Complex-Valued Correlation Increases Sensitivity and Specificity in the Analysis of Low Contrast-to-Noise fMRI Time-Series

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Synopsis

The standard in fMRI is a magnitude-only statistical analysis of the data, despite evidence of task related change in the phase time-series. This study demonstrates the increased sensitivity and specificity of implementing complex-valued correlation models for low magnitude and phase contrast-to-noise ratio (CNR) values in fMRI data sets.

Purpose & Background

The standard in fMRI is to discard the phase before the statistical analysis of the data, despite evidence of task related change in the phase time-series. With a real-valued isomorphism representation of Fourier reconstruction, correlation is computed in the temporal frequency domain with complex-valued (CV) time-series data, rather than with the standard of magnitude-only (MO) data. The purpose of this study is to demonstrate the increased statistical power of implementing complex-valued correlation models for low magnitude and phase contrast-to-noise ratio (CNR) values in fMRI data sets.

Methods

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To demonstrate the increased power of the CV correlation over MO correlation in functional MRI studies, a MATLAB simulation is run with a varying degree of magnitude contrast-to-noise ratio (CNR_{ρ}) and phase contrast-to-noise ratio ($\mathrm{CNR}_{\ddot{\imath}.}$). The SNR is defined as the mean magnitude signal over the standard deviation of the noise in the time-series, SNR = ρ/σ. For the CNR, the amplitude is defined as the difference between the baseline signal and the task related change in the signal for the magnitude and phase components of the time-series, A_{ρ} and $A_{\ddot{\imath}.}$, so $\mathrm{CNR}_{\rho} = A_{\rho}/\sigma$ and $\mathrm{CNR}_{\ddot{\imath}.} = A_{\ddot{\imath}.}/(\sigma/\rho)$. Since the standard deviation of a phase-only time-series is σ/ρ, the $\mathrm{CNR}_{\ddot{\imath}.}$ is proportional to the SNR. Typically in fMRI studies, the task related signal change in the magnitude corresponds to a 1-2% signal change, and the phase $\mathrm{CNR}_{\ddot{\imath}.}$ is approximately $\pi/36$ [1]. To compare MO and CV correlations, two 96 × 96 surfaces are generated with 600 time-points and standard normal random noise added to the real and imaginary channels. As visualized in Fig. 1, each voxel has an SNR between 0 and 50, and a task generated to represent a CNR_{ρ} between 0 and 1, and a $\mathrm{CNR}_{\ddot{\imath}.}$ between 0 and $\pi/36$. The MO and CV correlations are computed between the two time-series in each surface with equivalent parameter settings, so there is a 96 × 96 corresponding matrix for MO and CV.

Note, the CV correlation is computed in the Fourier frequency domain. The real-valued isomorphism representation of the inverse Fourier reconstruction operator is denoted Ω_T[2]. So, the voxel time-series for voxel j, y_j, is reconstructed from the temporal frequencies, $y_j = \Omega_T f_j$, and demeaned. Using the same notation for voxel k, the spatial covariance between voxels j and k is represented as $\mathrm{cov}(y_j, y_k) = 1/(2n)\, y_j^T y_k = 1/(2n)(\Omega_T \mathrm{f_j})^{\mathrm{T}} (\Omega_T f_k) = 1/4(f_j^T f_k)$. Note, the covariance corresponds to the jk^{th} entry in the voxel spatial covariance matrix, Σ, and is presented as summation of the overlap of the real and imaginary components of temporal frequencies. The diagonal matrix of spatial variances, D, is used to construct the spatial correlation, $R = D^{-1/2}\, \Sigma\, D^{-1/2}$. To compare the correlations between the MO and CV models, the Fisher-z transform, z, is computed and plotted for each time-series correlation, r, as $z = ½ \log((1+r)/(1-r))$.

Result

The simulation demonstrates the power of CV correlations over MO correlations for low magnitude contrast-to-noise time-series. Fig. 2 is the Fisher-*z* transform for the MO and CV correlations is computed for the surfaces generated with the parameters described in Fig. 1. Including the phase in the CV correlation calculation increases the sensitivity of the correlation value, as illustrated by comparing the top left corner of the Fisher-*z* maps. The additional task related information of the phase time-series improves the correlation calculation at low magnitude CNR values.

Conclusion

The simulation comparing decreasing CNR magnitude and phase values, illustrates the statistical power of CV correlations with a specific advantage for low CNR fMRI time-series. Including both magnitude and phase in spatial correlation computations more accurately identifies correlated task-activated regions in data sets with high noise variance. Identifying these correlations are important to preserve the signal of interest, in order to reduce the influence of processing or reconstruction induced correlations [3] on the statistical analysis of fMRI data.

Acknowledgements

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References

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Figures

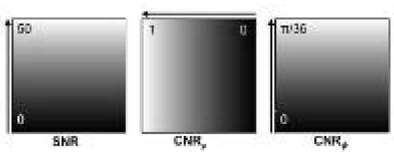

Figure 1. Surfaces representing the SNR, CNR_ρ, and CNR_ϕ parameters used to generate the simulated time-series.

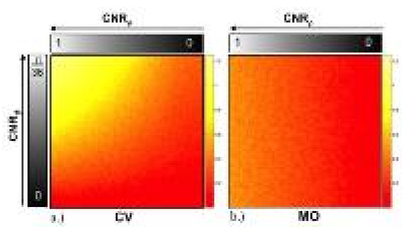

Figure 2. The Fisher-*z* transform of the MO and CV correlation.

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