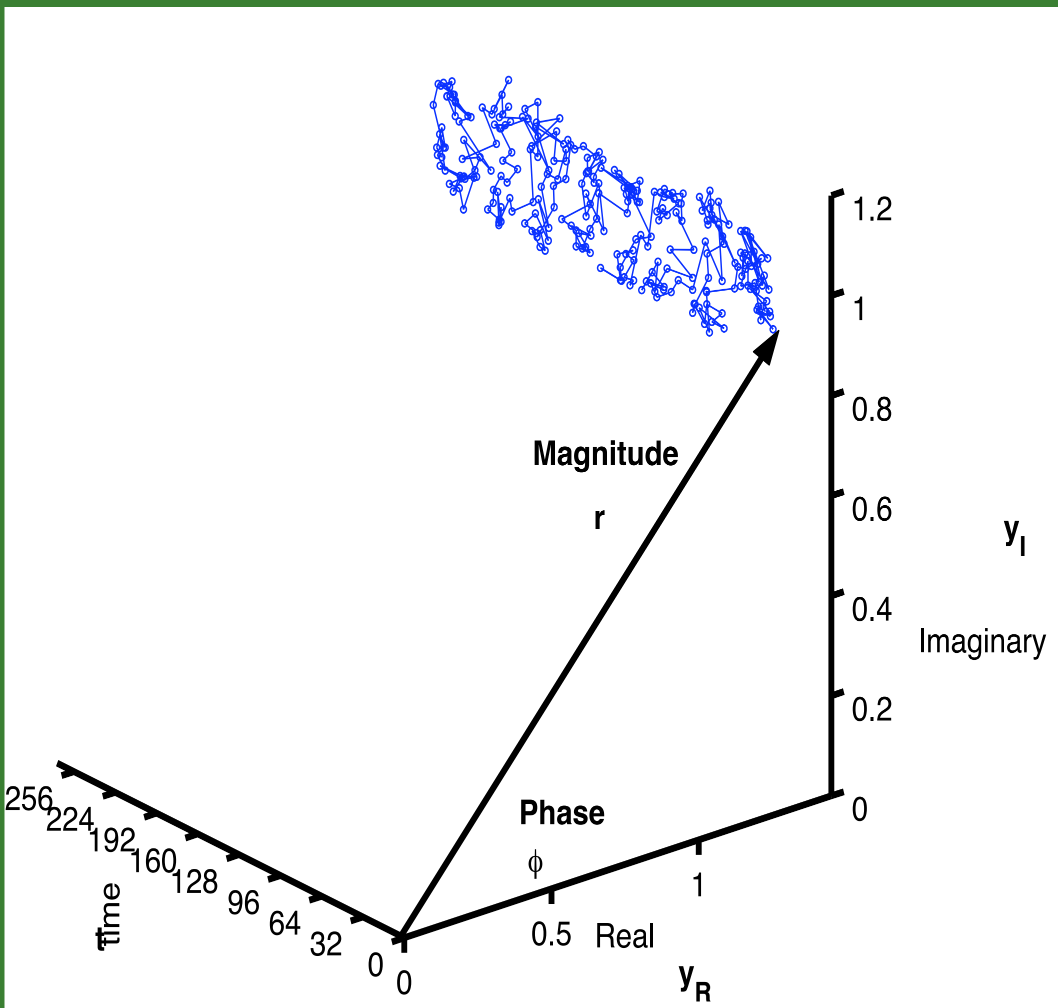
MO time courses only have vector length info!

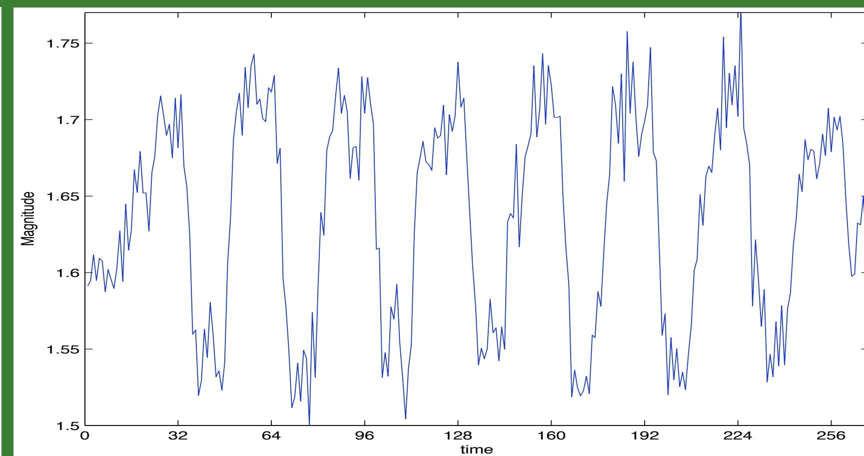
PO time courses only has vector angle info!

Real-Imaginary or Magnitude-Phase time courses have all info!

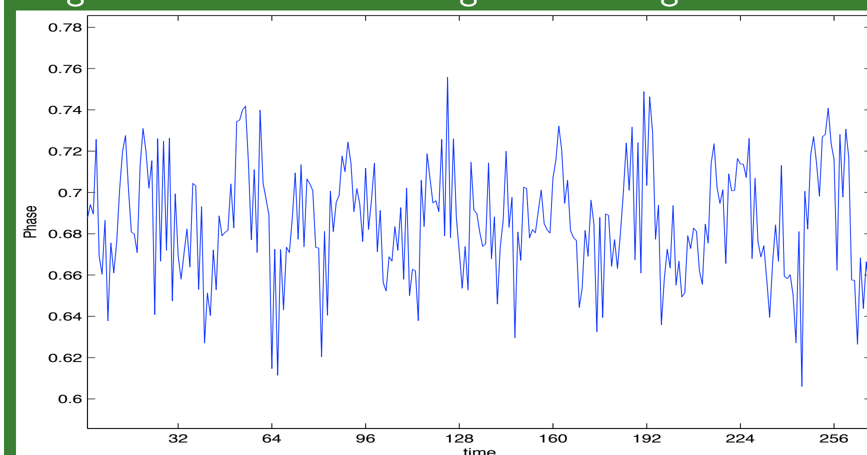
One Dimensional fMRI Time Series

Time series are complex-valued or bivariate with phase coupled means.





Magnitude: Task related magnitude changes!



Phase: Task related phase changes!

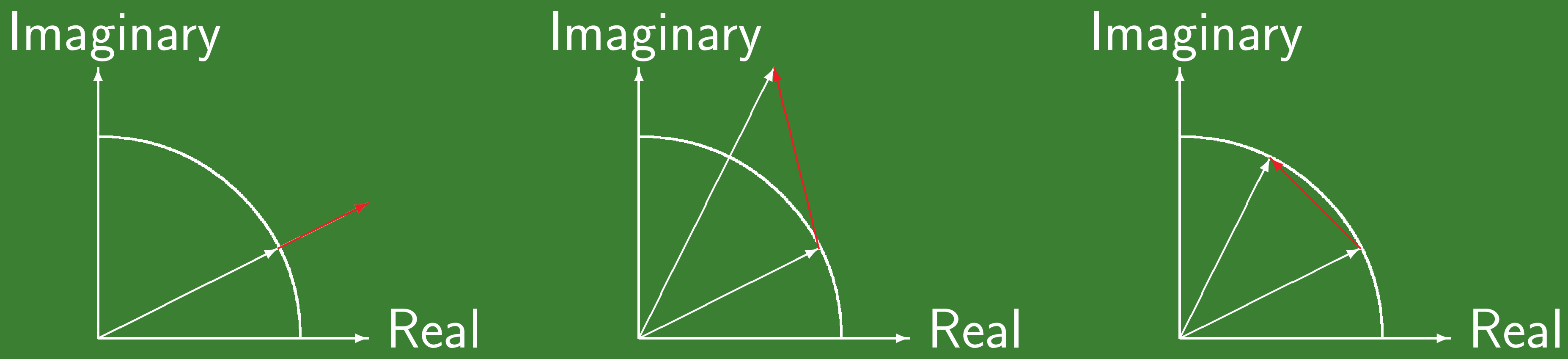
Real-Imaginary or Magnitude-Phase time courses have all info!

Recent work indicates that phase time courses may exhibit TRPCs

Menon, 2002; Hoogenrad et al., 1998; Borduka et al., 1999; Chow et al., 2006;

One Dimensional fMRI Time Series

Block-designed experiment: Off-On-Off-...-On-Off task

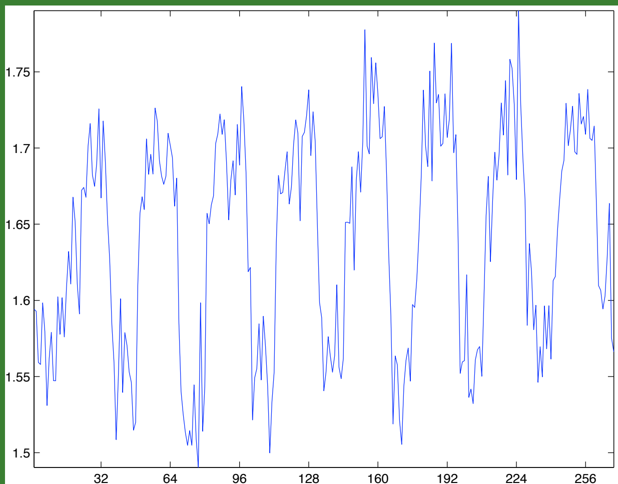


- Complex Magnitude w/ Constant Phase (CP) Activation[1,2]
- Complex Magnitude &/or Phase (CM) Activation[3]
- Real Magnitude-Only (MO/UP) Activation[4,5]
- Real Phase-Only (PO) Activation[6]

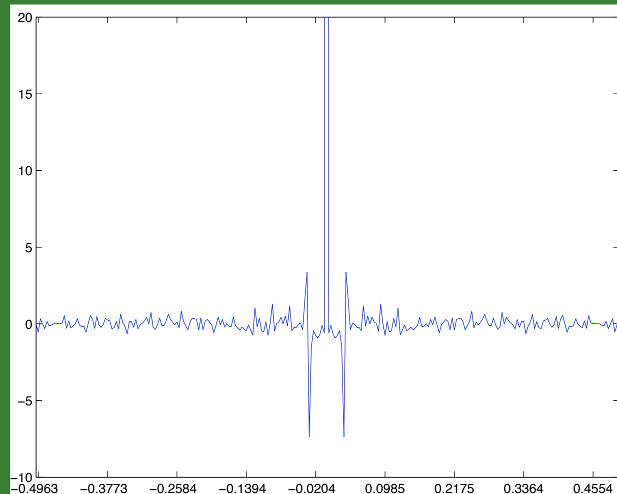
[1]Rowe and Logan: NeuroImage, 23:1078-1092, 2004.
[2]Rowe: NeuroImage 25:1124-1132, 2005a.
[3]Rowe: NeuroImage, 25:1310-1324, 2005b.
[4]Bandettini et al.: Magn Reson Med, 30:161-173, 1993.
[5]Friston et al.: Hum Brain Mapp, 2:189-210, 1995.
[6]Rowe, Meller, & Hoffmann: J Neuro Meth, in press, 2006.

One Dimensional fMRI Time Series

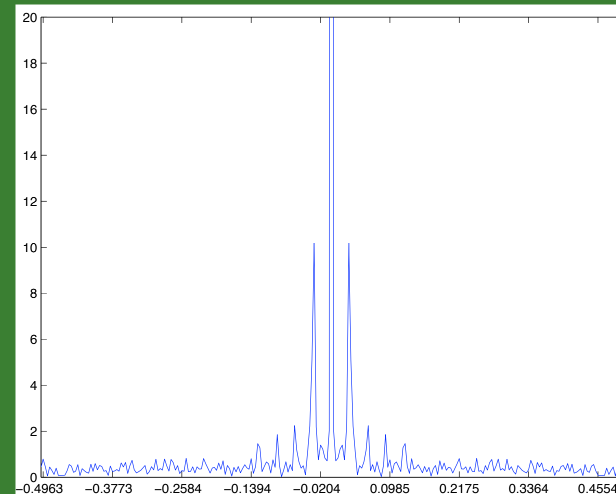
Let's consider the magnitude of the time series and its FT.



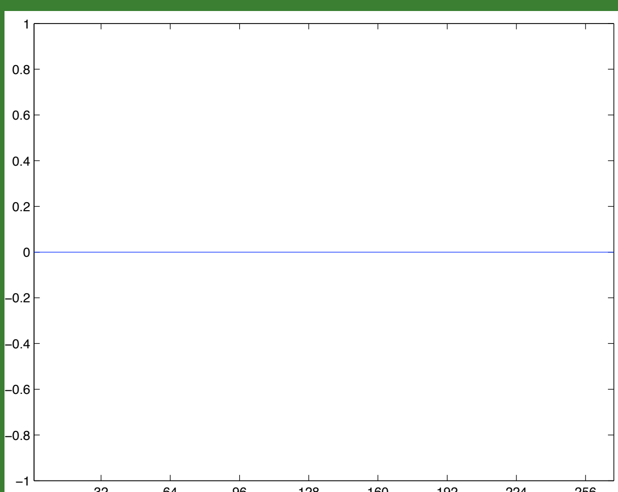
TS Mag.



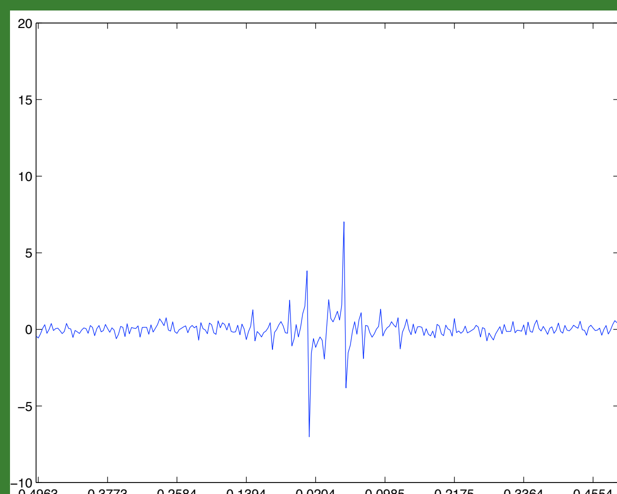
TS FT real



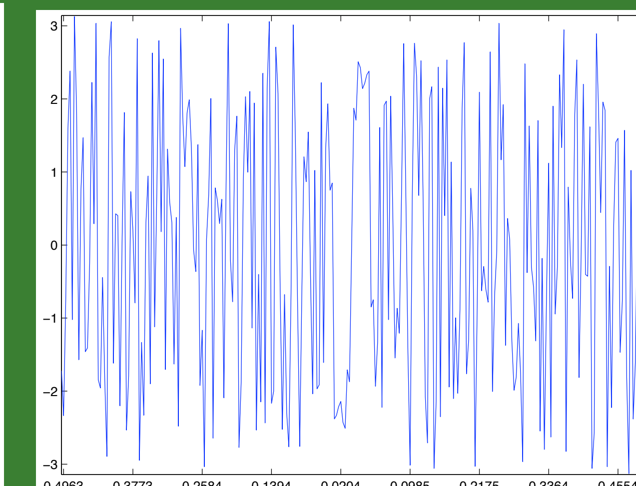
TS FT Mag.



TS Phase



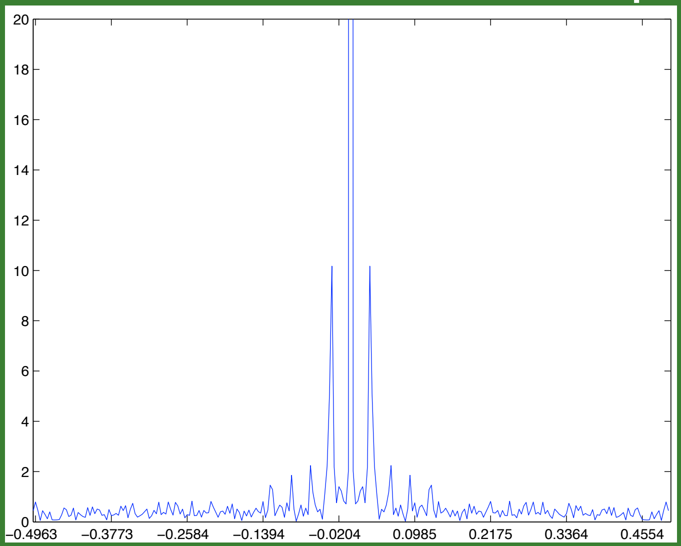
TS FT Imag



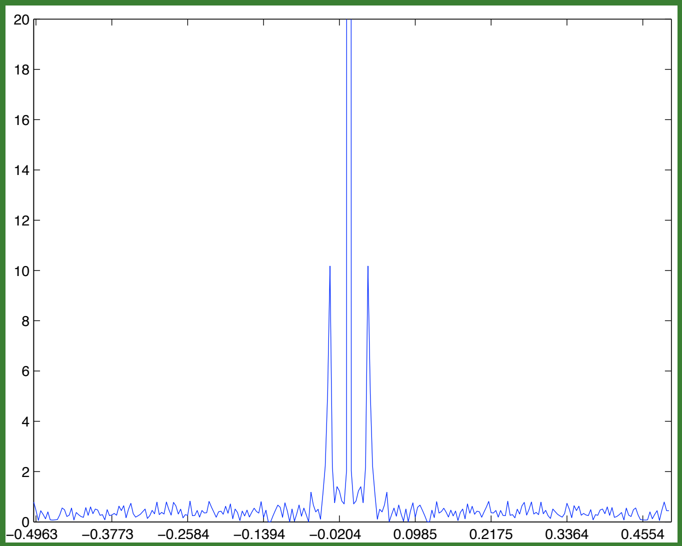
TS FT Phase

One Dimensional fMRI Time Series

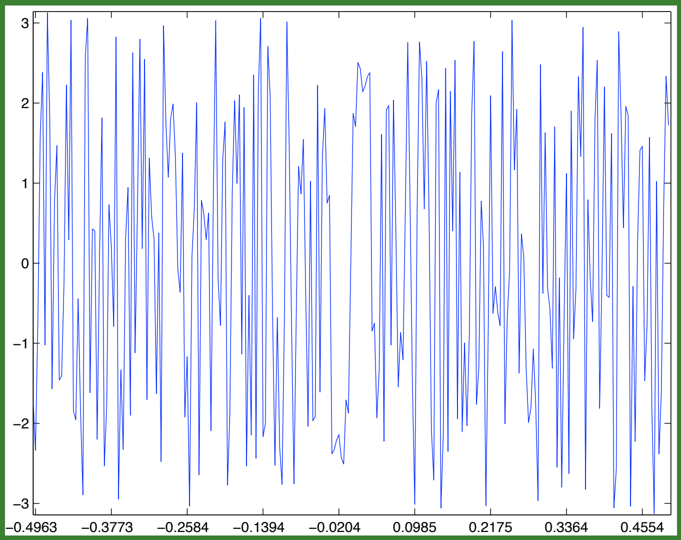
Let's filter some FT frequencies of the time series.



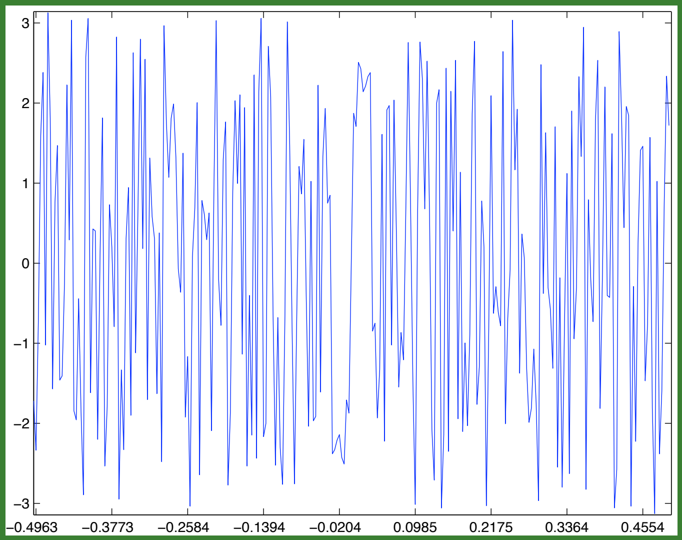
TS FT Mag.



TS FT Mag. filt



TS FT Phase

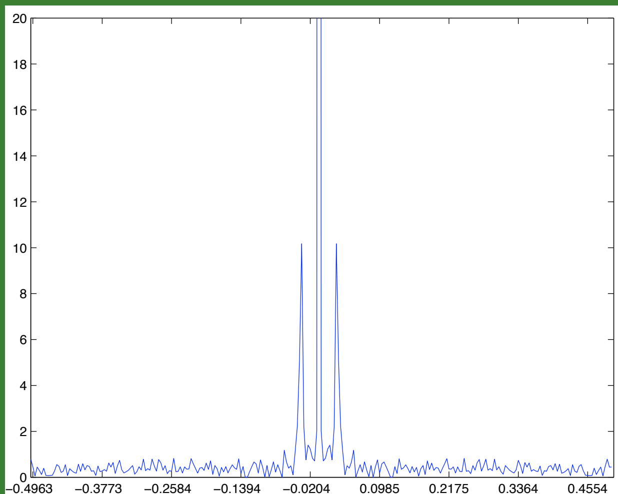


TS FT Phase

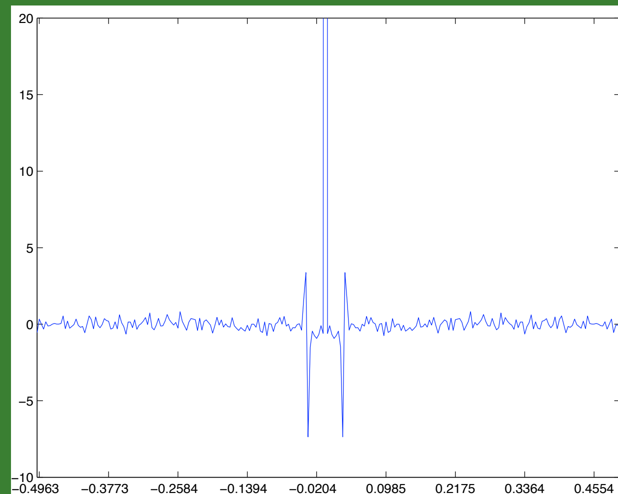
101,102,110,118,152,160,168,169

One Dimensional fMRI Time Series

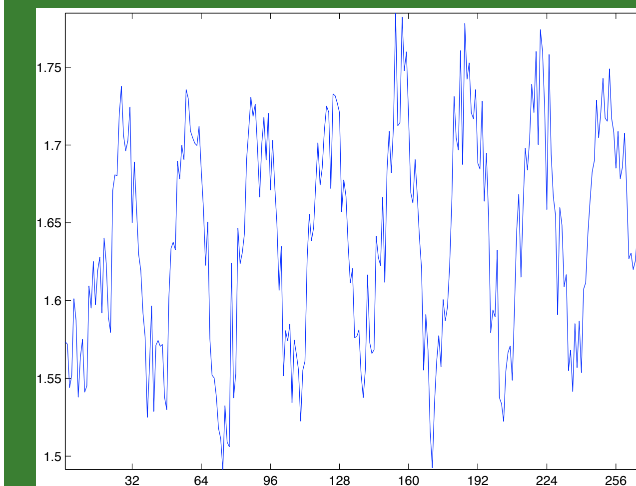
Let's filter some FT frequencies of the time series.



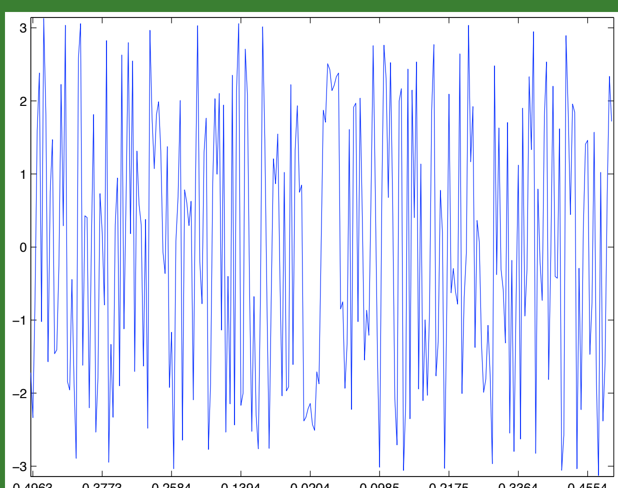
TS FT Mag.



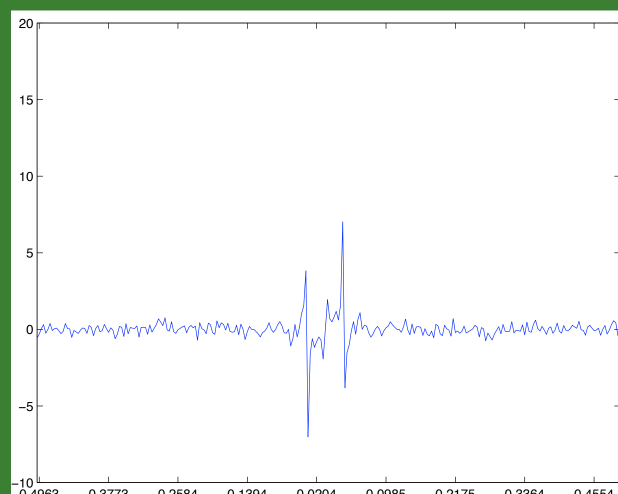
TS FT real



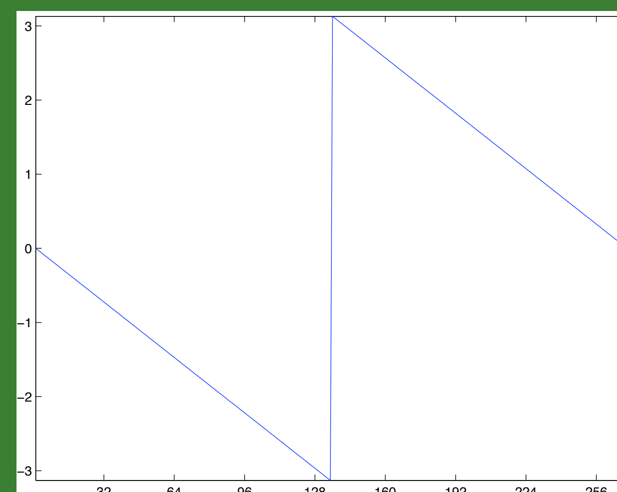
TS Mag.



TS FT Phase



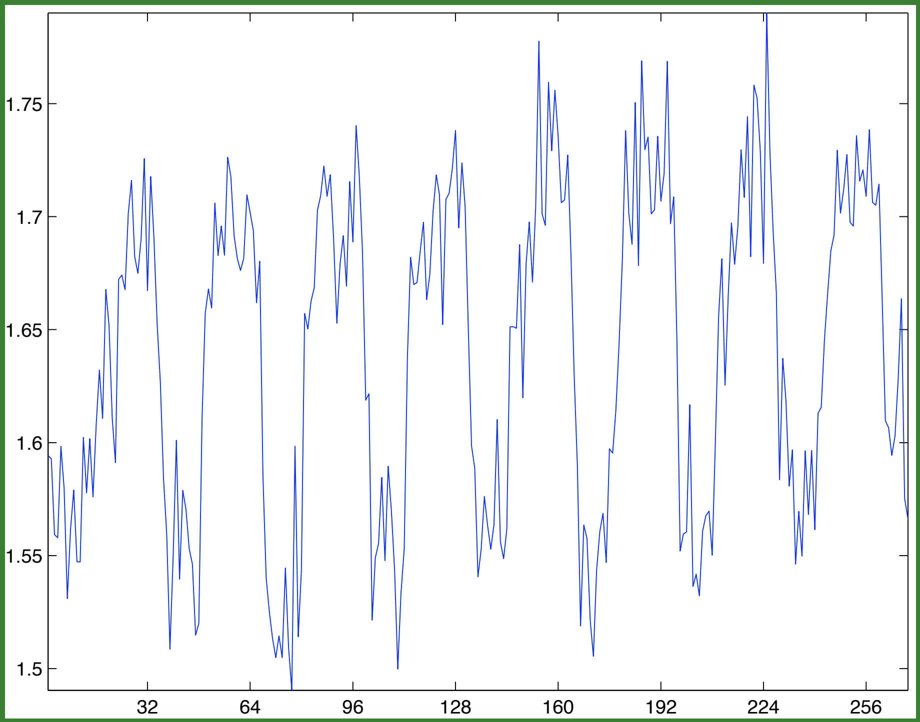
TS FT Imag



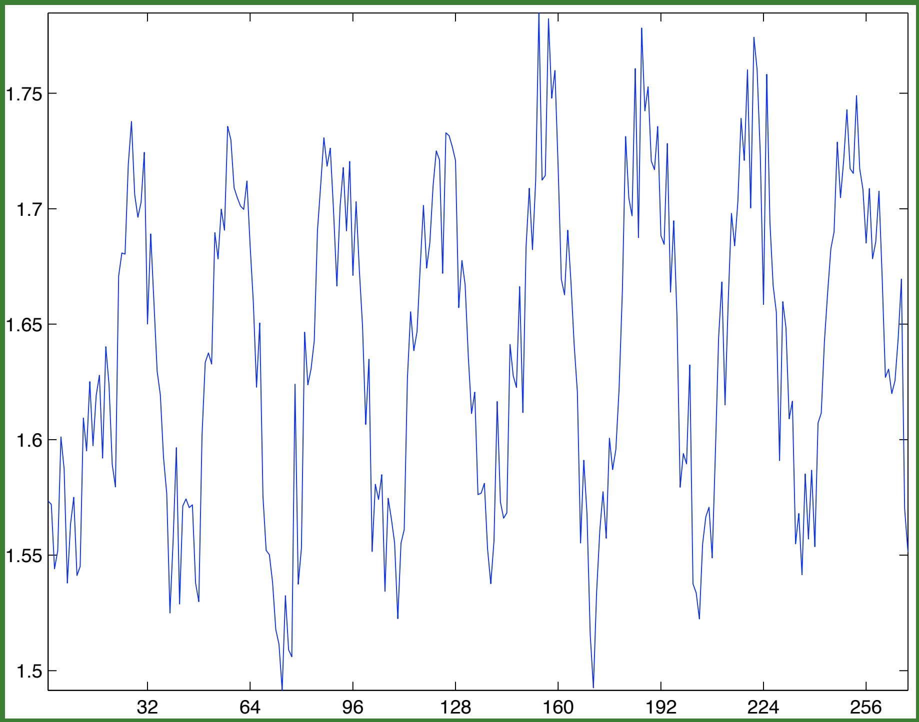
TS Phase

101,102,110,118,152,160,168,169

One Dimensional fMRI Time Series



Original Time Series



Filtered Time Series

This filtering will reduce your residual variance!
A smaller variance means larger activation statistics!
But you have changed the temporal autocorrelation!

FMRI time series statistics

Activation statistic (measure of association) is computed in every voxel.

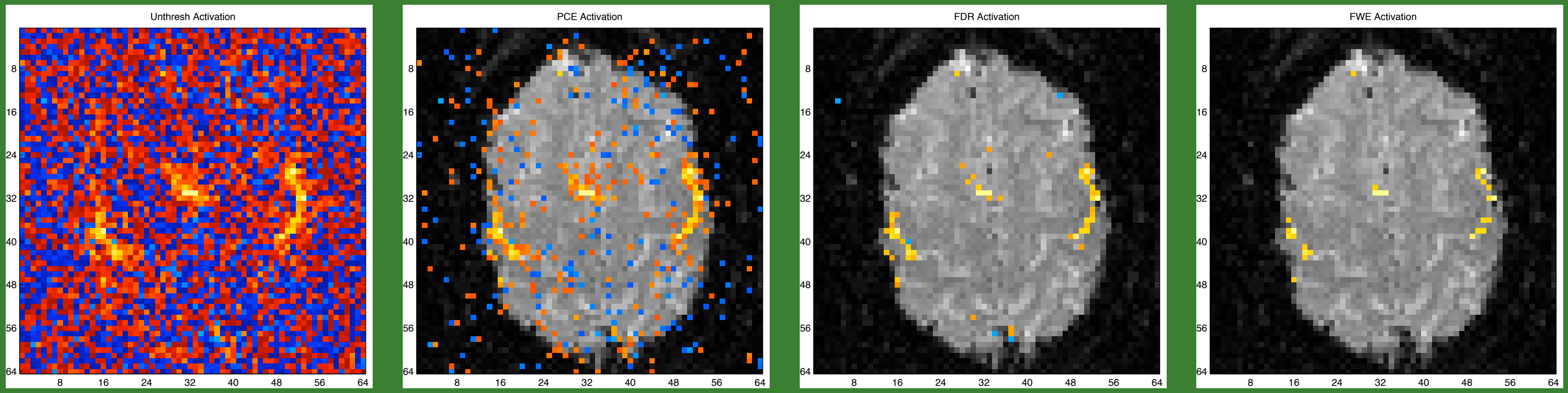
Many ways to compute activation statistics. Magnitude vs. Complex

Activation is another topic all to itself! Happy to return.

Need to separate activation signal from noise!

Thresholding and the multiple comparisons problem.

Thresholding is another topic all to itself!



Summary

➢ Introduction

➢ One Dimensional FT

- Time series constituents and Fourier spectrum.

➢ Two Dimensional FT

- An image constituents and Fourier spectrum.

➢ Two Dimensional MR Image Formation

- k-space and MR Image Reconstruction.

➢ One Dimensional fMRI time series

- fMRI time series Fourier spectrum and filtering.

➢ FMRI time series statistics

Thank You.