

Reconstructing an Increased Number of Simultaneously Excited fMRI Slices

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- Single-coil Aliasing Model
- The SPECS Model
- Incorporating Phase Shifted Acquisitions into SPECS
- fMRI Simulation
- Summary



Most multi-band models for accelerating fMRI data acquisition are faced by two limiting factors:

- Acceleration factor, A , limited to no more than the number of coils.
- The origin of a BOLD signal increase within one of several slices aliased together cannot be determined from one measurement.



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- Enables higher acceleration factors by improving the aliasing matrix rank
- Minimal induced correlations

¹Rowe et al. 2013



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^{2,3}Incorporation of phase shifted acquisitions into the SPECS model:

- Apply various phase shifts to different slices aliased together
- Multiple ways in which each voxel is aliased
- Improves power of detecting activation statistics

¹Rowe et al. 2013

²Rowe et al. 2014 (In prep.)

³Setsoy et al. MRM 2013



Single-coil aliasing

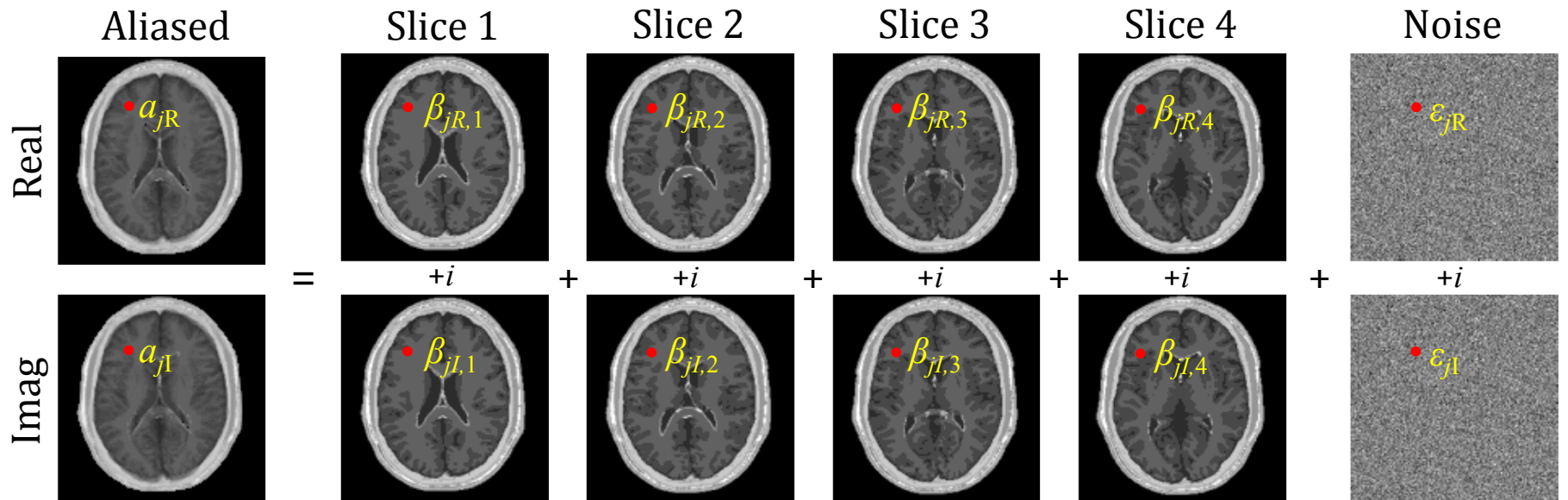
Assuming a homogeneous field, aliasing in a voxel, j , is

$$a_{jC} = (\beta_{jR,1} + \beta_{jR,2} + \dots + \beta_{jR,A}) + i(\beta_{jI,1} + \beta_{jI,2} + \dots + \beta_{jI,A}) + (\varepsilon_{jR} + i\varepsilon_{jI})$$

$$= a_{jR} + ia_{jI}$$

$$E[\varepsilon_j] = 0$$

$$\text{cov}(\varepsilon_j) = \sigma^2 I_2$$





Single-coil aliasing



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$$= a_{jR} + ia_{jI}$$

In real-valued matrix form:

$$\begin{pmatrix} a_{jR} \\ a_{jI} \end{pmatrix}_{2 \times 1} = \begin{pmatrix} 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 1 \end{pmatrix}_{2 \times 2A} \begin{pmatrix} \beta_{jR,1} \\ \vdots \\ \beta_{jR,A} \\ \beta_{jI,1} \\ \vdots \\ \beta_{jI,A} \end{pmatrix}_{2A \times 1} + \begin{pmatrix} \varepsilon_{jR} \\ \varepsilon_{jI} \end{pmatrix}_{2 \times 1}$$

Aliased image
Aliasing matrix
True un-aliased images
Measurement error

$$a = (I_2 \otimes X_A) \beta + \varepsilon$$



Single-coil aliasing



At this point, our initial aliasing model is of the form

$$a = \left(I_2 \otimes X_A \right) \beta + \varepsilon = X \beta + \varepsilon$$

The goal is to estimate the true un-aliased voxel values

$$\hat{\beta} = (X' X)^{-1} X' a$$



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When aliasing A slices, the matrix, X , has 2 equations and $2A$ unknowns.

- $(X'X)$ is not square, invertible or of full rank.

We can improve the rank of X by adding $2(A-1)$ more rows.



Artificial aliasing

If $X = (I_2 \otimes X_A)$, where $X_A = [1, 1, \dots, 1]$, we can add $A-1$ rows to X_A to make $\begin{bmatrix} X_A \\ C \end{bmatrix}$ square, invertible and full in rank

Orthogonal Coefficients

$$A=2: C = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

Hadamard Coefficients

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$$A=3: C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

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$$A=3: C = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A=4: C = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Hadamard Coefficients

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$$A=4: C = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$



Artificial aliasing

Adding rows to X gives us additional ways in which the true voxel values in β could be “aliased”.

Acquired
aliased
voxels

$$\begin{pmatrix} a_{jR} \\ ?? \\ a_{jI} \\ ?? \end{pmatrix} = \begin{pmatrix} X_A & 0 \\ C & \\ 0 & X_A \\ & C \end{pmatrix} \beta + \begin{pmatrix} \epsilon_{jR} \\ ?? \\ \epsilon_{jI} \\ ?? \end{pmatrix}$$

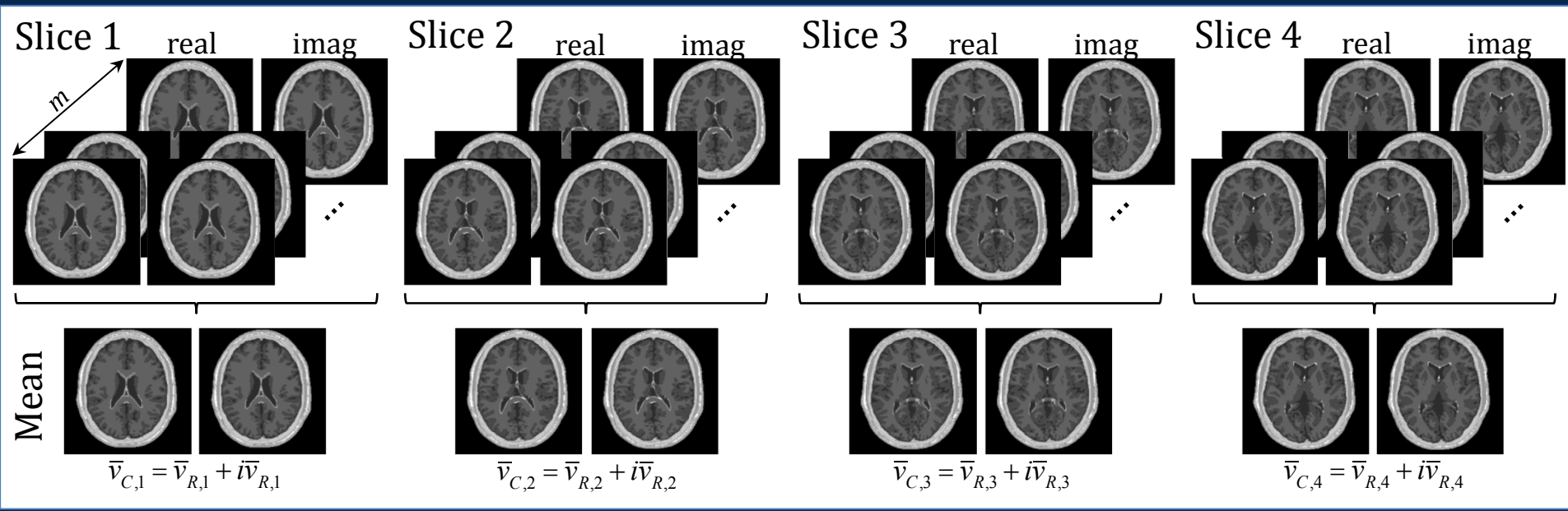
Artificially aliased voxels

If we acquire calibration data for each slice, we can use C to artificially alias the slices.



Single-coil calibration data

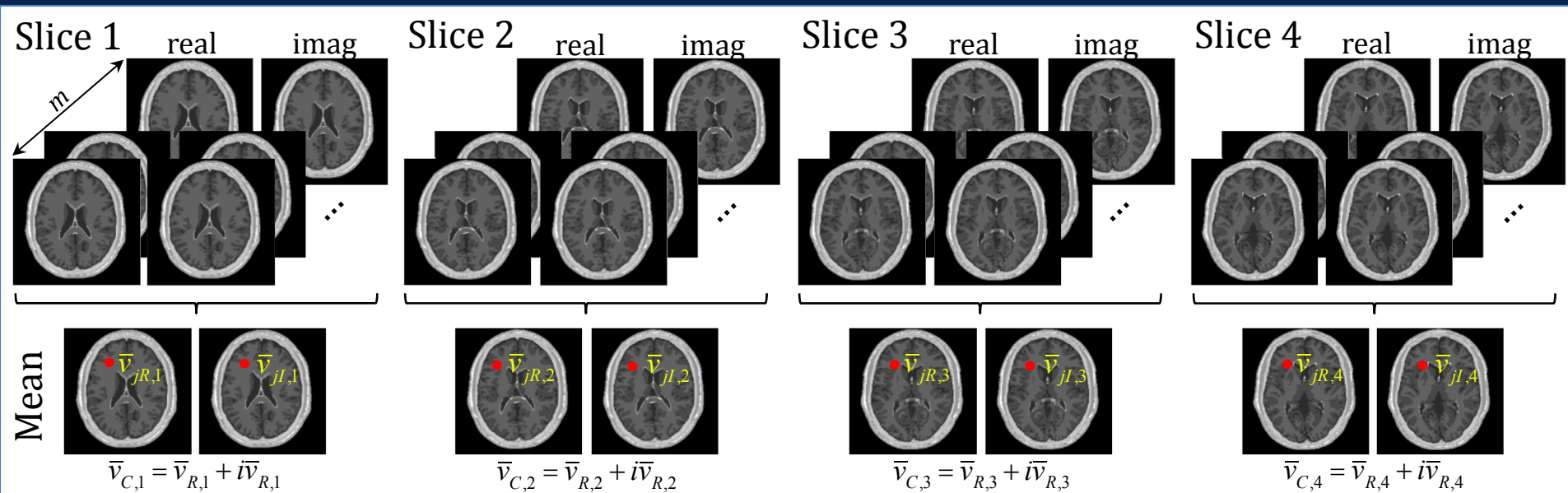
Acquire a time series of m fully sampled complex-valued images for each slice:





Single-coil calibration data

Acquire a time series of m fully sampled complex-valued images for each slice:



For a voxel j in the mean calibration images, we can construct a mean vector

$$\bar{\mathbf{v}}_{jC} = \left[\bar{\mathbf{v}}_{jC,1}, \bar{\mathbf{v}}_{jC,2}, \bar{\mathbf{v}}_{jC,3}, \bar{\mathbf{v}}_{jC,4} \right]^T$$



Incorporating artificially aliased mean calibration voxel values into the model:

Acquired aliased voxels

Artificially aliased voxels

$$\begin{pmatrix} a_{jR} \\ C\bar{v}_{jR} \\ a_{jI} \\ C\bar{v}_{jI} \end{pmatrix} = \begin{pmatrix} X_A & 0 \\ C & \\ 0 & X_A \\ & C \end{pmatrix} \beta + \begin{pmatrix} \varepsilon_{jR} \\ C\eta_{jR} \\ \varepsilon_{jI} \\ C\eta_{jI} \end{pmatrix}$$

$$\begin{matrix} a \\ 2A \times 1 \end{matrix} = \begin{matrix} X \\ 2A \times 2A \end{matrix} \begin{matrix} \beta \\ 2A \times 1 \end{matrix} + \begin{matrix} \text{error} \\ 2A \times 1 \end{matrix}$$

The invertible aliasing matrix allows us to estimate the un-aliased voxel values through a simple inverse: $\hat{\beta} = X^{-1}y$



To avoid inducing correlations into the un-aliased voxels through the SPECS model, a bootstrapping approach can be used.

Consider the contrast matrices for $A=4$:

Orthogonal Coefficients

$$A=4: C = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

Assume the covariance of each calibration image is

$$\text{cov}(v_{j,\text{coil}}) = \sigma^2$$

Hadamard Coefficients

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Row sum-of-squares (M)

$$A=4: C = \begin{bmatrix} -3 & -1 & 1 & 3 \\ 1 & -1 & -1 & 1 \\ -1 & 3 & -3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 20 \\ 4 \\ 20 \end{bmatrix}$$

Assume the covariance of each calibration image is

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$$A=4: C = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$$

Averaging M random calibration images with rows of the matrix C will have a covariance

$$\text{cov}(\bar{v}_{j,coil}) = \sigma^2$$



With the mean and covariance of the data vector,

$$E[a_j] = [a_{jR0}, C\bar{v}_{jR0}, a_{jI0}, C\bar{v}_{jI0}]^T, \quad \Sigma = \text{cov}(a_j) = \sigma^2 I_{2A}$$

the mean and covariance of images reconstructed with a model that employs the bootstrapping approach are

$$E[\hat{\beta}_j] = X^{-1} E[a_j], \quad \text{cov}(\hat{\beta}_j) = \sigma^2 \left(X^{-1} \right) I_{2A} \left(X^{-1} \right)^T$$

Since the aliasing matrix, X , is orthogonal, the theoretical correlation structure induced in $\hat{\beta}_j$ by SPECS is identity.



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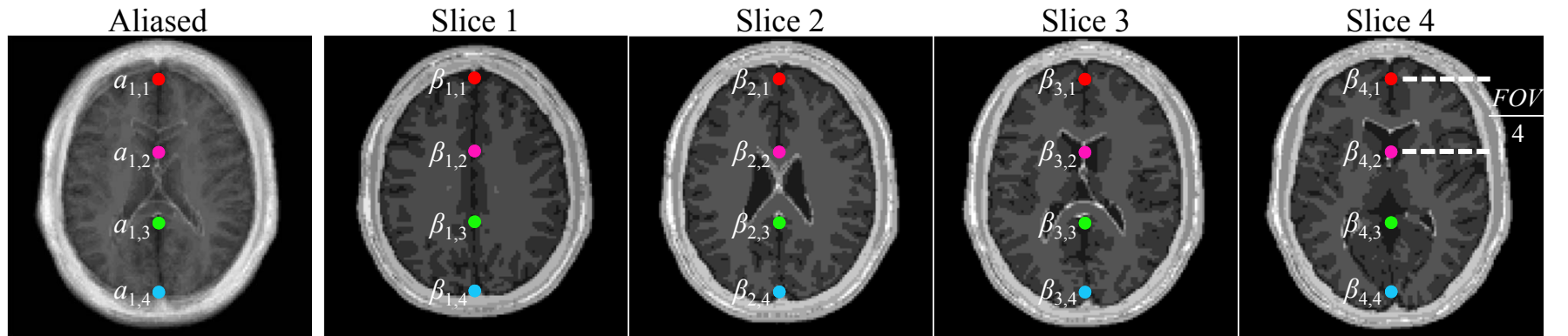
Note: If there is a BOLD signal increase (activity) in one of the aliased slices, one cannot determine the origin of the signal increase (which slice) with only one measurement.



Shifted Acquisitions



For a “uniform” acquisition of $N_s=4$ slices, we can consider a vector of 4 aliased voxels (positioned FOV/4 apart) at once:



$$\begin{bmatrix} a_{1,1} \\ a_{1,2} \\ a_{1,3} \\ a_{1,4} \end{bmatrix} = a_1 = X_{A1} \beta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \beta_{2,1} \\ \beta_{2,2} \\ \beta_{3,1} \\ \beta_{3,2} \\ \beta_{4,1} \\ \beta_{4,2} \\ \beta_{4,3} \\ \beta_{4,4} \end{bmatrix}$$

This is exactly the same as before, only we are now looking at more than one voxel at a time.

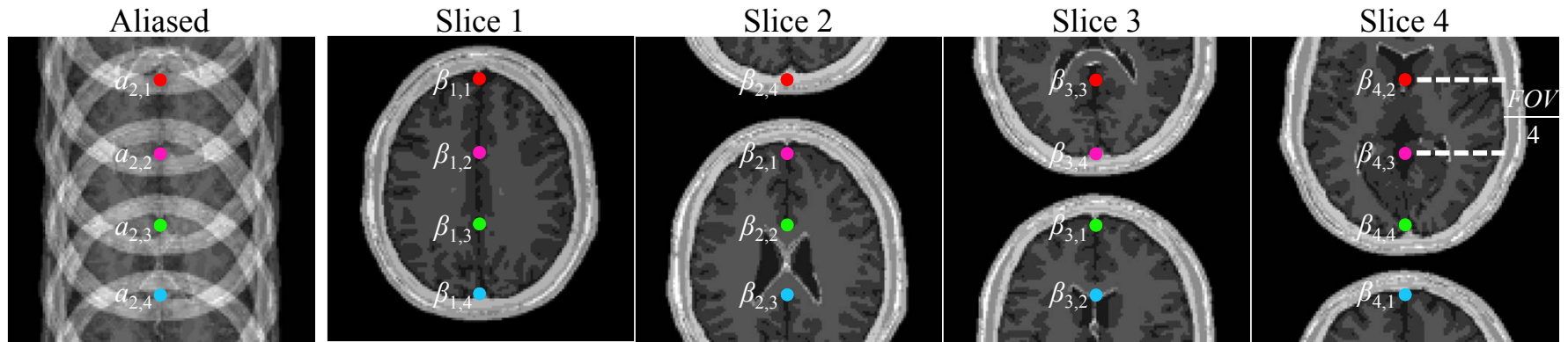
$$[1 \text{ Eq with 4 unknowns}] \rightarrow [4 \text{ Eqs with 16 unknowns}] \rightarrow A=4$$



Shifted Acquisitions



If we shift slices $j=1, 2, \dots, N_s=4$ in PE by: $(j-1)\frac{FOV}{4}$



$$\begin{bmatrix} a_{2,1} \\ a_{2,2} \\ a_{2,3} \\ a_{2,4} \end{bmatrix} = a_2 = X_{A2} \beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

Now have a new way in which the 16 voxels in β can be aliased.

When combined with the uniform aliasing scheme we have:

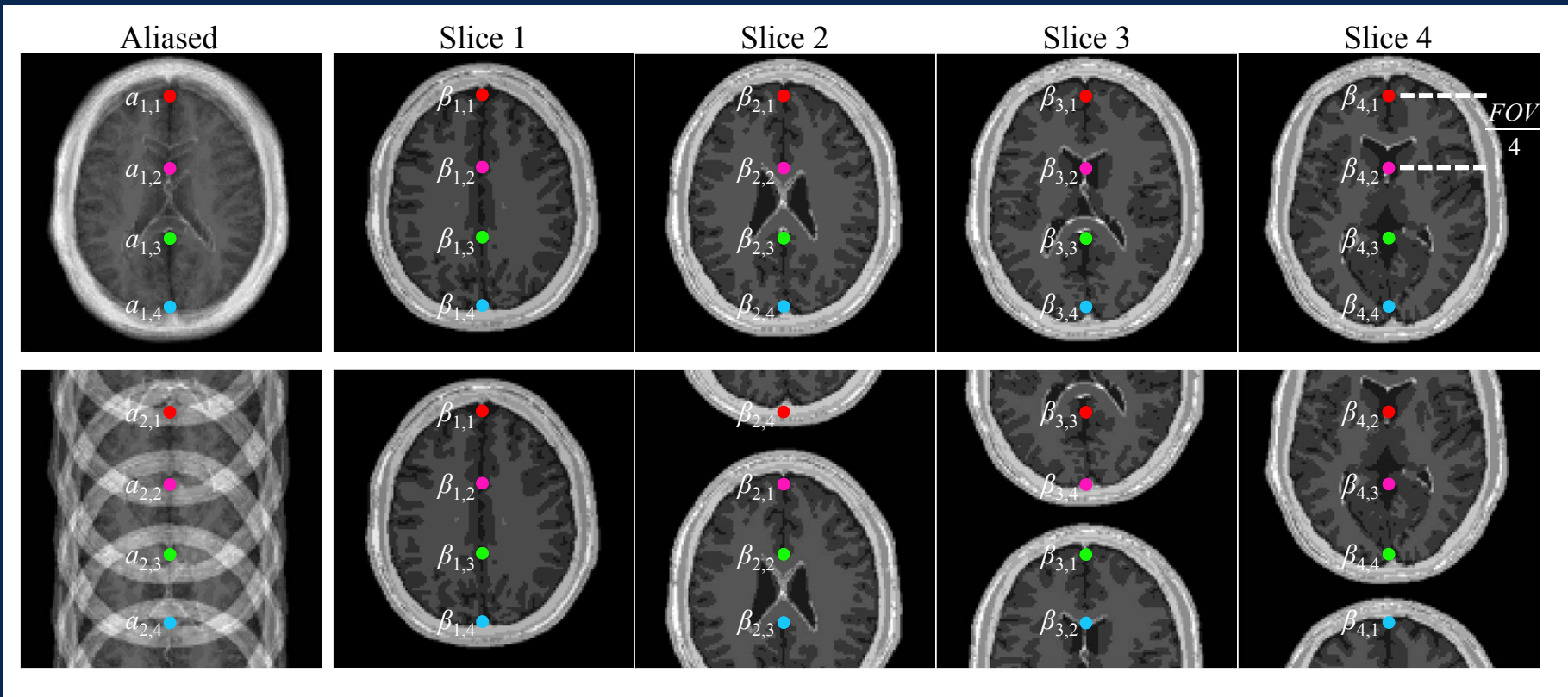
$$[1 \text{ Eq with 4 unknowns}] \rightarrow [8 \text{ Eqs with 16 unknowns}] \rightarrow A=2$$



Shifted Acquisitions



Consider the two aliasing patterns at once in a single system



[8 Eqs with 16 unknowns] $\rightarrow A=2$



Shifted Acquisitions

Consider the two aliasing patterns at once in a single system

$$\begin{bmatrix} a_{1,1} \\ a_{1,2} \\ a_{1,3} \\ a_{1,4} \end{bmatrix} = a_1 = X_{A1} \beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

$$\begin{bmatrix} a_{2,1} \\ a_{2,2} \\ a_{2,3} \\ a_{2,4} \end{bmatrix} = a_2 = X_{A2} \beta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{1,1} \\ \beta_{1,2} \\ \vdots \\ \beta_{2,1} \\ \vdots \\ \beta_{4,4} \end{bmatrix}$$

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To incorporate the phase shifted acquisitions into the SPECS model, denote the j^{th} column of the coefficient matrix, C , by C_j to form the new coefficient matrix:

$$C \rightarrow \begin{bmatrix} X_1 \otimes C_1 & X_2 \otimes C_2 & X_3 \otimes C_3 & X_4 \otimes C_4 \end{bmatrix}$$

[8 Eqs with 16 unknowns] $\rightarrow A=2$



Shifted SPECS Model



The original SPECS model is thus updated in three ways:

$$\begin{pmatrix} a_R \\ C\bar{v}_R \\ a_I \\ C\bar{v}_I \end{pmatrix} = \begin{pmatrix} X_A & 0 \\ C & \beta + noise \\ 0 & X_A \\ C & \end{pmatrix}$$

1) New β vector with values from the N_S rows (FOV/ N_S apart) in the N_S slices

3) Artificially shift and artificially alias random reference images in accordance with shifted aliasing schemes.

$$\text{Acceleration, } A = \frac{N_S}{N_{acq}}$$

2) New aliasing matrix

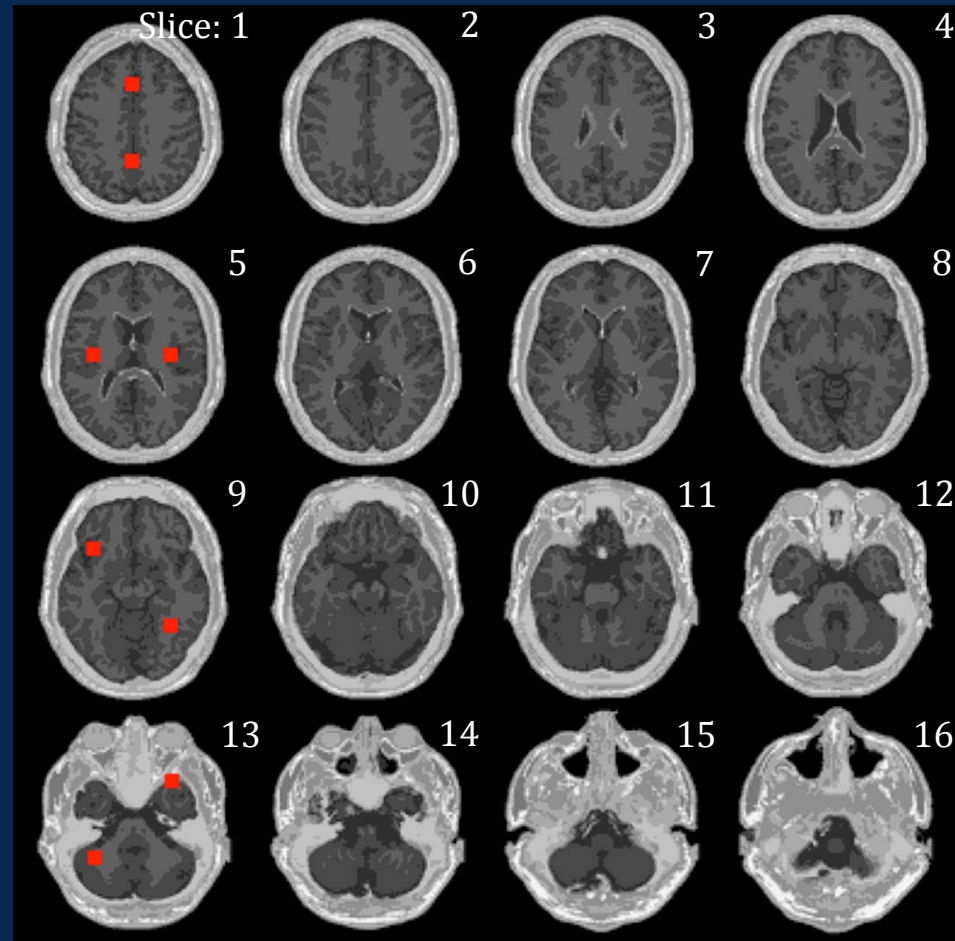
$$\begin{bmatrix} X_A \\ C \end{bmatrix}_{2N_S \times 2N_S} \rightarrow \begin{bmatrix} X_{A1} \\ \vdots \\ X_{AN_{acq}} \\ (X_1 \otimes C_1), (X_2 \otimes C_2), \dots, (X_{N_S} \otimes C_{N_S}) \end{bmatrix}_{(2N_S^2 N_{acq} \times 2N_S^2)}$$



Simulation – fMRI Setup

Simulated an fMRI study in $N_s=16$ (96×96) slices

- Max magnitude/SNR = 50
- Phase varied linearly from 0 to π over slices.
- Added $N(0,1)$ noise
- BOLD activity simulated in
 - Slices: 1, 5, 9 and 13
 - 20 epoch block design
 - 15 TRs “on”, 15 TRs “off”
- CNR = 1
- 500 reference images for each slice (non-task)





Simulation - Aliasing

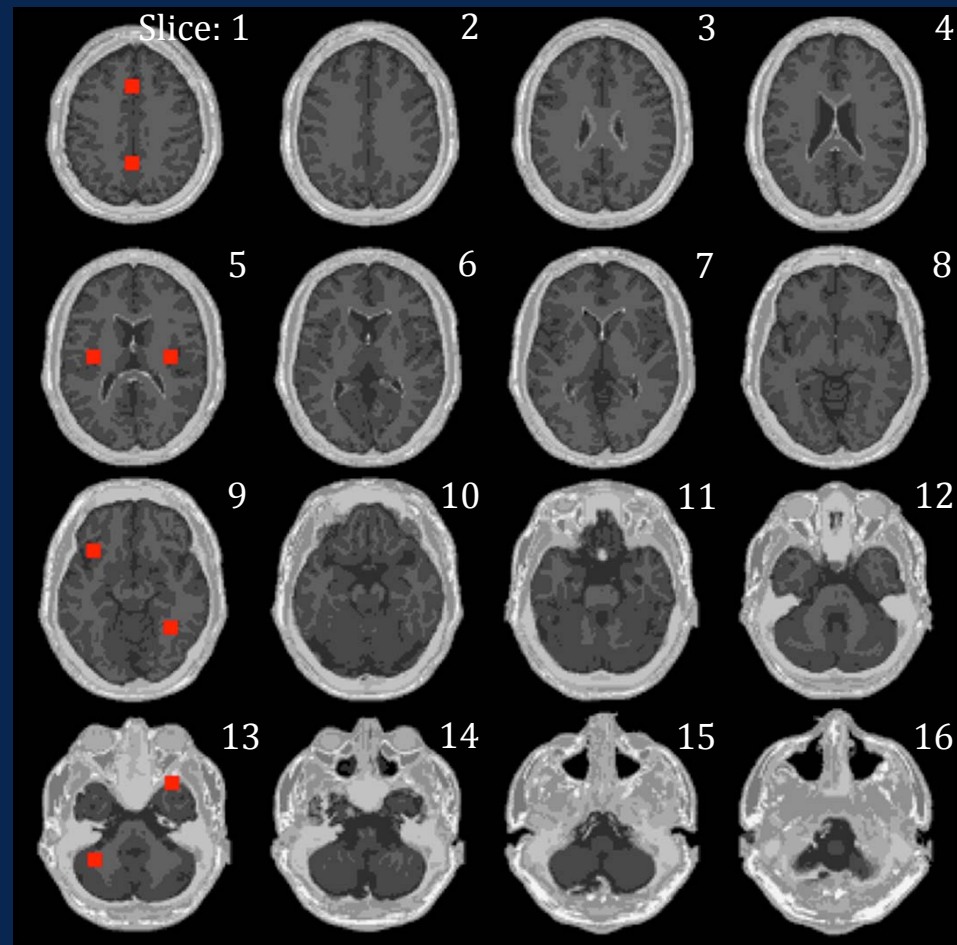


Five data sets simulated with N_{acq} acquisitions and acceleration factors, A , as follows:

- $A=16, (N_{acq}=1)$
- $A=8, (N_{acq}=2)$
- $A=4, (N_{acq}=4)$
- $A=2, (N_{acq}=8)$
- $A=1, (no\ aliasing)$

For each of $acq=1, 2, \dots, N_{acq}$ acquisitions, shifts were performed on slice $j=1, 2, \dots, 16$

by: $(acq - 1)(j - 1) \frac{FOV}{16}$

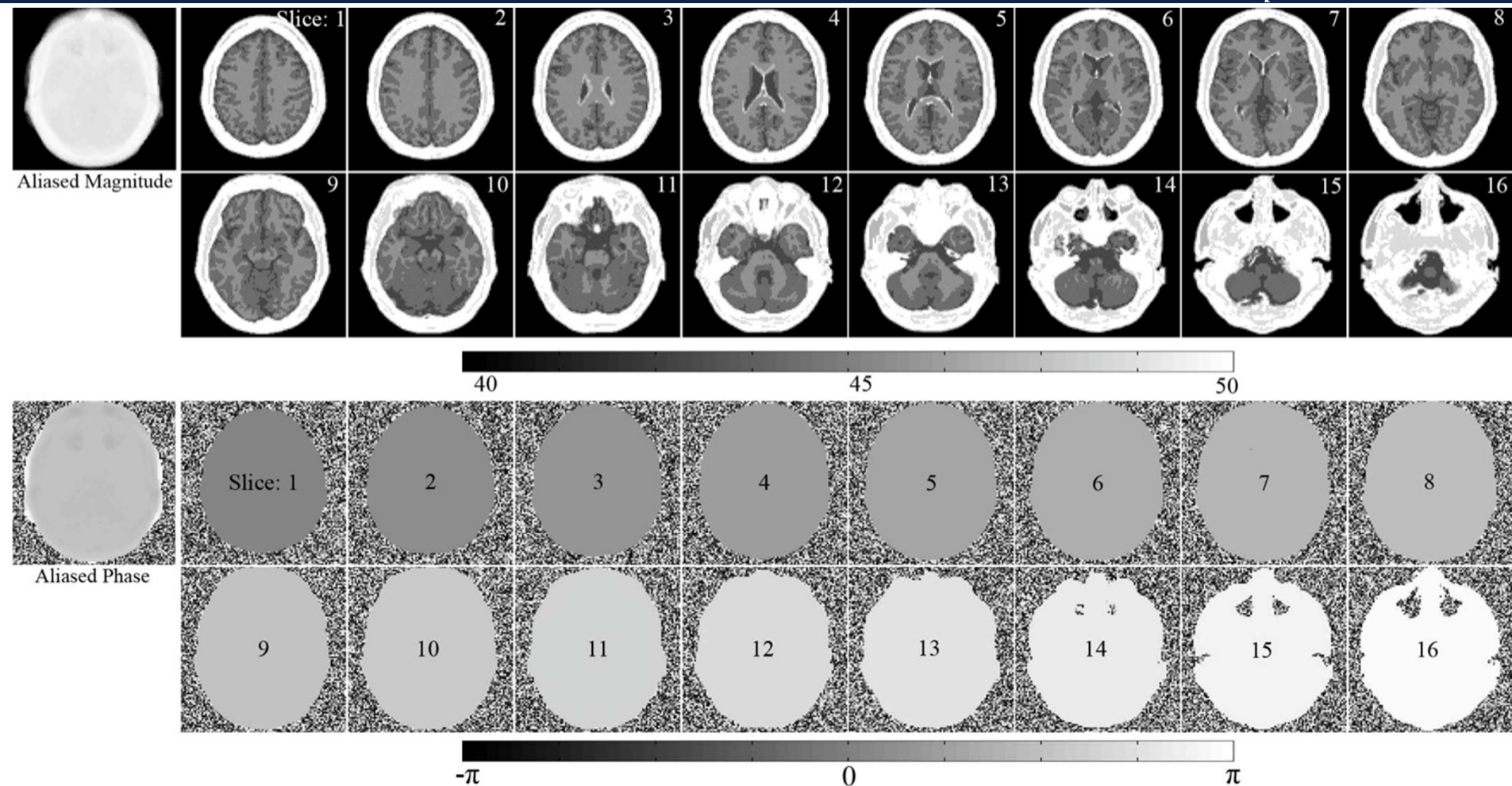




Simulation - Results



Magnitude and phase images for all reconstructed data sets are indistinguishable, and are thus presented for $A=16$ ($N_{acq}=1$).

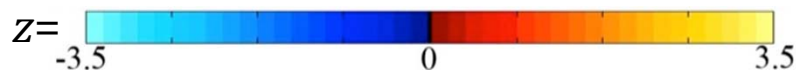
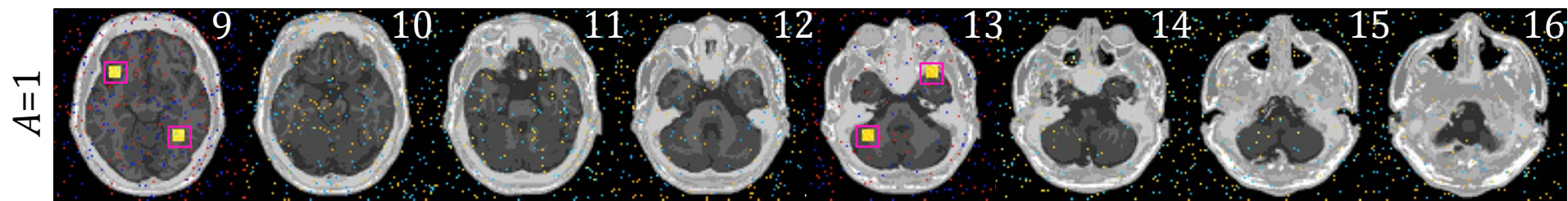
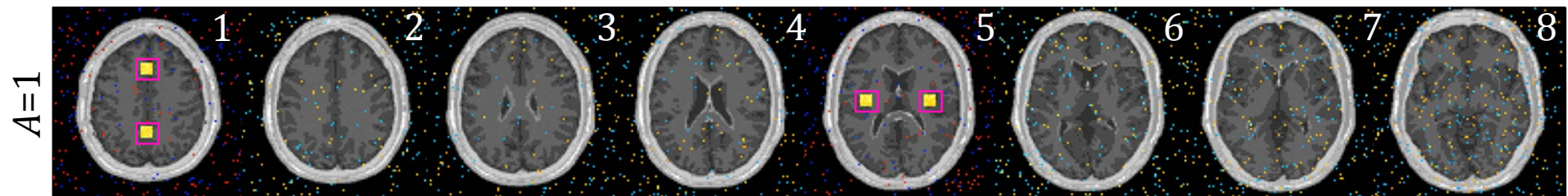




fMRI Results ($A=1, N_{acq}=16$)



After separation, activation should only be present within the ROIs, otherwise denote false-positives from residual aliasing.

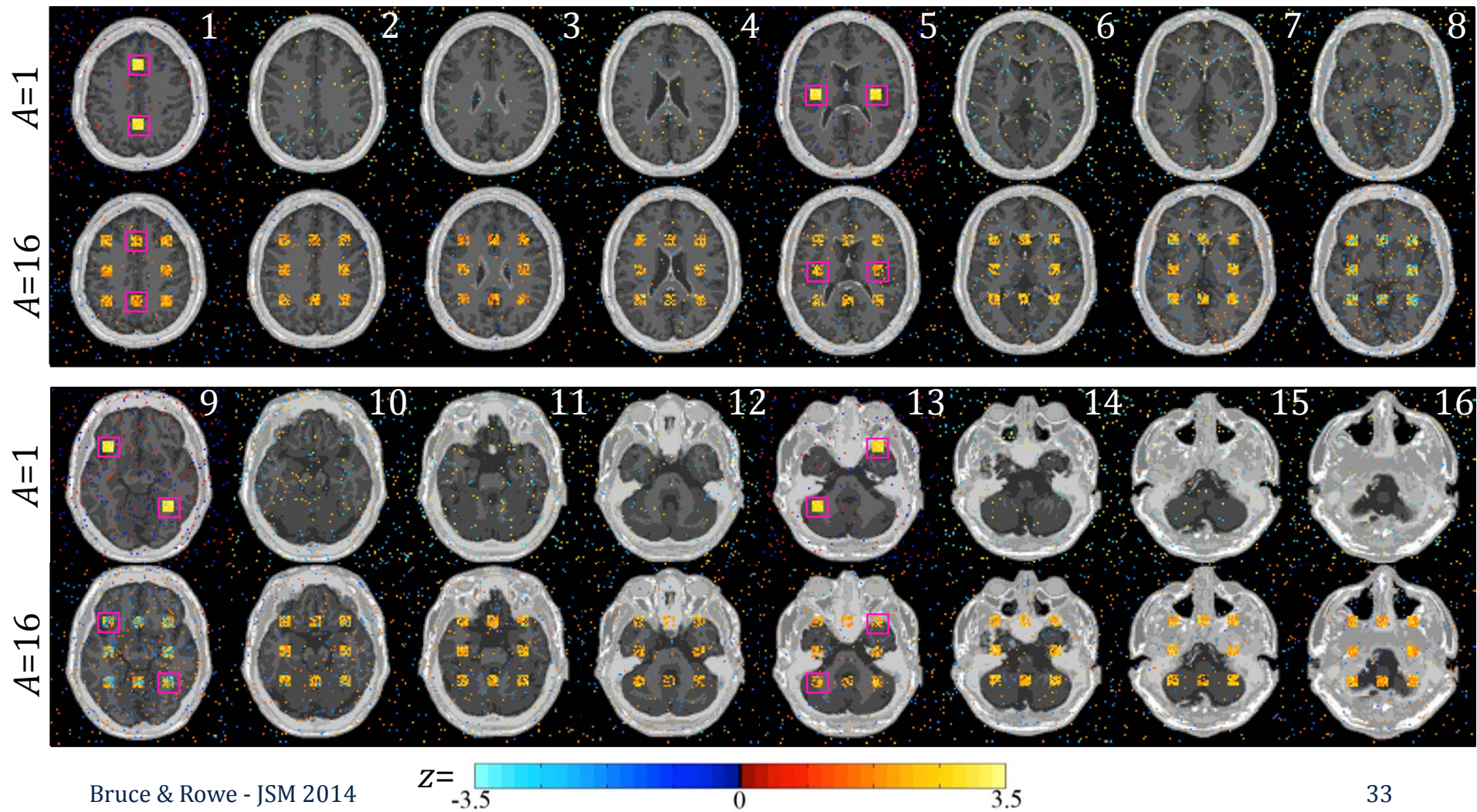




fMRI Results ($A=16, N_{acq}=1$)



After separation, activation should only be present within the ROIs, otherwise denote false-positives from residual aliasing.

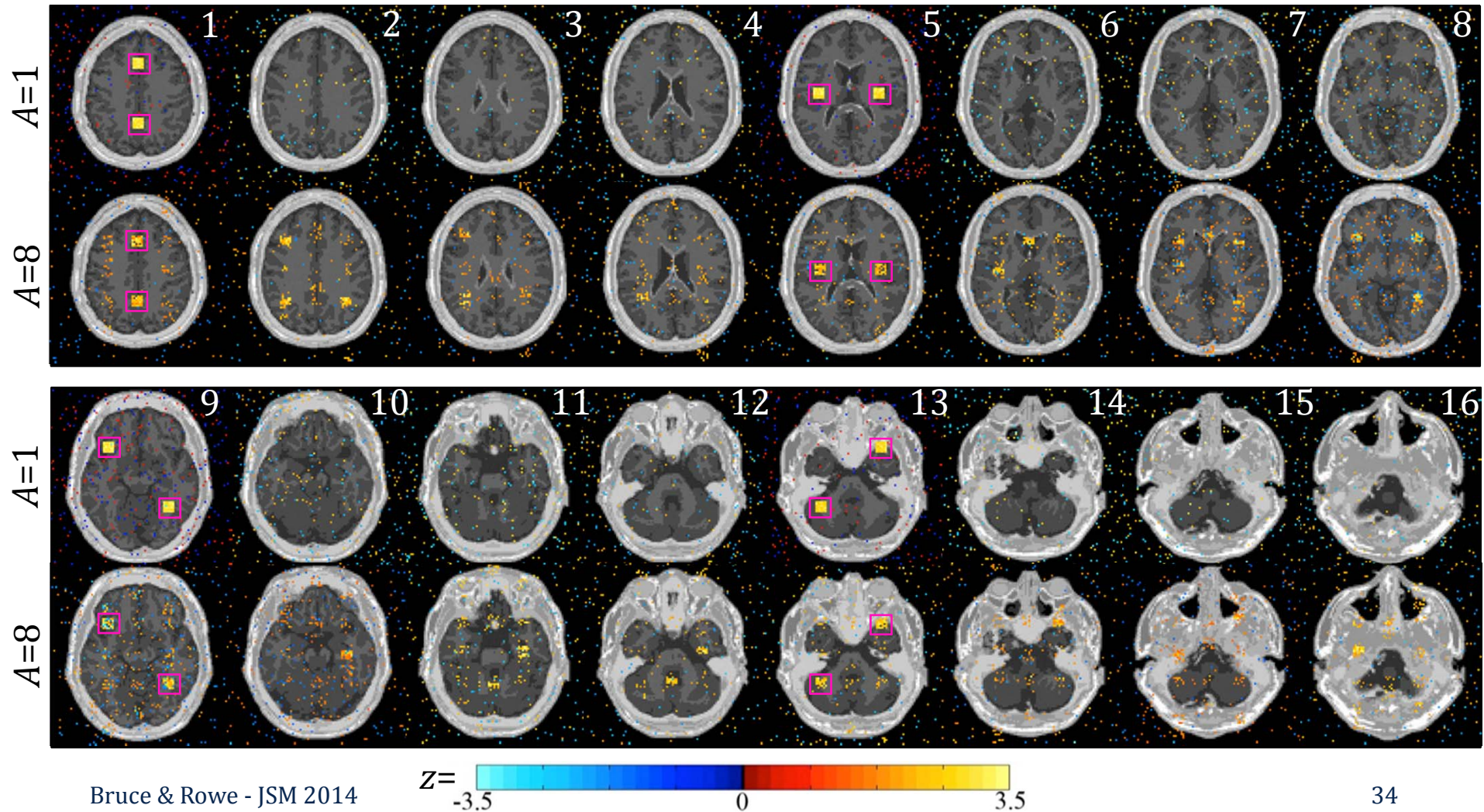




fMRI Results ($A=8, N_{acq}=2$)



After separation, activation should only be present within the ROIs, otherwise denote false-positives from residual aliasing.

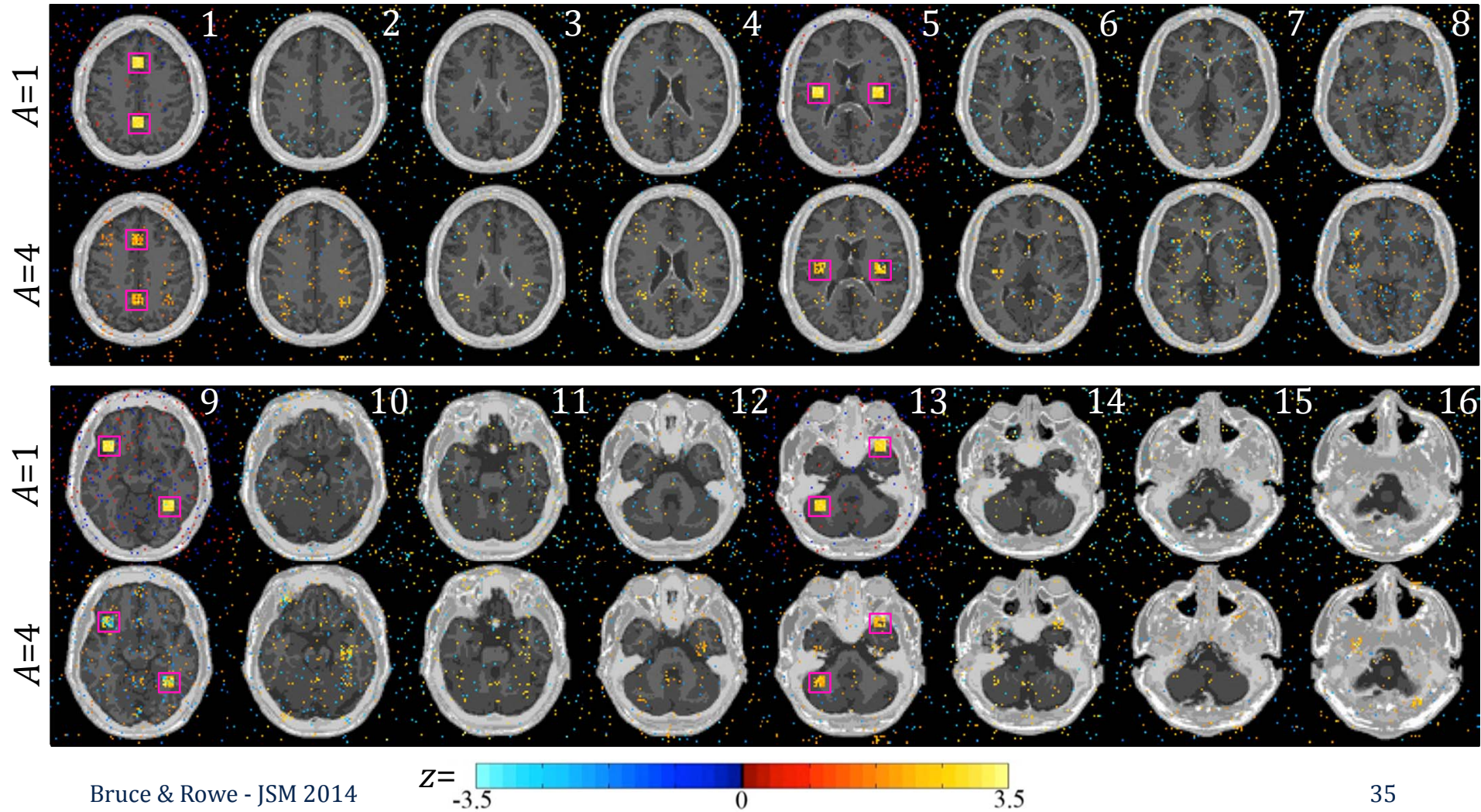




fMRI Results ($A=4, N_{acq}=4$)



After separation, activation should only be present within the ROIs, otherwise denote false-positives from residual aliasing.

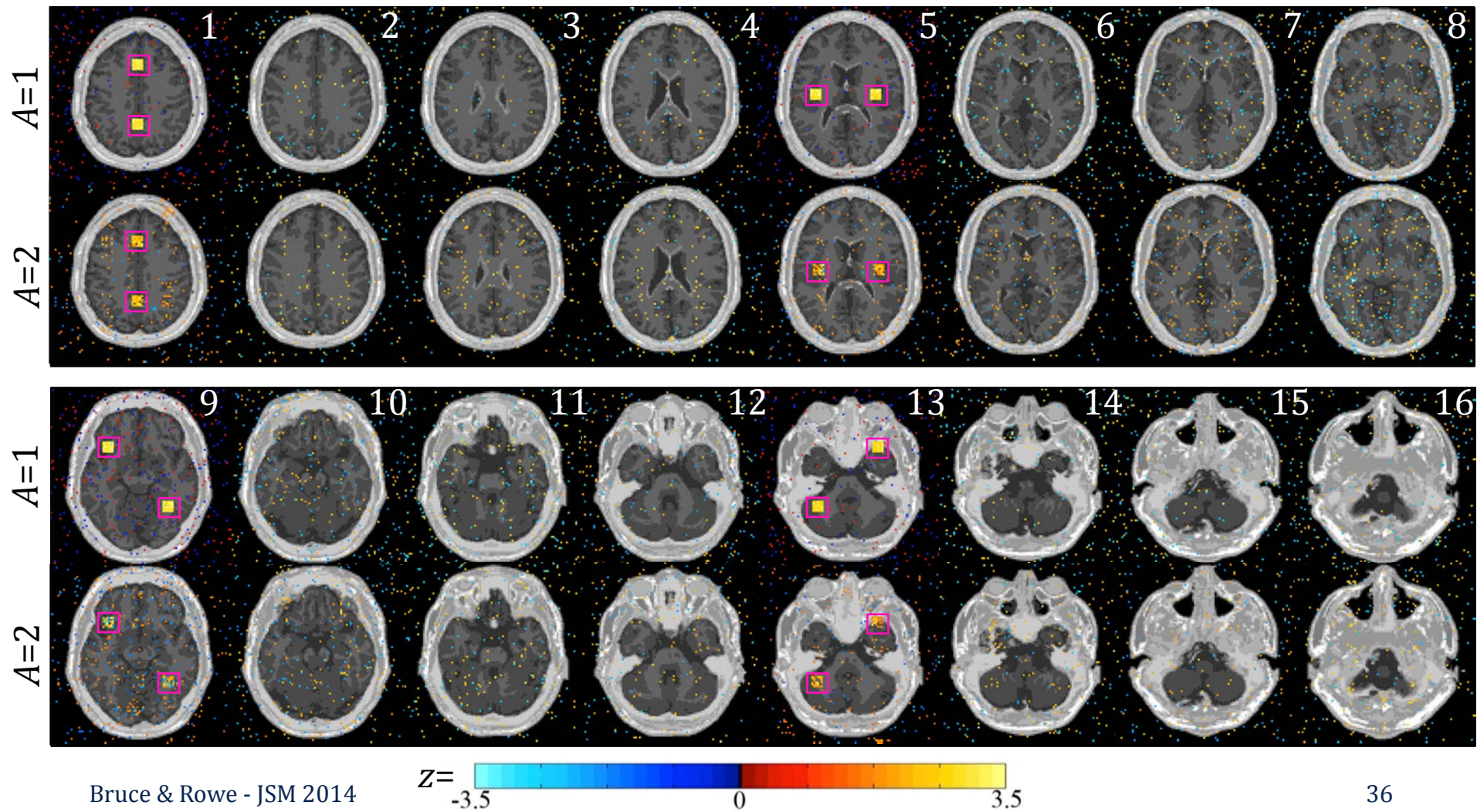




fMRI Results ($A=2, N_{acq}=8$)



After separation, activation should only be present within the ROIs, otherwise denote false-positives from residual aliasing.





SPECS incorporates coefficients of orthogonal polynomials and calibration images into the original aliasing model:

- Improves the rank of the aliasing matrix
- Induces no artificial correlation when using Hadamard coefficients
- Acceleration factors can exceed the number of coils (in this case one)

Incorporating multiple phase shifted acquisitions into SPECS:

- Provides multiple ways in which each voxel can be aliased
- Increases the power of detecting activation statistics
- Enables up to four-fold acceleration of fMRI data acquisition with only a single coil



Thank you

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