

Processing Induced Voxel Correlation in SENSE



FMRI Via the AMMUST Framework

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Synopsis

Many preprocessing steps are applied prior to fMRI statistical analysis and their effects ignored. Nencka et al. (2009) presented the AMMUST- k model to examine preprocessing and reconstruction induced correlation. We extend the AMMUST- k model to include the SENSE multi coil image reconstruction method. We find induced correlations between a voxel and its unfolded counterparts. This body of work has null hypothesis baseline fMRI implications.

Introduction

In fMRI, images of objects are Fourier encoded [1] similar to the Fourier transform in Figure 1,

$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) * (V_R + iV_I) * (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T = (F_R + iF_I) \quad (1)$$

Figure 1. Fourier encoded image of an objects.

the image of an object is reconstructed via the inverse Fourier transform as in Figure 2.

$$(\Omega_{yR} + i\Omega_{yI}) * (F_R + iF_I) * (\Omega_{xR} + i\Omega_{xI})^T = (V_R + iV_I) \quad (2)$$

Figure 2. Image reconstruction via inverse FT.

the inverse FT process in Figure 2 can be represented with a real-valued isomorphism [2]

$$v = O_I * \Omega_a * O_k * f \quad (3)$$

Figure 3. Alternative image reconstruction via inverse FT.

as in Figure 3 when O_k and O_I are identity matrices and $\Omega_a = \Omega$ however preprocessing in k -space can be performed with O_k and in image space with O_I and adjusting for T_2^* and $-\Delta B_0$ through Ω_a . It was shown that the reconstructed voxels covariance matrix Σ can be represented as

$$\text{cov}(v) = (O_I \Omega_a O_k) \Gamma (O_k^T \Omega_a^T O_I^T) = \Sigma \quad (4)$$

where Γ is the covariance matrix for the spatial frequencies [3]. With $\Gamma = I$, it was shown that preprocessing and reconstruction operations can induce spatial correlation between voxels [3].

Methods

Equation 3 is generalized to include SENSE multi coil image reconstruction

$$v_{SE} = (S^H \Psi_{SE}^{-1} S)^{-1} S^H \Psi_{SE}^{-1} a \quad (5)$$

where in a given voxel, S_C is the complex-valued coil sensitivity, Ψ_C is the complex-valued noise covariance matrix [5], and a_C is the vector of complex-valued aliased voxel measurements [4]. This generalization is

$$v = P_2 U P_1 O_I \Omega_a O_k f \quad (6)$$

Figure 4. SENSE Image reconstruction with processing.

where f_j is the aliased spatial frequency vector for coil j , O_{kj} the operations on aliased spatial frequency vector for coil j , Ω_{aj} the adjusted reconstruction operator for aliased spatial frequency vector for coil j , O_{Ij} the image space operations on the reconstructed image vector from the aliased spatial frequency vector for coil j , P_1 is a permutation that reorders values from by coil image to by voxel, u_q denotes an isomorphism matrix representation for a SENSE reconstruction of voxel q , p is the total number of voxels, P_2 is a permutation that reorders values from by unaliased voxel to unaliased image, and O_I are the image space operations on the combined reconstructed image vector. If Equation 6 is written as $v = Of$, then the covariance between voxels is $\Sigma = OO^T$, with an identity spatial frequency correlation. The correlation matrix R can be found.

Results

A true noiseless vector of $n_c=4$ aliased image spatial frequencies was generated for a 96×96 Shepp-Logan phantom image scaled by 50 with an AP reduction of $R=3$. The coil covariance matrix is $\text{real}(\Psi_C) = \text{imag}(\Psi_C) = [1, \rho, \rho^2, \rho; \rho, 1, \rho, \rho^2; \rho^2, \rho, 1, \rho; \rho, \rho^2, \rho, 1]$ where $\rho=0.33$. In reconstruction, O_{kj} included apodization of each image with a FWHM=2 voxels, $\Omega_{aj} = \Omega$, u_j contains the true sensitivity and coil covariance matrix, while $O_I = I$. The induced correlation image for magnitude squared data for the center voxel is in Figure 5 (left) overlaid upon the reconstructed mean image. It is apparent that there is induced local correlation from apodization and induced correlation of the center voxel of interest with two others regions from the SENSE unfolding procedure. In Figure 5 (right) the voxel standard deviation is displayed with true value being one.

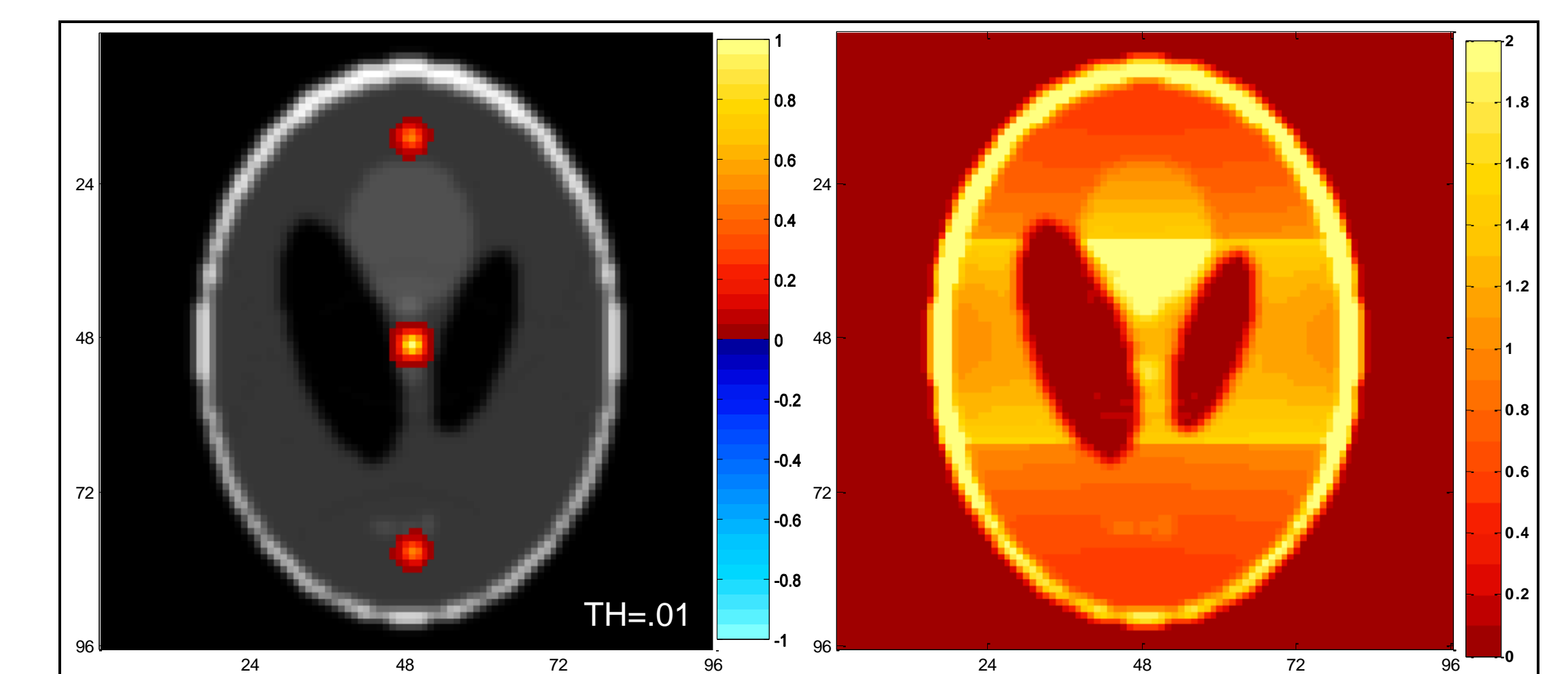


Figure 5. Induced correlation for the center voxel (left) and voxel standard deviations (right).

Discussion

Previous work that theoretically describes induced correlation between image voxels from spatial preprocessing and reconstruction operations has been summarized [3]. This previous work has been extended to include the SENSE multi coil image reconstruction method [4]. This has null hypothesis fMRI connectivity implications as the no connectivity scenario is not for no spatial correlation but is rather for the spatial correlation induced by preprocessing.

References

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