Spatial Correlations Induced During fcMRI and fMRI Preprocessing Decomposed into Second Order Temporal Frequency Bands

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INTRODUCTION

- It is well known that removing noise within an acquired signal, with spatial and temporal operations, and image reconstruction induces non-biological correlations [1,2].
- This study describes the impact of applying processing operators using linear combinations of second order voxel temporal frequencies.
- To illustrate the benefit this framework, assume *voxel a* and *voxel c* are correlated, *voxel b* and *voxel c* are correlated, but *voxel a* and *voxel b* are not correlated. The spatial correlation between *voxel a* and *voxel c* arises from overlapping frequency content between their two temporal frequency spectrums. Similar reasoning is used to discuss the correlation between *voxel b* and *voxel c,* and the lack of correlation between *voxel a* and *voxel b*.
- Before processing *voxel a* and *voxel b* share no overlapping frequency content, after spatial preprocessing the two voxels now have overlapping temporal frequency content, thus a spatial correlation is artificially induced.

THEORY

The complex-valued image, V_C, is reconstructed with $V_C = \Omega_C F_C \Omega_C^T$, where Ω_C is the complex-valued inverse Fourier transform matrix, and F_C are the complex-valued spatial frequencies; V is a real-valued image representation of V_C, and v is a vectorized version of V, ordered by image [3]. A permutation matrix, P, reorders the elements of vector v by voxel,

$$y = Pv \tag{1}$$

and the vector y is Fourier transformed into the frequency domain,

$$f = (I_p \otimes \Omega_T)Pv \tag{2}$$

Spatial Correlation Frequency Representation

The spatial covariance and correlation between any two voxels, j and k, where y_k and y_k, f_j and f_k, correspond two the jth and kth $2n\times 1$ elements in y and f, using Fourier transform identities, are expressed as,

$$\operatorname{cov}(y_j, y_k) = \tilde{f}_{jR}\tilde{f}'_{kR} + \tilde{f}_{jI}\tilde{f}'_{kI} \qquad \operatorname{corr}(y_j, y_k) = \frac{\tilde{f}_{jR}\tilde{f}'_{kR} + \tilde{f}_{jI}\tilde{f}'_{kI}}{\sqrt{(\tilde{f}_{jR}\tilde{f}'_{jR} + \tilde{f}_{jI}\tilde{f}'_{jI})(\tilde{f}_{kR}\tilde{f}'_{kR} + \tilde{f}_{kI}\tilde{f}'_{kI})}} \tag{3,4}$$

The covariance is expanded for all voxels, with an expected value representation, such that the entry in jth row and kth column is the expected value of Eqn. 3

$$\mathbf{E}[\Sigma_{\tilde{f}}] = \frac{1}{2}\begin{bmatrix} \frac{1}{n}\mu'_{\tilde{y}1}\mu_{\tilde{y}1} + tr(\bar{\Omega}_T \Sigma_{\tilde{f}1} \bar{\Omega}'_T) & \cdots & \frac{1}{n}\mu'_{\tilde{y}1}\mu_{\tilde{y}p} + tr(\bar{\Omega}_T \Sigma_{\tilde{f}1,p} \bar{\Omega}'_T) \\ \vdots & \ddots & \vdots \\ \frac{1}{n}\mu'_{\tilde{y}p}\mu_{\tilde{y}1} + tr(\bar{\Omega}_T \Sigma_{\tilde{f}p,1} \bar{\Omega}'_T) & \cdots & \frac{1}{n}\mu'_{\tilde{y}p}\mu_{\tilde{y}p} + tr(\bar{\Omega}_T \Sigma_{\tilde{f}p} \bar{\Omega}'_T) \end{bmatrix} \tag{5}$$

Define $D_{\Sigma\tilde{f}}$ as the diagonal elements of $E[\Sigma_{\tilde{f}}]$, then the spatial correlation is

$$R_{\Sigma\tilde{f}} = D_{\Sigma\tilde{f}}^{-1/2}\mathbf{E}[\Sigma_{\tilde{f}}]D_{\Sigma\tilde{f}}^{-1/2} \tag{6}$$

Operator Application

With the notation used to define Eqn. 2, if each image a spatial smoothing operator with a Gaussian kernel, S_m

$$\tilde{f} = (I_p \otimes \bar{\Omega}_T)P(I_{2n} \otimes S_m)\tilde{v} \tag{7}$$

The operations applied to the temporal frequencies are $O_f = (I_{2n} \otimes S_m \Omega_p)$ such that each processed voxel time series is represented

$$\tilde{v}_s = P(I_{2n} \otimes S_m \Omega_p)\tilde{f} = PO_f\tilde{f} \tag{8}$$

and the $2pn\times 2pn$ spatiotemporal covariance matrix is defined as

$$\mathbf{cov}[\tilde{f}] = \Sigma_{\tilde{f}}, \tag{9}$$

and the covariance matrix with processing operators is defined

$$\mathbf{cov}[\tilde{f}_S] = O_f \Sigma_{\tilde{f}} O'_f. \tag{10}$$

Frequency Bands

In fcMRI, temporal frequencies < 0.8 Hz are of interest. To understand the contribution each temporal frequency band yields to spatial correlation, consider the notation,

$$r_b = \frac{\tilde{f}_{jR_b}\tilde{f}'_{kR_b} + \tilde{f}_{jI_b}\tilde{f}'_{kR_b}}{\sqrt{\sum_{i=1}^{p_b}(\tilde{f}_{jR_i}\tilde{f}_{jR_i} + \tilde{f}_{jI_i}\tilde{f}_{jI_i})\sum_{i=1}^{p_b}(\tilde{f}_{kR_i}\tilde{f}_{kR_i} + \tilde{f}_{kI_i}\tilde{f}_{kI_i})}} \tag{11}$$

If p_b is the total number of bands, the spatial covariance matrix can be written as a summation of covariance of each band

$$\mathbf{cov}[\tilde{f}] = \Sigma_{\tilde{f}} = \Sigma_{\tilde{f}_1} + \cdots + \Sigma_{\tilde{f}_{pb}} \tag{12}$$

and the spatial covariance for the processed images are represented

$$\mathbf{cov}[\tilde{f}_S] = O_f(\Sigma_{\tilde{f}_1} + \cdots + \Sigma_{\tilde{f}_{pb}})O'_f. \tag{13}$$

The diagonal of the $p\times p$ processed spatial covariance matrix, $O_f \Sigma_{\tilde{f}} O'_f$, is defined as $D_{\Sigma\tilde{f}}$, and the spatial covariance matrix can be represented in terms of processed frequency bands as

$$R_b = D_{\Sigma\tilde{f}_S}^{-1/2} O_f(\Sigma_{\tilde{f}_1} + \cdots + \Sigma_{\tilde{f}_{pb}})O'_f D_{\Sigma\tilde{f}_S}^{-1/2} \tag{14}$$

METHODS

Dividing fMRI and fcMRI data sets into physiologically meaningful bands with voxels chosen to demonstrate the advantage of the described framework.

Scenario 1: fcMRI

- Voxels are chosen such that voxel 1 & 2 are neighbors, voxel 2 & voxel 3 have no initial regional or functional relationship, and voxel 1 & voxel 3 are initially functionally related.
- Bands (Hz): 0.0007-0.0326, 0.0340-0.0660, 0.0674-0.0993, 0.1007-0.2326, 0.2340-0.3660, 0.3674-0.4993

Scenario 2: fMRI

- Voxels are chosen such that voxel 4 & 5 are neighbors, voxel 5 & voxel 6 have no initial regional or functional relationship, and voxel 4 & voxel 6 are initially functionally related.
- Bands (Hz): 0.0009-0.0493, 0.0507- 0.0993, 0.1007-0.1993, 0.2007-0.2993, 0.3007-0.3993, 0.4007-0.4993

RESULTS & DISCUSSION

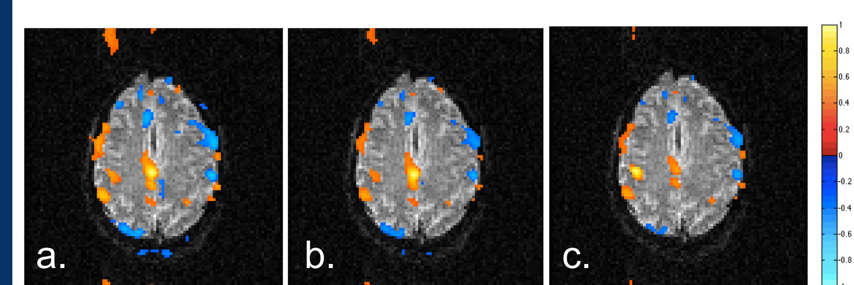

Figure 1. fcMRI total correlation maps for voxel 1 (a.), voxel 2 (b.), voxel 3 (c.).

Table 1. *Scenario 1: Spatial Correlation By Band*

	Voxel 1 & 2		Voxel 2 & 3		Voxel 1 & 3	
	Before	After	Before	After	Before	After
B1	0.0112	0.1587	-0.0162	0.1150	0.0516	0.1333
B2	-0.0128	0.1341	-0.0033	0.0866	0.0588	0.0992
B3	-0.0033	0.0779	-0.0069	0.0558	0.0353	0.0660
B4	-0.0051	0.2185	-0.0023	0.1116	0.0729	0.1218
B5	-0.0303	0.1723	-0.0039	0.0662	0.0476	0.0857
B6	0.0101	0.1569	-0.0132	0.0475	0.0450	0.0511
Total	**-0.0303**	**0.9183**	**-0.0458**	**0.4827**	**0.3111**	**0.5571**

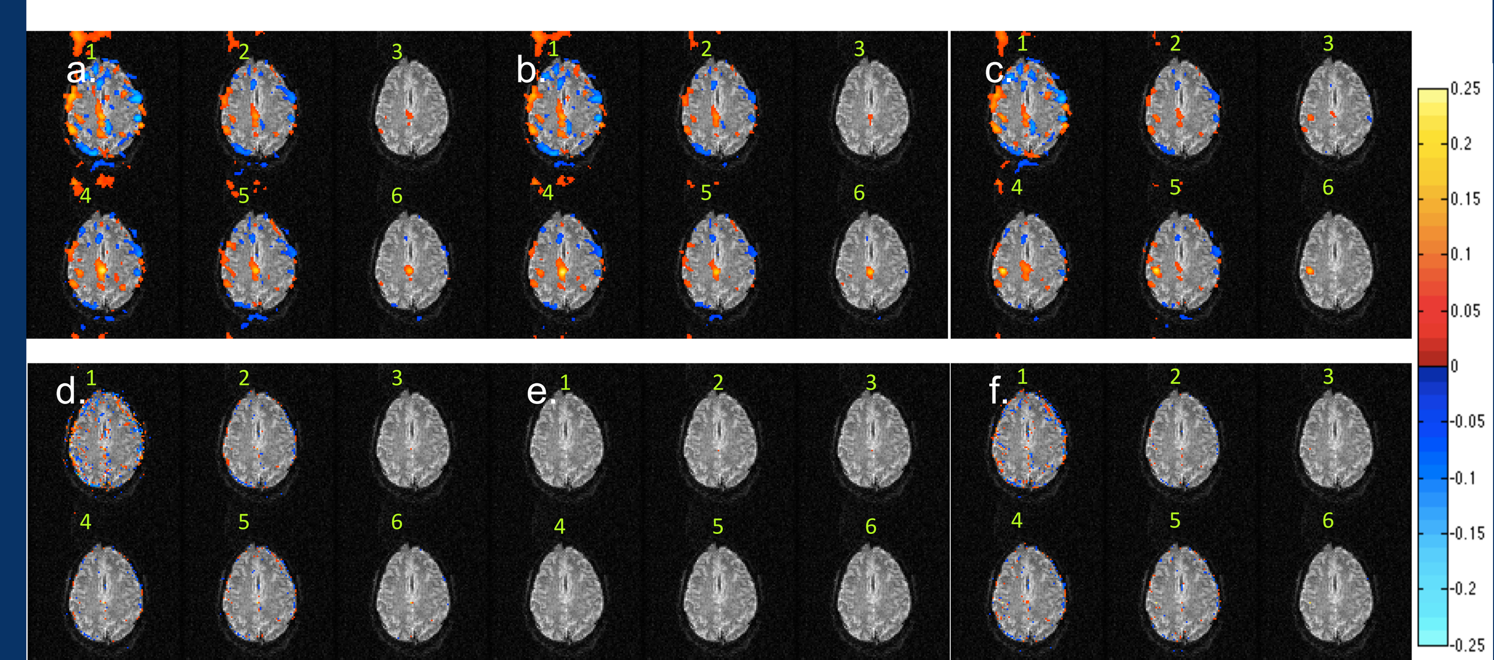

Figure 2. fcMRI correlation maps based on Eqn. 14 by band for after and before applying spatial smoothing for voxel 1 (a.) and (d.), voxel 2 (b.) and (e.), voxel 3 (c.) and (f.), respectively.

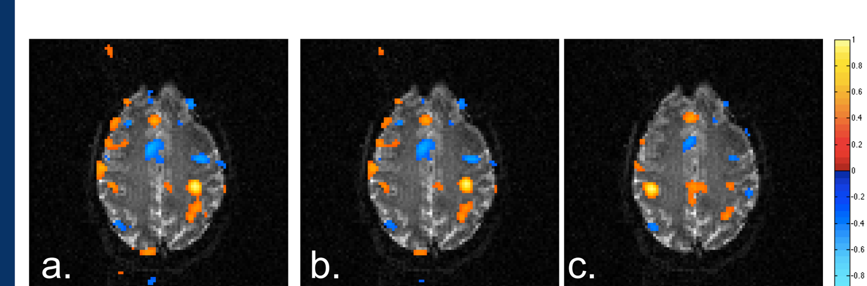

Figure 3. fMRI total correlation maps for voxel 4 (a.), voxel 5 (b.), voxel 6 (c.).

Table 2. *Scenario 2: Spatial Correlation By Band*

	Voxel 4 & 5		Voxel 5 & 6		Voxel 4 & 6	
	Before	After	Before	After	Before	After
B1	0.0696	0.2142	0.0451	0.1306	0.1119	0.1303
B2	0.0302	0.1496	0.0339	0.0825	0.0492	0.0878
B3	0.0342	0.1596	0.0291	0.0773	0.0400	0.0766
B4	0.0158	0.1380	0.0094	0.0524	0.0326	0.0449
B5	0.0419	0.1399	0.0316	0.0524	0.0405	0.0537
B6	0.0133	0.1221	0.0207	0.0510	0.0352	0.0597
Total	**0.2051**	**0.9234**	**0.1698**	**0.4463**	**0.3094**	**0.4531**

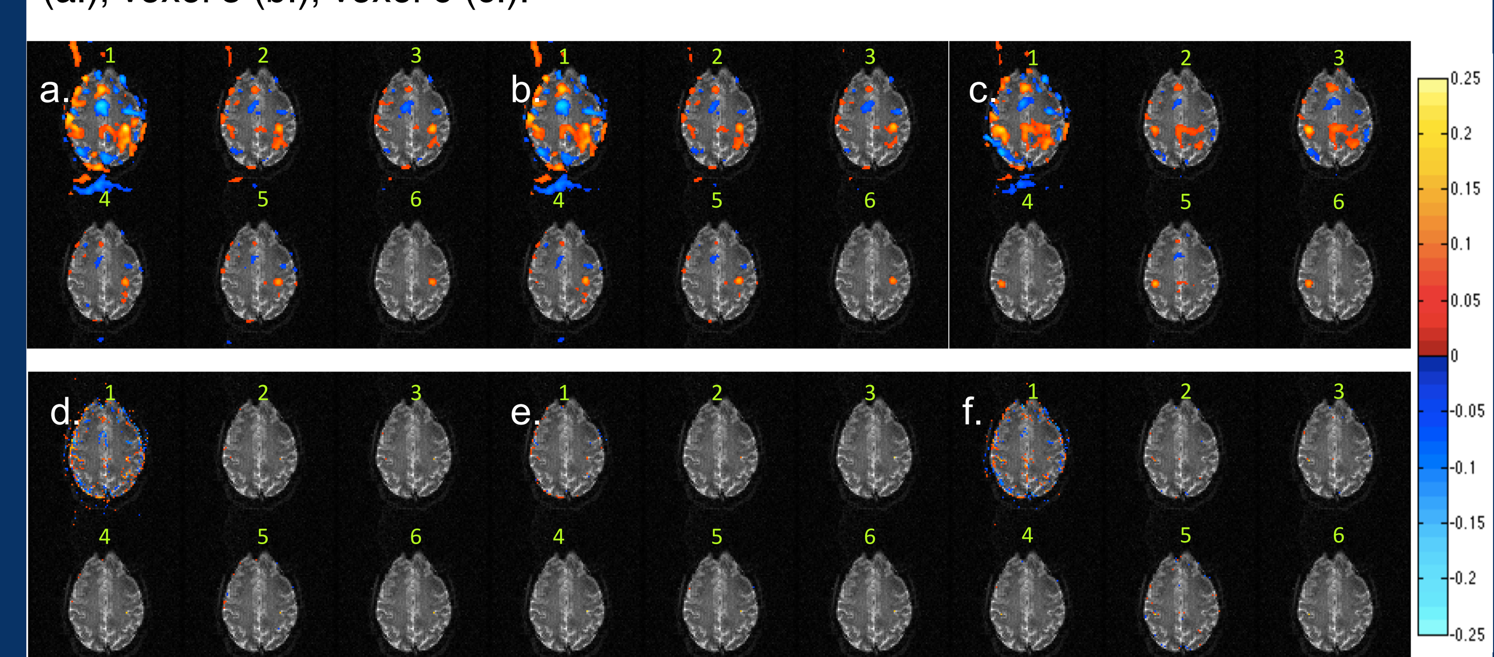

Figure 4. fMRI correlation maps based on Eqn. 14 by band for after and before applying spatial smoothing for voxel 4 (a.) and (d.), voxel 5 (b.) and (e.), voxel 6 (c.) and (f.), respectively.

In addition to analyzing the spatial correlation maps, observing the spectral decomposition for each voxel of interest, provides visual insight to the potential temporal frequency contribution for each voxel towards the spatial correlation with the following equation:

$$r_i = \frac{f_{jR_i}^2 + f_{jI_i}^2}{\sum_{i=1}^{n/2}(f_{jR_i}^2 + f_{jI_i}^2)} \tag{15}$$

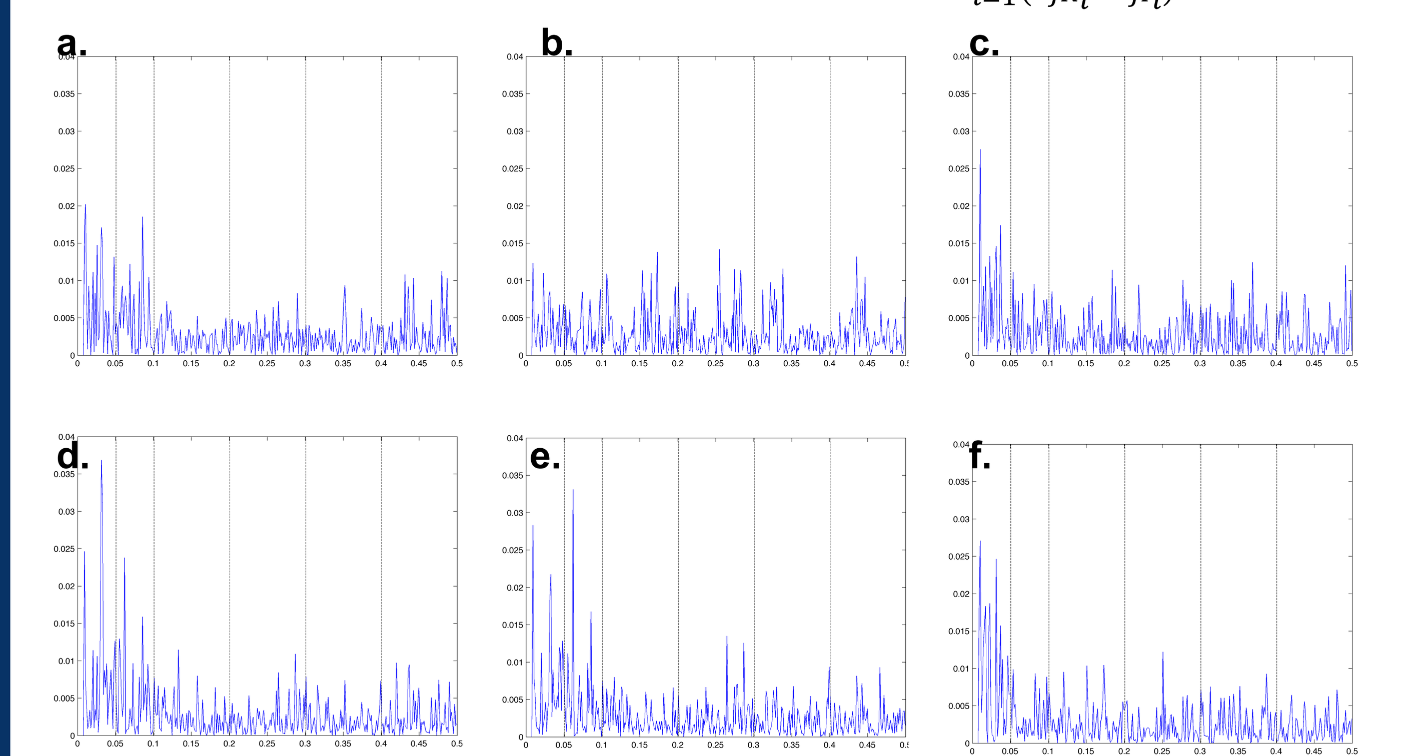

Figure 5. fcMRI spectral decomposition based on Eqn. 15 by band for before and after applying spatial smoothing for voxel 1 (a.) and (d.), voxel 2 (b.) and (e.), voxel 3 (c.) and (f.), respectively. Vertical lines correspond to band divisions.

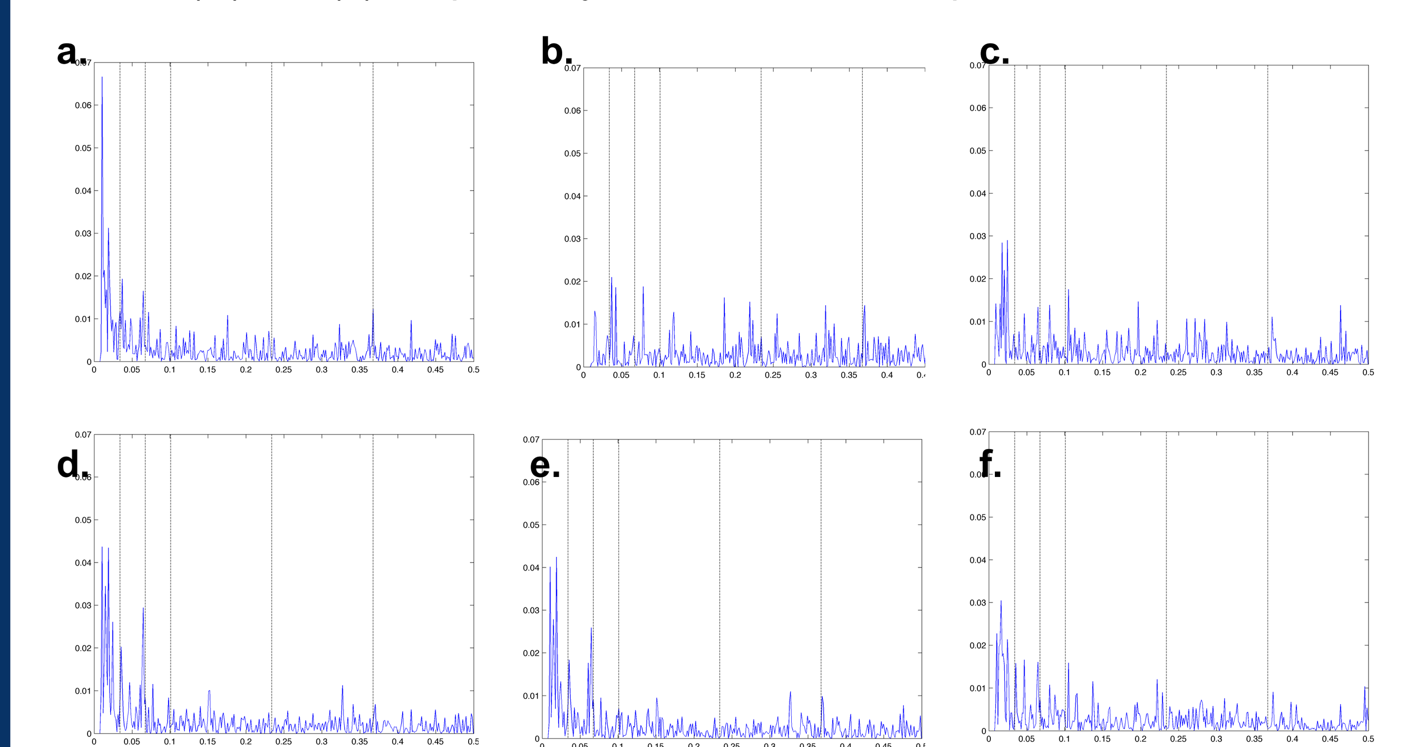

Figure 6. fcMRI spectral decomposition based on Eqn. 15 by band for before and after applying spatial smoothing for voxel 4 (a.) and (d.), voxel 5 (b.) and (e.), voxel 6 (c.) and (f.), respectively. Vertical lines correspond to band divisions.

In both scenarios, the impact of spatial filtering is most significantly observed between, voxel 2 and 3, and voxel 5 and 6, which are initially regionally and biologically uncorrelated. As expected, the neighboring voxels experience the largest increase in artificial spatial correlation; and, the initially biologically related voxels see in an increase in spatial correlation, although not as significant as the other cases.

The first two bands in Fig. 5 represent the functional connectivity range of spectral decomposition for each voxel temporal frequency spectrum, all three voxels show increased r_i values in those bands, and these contributions are confirmed in Table 1. Most notably, voxel 2, appears to have several artificial r_i peaks after processing. In Fig. 6, the peak corresponding to activation is at 0.0312 Hz, in the first band, voxel 5, which is initially inactive, experiences the largest induced correlation in this band with voxel 4 and voxel 6, both initially active. Voxel 4 and voxel 6, show increased correlation in this band, although not nearly to the extent as to voxel 5.

The true spatial correlation matrix was derived such that when spatial and temporal operators are applied to the data, the expected value of the spatial covariance and spatial correlation matrices can be quantitatively determined in terms of their temporal frequencies. The results demonstrate the advantageous framework and the benefit to clinical interpretations.

REFERENCES

[1] Nencka et al., 2009 [2] Bruce et al., 2011 [3] Rowe et al., 2007