Image Space Correlations Induced by K-Space Processes

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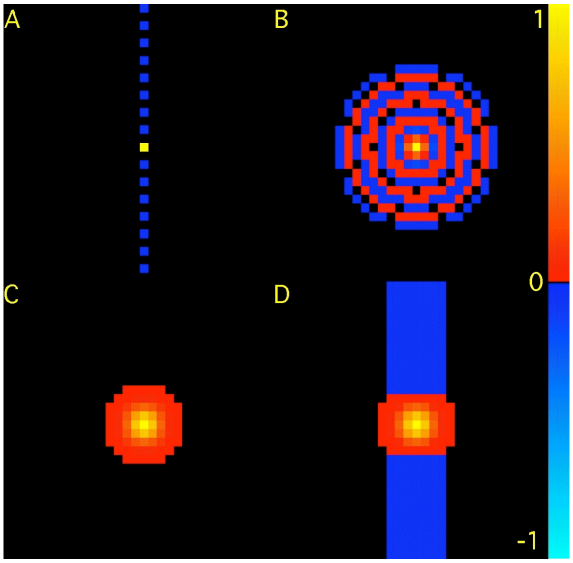
The collected MRI signal and noise, $S'+\eta'$, can be the complex-valued m-by-n 2D Fourier transform of the m-by-n image, ρ'. Complex-valued Fourier reconstruction can be performed through a real-valued isomorphism such that $\rho=\Omega S$.[1] S is a stack of the m rows of n real columns above a stack of the m rows of n imaginary columns, a 2mn x 1 vector. Ω is a Fourier matrix, and ρ is the reconstructed image in the same format as S. With this parameterization, it is possible to consider the effects of k-space operations on image-space correlations. If the acquired signal has covariance matrix Γ, the reconstructed signal has covariance matrix $\Omega\Gamma\Omega^T$, where ^{T} denotes transposition. Any linear k-space operation, O, can be represented as $R=\Omega OS$. Multiple linear steps are performed to reach the pre-processed signal S. The raw signal, Σ, is a vector of alternating real/imaginary points along the EPI trajectory. Multiplication of Σ with a reordering matrix, r, stacks the real above the imaginary observations. Subsequent multiplication with a censoring matrix, C, removes the points during the EPI blips. An alternating row reversal matrix, P, reverses the lines acquired with negative read gradients. Without any pre-processing, the reconstructed data is the result of multiple linear operations: $R=\Omega PCr\Sigma$. Without loss of generality, Σ can be considered as uncorrelated noise with zero mean and identity covariance. The induced image-space covariance from an operation on such data is thus $\Omega OO^T\Omega^T$. If no k-space operation is performed, there is no image-space covariance induced through reconstruction as $\Omega\Omega^T$ is the identity. The results of P, C, r, and alternating line shifts yield negligible image-space correlations. We developed matrix representations to consider the image-space correlations induced by various k-space processes. Common operations yielding interesting results include homodyne reconstruction, apodization with a Tukey window, and Gaussian smoothing. If zeros are inserted as place holders into S where no data are collected, the homodyne matrix, H, yields a homodyne reconstruction of data upon multiplication with S. Apodization and smoothing both involve pointwise multiplication of the k-space data with appropriate windowing functions. Thus, the matrices representing these operations are diagonal with the elements corresponding to the k-space window values. Assuming a matrix dimension of 32, a Tukey window with a radius of 12 pixels and width of 4 pixels, and a Gaussian smoothing kernel FWHM of 1.17 pixels, theoretical image-space correlations were computed. The resulting image-space correlations for the central pixel resulting from these are shown in Figures A, B and C, respectively. The image-space correlation from the three processes performed sequentially is shown in Figure D. Such k-space operations clearly yield non-negligible image-space correlations. Apodization is especially interesting as it is often performed as a necessary step in the reconstruction of non-Cartesian k-space acquisitions.[2] With conclusions based upon image space correlations, one must account for such pre-processing induced correlations. This framework allows the quantification of such correlations.

(1)Rowe, D.B. et al. J. Neur. Meth. 159: 361-369. (2) Bernstein, M.A. et al. Handbook of MRI Pulse Sequences Elsevier, 2004.



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