SENSE Induced Correlations are used to Optimize RF Coil Design for Specific fcMRI Studies

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Target Audience: MR Hardware Engineers

Background and Purpose: With the advent of parallel MRI techniques such as SENSitivity Encoding (SENSE),[1] much attention has been placed on the optimization of RF coil design in an effort to improve reconstructed images through advancements in hardware. The overlapping of coil magnetic fields (B-fields) results in an amplification of noise in the reconstructed images, and is almost exclusively measured using the geometry factor (*g*-factor), which is directly proportional to the SNR in the SENSE reconstructed images. In recent studies, inverse methods of achieving optimal RF coil design have been developed,[2] where a cost function, defined by the *g*-factor in a region of interest (ROI), is minimized. However, it has also recently been shown that the un-aliasing process in the SENSE model induces an artificial correlation between the previously aliased voxels.[3] This correlation is of no biological origin and can have detrimental fcMRI implications. The cost function would thus be more appropriately defined to minimize both the *g*-factor within a ROI as well as the correlations induced between the ROI and regions with which it was previously aliased. The goal of this study is to observe the change in the correlations induced by the SENSE model into a ROI with variation in coil geometry.

Methods: A 60×60×60 voxel FOV was simulated with an oval phantom. The ROI in Fig. 1a was positioned to encompass areas both with and without aliasing after a SENSE reconstruction using a reduction factor of R=2. Changes in a butterfly shaped coil geometry in Fig. 1b were performed by varying the inner and outer lengths, l_{in} and l_{out}, from 27 to 45 cm. The center angle, ϕ_i, of each coil i=[1:4] in Fig. 1c were rotated by 180°, starting with coil 1 centered at $\pi/4$. The coil radius was r=18 cm, the angle between the outer edges was $\delta_i=3\pi/8$, the angle between the inner edges was $\theta_i= \pi/8$ and the gap between coils were held constant at $\Delta=\pi/8$. In this study, the Biot-Savart law was used to estimate the B-fields over the entire volume. For subsequent studies, full-wave simulations will be carried out using the HFSS software package for more realistic B-field estimates. The coil covariance, Ψ, was estimated using an inner product of the coil B-fields within the phantom. With a matrix of N_C=4 coil sensitivities in the R=2 aliased folds, S, the covariance induced solely by the un-aliasing of a single aliased voxel with the SENSE model is $\mathrm{cov}_{SE}=UU^H$, where $U=(S^H\ \Psi^{-1}S)^{-1}S^H\ \Psi^{-1}$. The induced correlation is derived by $\mathrm{corr}_{SE}=D(\mathrm{cov}_{SE})D^H$, where D is a diagonal matrix with entries from $diag(\mathrm{cov}_{SE})^{-½}$. As both S and Ψ are complex-valued, the correlation induced between the real parts (equivalently between the imaginary parts) of the un-aliased voxels is denoted corr_{RR}, while the correlation induced between the real/imaginary parts of the un-aliased voxels is denoted corr_{RI}.

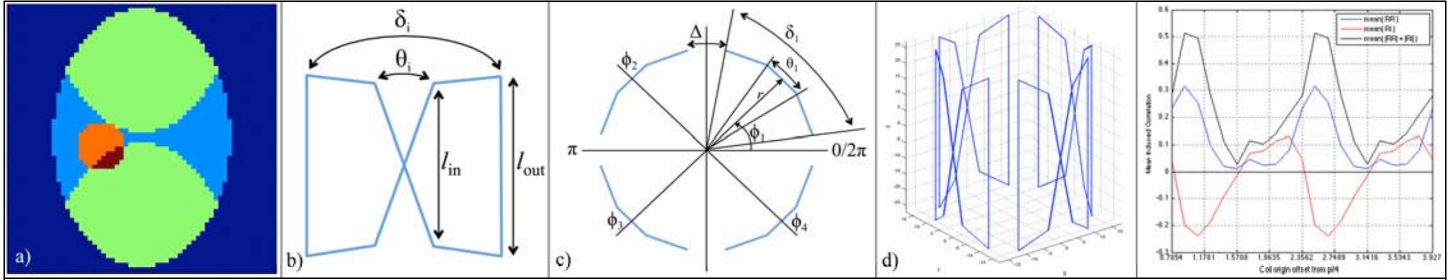

Figure. 1. The **a)** Phantom, folds (green areas) for R=2, and circular ROI (orange-red circle), **b)** coil vertices, **c)** defining angles of coil array, **d)** coil layout for optimal corr_{RR} and corr_{RI}, and **e)** $mean(\mathrm{corr}_{RR})$ (blue), $mean(\mathrm{corr}_{RI})$ (red), and $mean(|\mathrm{corr}_{RR}|+|\mathrm{corr}_{RI}|)$ (black) in the ROI with choice of ϕ_1.

Results & Discussion: The coil layout that jointly minimizes corr_{RR} and corr_{RI} from aliased voxels induced into the ROI is presented in Fig. 1d, where l_{out}=45 cm, l_{in}=39.6 cm. As shown in Fig. 1e, when rotating the array by 180°, the correlations are minimized when the centers of the coils were aligned with multiples of $\pi/2$. However, although not presented here, the centers of the coils did not align themselves with $\pi/2$ when corr_{RR} and corr_{RI} were optimized individually, and thus the assumption that coil geometries are minimized for every ROI when symmetric cannot be made. The ROI's position relative to each coil plays a key role in determining the optimal layout for minimizing both the g-factor and the induced correlation. It is for this reason that the off center ROI in Fig. 1a was selected.

Conclusion: While the *g*-factor is a useful metric in determining the amplification of noise within a ROI in the SENSE reconstructed images as a result of the overlapping B-fields, it does not provide a measure of correlations induced into the ROI by the SENSE model. As the SENSE model uses sensitivities that are dependent on the coil geometry, the correlations induced into a ROI have been shown to vary with changes in the RF coil geometry. Specific coils can be designed for fcMRI studies by developing a cost function that uses a least squares estimation to minimize both the g-factor and the SENSE induced correlations with changes in parameters: N_C, R, l_{out}, l_{in}, ϕ_i, δ_i, θ_i, and Δ_{ij} (the gaps between coils i-j). For a specific ROI, a design of this kind would both maximize the SNR in areas of aliasing and minimize the potential for Type I&II errors in fcMRI studies resulting from changes to the covariance of the data. The proof of concept has been demonstrated here with an N_C=4 element coil array. However, the technique can be applied to coil arrays with more elements, in which each element is independently optimized to maximize the imaging performance in any selected ROI.

References: **1.** KP Pruessmann et al. MRM 42:952-962, 1999. **2.** LT Muftuler et al. Phys. Med. Biol. 51:6457-6469, 2006. **3.** IP Bruce et al. MRI 30:1143-1166, 2011.

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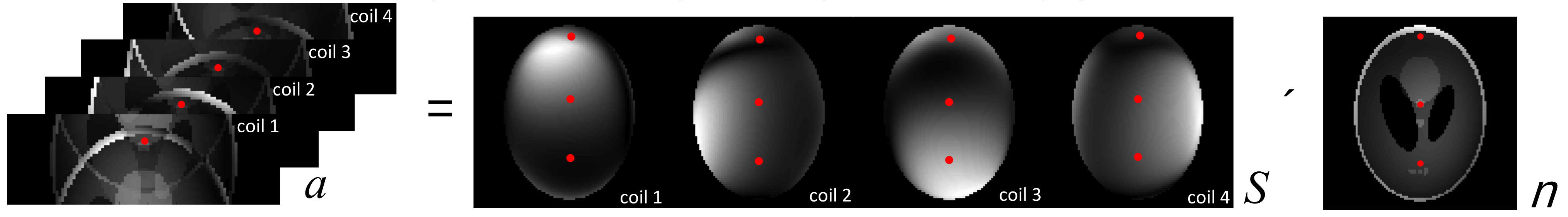
INTRODUCTION

The SENSE parallel MRI model (1) is based on the notion that aliased voxel values acquired by N_C receiver coils, a, are a linear combination of the R true un-aliased voxel values, v, with an $N_C \times R$ coefficient matrix, S, derived from coil magnetic field (B-field) sensitivity profiles, $a=Sv$.



The un-aliased voxel vector is thus derived through a complex-valued weighted least squares estimation, $v = [(S^H \Psi^{-1} S)^{-1} S^H \Psi^{-1}] a = Ua$, where H denotes the Hermitian, and Ψ is the covariance between coils. In most coil design studies, the geometry factor, $g_j = \sqrt{(S^H \Psi^{-1} S)^{-1}_{j,j} (S^H \Psi^{-1} S)_{j,j}}$, is a measure of the amplification of noise in a voxel j, and is typically the de facto metric for assessing the geometry of an array to be used for SENSE imaging.

Recently (2), it has been shown that through the SENSE unfolding matrix, U, there is an artificial correlation induced between the voxels in v, derived by $\mathrm{corr}_{SE} = D(UU^H)D^H$, where D is a diagonal matrix with entries from $diag(UU^H)^{-1/2}$. The correlation induced by the SENSE model is of no biological origin, and can thus influence functional connectivity studies where the null hypothesis assumes no correlation between voxels.

Both g and U (and thus corr_{SE}) are solely dependent on S and Ψ, properties of the coil B-fields, and thus the coil layout itself. The aim of this study is therefore to develop a framework for optimizing RF coil design for fcMRI studies that focus on a particular ROI by simultaneously minimizing both g (to reduce the amplification of noise in the ROI) and corr_{SE} (to reduce potential Type I errors caused by the SENSE unfolding).

METHODS

An array of N_C=8 axially symmetric coils was generated 1500 times by randomly shifting the vertices of each coil from an initial, symmetric location. In each iteration, the parameters that define the layout of each coil, ξ=[1:8], in Fig. 1a varied by

- All coil radii set to r=14 cm.
- l_{out} varied between 75-125% of 2r.
- l_{in} varied between 75-125% of 2r.
- δ_ξ varied between 50-120% of $2\pi/N_C$.
- θ_ξ varied between 10-60% of δ_ξ.

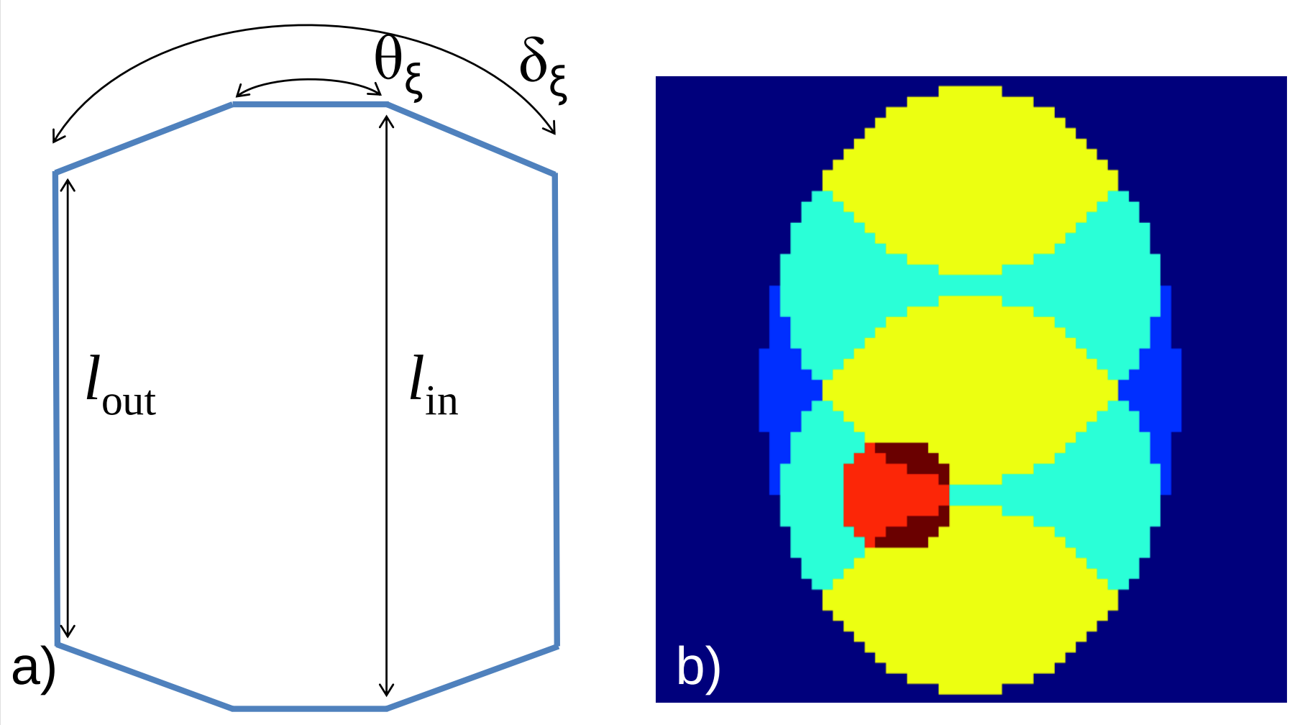


Figure 1: a) coil parameters and b) ROI in an oval phantom.

An upper limit on the gap between coils was set to 3% of the sum of δ_ξ from the adjacent coils. For each of the 1500 configurations, B-fields were simulated over a 60×60×60 voxel FOV with the oval phantom in Fig. 1b using Biot-Savart. The red/orange circular ROI in Fig. 1b was positioned to encompass areas of both two and three fold aliasing in a SENSE reconstruction with R=3, extended throughout the middle 80% of the FOV, and is off center to replicate an ROI in the brain that is not in the center.

RESULTS AND DISCUSSION

To measure the SENSE induced correlation, the determinant was taken of a correlation matrix that was formed by placing a real-valued representation of corr_{SE} for each voxel in the ROI along the diagonal. An ideal coil layout would have both $g = \det[\mathrm{corr}_{SE}] = 1$. In each iteration, $\det[\mathrm{corr}_{SE}]$ and the mean g-factor in the ROI were calculated for the given array configuration, and layout with the closest g to one is presented in Fig. 2, the array with the closest $\det[\mathrm{corr}_{SE}]$ to 1 is in Fig. 3, while the array in which both g and $\det[\mathrm{corr}_{SE}]$ were simultaneously closest to 1 is in Fig. 4.

<u>Full array</u> <u>Unrolled array</u> <u>*g*-factor map</u>

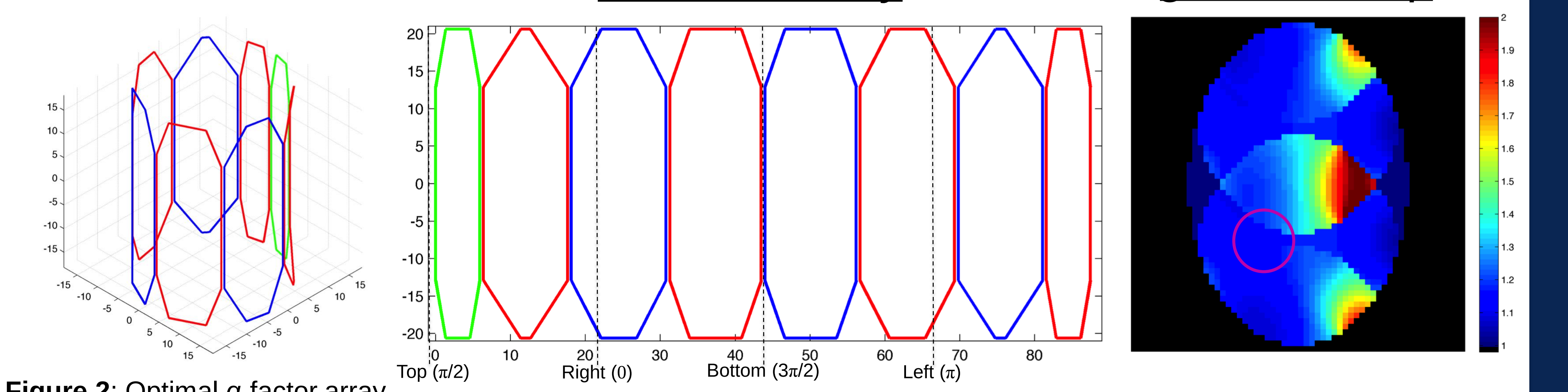


Figure 2: Optimal *g*-factor array

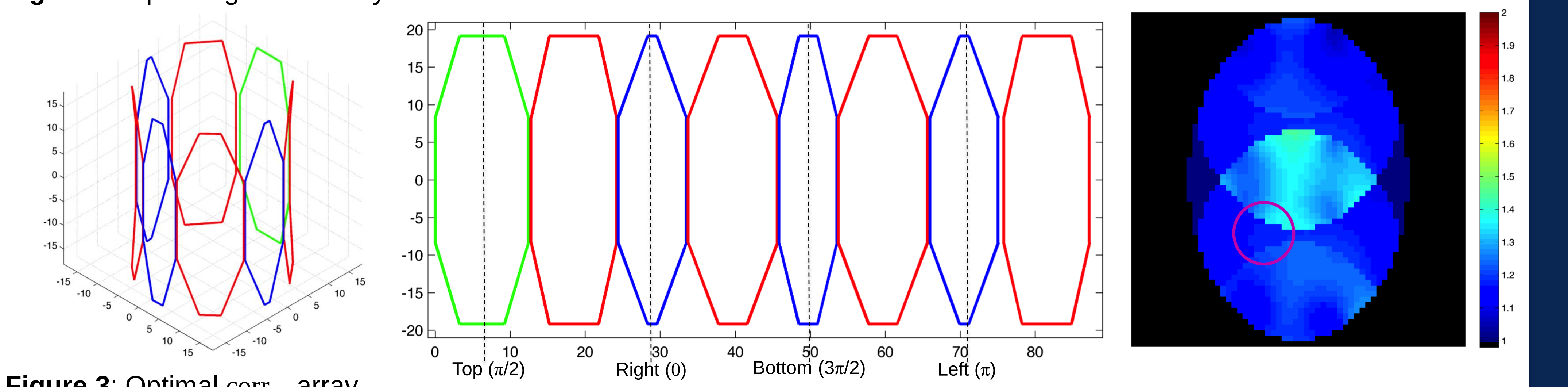


Figure 3: Optimal corr_{SE} array

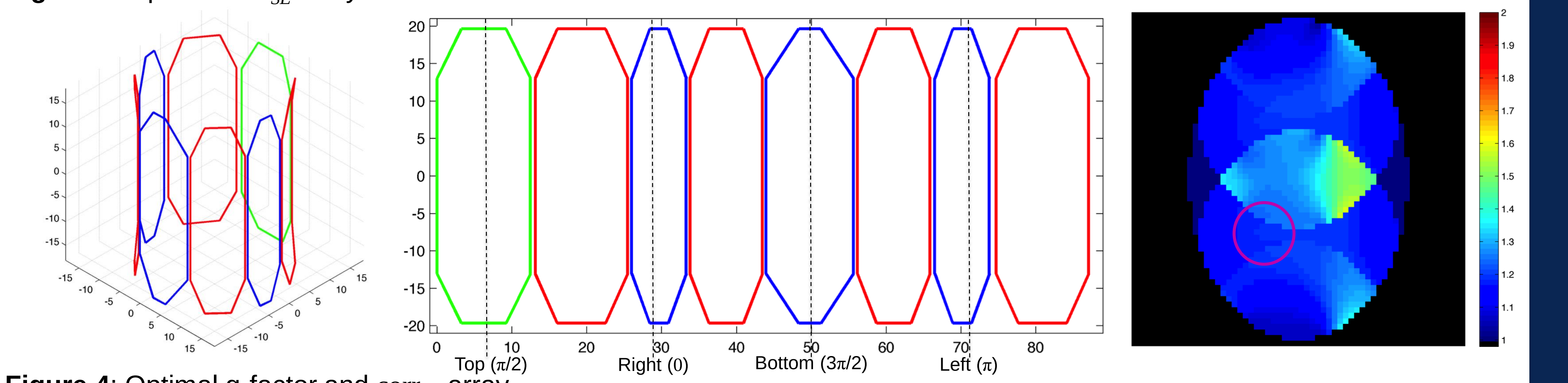


Figure 4: Optimal g-factor and corr_{SE} array

It is apparent that a coil optimized for the g-factor, as is common in many RF coil design studies, does not necessarily exhibit an optimal SENSE induced correlation between voxels, while a coil optimized with corr_{SE} as an optimization criteria not only exhibit a lower induced correlation, but also a more uniform g-factor. As the g-factor measures the amplification of noise in SENSE reconstructed images, and corr_{SE} measures the correlation induced between previously aliased voxels that could influence fcMRI conclusions, a cost function that optimizes both metrics is more appropriate for coils that might be used in fcMRI studies.

REFERENCES

(1) KP Pruessmann et al. SENSE: Sensitivity Encoding for fast MRI. MRM 42:952-962, 1999.

(2) IP Bruce et al. A statistical examination of SENSE image reconstruction via an isomorphism representation. MRI 30:1143-1166, 2011.