Continuous Probability Functions
(Cont.)

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Outline

- **Continuous Student-t Distribution**
  PDF, Moments, CDF, Matlab
- **Continuous Cauchy Distribution**
  PDF, Moments, CDF, Matlab
- **Continuous F Distribution**
  PDF, Moments, CDF, Matlab

Continuous Distributions

Student-t:

A random variable x has a continuous Student-t distribution, $x \sim t(\nu)$ if

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}, \quad x \in \mathbb{R}$$

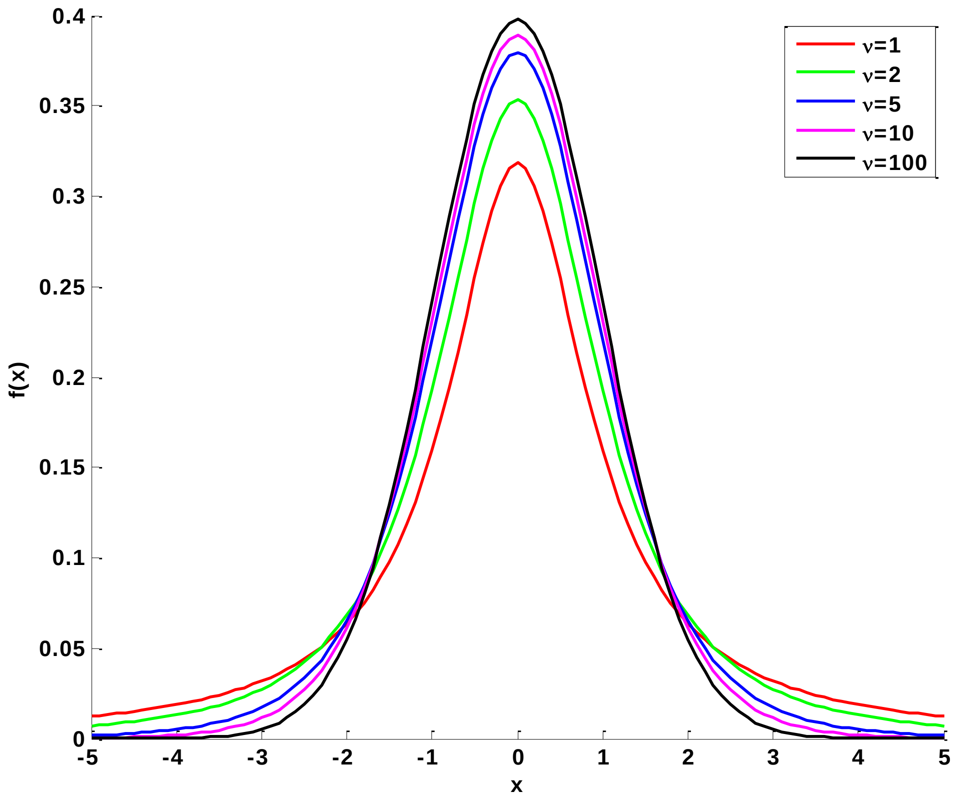
where $\nu = 1, 2, \ldots$.

Continuous Distributions

Student-t:

$$f(x\mid\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
x=(-5:.1:5)';
nu=[1,2, 5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tpdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])
```



Continuous Distributions

Student-t:

$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

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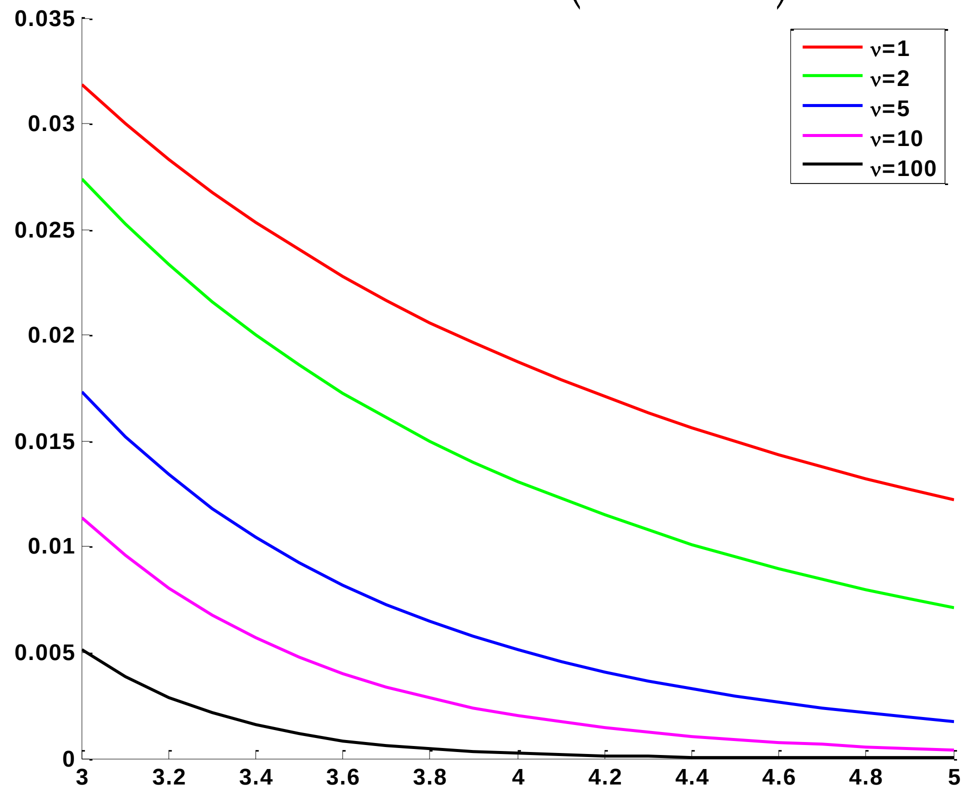
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])
```



Continuous Distributions

Student-t:

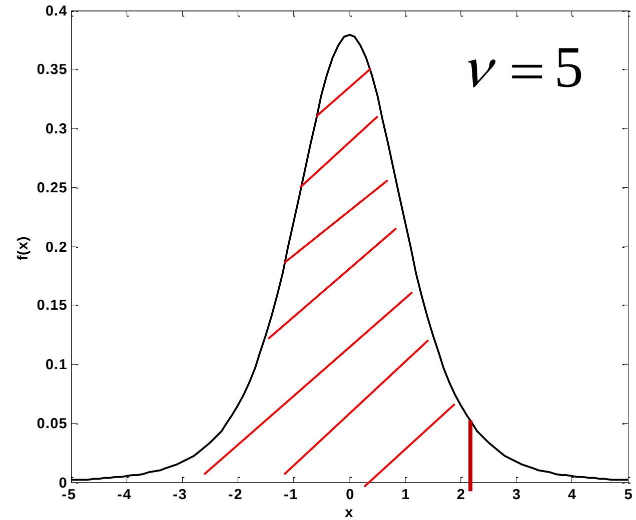
The CDF of the continuous Student-t distribution is

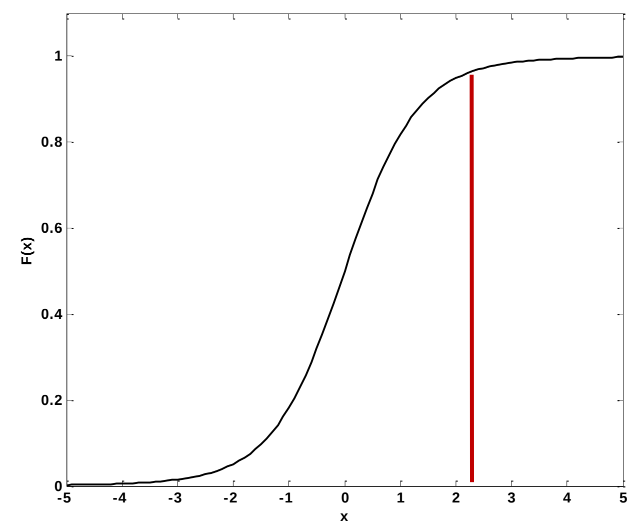
$$F(x|\theta) = \int_{t=-\infty}^{x} f(t|\theta)dt$$

$$F(x|\nu) = \int_{t=-\infty}^{x} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{(1+\frac{1}{\nu}t^2)^{(\nu+1)/2}} dt$$

$$= \frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \frac{{}_2F_1\left(\frac{1}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{x^2}{\nu}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}$$

Where ${}_2F_1$ is the hypergeometric function





Continuous Distributions

Student-t:

$$f(x \mid \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
x=(-5:.1:5)';
nu=[1,2,5,10,100];,
figure(1)
hold on
for count=1:length(nu)
    y = tcdf(x,nu(count));
    if count==1
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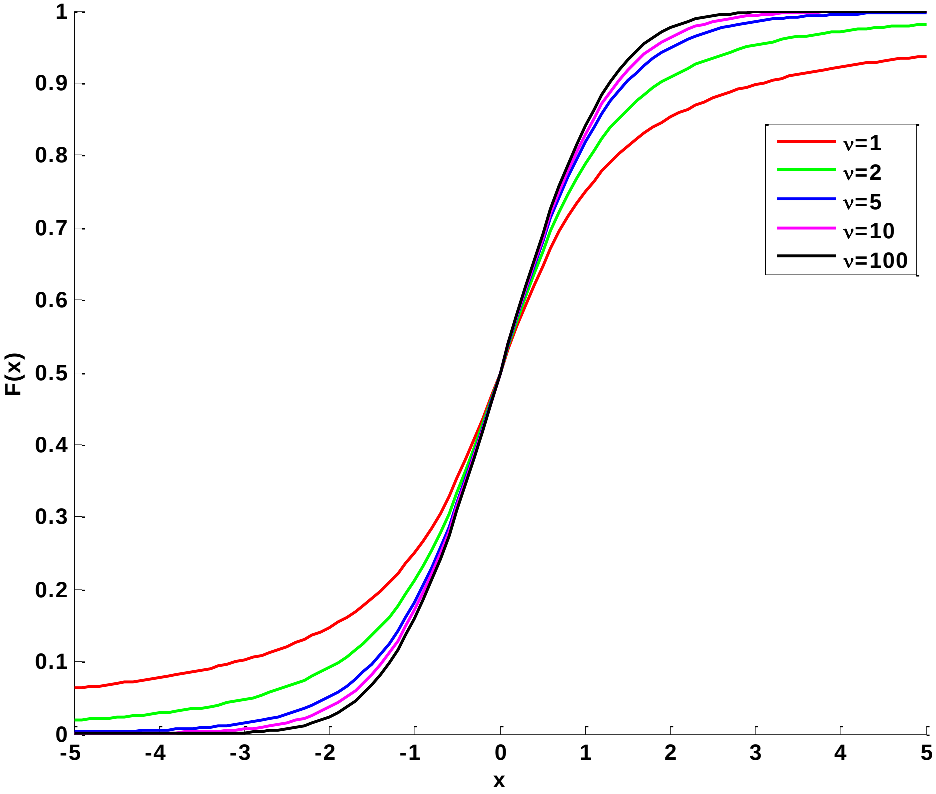
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])
```



Continuous Distributions

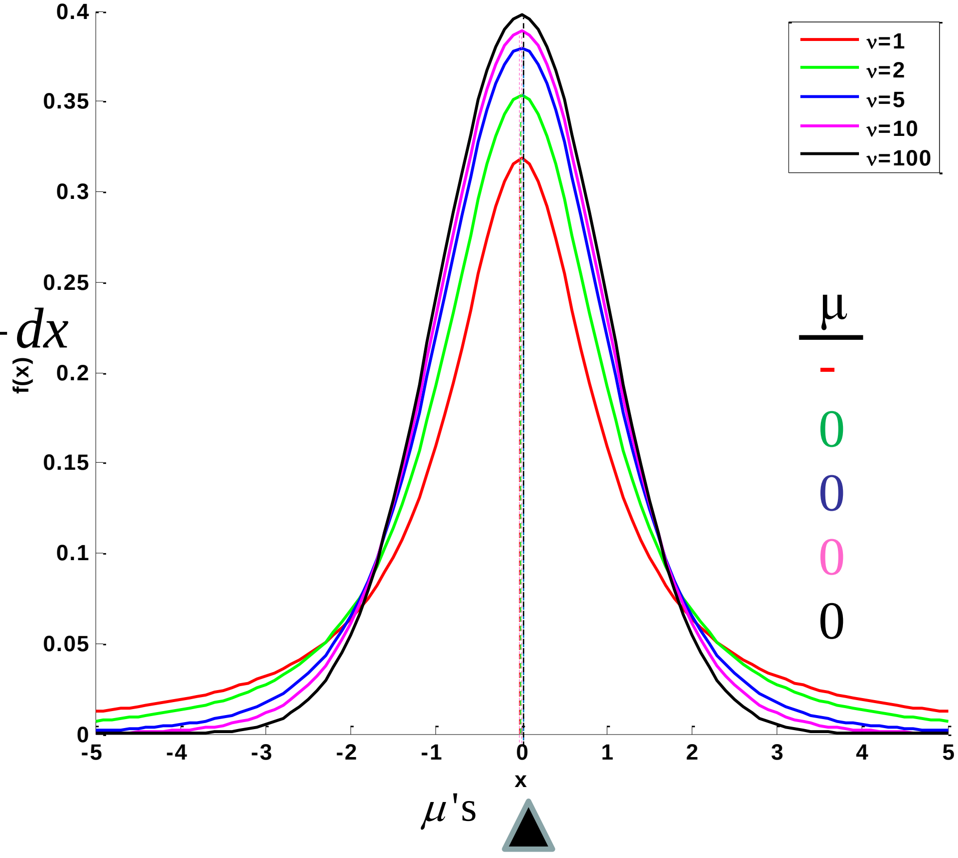
Student-t:

It can be shown that

$$\mu = \int_x x f(x \mid \theta) dx$$

$$= \int_{x=-\infty}^{\infty} x \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}} dx$$

$$= 0 \qquad \nu > 1$$



Continuous Distributions

Student-t:

It can be shown that

median

$$\int_{x=-\infty}^{\tilde{x}} f(x\,|\,\theta)dx = \frac{1}{2}$$

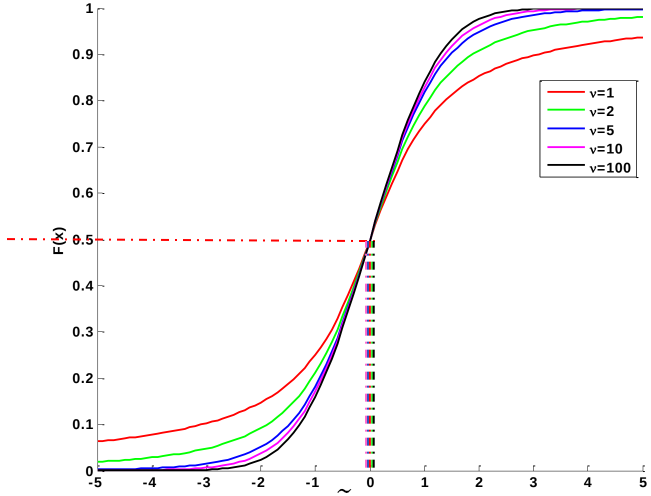
$$\tilde{x} = 0$$

mode

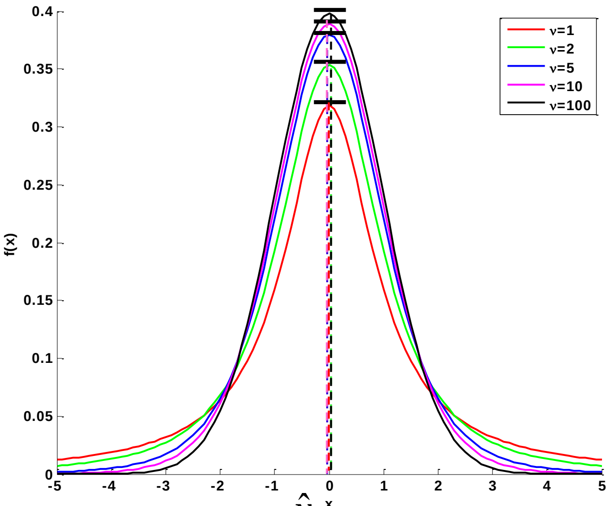
$$\frac{\partial}{\partial x} f(x\,|\,\theta)\bigg|_{\hat{x}} = 0$$

$$\hat{x} = 0$$

1/2



$\tilde{x}$



$\hat{x}$

$\tilde{x}$
0
0
0
0
0

$\hat{x}$
0
0
0
0
0

Continuous Distributions

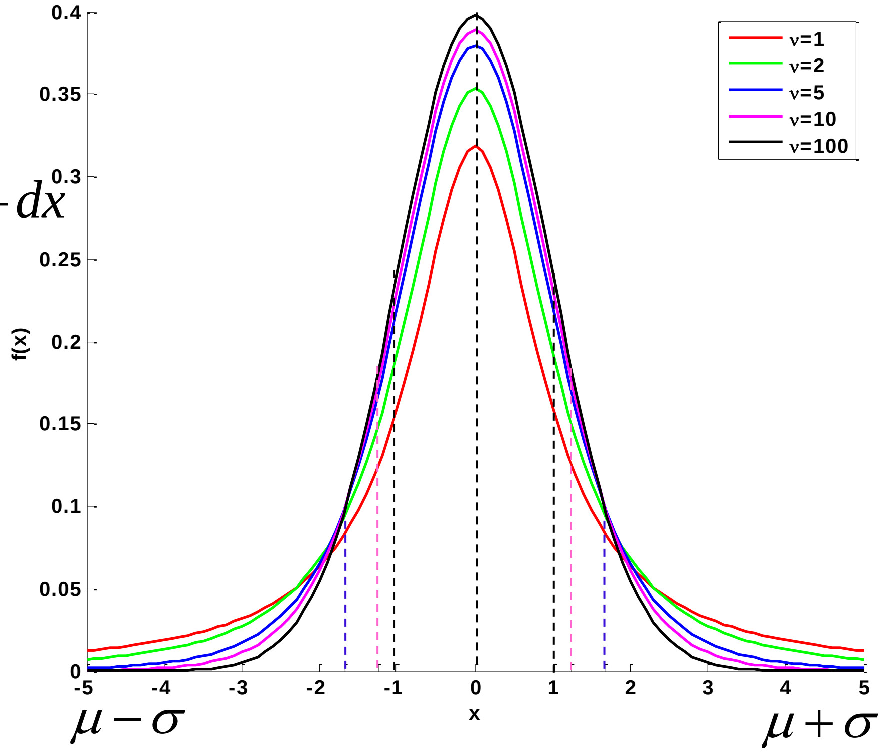
Student-t:

that

$$\sigma^2 = \int_x (x-\mu)^2 f(x|\theta)dx$$

$$= \int_{x=-\infty}^{\infty} (x-\mu)^2 \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}} dx$$

$$= \frac{\nu}{\nu-2}$$



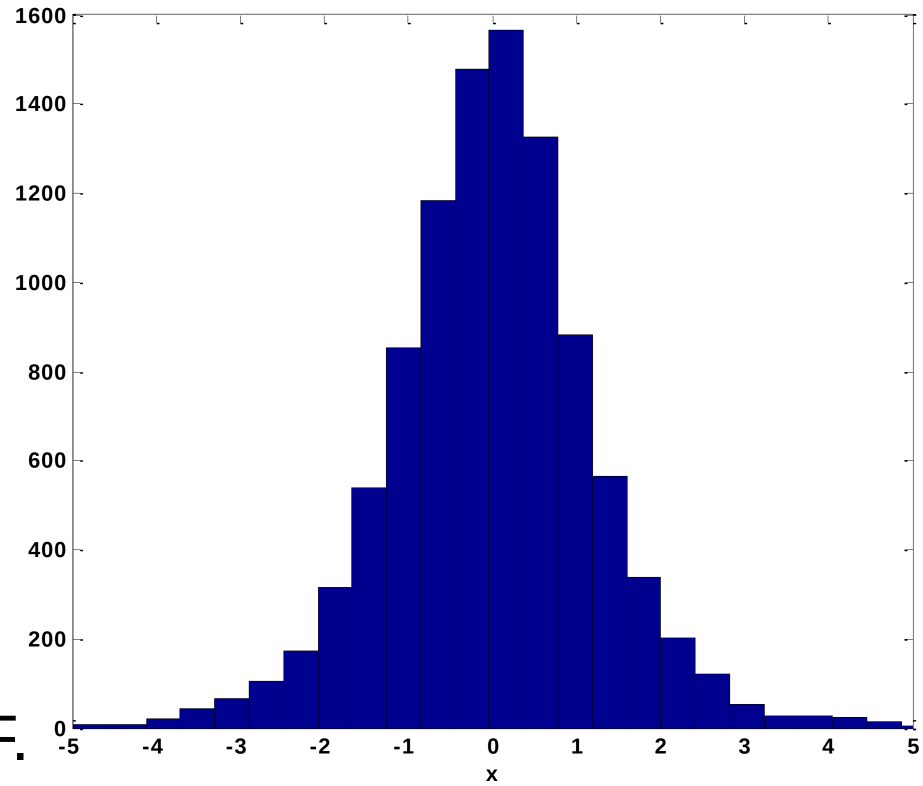
Continuous Distributions

Student-t:

$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
nu=5;,num=10^4;
x=trnd(nu,num,1);
mean(x)
var(x)
hist(x,50), xlim([-5 5])
```

	True	Simulated
μ	0	-0.0062
σ^2	1.6667	1.6318



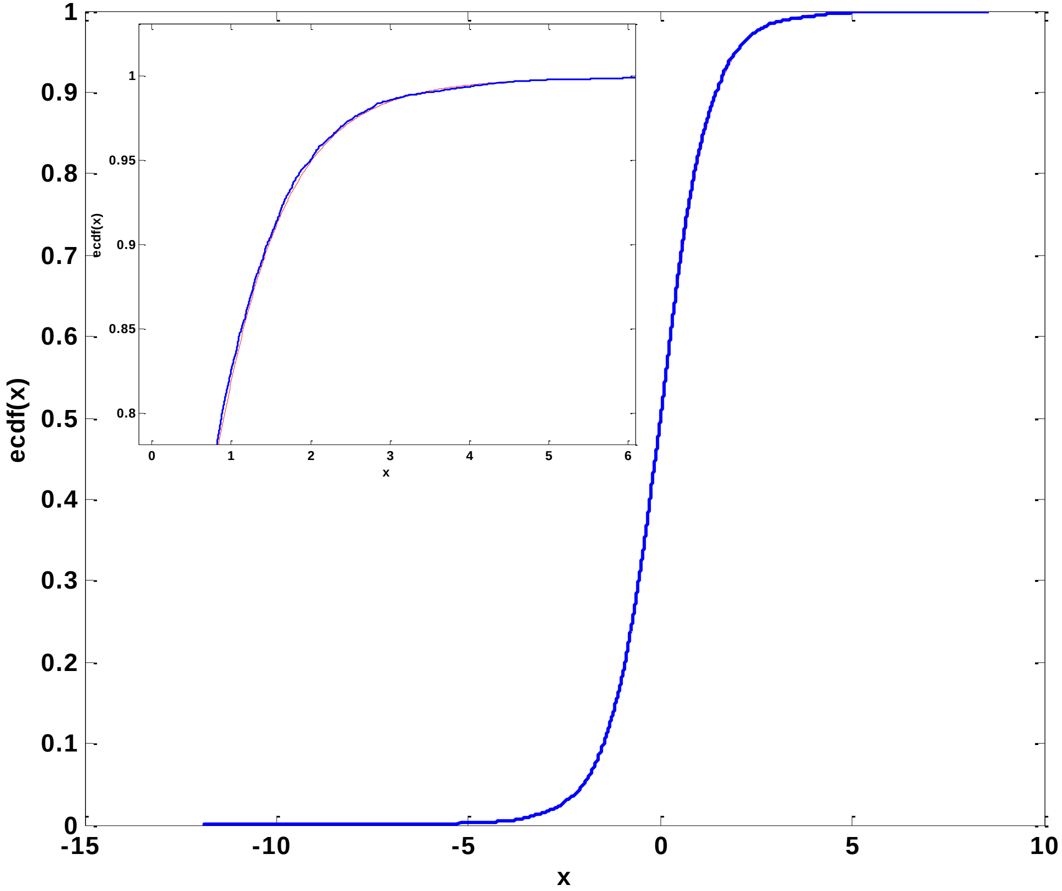
Can also find and plot ECDF.

Continuous Distributions

Student-t:

$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
nu=5;
y=tcdf((-5:.01:5),nu);
plot((-5:.01:5),y, 'r')
hold on
[F,xx]=ecdf(x);
stairs(xx,F,'LineWidth',2)
```



Continuous Distributions

Student-t:

The Student-t Distribution, can be generalized to have location and scale parameters, so that

$x \sim t(\nu,\delta,\tau)$ if

$$f(x|\nu,\delta,\tau)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{(\tau^2)^{-1/2}}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}\left(\frac{x-\delta}{\tau}\right)^2\right)^{(\nu+1)/2}},\ x\in\mathbb{R}$$

where $\nu=1,2,\ldots$.

Continuous Distributions

Cauchy (Lorentzian):

A random variable x has a continuous (standard) Cauchy distribution, x~Cauchy if

$$f(x|\nu)=\frac{1}{\pi}\frac{1}{1+x^2}\ ,\ \text{where, } x\in\mathbb{R}.$$

Cauchy distribution is a special case of the Student-t distribution with $\nu=1$.

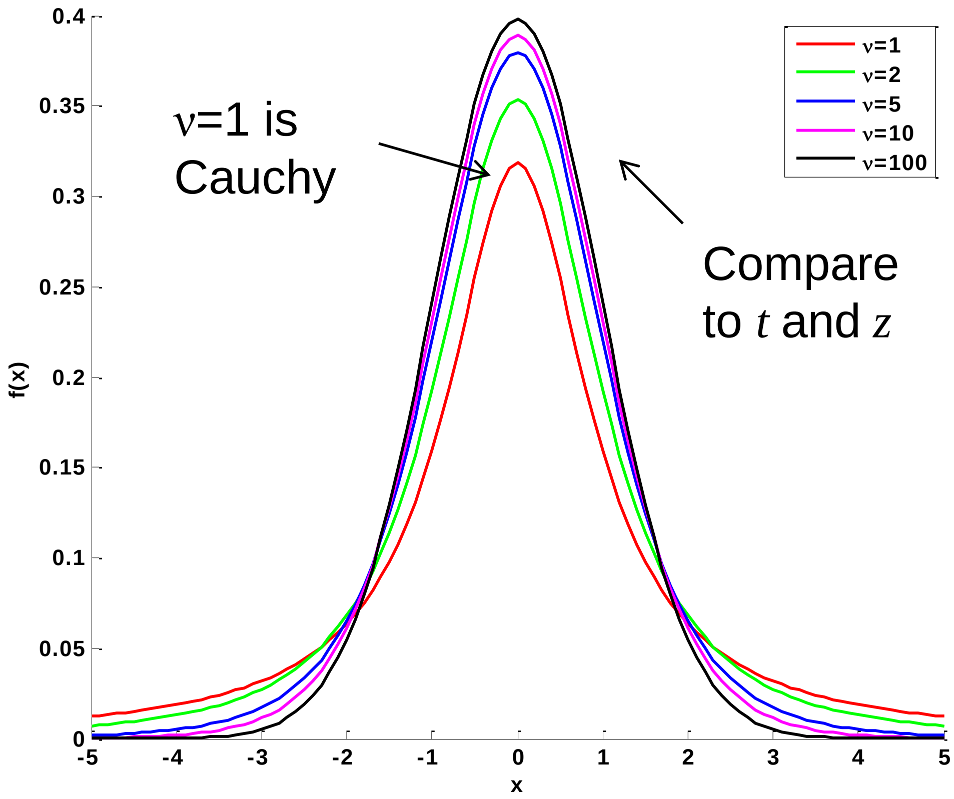
$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

Continuous Distributions

Student-t (Cauchy):

$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
x=(-5:.1:5)';
nu=[1,2,5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tpdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])
```



Continuous Distributions

Student-t (Cauchy):

$$f(x|\nu)=\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\sqrt{\nu\pi}}\frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

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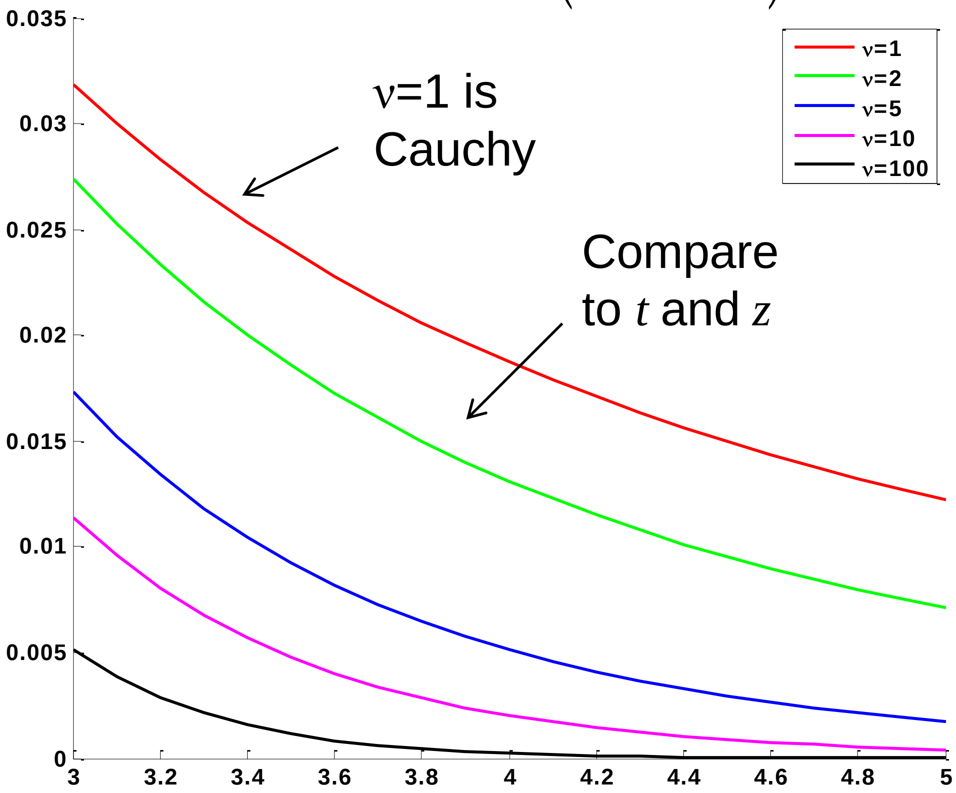
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])
```



Continuous Distributions

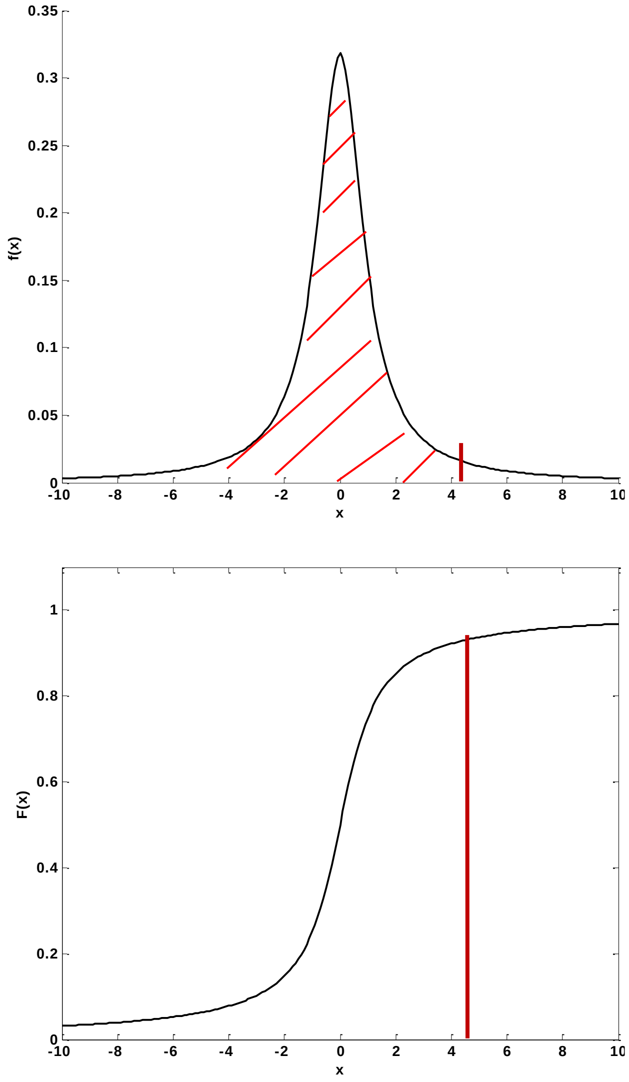
Cauchy:

The CDF of the continuous Cauchy distribution is

$$F(x|\theta) = \int_{t=-\infty}^{x} f(t|\theta)dt$$

$$F(x) = \int_{t=-\infty}^{x} \frac{1}{\pi}\frac{1}{1+t^2}dt$$

$$= \frac{1}{2}+\frac{1}{\pi}\mathrm{a}\tan(x)$$



Continuous Distributions

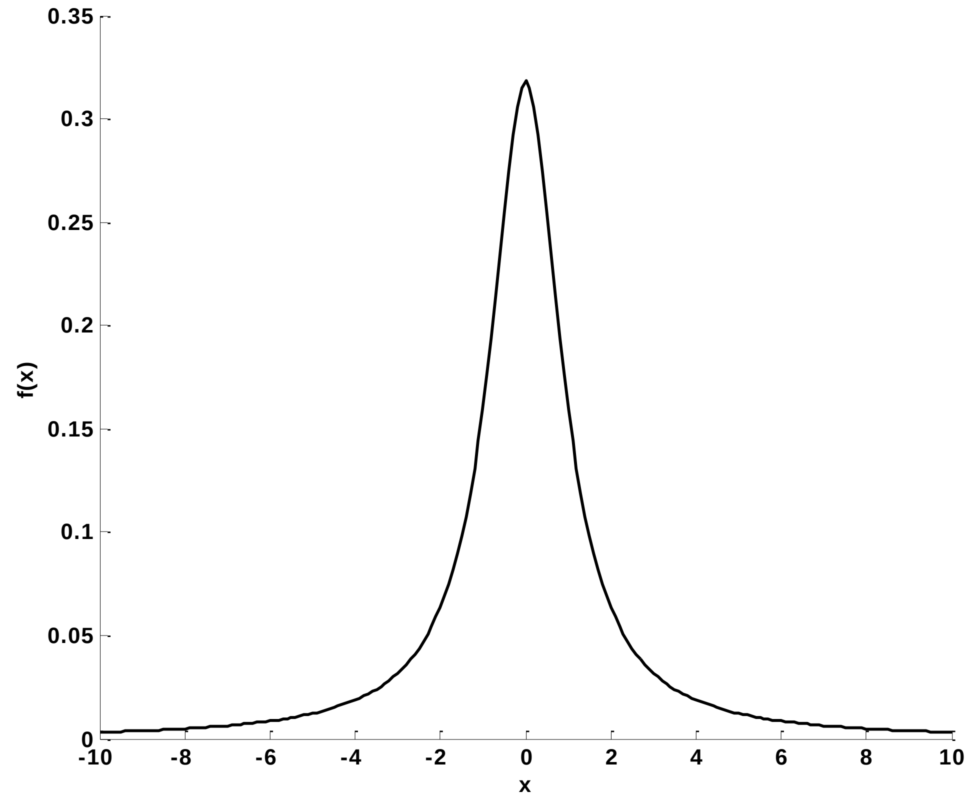
Cauchy:

It can be shown that

$$\mu = \int_x x f(x|\theta) dx$$

$$= \int_{x=-\infty}^{\infty} x \frac{1}{\pi} \frac{1}{1+x^2} dx$$

$=$ not defined



Continuous Distributions

Cauchy:

It can be shown that

median

$$\int_{x=-\infty}^{\tilde{x}} f(x\,|\,\theta)dx = \frac{1}{2}$$

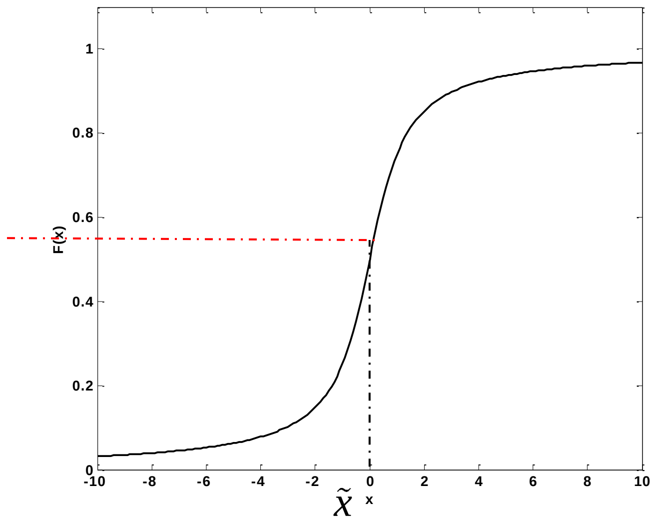
$$\tilde{x} = 0$$

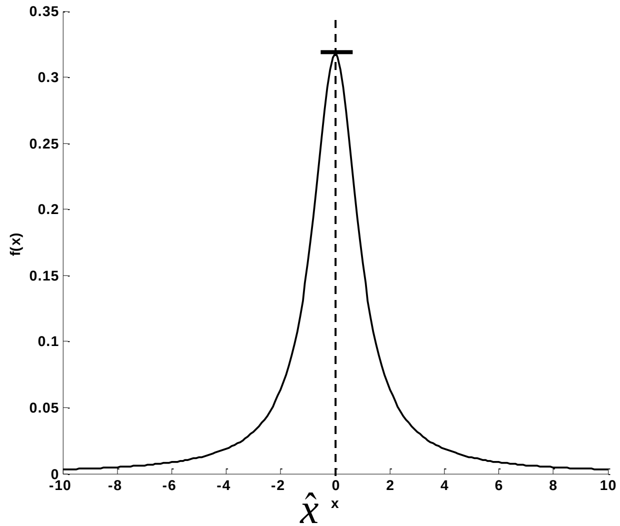
mode

$$\frac{\partial}{\partial x} f(x\,|\,\theta)\Big|_{\hat{x}} = 0$$

$$\hat{x} = 0$$

1/2





Continuous Distributions

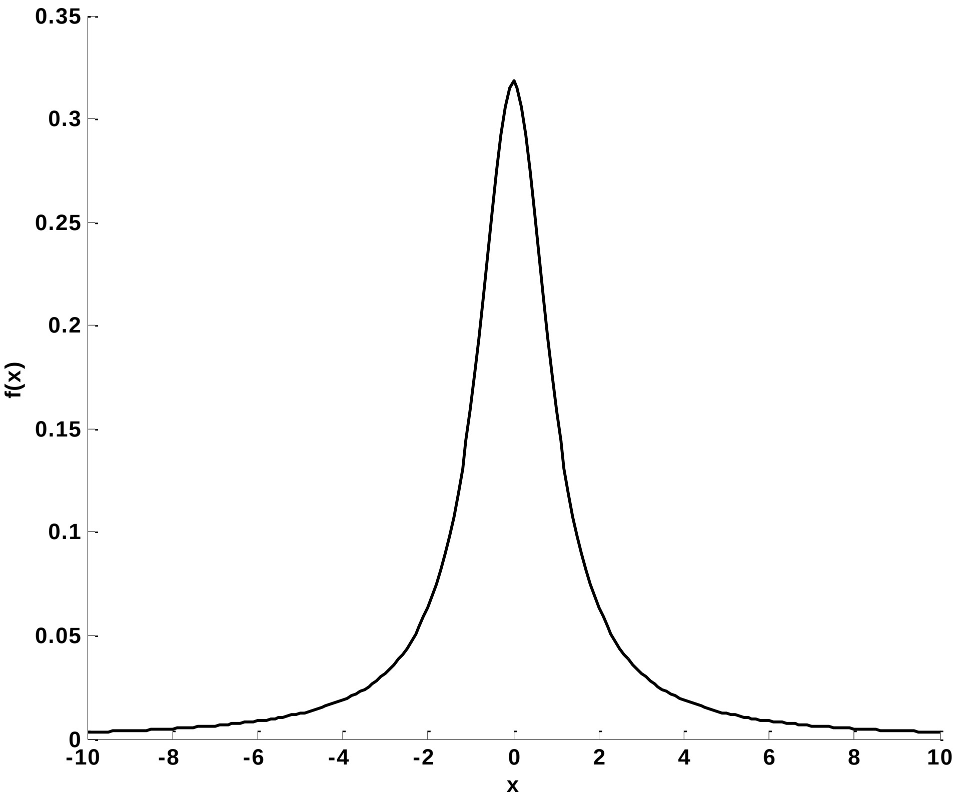
Cauchy:

that

$$\sigma^2 = \int_x (x-\mu)^2 f(x|\theta)dx$$

$$= \int_{x=-\infty}^{\infty} (x-\mu)^2 \frac{1}{\pi}\frac{1}{1+x^2}dx$$

$$= \text{not defined}$$



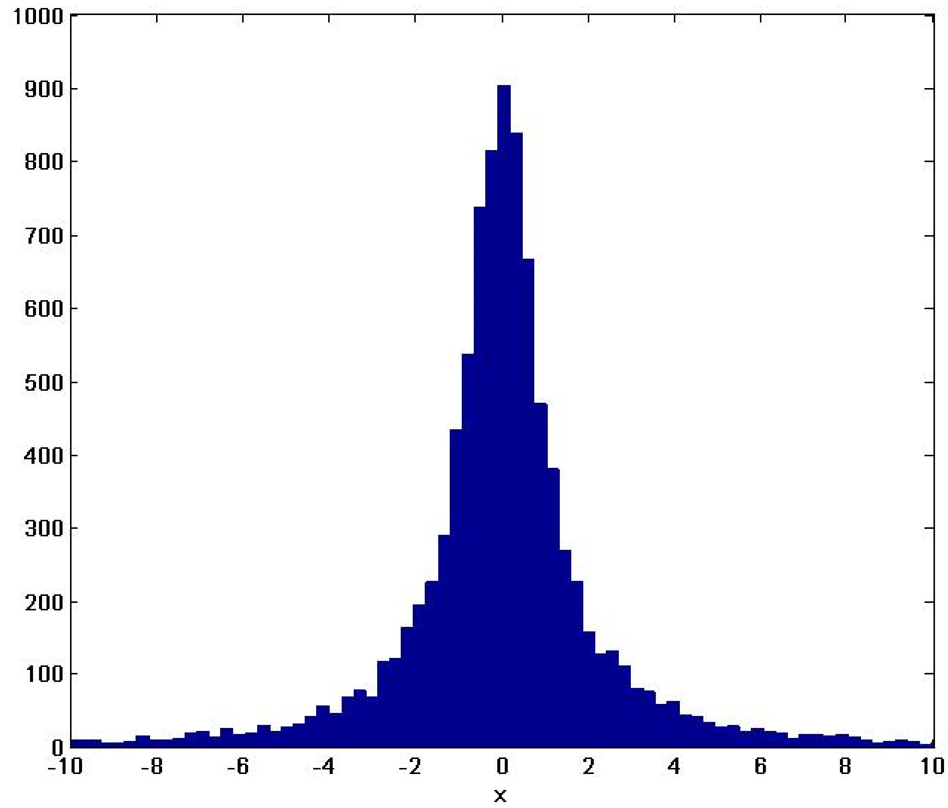
Continuous Distributions

Cauchy:

$$f(x)=\frac{1}{\pi}\frac{1}{1+x^2}$$

```
nu=1;,num=10^4;
x=trnd(nu,num,1);
mean(x)
var(x)
hist(x,10^5), xlim([-10 10])
```

	True	Simulated
μ	-	4.2813
σ^2	-	9.3156×10^4



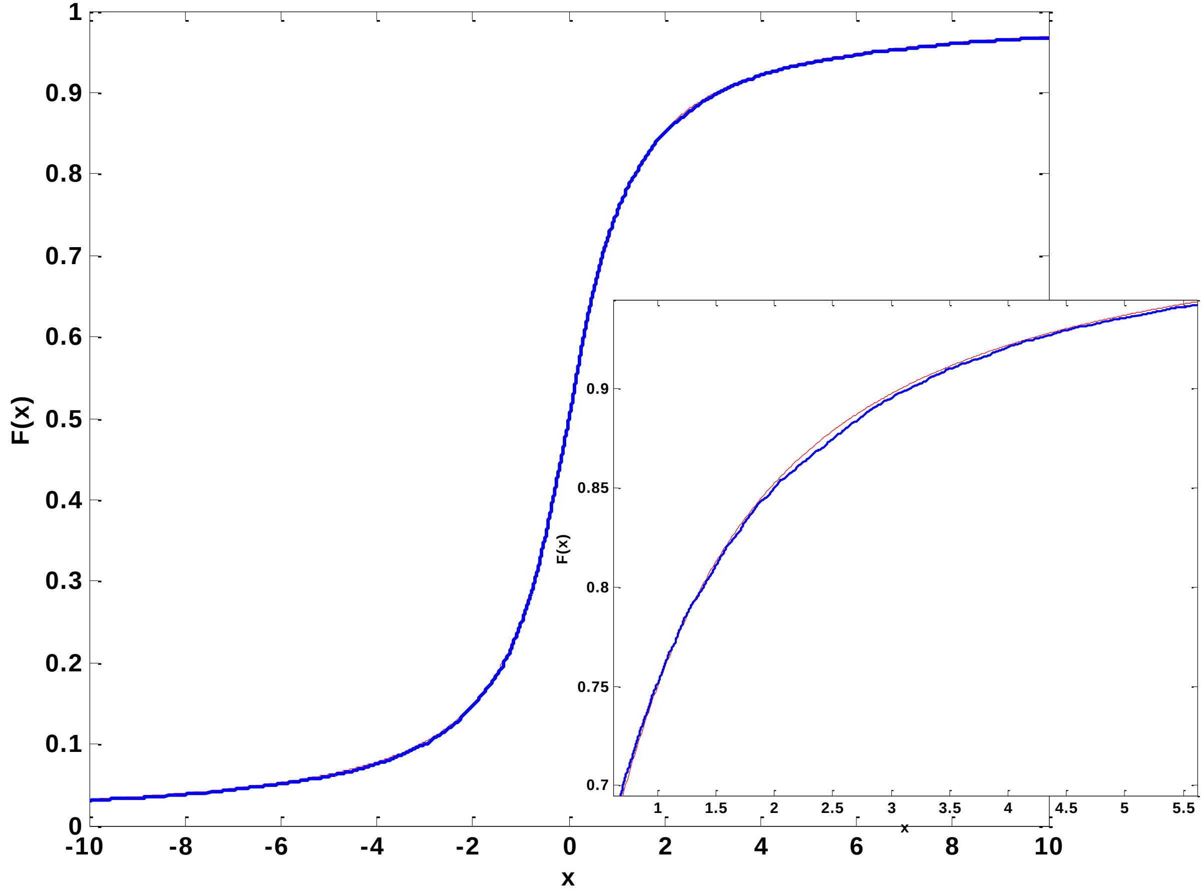
Can also find and plot ECD

Continuous Distributions

Cauchy:

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

```
nu=1;
y=tcdf((-10:.01:10),nu);
plot((-10:.01:10),y, 'r')
hold on
[F,xx]=ecdf(x);
stairs(xx,F,'LineWidth',2)
axis([-10 10 0 1])
```



Continuous Distributions

F:

A random variable x has a continuous F distribution, $x \sim F(\nu_1,\nu_2)$ if

$$f(x|\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}} \quad , \qquad x>0$$

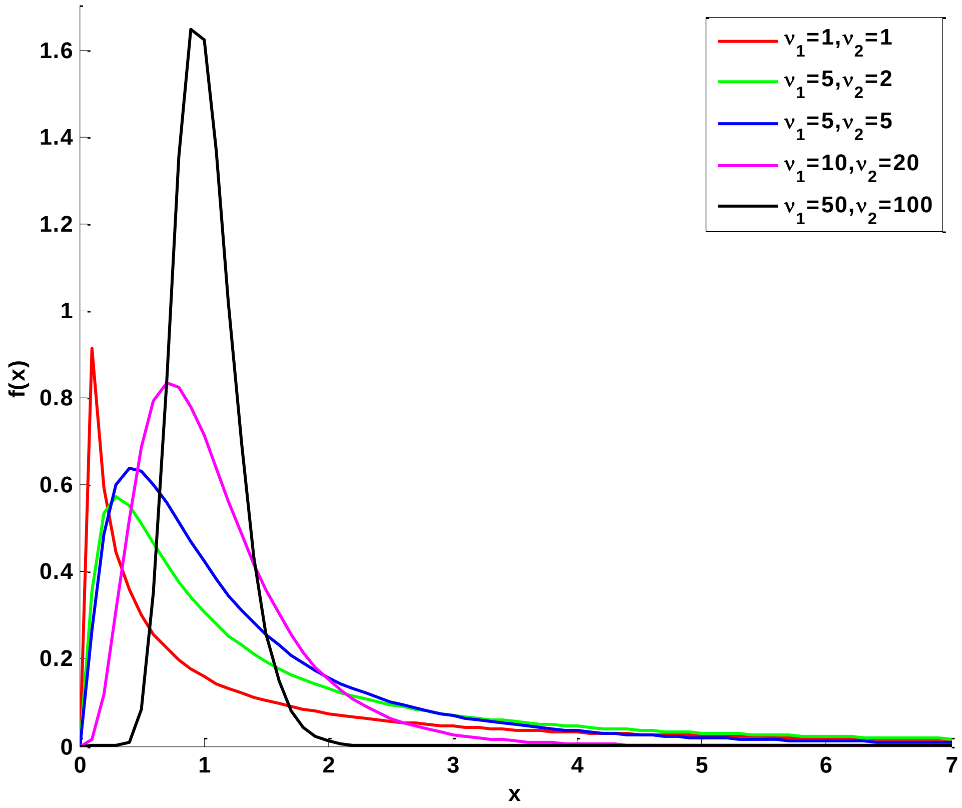
where $\nu_1,\nu_2=1,2,\ldots$.

Continuous Distributions

F:

$$f(x\,|\,\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

```
x=(0:.1:15)';
nu1=[1,5,5,10,50];, nu2=[1,2, 5,20,100];
figure(1)
hold on
for count=1:length(nu1)
    y = fpdf(x,nu1(count),nu2(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([0 7]), ylim([0 1.7])
```



Continuous Distributions

F:

$$f(x|\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

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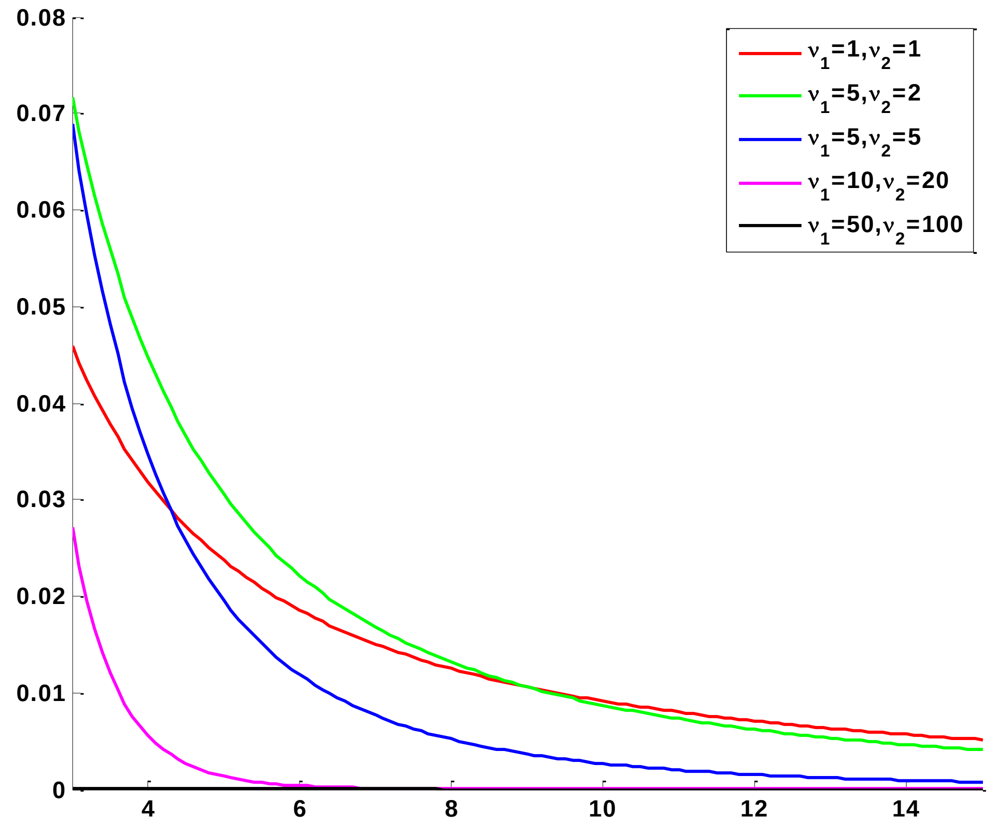
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([3 15]),ylim([0 .08])
```



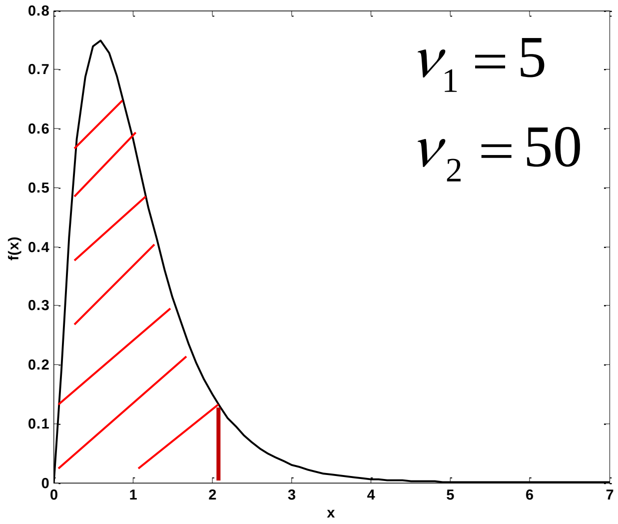
Continuous Distributions

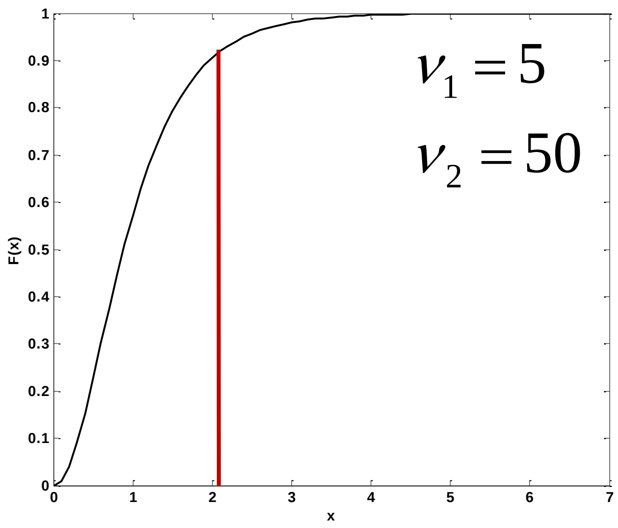
F:

The CDF of the continuous F distribution is

$$F(x|\theta) = \int_{t=-\infty}^{x} f(t|\theta)dt$$

$$F(x|\nu_1,\nu_2) = \int_{t=-\infty}^{x} \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \frac{t^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}t\right)^{(\nu_1+\nu_2)/2}} dt$$





Continuous Distributions

F:

$$f(x|\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

```
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nu1=[1,5,5,10,50];, nu2=[1,2,5,20,100];
figure(1)
hold on
for count=1:length(nu1)
     y = fcdf(x,nu1(count),nu2(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
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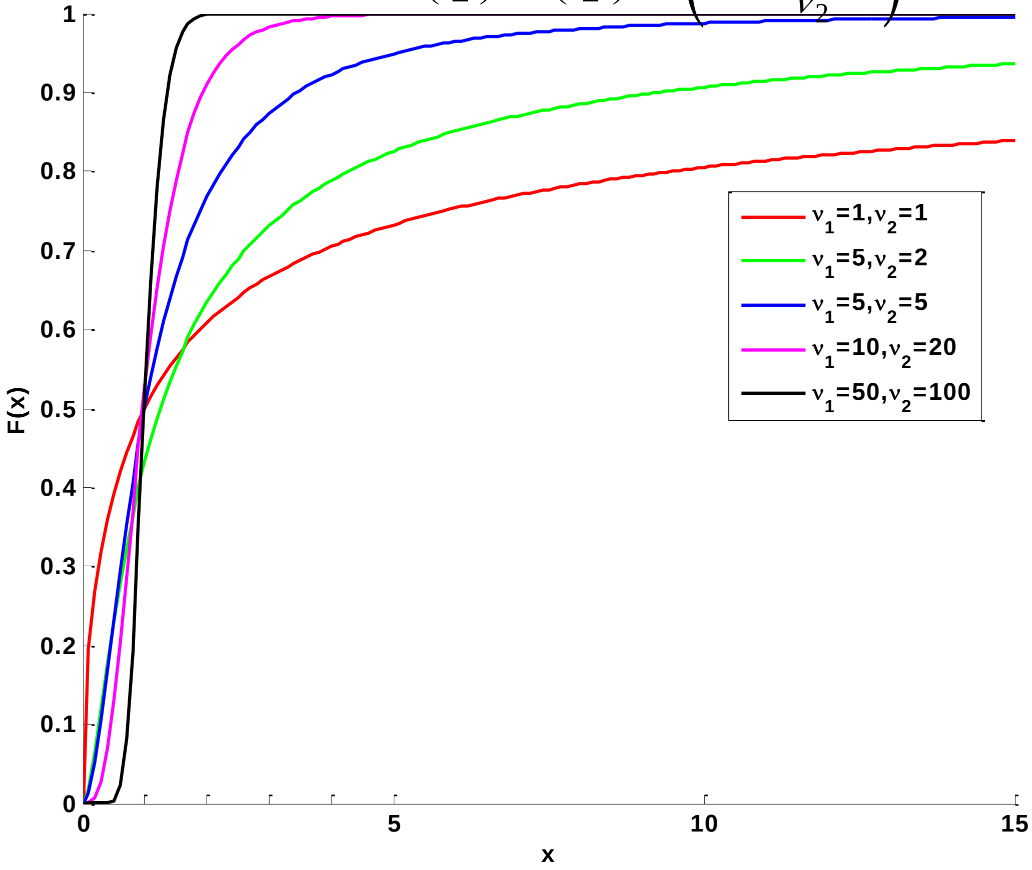
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([0 15])
```



Continuous Distributions

F:

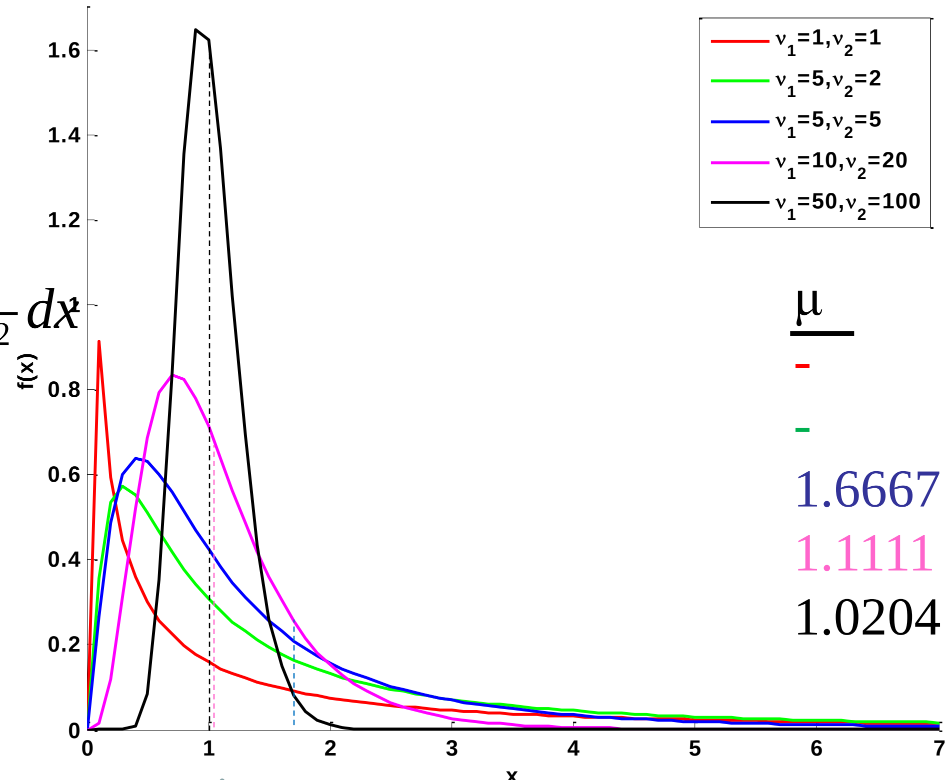
$$f(x \mid \nu_1,\nu_2) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

It can be shown that

$$\mu = \int_x x f(x \mid \theta)\,dx$$

$$= \int_{x=-\infty}^{\infty} x \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1+\frac{1}{\nu}x^2\right)^{(\nu+1)/2}}\,dx$$

$$= \frac{\nu_2}{\nu_2-2} \qquad \nu_2 > 2$$



μ's

Continuous Distributions

F:

It can be shown that

median

$$\int_{x=-\infty}^{\tilde{x}} f(x|\theta)dx = \frac{1}{2}$$

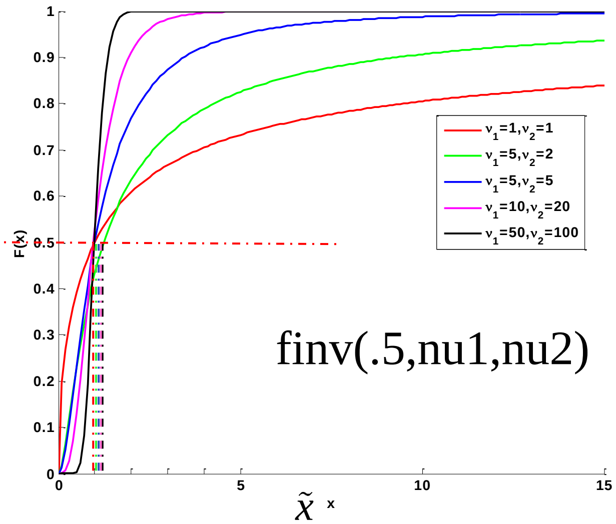
$$\tilde{x} = \text{no closed form}$$

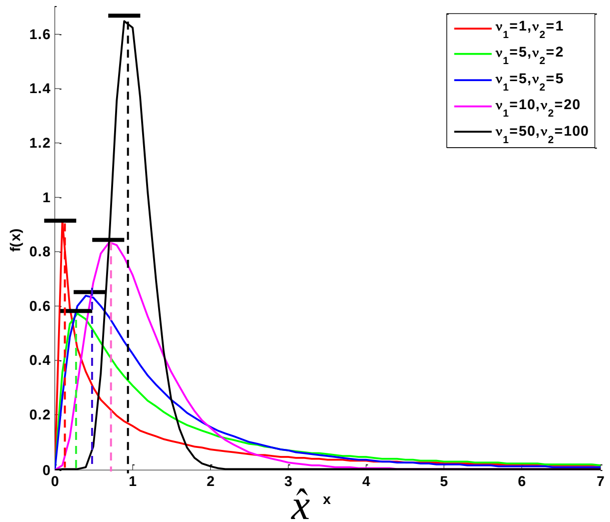
mode

$$\frac{\partial}{\partial x} f(x|\theta)\bigg|_{\hat{x}} = 0$$

$$\hat{x} = \frac{\nu_1 - 2}{\nu_1}\frac{\nu_2}{\nu_2+2} \qquad \nu_1 > 2$$

1/2





$\tilde{x}$	$\hat{x}$
1.0000	-
1.2519	.3000
1.0000	.4286
0.9663	.7273
0.9933	.9412

Continuous Distributions

F:

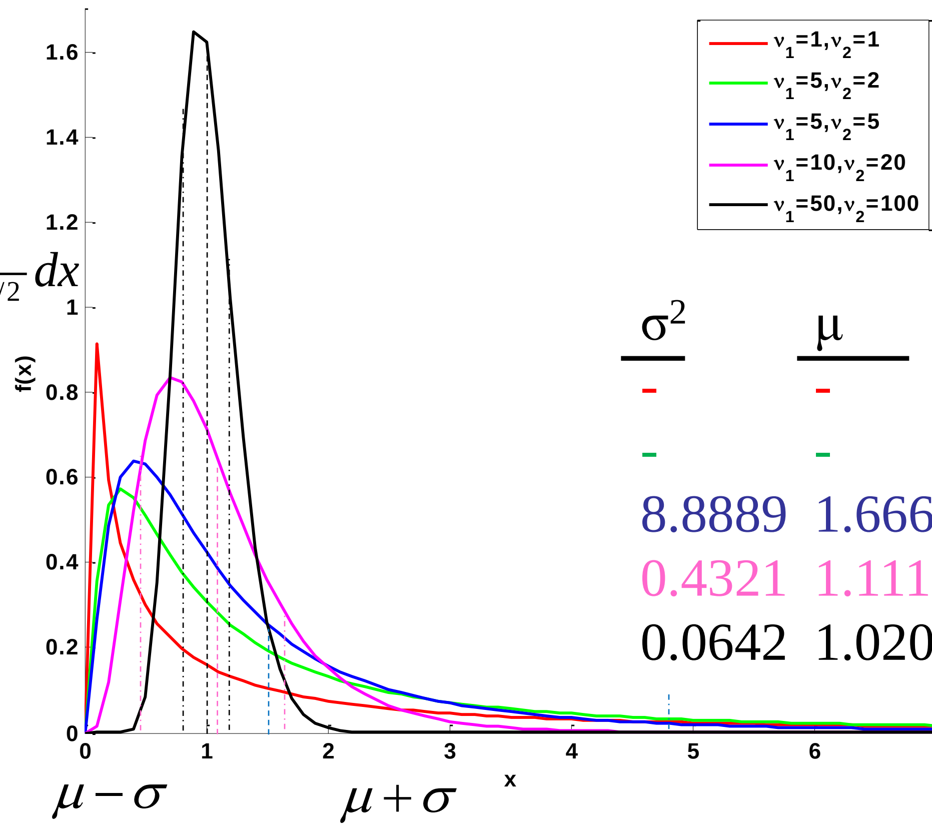
that

$$\sigma^2 = \int_x (x-\mu)^2 f(x|\theta)dx$$

$$= \int_{x=-\infty}^{\infty} (x-\mu)^2 \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}} dx$$

$$= \frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$$

$$\nu_2 > 4$$



σ²	μ
-	-
-	-
8.8889	1.6667
0.4321	1.1111
0.0642	1.0204

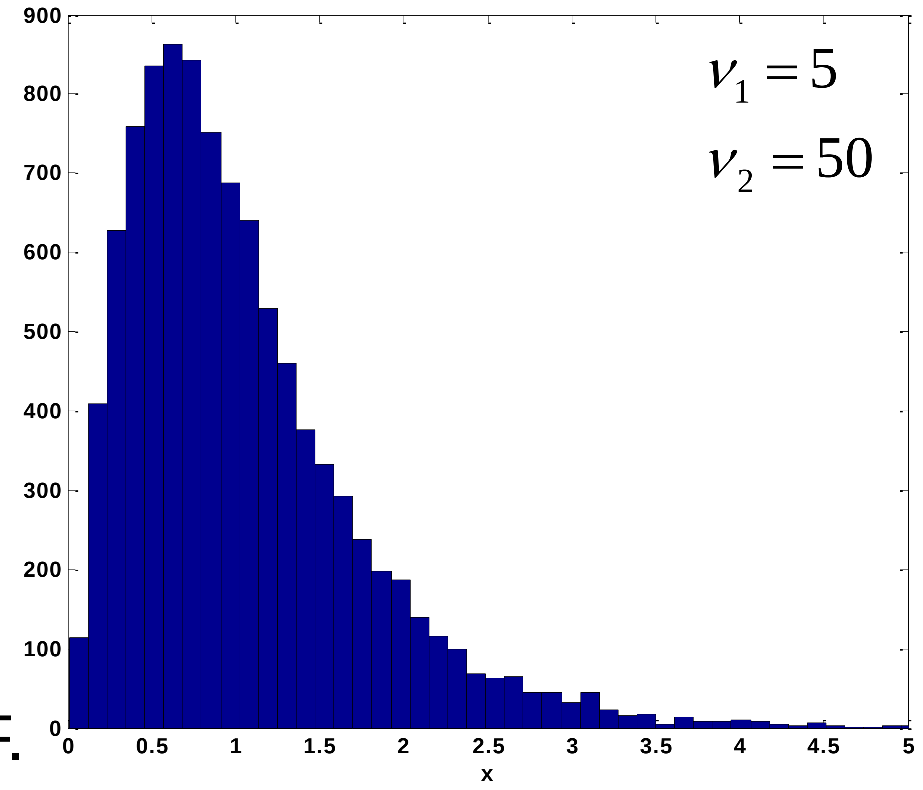
Continuous Distributions

F:

$$f(x|\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

```
nu1=5;,nu2=50;,num=10^4;
x=frnd(nu1,nu2,num,1);
mean(x)
var(x)
hist(x,50), xlim([0 5])
[mu,sigma2] = fstat(nu1,nu2)
```

	True	Simulated
μ	1.0417	1.0373
σ^2	0.5001	0.4859



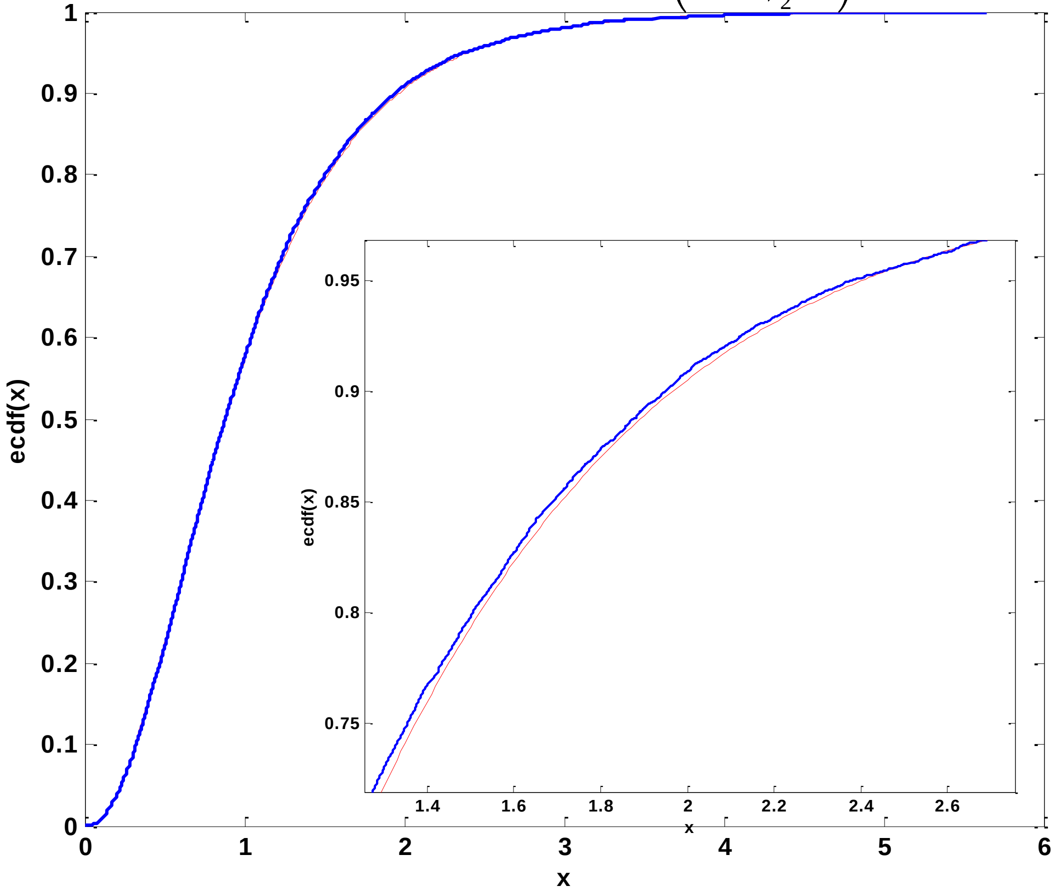
Can also find and plot ECDF.

Continuous Distributions

F:

$$f(x\,|\,\nu_1,\nu_2)=\frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}x\right)^{(\nu_1+\nu_2)/2}}$$

```
nu1=5;,nu2=50;,num=10^4;
y=fcdf((0:.01:5),nu1,nu2);
plot((0:.01:5),y, 'r')
hold on
[F,xx]=ecdf(x);
stairs(xx,F,'LineWidth',2)
```



Homework 5:

1) Let $x \sim t(5)$, using pencil and paper, find

   $P(\mu - \sigma \leq x \leq \mu + \sigma)$.

2) Show by pencil and paper that the variance of the Student-t distribution is $\sigma^2 = \dfrac{\nu}{\nu - 2}$.

3) Numerically integrate Student-t pdf with rectangles to find the 99th percentile. i.e. find x_0 such that

   $P(x \leq x_0) = .99$ for $\nu = 5$.

Homework 5:

4) Generate 10^6 Student-t $\nu = 5$ random variables.
Empirically determine the 99th percentile.
i.e. Find the $.99*10^6$ largest value x_0.
such that $P(x \leq x_0) \approx .99$.
Compare values to 3) and to value from t-table.

Homework 5:

5) Make a histogram of the random variables in 4) with at least 50 bins.

6) Make a empirical CDF from the values in 4).

Homework 5:

$$f(x)=\frac{1}{\pi}\frac{1}{1+x^2}$$

7) Let x~Cauchy distributed. Derive the distribution of $y=\sigma x+\mu$.

8) Numerically integrate the Cauchy PDF with rectangles to find the 99th percentile. i.e. find x_0 such that $P(x\leq x_0)\approx .99$. Compare to the exact percentile.

$$F(x)=\frac{1}{2}+\frac{1}{\pi}\mathrm{a}\tan(x)$$

9) Generate 10^6 Cauchy random variates.

a) Make a histogram, 50 bins.

b) Compute sample mean and variance.

c) Multiply each random variate by 2 and add 5.

d) Make a histogram.

e) Compute sample mean and variance.

Homework 5:

10) Generate 10^6 Cauchy random variables.

Empirically determine the 50th and 99th percentiles.

i.e. Find the $.50*10^6$ and $.99*10^6$ largest values

x_0 such that $P(x \leq x_0) \approx .5$ and $P(x \leq x_0) \approx .99$.

Compare values to value 8.

Homework 5:

11) Write a Matlab program to numerically differentiate the F distribution and estimate the mode. $\nu_1 = 5, \nu_2 = 50$ $\Delta x = 1/100$

12) Generate 10^6 F distributed random variables. $\nu_1 = 5$ $\nu_2 = 50$
Empirically determine the 50th and 99th percentiles.
i.e. Find the $.50*10^6$ and $.99*10^6$ largest values x_0 such that $P(x \le x_0) \approx .5$ and $P(x \le x_0) \approx .99$.
Compare first value to 11 and second value to table.