Complex-Valued Voxel Thresholding Increases Image Contrast in SWI

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Synopsis

It is often desirable to threshold signal plus noise voxels from pure noise voxels. Generally thresholding utilizes only the magnitude of the images. More recently a method has used both the magnitude and phase. This work is an extension and uses the normality of the real and imaginary values with phase coupled means. A statistic is derived that is F-distributed in large samples. In small samples Monte-Carlo critical values can be used. We apply this method to SWI images and show increased image contrast while it is found to be more robust to phase variations from unwanted field inhomogeneity effects.

Introduction

In MRI it is often desirable to threshold voxels that contain signal from tissue along with measurement noise from those that contain purely measurement noise. Generally this thresholding utilizes only the magnitude portion of the images. Recently methods have been developed that utilize both the magnitude and phase for thresholding voxels [1]. This manuscript is an extension of that work and uses the bivariate normality of the real and imaginary values with phase coupled means. A likelihood ratio statistic is derived that simplifies to a more familiar form that is F-distributed in large samples. In small samples, critical values from Monte Carlo simulation can be used to threshold this statistic with the proper Type I and Type II error rates. This method is applied to magnetic resonance susceptibility weighted images (SWI) and shown to produce increased image contrast.

Theory

In a voxel, the observed complex-valued data is described as $y_R=\rho\cos\theta+\varepsilon_R$ and $y_I=\rho\sin\theta+\varepsilon_I$ where y_R and y_I are the measurements for the real and imaginary parts, ε_R and ε_I are the error terms for the real and imaginary parts, while ρ and θ are the population magnitude and phase. Assuming that ε_R and ε_I are normally distributed with a mean of 0 and variance σ^2 [2], the joint probability distribution of the bivariate voxel observation (y_R,y_I) can be found then converted to polar coordinates to find the joint distribution of the observed magnitude and phase (m,φ) [3]. We would like to determine if the observed magnitude and phase in a voxel are signal or if they are noise. Given measurements $(m_1,\varphi_1),\ldots,(m_n,\varphi_n)$ from $p(m,\varphi)$ the likelihood can be determined. Each voxel and its 8 neighbors (n=9) is used to estimate its magnitude and phase with image wrap around. This voxel separation procedure can be achieved by testing H_0: $\rho=0$, $\theta=0$ vs. H_1: $\rho>0$, $\theta\neq0$ with a likelihood ratio test. Under H_0 and H_1 the MLEs are

$$\tilde{\rho}=0,\tilde{\theta}=0,\ \ \tilde{\sigma}^2=\frac{1}{2n}\sum_{i=1}^{n}\left(y_{Ri}^2+y_{Ii}^2\right)$$

and

$$\hat{\rho}=\left[(\bar{y}_R)^2+(\bar{y}_I)^2\right]^{\frac{1}{2}},\ \ \hat{\theta}=\tan^{-1}\left[\sum_{i=1}^{n}y_{Ii}\Big/\sum_{i=1}^{n}y_{Ri}\right],\ \ \hat{\sigma}^2=\frac{1}{2}\sum_{i=1}^{n}\left(y_{Ri}^2+y_{Ii}^2\right)-\frac{1}{2}\left[(\bar{y}_R)^2+(\bar{y}_I)^2\right]\ .$$

A formal statistic can be derived from the likelihood ratio and a statistical hypothesis test performed on the population magnitude and phase.

Applying this procedure here, the test statistic is $F=(x_1/2)/(x_2/2n)$ where $x_1=n[(\bar{y}_R)^2+(\bar{y}_I)^2]/\sigma^2$ and $x^2=[\Sigma y_{Ri}+\Sigma y_{Ii}]/\sigma^2$. Further, one can show that x_1 and x_2 are χ^2 distributed with 2 and 2n degrees of freedom. The test statistic denoted by F is found by dividing these by their degrees of freedom and taking ratio. Since x_1 and x_2 are χ^2 distributed, $E(x_1)=2$, $E(x_2)=2n$, $\text{var}(x_1)=4$, $\text{var}(x_2)=4n$. The covariance can be found as $\text{cov}(x_1,x_2)= E(x_1\cdot x_2)-E(x_1)E(x_2)$, where $E(x_1\cdot x_2)=4n+4$. The correlation between x_1 and x_2 is now $\text{cor}(x_1,x_2)= \text{cov}(x_1,x_2)/(\text{var}(x_1)\text{var}(x_2))^{½}$, which is $1/\sqrt{n}$. This correlation tends to zero in large samples, the F statistic becomes F distributed and F critical values can be used. However, critical values for small n can be achieved by way of Monte Carlo simulation.

Susceptibility weighted imaging (SWI) MRI data [4] is used to test the noise removal procedure both in magnitude and phase. A SWI brain volume was acquired on a 3 T Siemens Trio with a matrix size of 352×512, FOV of 176 mm × 256 mm, an in-plane resolution of 0.5×0.5 mm^2, TR=26 ms, TE= 15 ms, and flip angle (FA) = 11° [5].

Results

A map of these F statistics is in Fig. 1 and a histogram is in Fig. 2. Table 1 contains significance values α and critical values F_α for n=9. Intermediate values can be interpolated. The F statistic map in Fig. 1 is thresholded for α = 0.05 and α = 0.05/512/352 from Table 2. A zero/one mask from thresholded F statistics is applied to the original magnitude and phase images. Thresholded original images are presented in Fig. 3. The magnitude and phase of thresholded voxels are set to zero (for display they are set to $-\pi$). The first two images of Fig. 3 are thresholded at F=2.8102 while the second two are thresholded at F=7.5869. A vertical line is drawn in Fig. 2 for these threshold values. In Fig. 2, it is obvious that there are F statistics from two different distributions. The first distribution is for large F statistic values that tapers for smaller F statistic values. The second distribution is for smaller F statistic values that tapers for larger F statistic values.

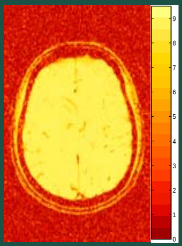

Fig. 1: F statistic map.

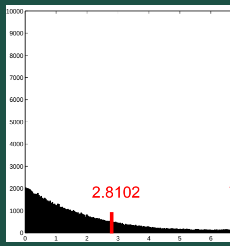

Fig. 2: F statistic histogram.

α	F_α
0.05	2.8102
0.01	3.9377
0.001	5.1991
0.0001	6.1512
0.00001	6.8678
0.000001	7.3911
0.05/256/256	7.5627
0.05/512/352	7.5869
0.05/512/384	7.7051
0.05/512/512	7.7575

Table 1: Significance & critical values.

Magnitude α=0.05. Phase α=0.05. Magnitude α=0.05 Bonf. Phase α=0.05 Bonf.

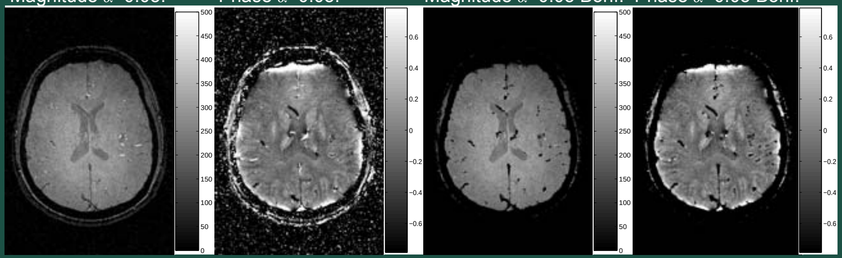

Figure 3: Thresholded magnitude and phase images.

Note in Fig. 3 that as the false positive rate decreases, the number of voxels outside of the head decreases and more voxels within the head are also eliminated. This is due to the Type I and Type II error rates. It is apparent that the magnitude α=0.05 Bonferroni image in Fig. 3 shows similar anatomy to the phase α=0.05 Bonferroni image in Fig. 3 indicating similar biological information.

Discussion

A magnitude and phase statistical thresholding procedure based upon a likelihood ratio test was presented. It was shown through Monte Carlo simulation that this method operates according to its theoretical statistical properties in terms of both false positives and false negatives. This statistical thresholding method was successfully applied to real human SWI data and shown to produce increased image contrast by eliminating false positives. It can also be seen that this new approach is more robust to variations in phase caused by unwanted field inhomogeneity effects.

References

[1] Pandian D, et al. 2008 JMRI 28:727-35. [2] Macovski A 1998 MRI 38:494-7. [3] Rowe DB, Logan BR 2004 NIMG 23:1078-92. [4] Haacke EM et al. 2004 MRM 52:612-8. [5] Haacke EM et al. 1999 John Wiley.

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