Complex-Valued Voxel Thresholding Increases Image Contrast in SWI

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Introduction: In MRI it is often desirable to threshold voxels that contain signal from tissue along with measurement noise from those that contain purely measurement noise. Generally this thresholding utilizes only the magnitude portion of the images. Recently methods have been developed that utilize both the magnitude and phase for thresholding voxels [1]. This manuscript is an extension of that work and uses the bivariate normality of the real and imaginary values with phase coupled means. A likelihood ratio statistic is derived that simplifies to a more familiar form that is F-distributed in large samples. In small samples, critical values from Monte Carlo simulation can be used to threshold this statistic with the proper Type I and Type II error rates. This method is applied to magnetic resonance susceptibility weighted images (SWI) and shown to produce increased image contrast.

Theory: In a voxel, the observed complex-valued data can be described as $y_R=\rho\cos\theta+\varepsilon_R$ and $y_I=\rho\sin\theta+\varepsilon_I$ where y_R and y_I are the measurements for the real and imaginary parts, ε_R and ε_I are the error terms for the real and imaginary parts, while ρ and θ are the population magnitude and phase. Assuming that ε_R and ε_I are normally distributed with a mean of 0 and variance σ^2 [2], the joint probability distribution of the bivariate voxel observation (y_R,y_I) can be found then converted to polar coordinates to find the joint distribution of the observed magnitude and phase (m,φ) [3]. We would like to determine if the observed magnitude and phase in a voxel are signal or if they are noise. Given measurements $(m_1,\varphi_1),\ldots,(m_n,\varphi_n)$ from $p(m,\varphi)$ the likelihood can be determined. Each voxel and its 8 neighbors (n=9) is used to estimate its magnitude and phase with image wrap around. This voxel separation procedure can be achieved by testing H_0: $\rho=0$, $\theta=0$ vs H_1: $\rho>0$, $\theta\neq0$ with a likelihood ratio test. Under H_0 and H_1 the MLEs are

$$\tilde{\rho}=0,\tilde{\theta}=0,\ \tilde{\sigma}^2=\frac{1}{2n}\sum_{i=1}^{n}\left(y_{Ri}^2+y_{Ii}^2\right) \text{ and } \hat{\rho}=\left[(\bar{y}_R)^2+(\bar{y}_I)^2\right]^{\frac{1}{2}},\ \hat{\theta}=\tan^{-1}\left[\sum_{i=1}^{n}y_{Ii}\Big/\sum_{i=1}^{n}y_{Ri}\right],\ \hat{\sigma}^2=\frac{1}{2}\sum_{i=1}^{n}\left(y_{Ri}^2+y_{Ii}^2\right)-\frac{1}{2}\left[(\bar{y}_R)^2+(\bar{y}_I)^2\right].$$

A formal statistic can be derived from the likelihood ratio and a statistical hypothesis test performed on the population magnitude and phase parameters. Applying this procedure here, the test statistic is $F=(x_1/2)/(x_2/2n)$ where $x_1=n[(\bar{y}_R)^2+(\bar{y}_I)^2]/\sigma^2$ and $x_2=[\Sigma y_{Ri}+\Sigma y_{Ii}]/\sigma^2$. Further, one can show that x_1 and x_2 are χ^2 distributed with 2 and $2n$ degrees of freedom. The test statistic denoted by F is found by dividing these by their degrees of freedom and taking ratio. Since x_1 and x_2 are χ^2 distributed, $E(x_1)=2$, $E(x_2)=2n$, $\mathrm{var}(x_1)=4$, $\mathrm{var}(x_2)=4n$. The covariance can be found as $\mathrm{cov}(x_1,x_2)=E(x_1\cdot x_2)-E(x_1)E(x_2)$, where $E(x_1\cdot x_2)=4n+4$. The correlation between x_1 and x_2 is now $\mathrm{cor}(x_1,x_2)=\mathrm{cov}(x_1,x_2)/(\mathrm{var}(x_1)\mathrm{var}(x_2))^{1/2}$, which is $1/\sqrt{n}$. This correlation tends to zero in large samples, the F statistic becomes F distributed and F critical values can be used. However, critical values for small n can be achieved by way of Monte Carlo simulation.

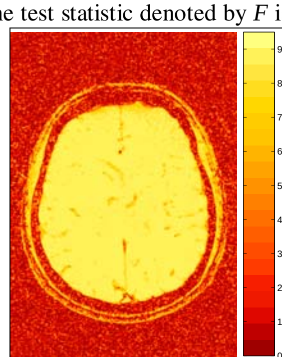

Figure 1: F statistic map.

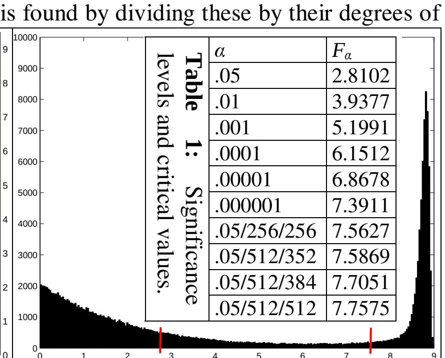

Table 1: Significance levels and critical values.

α	F_α
.05	2.8102
.01	3.9377
.001	5.1991
.0001	6.1512
.00001	6.8678
.000001	7.3911
.05/256/256	7.5627
.05/512/352	7.5869
.05/512/384	7.7051
.05/512/512	7.7575

Figure 2: Computed F statistic histogram.

Results: Susceptibility weighted imaging (SWI) MRI data [4] is used to test the noise removal procedure both in magnitude and phase. A SWI brain volume was acquired on a 3T Siemens Trio with a matrix size of 352×512, FOV of 176 mm × 256 mm, an in-plane resolution of 0.5×0.5 mm^2, TR=26 ms, TE= 15 ms, and flip angle (FA) = 11^o [5].

The map of these F statistics is presented in Fig. 1 and a histogram is presented in Fig. 2. Inset in Figure 2 is Table 1 containing significance values α and corresponding critical values F_{α}. for n=9. The F statistic map in Fig. 1 is thresholded with the critical values in Table 2 for α = 0.05, and 0.05/512/352. A zero-one mask is produced from the thresholded F statistics then applied to the original magnitude and phase images. Thresholded original observed images are presented in Fig. 3. The magnitude and phase of thresholded voxels are set to zero but for display the thresholded phase voxel values are set to $-\pi$. In he top row of Fig. 3 the images are thresholded at F=2.8102 while for the bottom row the images are thresholded at F=7.7575. A vertical line can be drawn in Fig. 2 for each of these threshold values. In Fig. 2, it is obvious that there are F statistic values from two different distributions. The first distribution is for large values of the F statistic (corresponding to large magnitude and or phase voxels) that tapers for smaller F statistic values. The second distribution is on the smaller side for smaller F statistic values (corresponding to small magnitude and or phase voxels) that tapers for larger F statistic values. Note in Fig. 3 that as the false positive rate decreases from the top row to the bottom row, the number of voxels outside of the head decreases and more voxels within the head are also eliminated. This phenomenon is due to the relationship between Type I and Type II error rates. It is apparent that the magnitude image in Fig. 3 bottom row shows similar anatomy to the phase image in Fig. 3 bottom row indicating similar biological information.

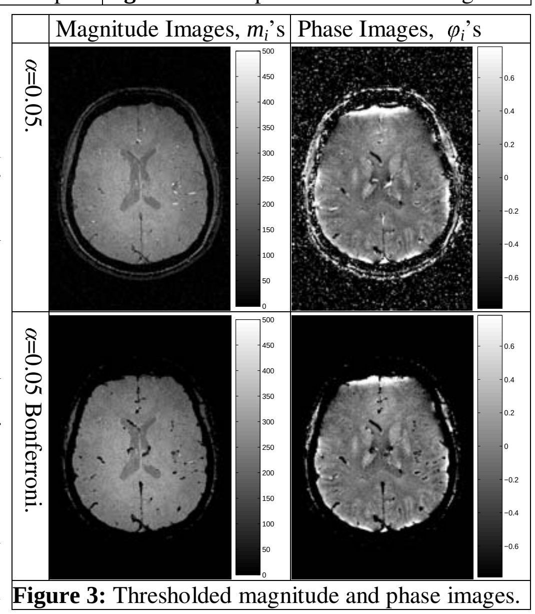

Figure 3: Thresholded magnitude and phase images.

Discussion: A magnitude and phase statistical thresholding procedure based upon a likelihood ratio test was presented. It was shown through Monte Carlo simulation that this method operates according to its theoretical statistical properties in terms of both false positives and false negatives. This statistical thresholding method was successfully applied to real human SWI data and shown to produce increased image contrast by eliminating false positives. It can also be seen that this new approach is more robust to variations in phase caused by unwanted field inhomogeneity effects.

References: 1. Pandian D, et al., 2008. JMRI 28:727-735. **2.** Macovski A, 1998. MRI 38:494-497. **3.** Rowe DB, Logan BR, 2004. NIMG 23:1078-1092. **4.** Haacke EM, et al., 2004. MRM 52:612-618. **5.** Haacke EM, et al., 1999. John Wiley and Sons.

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