Incorporating Relaxivities to More Accurately Reconstruct Magnetic Resonance Images

M. Muge Karaman and Daniel B. Rowe
Department of Mathematics, Statistics, and Computer Science, Marquette University Milwaukee, WI, USA

Synopsis

In fMRI, spatial frequency measurements are subject to the Fourier encoding anomalies; intra-acquisition decay (T_2*), longitudinal relaxation time (T_1) and a phase determined by the magnetic field inhomogeneity (ΔB). Thus, the resulting image can include some artificial effects resulting from the Fourier anomalies. Nencka et al. [1] presented the AMMUST-*k* model to examine preprocessing and the two Fourier anomalies (T_2* and ΔB) induced correlation. We extend the AMMUST framework to incorporate anomalies, T_2*, ΔB and T_1, into the Fourier reconstruction process to correct their effects. The exact image-space means, variances, and correlations are theoretically computed by implementing the new extended linear framework.

Introduction

In fMRI, the signal is ideally the Fourier transform (FT) of the proton spin density (ρ) weighted by the Fourier anomalies: T_2*, ΔB and T_1 [4].

$$s(k_x,k_y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\rho(x,y)\left(1-e^{-TR/T_1(x,y)}\right)e^{-TE/T_2^*(x,y)}e^{i\gamma\Delta B(x,y)TE}e^{-i2\pi(k_x x+k_y y)}dxdy \quad (1)$$

ρ(x,y), $T_1(x,y)$ and T_2*(x,y) are tissue dependent.

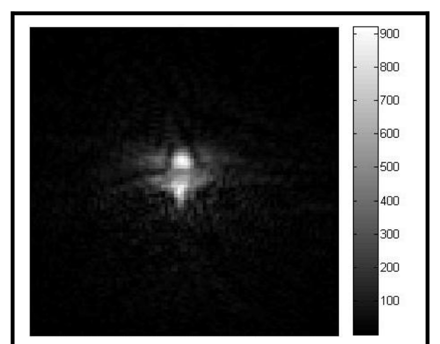
Fig. 1. Spatial frequencies.

The signal in Fig. 1 is complex-valued with a real and imaginary component,

$$s(k_x k_y)=s_R(k_x,k_y)+is_I(k_x,k_y),$$

and thus the image of an object, y, is reconstructed via the inverse Fourier transform (IFT) through a real-valued isomorphism as in Figures 2 and 3 [2].

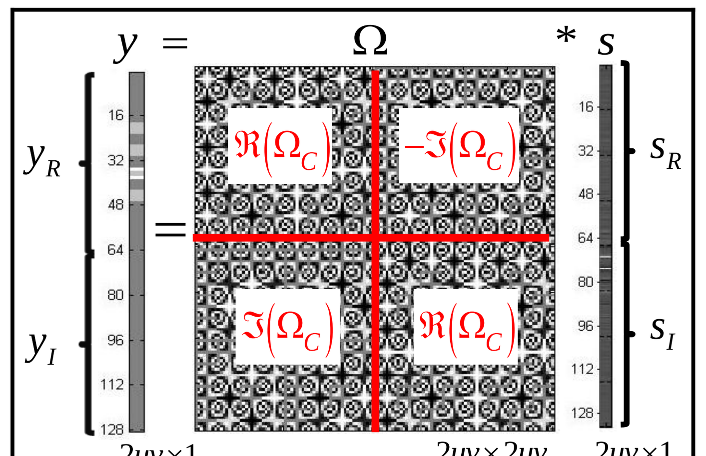
Fig. 2. Fourier Reconstruction

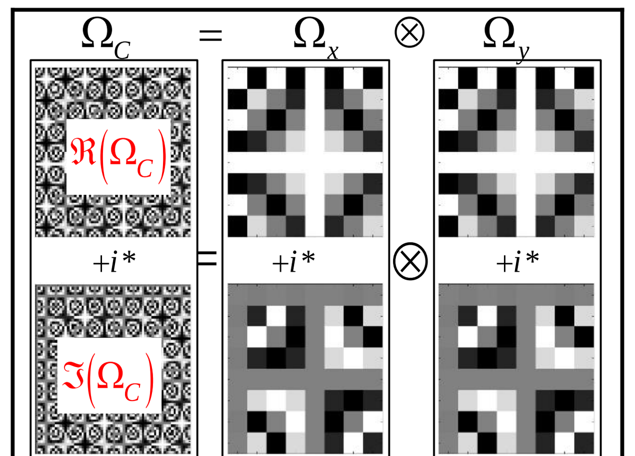
Fig. 3. Ω_C matrix

Similarly, the k-space observation, s, is encoded via the FT operator, $s=\bar{\Omega}y$.

FT and IFT operators are the inverses: $\Omega\bar{\Omega}=I$.

Methods

We incorporate the exponential terms of T_2* and ΔB as well as T_1 into the FT operator, $\bar{\Omega}$, that can be written as,

$$\bar{\Omega}=\begin{pmatrix}\Re(\bar{\Omega}_C) & -\Im(\bar{\Omega}_C)\\ \Im(\bar{\Omega}_C) & \Re(\bar{\Omega}_C)\end{pmatrix}, \text{ where } \bar{\Omega}_C=\bar{\Omega}_x\otimes\bar{\Omega}_y.$$

The modified FT operator, $\bar{\Omega}_b$, which includes T_2^*, ΔB and T_1 effects can be created by multiplying the Kronecker product, $\bar{\Omega}_C$, by the term, E:

$$E\left(k_x,k_y,x,y\right)=e^{-t\left(k_x,k_y\right)/T_2^*(x,y)}e^{i\gamma\Delta B(x,y)t(k_x,k_y)}\left(1-e^{-TR/T_1(x,y)}\right). \quad (2)$$

Thus, the modified Kronocker product matrix is found as,

$$\bar{\Omega}_{C,b}=\left(\bar{\Omega}_x\otimes\bar{\Omega}_y\right).*E\left(k_x,k_y,x,y\right), \quad (3)$$

and finally the modified FT operator, $\bar{\Omega}_b$, can be written as,

$$\bar{\Omega}_b=\begin{pmatrix}\Re(\bar{\Omega}_{C,b}) & -\Im(\bar{\Omega}_{C,b})\\ \Im(\bar{\Omega}_{C,b}) & \Re(\bar{\Omega}_{C,b})\end{pmatrix}. \quad (4)$$

We develop the modified IFT operator, Ω_b, to account for the Fourier anomalies by simply taking the inverse of modified FT operator, $\Omega_b=\bar{\Omega}_b^{-1}$.

The regular IFT operator, Ω, and the modified IFT operators, Ω_a and Ω_b, are given in Figs. 4a-c, respectively. Ω_a includes only T_2* and ΔB effects; Ω_b includes T_2*, ΔB and T_1 effects.

Thus, y can be written as, $y=\bar{\Omega}_{a/b}s$. (5)

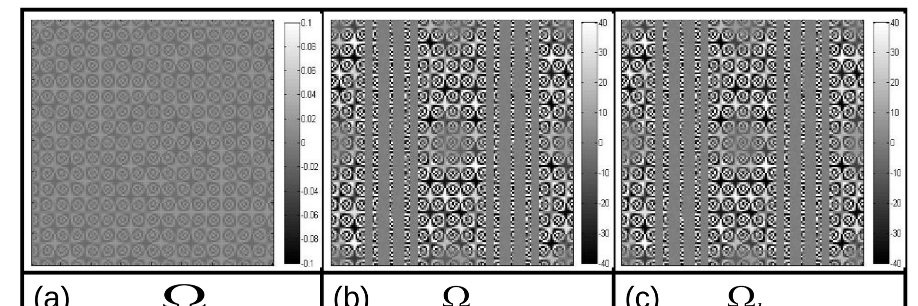

Fig. 4. IFT Operators

We also consider the preprocessing operators, O_I and O_K, introduced in [1]. Finally, Eq. (5) can be generalized to;

$$y=S_m\Omega_{a/b}AFHP_R^{-1}\Omega_{row}^{-1}\Phi\Omega_{row}P_RP_CRCs=O_I\Omega_{a/b}O_Ks. \quad (6)$$

Theoretical Illustration Results

A true noiseless image of spatial frequencies was generated for a 96×96 Shepp-Logan phantom for theoretical illustration [3]. T_1 and T_2^* maps were considered as the true relaxation parameters of Shepp-Logan phantom at 3.0 T. The B field inhomogenity, ΔB, was modeled as a linear gradient ranging from 0 to 2.5×10^{-6} T.

True magnitude and phase images are shown in Figs. 5a and b. The results on the magnitude and phase images in light of T_2* and ΔB effects; and in light of T_2*, ΔB and T_1 effects are given in Figs. 6 and 7, respectively. The first columns (a and b) of Figs. 6 and 7 show the results that were calculated by using the standard IFT operator, Ω; whereas the second columns (c and d) show the ones that were calculated through the modified IFT operators, Ω_a and Ω_b.

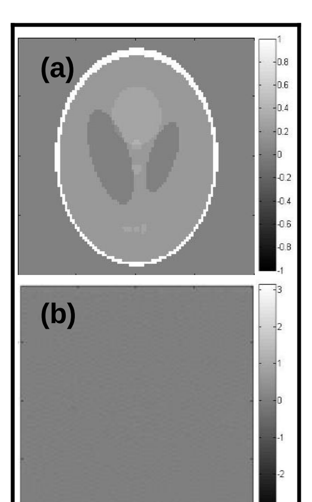
Fig. 5. True maps

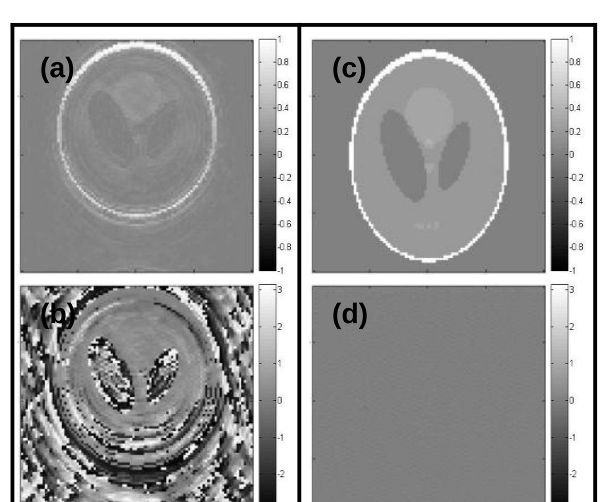
Fig. 6. T_2* and ΔB effects on images

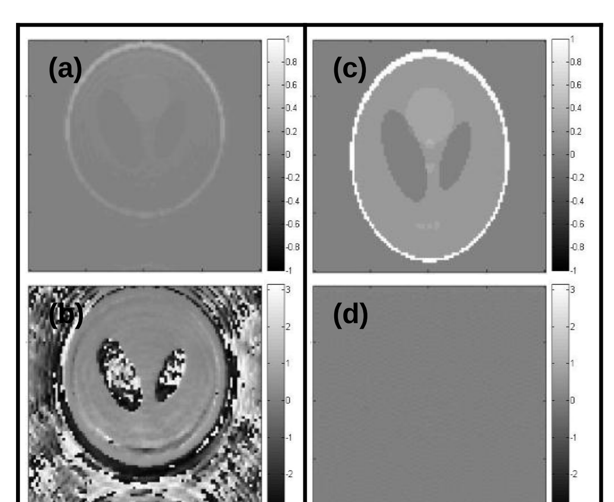
Fig. 7. T_2*, ΔB and T_1 effects on images

Ignoring Fourier encoding anomalies produces shifting and blurring in the reconstructed magnitude and phase images.

Correlation Results

The reconstructed voxels covariance matrix Σ can be represented as

$$\mathrm{cov}(\nu)=(O_I\Omega_{a/b}O_k)\Gamma(O_k^T\Omega_{a/b}^TO_I^T)=\Sigma, \quad (7)$$

where Γ is the covariance matrix for the spatial frequencies [1].

We present the correlation map for the centered voxel when Ω_b is considered and $\Gamma=I$. Fig. 8 shows the true and induced magnitude-squared correlation maps, respectively.

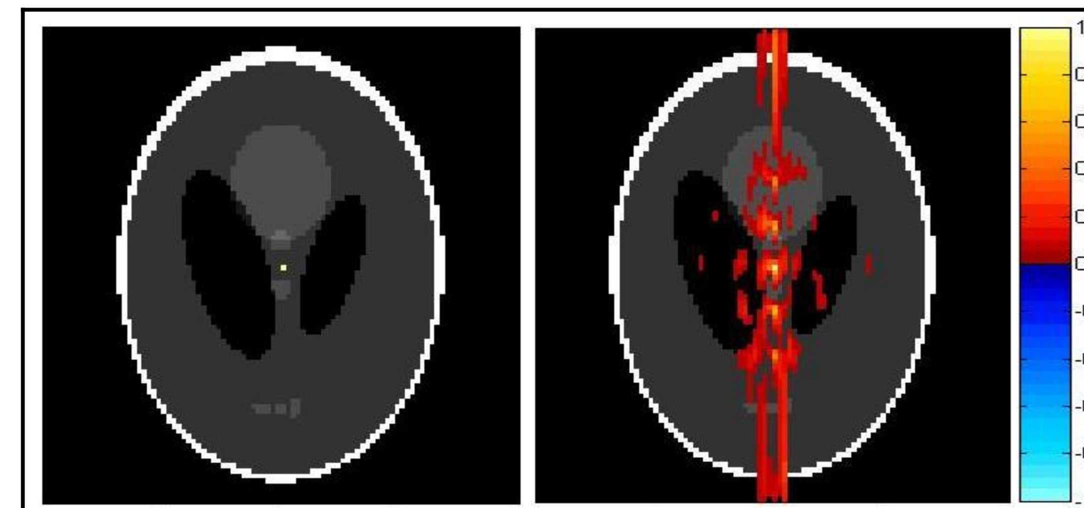
Fig. 8. True (left) and induced (right) magnitude-squared correlation maps.

Discussion

We have extended the previous study [1] by developing a linear operator for the Fourier encoding anomalies which stands for the T_1 recovery term that was neglected before, as well as the other two anomalies. The theoretical results illustrate that the Fourier encoding anomalies affect computed mean images; and the voxel correlations. It has been observed that longitudinal relaxation time causes blurring in the image means as magnetic field inhomogeneities cause image shifting and blurring. The modified Fourier reconstruction operators which include these parameters successively remove the artificial effects caused by the anomalies.

References

[1] Nencka et al. JNM 181:268–282, 2009.
[2] Rowe et al. JNM 159:361–369, 2007.
[3] Gach et al. ICSENG 19:521-526.
[4] Haacke et al. John Wiley and Sons, 1999.

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