The Effect of Detrending When Computing Regression Coefficients in Block Design fMRI

S.W. Morgan, D. B. Rowe, Biophysics Research Institute, Medical College of Wisconsin, WI, USA

Abstract: Detrending is a method used when applying linear regression to determine functional brain activation. It is performed to eliminate an overall mean and linear time drift in the signal. However, the resulting regression coefficients are not identical to those computed using a multiple linear regression method. In this study, these two methods are compared in a simulation and in a real fMRI bilateral finger tapping experiment. The result is that the correct determination of the regression coefficients crucially depends upon the chosen reference function.

Introduction: When fitting the fMRI signal to an idealized reference function such as a square wave, the method of detrending plus simple linear regression is often used where an estimated linear trend is subtracted and the difference fit to the reference function. The multiple linear regression alternative (1), (2) takes no such step.

Mathematics: The determination of whether functional activation has occurred is based upon the regression coefficients from the fit of the BOLD signal to an idealized reference function (3), (4). The equation to be fit for the multiple regression technique for each voxel is

$$y = X\beta + \varepsilon. \qquad [1]$$

In Eq. 1, y is a vector containing the observed signal for each of n time points. The design matrix X is n by 3. The design matrix has columns which consist of an n dimensional column of ones, the first n counting numbers centered about their mean, and the n dimensional reference function vector. The matrix containing the first two column vectors is denoted by x_1 and the last column vector (the reference function) by x_2. The vectors β and ε are 3x1 and nx1. They contain the coefficient and error vectors respectively. The multiple regression method estimates the coefficients from Eq. 1 above, giving the estimated coefficient $\beta_{2,m}$, for x_2, by the equation

$$\beta_{2,m} = \{-(x_2'x_2)^{-1}x_2'x_1[x_1'x_1-x_1'x_2(x_2'x_2)^{-1}x_2'x_1]^{-1}x_1'+[x_2'x_2-x_2'x_1(x_1'x_1)^{-1}x_1'x_2]^{-1}x_2'\}y. \qquad [2]$$

The coefficient $\beta_{2,d}$ for the detrend plus simple linear regression method is obtained by first subtracting the linear trend and forming the equation

$$y-x_1\beta_{1,d} = x_2\beta_2+\delta \qquad [3]$$

where δ is another n by 1 error vector, so that the estimated coefficient for the detrend case is

$$\beta_{2,d} = \{(x_2'x_2)^{-1}x_2'-(x_2'x_2)^{-1}x_2'x_1(x_1'x_1)^{-1}x_1'\}y. \qquad [4]$$

It is easily seen that these are not identical.

Simulation: The simulation formulated the design matrix $X=(x_1,x_2)$ where x_1 contained a column of ones and a column of the counting numbers from 1 to 128 centered about their mean of 64.5. The third column, x_2 was a square wave of period 16 time points. The coefficient vector was chosen to be $\beta'=[2,10,3]$. Then error was randomly generated, using a random normal centered about zero. The values for y were found since $X\beta$ and ε were known and detrending followed by simple linear regression and multiple linear regression were performed. The detrend method never found the true coefficient for the reference function. However, if no error was present, the multiple linear regression method found the exact coefficient if it used the same reference function as was used in computing y. If there was error, both methods differed from the true value of the coefficient but not systematically. The detrend plus simple linear regression method always gave the same result (but not necessarily close to the correct one) for a given y, no matter what reference function was used. However, if the reference function used to compute y was not centered on zero, the detrend method never gave the correct coefficient, and was approximately one-half the true value (see Table 1). The linear trend made no difference on the coefficients whether it was centered around zero or not. If the reference function used to compute the coefficient was different than the one used to compute y, the multiple regression method gave an incorrect value for the coefficient. In all cases, the multiple regression coefficient fit to a noncentered reference function was always approximately twice the value of the other coefficients. If the square wave used to compute y used values of -1 and 1, the multiple linear regression coefficient would be approximately 6 for the 0,1 square wave multiple linear regression case. The other coefficients would be approximately 3 for the same β used in Table 1.

	-1,1	**0,1**
Detrending plus Simple Regression	1.5575	1.5575
Multiple Linear Regression	1.5760	3.1519

Table 1. Multiple linear regression and detrending plus simple linear regression coefficients for a simulation computing y (with error) using a 0,1 square wave reference function and whose true coefficient is 3.

FMRI Task: The fMRI task used was bilateral finger tapping at 3 T, using gradient echo EPI. Five axial slices were imaged, with 1.7 mm cubic voxels, and a 96 by 96 matrix with a 16.32 cm field-of-view. TE was 41.6 msec and TR was 1000 msec. The finger tapping task was 20 sec on and 20 sec off per block. The total time of the task was 340 seconds, so finger tapping was off first and off last. The linear trend was centered about zero. This yielded an average difference between the two methods in the reference function coefficient of 0.35% if the reference function was centered around zero. If the square wave used values of 0 and 1, the average difference was 88% (compare to Table 1 for the simulation). This difference does not depend upon whether or not the data was 2D registered or not.

Discussion: Detrending plus simple linear regression will not make significant difference if the data is fit to a reference function centered around 0. However, the coefficient will vary if the reference function is made of 0's and 1's. The true coefficient is guaranteed to be found only if the full regression method is used and if the correct reference function is also used. The detrend plus simple regression method will be consistent but will only give the correct result if the BOLD signal originates from a reference function that is centered about zero. It is not possible to determine whether the reference function centered about zero or not centered around zero should be used. However, since the coefficients are never exactly the same for the multiple linear regression or detrend plus simple linear regression methods, the t-statistics should also be different.

References

1. Press S.J., *Applied multivariate analysis*. Malabar, FL: Robert E. Krieger Publishing, 1982.
2. Myers, R.H. *Classical and modern regression with applications*. Pacific Grove, CA: Duxbury Press, 1990.
3. Bandettini, P.A., Jesmanowicz, A., Wong, E.C., Hyde, J.S., *Magn. Reson. Med.* 30, 161, 1993.
4. Cox, R.W., Jesmanowicz, A., Hyde, J.S., *Magn. Reson. Med.* 33, 230, 1995.