

# Continuous Probability Functions (Cont.)

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# Outline

- **Continuous Student-t Distribution**  
**PDF, Moments, CDF, Matlab**
- **Continuous Cauchy Distribution**  
**PDF, Moments, CDF, Matlab**
- **Continuous F Distribution**  
**PDF, Moments, CDF, Matlab**

# Continuous Distributions

## Student-t:

A random variable  $x$  has a continuous Student-t distribution,  $x \sim t(\nu)$  if

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}, \quad x \in \mathbb{R}$$

where  $\nu = 1, 2, \dots$ .

# Continuous Distributions

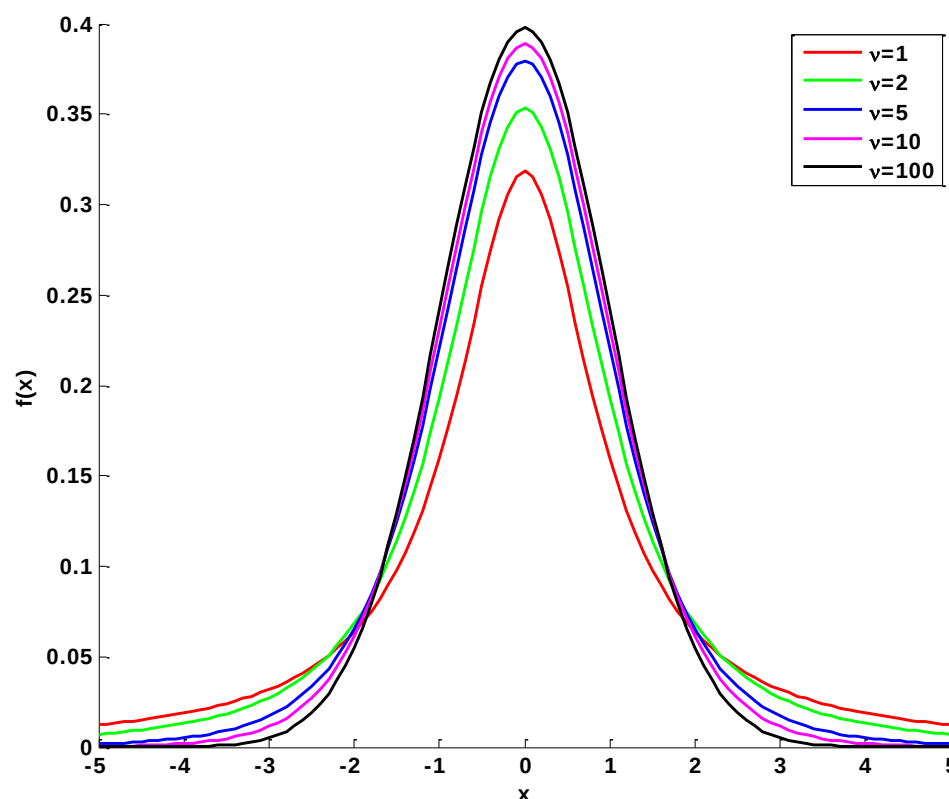
## Student-t:

```

x=(-5:.1:5)';
nu=[1,2, 5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tpdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])

```

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$



# Continuous Distributions

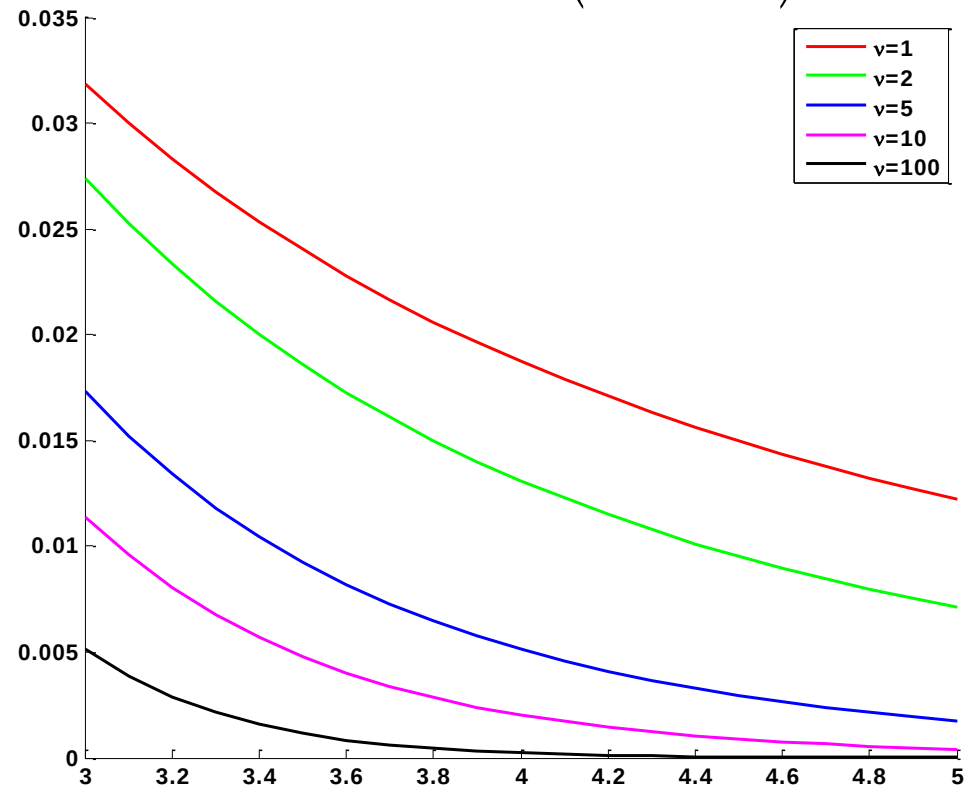
## Student-t:

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nu=[1,2, 5,10,100];
figure(1)
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    y = tpdf(x,nu(count));
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    elseif count==2
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    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])

```

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$



# Continuous Distributions

## Student-t:

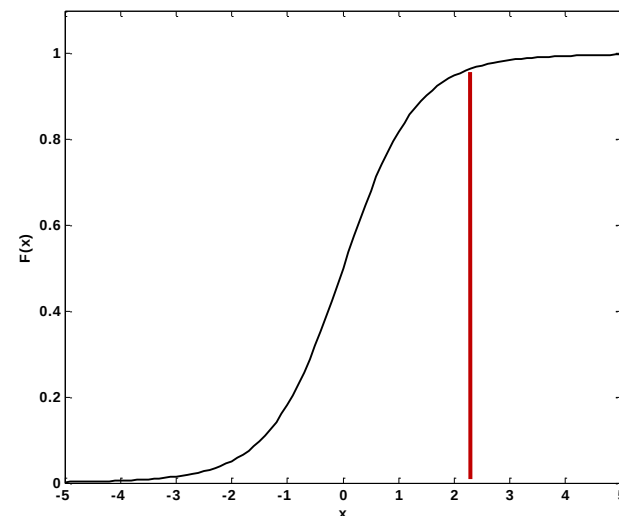
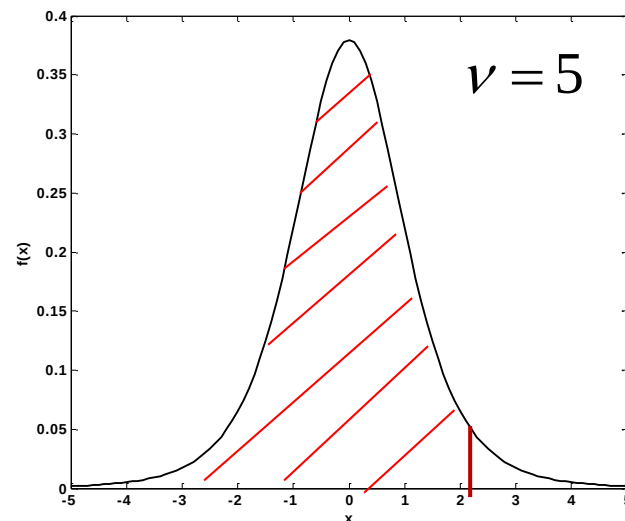
The CDF of the continuous Student-t distribution is

$$F(x|\theta) = \int_{t=-\infty}^x f(t|\theta)dt$$

$$F(x|\nu) = \int_{t=-\infty}^x \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}t^2\right)^{(\nu+1)/2}} dt$$

$$= \frac{1}{2} + x\Gamma\left(\frac{\nu+1}{2}\right) \frac{{}_2F_1\left(\frac{1}{2}, \frac{\nu+1}{2}; \frac{3}{2}; -\frac{x^2}{\nu}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}}$$

Where  ${}_2F_1$  is the hypergeometric function



# Continuous Distributions

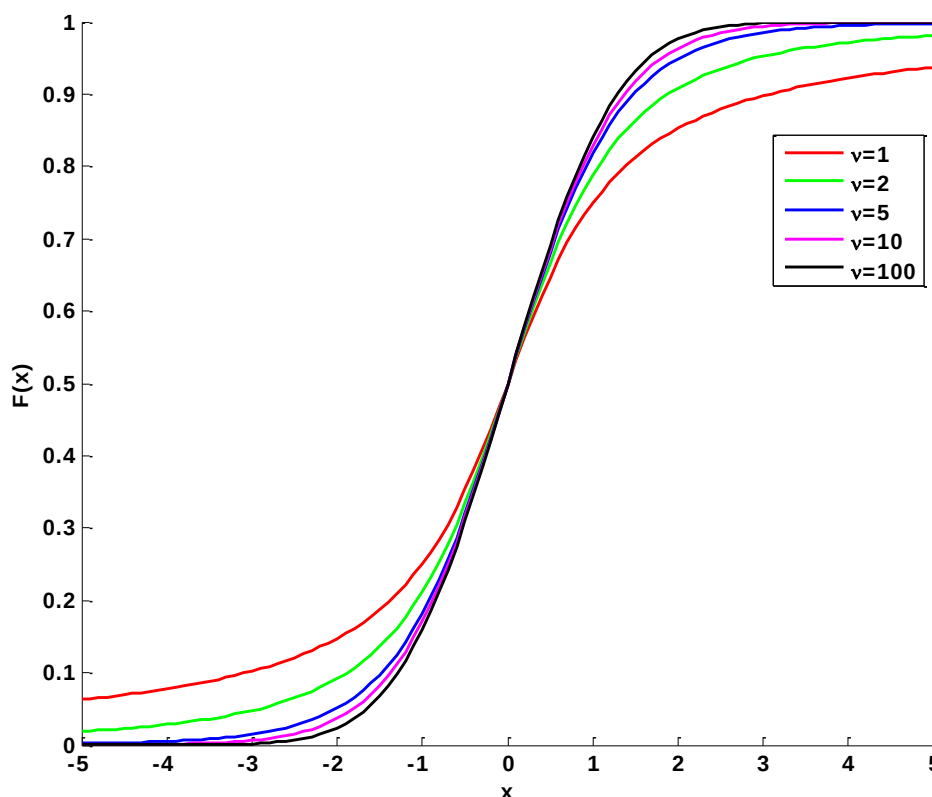
## Student-t:

```

x=(-5:.1:5)';
nu=[1,2,5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tcdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])

```

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$



# Continuous Distributions

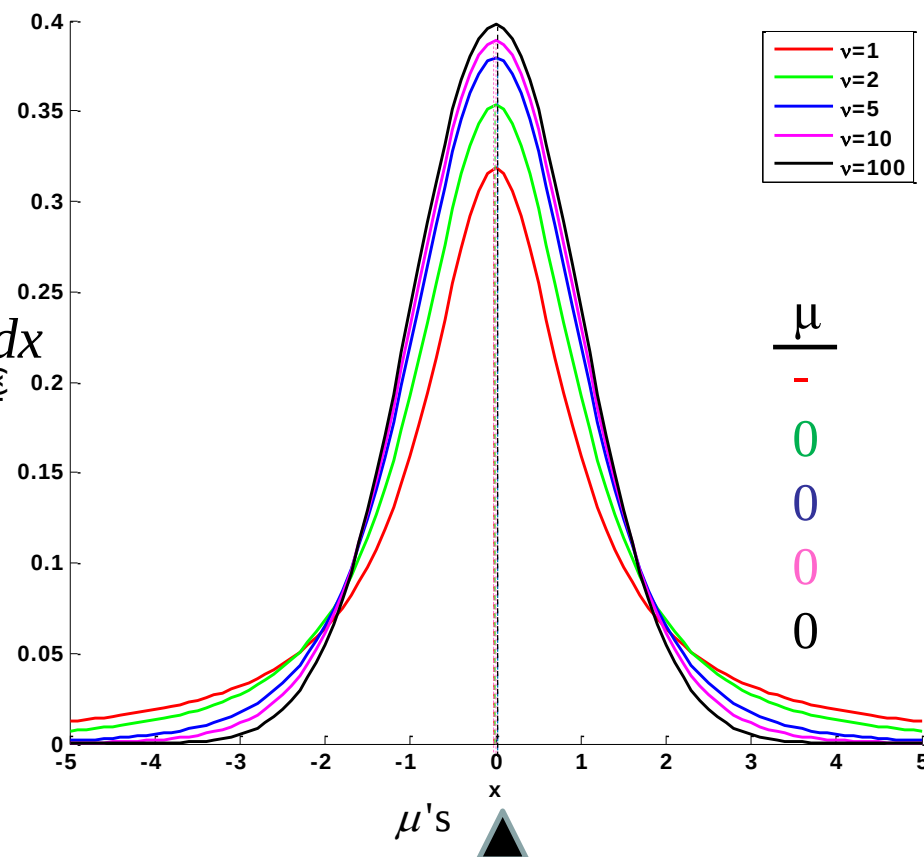
## Student-t:

It can be shown that

$$\mu = \int_{-\infty}^{\infty} xf(x|\theta)dx$$

$$= \int_{-\infty}^{\infty} x \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}} dx$$

$$= 0 \quad \nu > 1$$



# Continuous Distributions

## Student-t:

It can be shown that

median

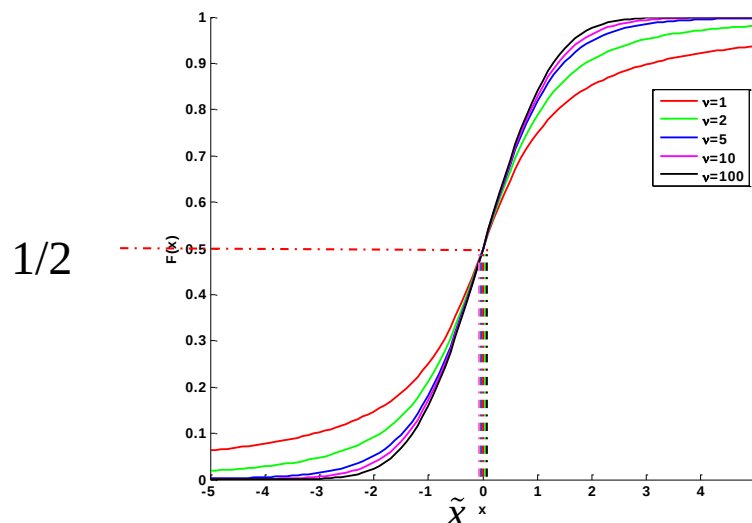
$$\int_{x=-\infty}^{\tilde{x}} f(x | \theta) dx = \frac{1}{2}$$

$$\tilde{x} = 0$$

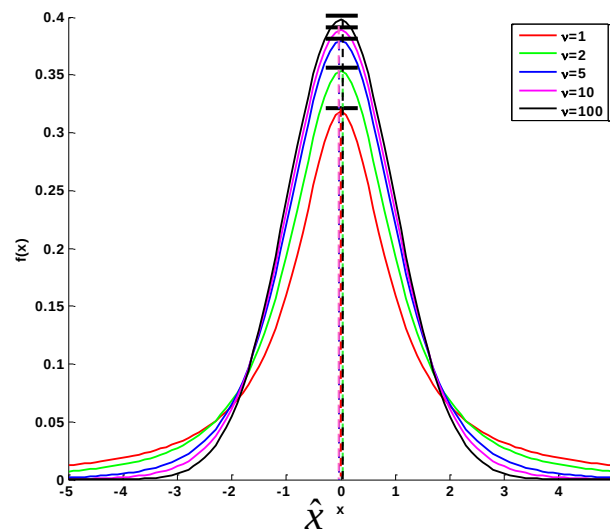
mode

$$\left. \frac{\partial}{\partial x} f(x | \theta) \right|_{\hat{x}} = 0$$

$$\hat{x} = 0$$



$$\frac{\tilde{x}}{0}$$



$$\frac{\hat{x}}{0}$$

# Continuous Distributions

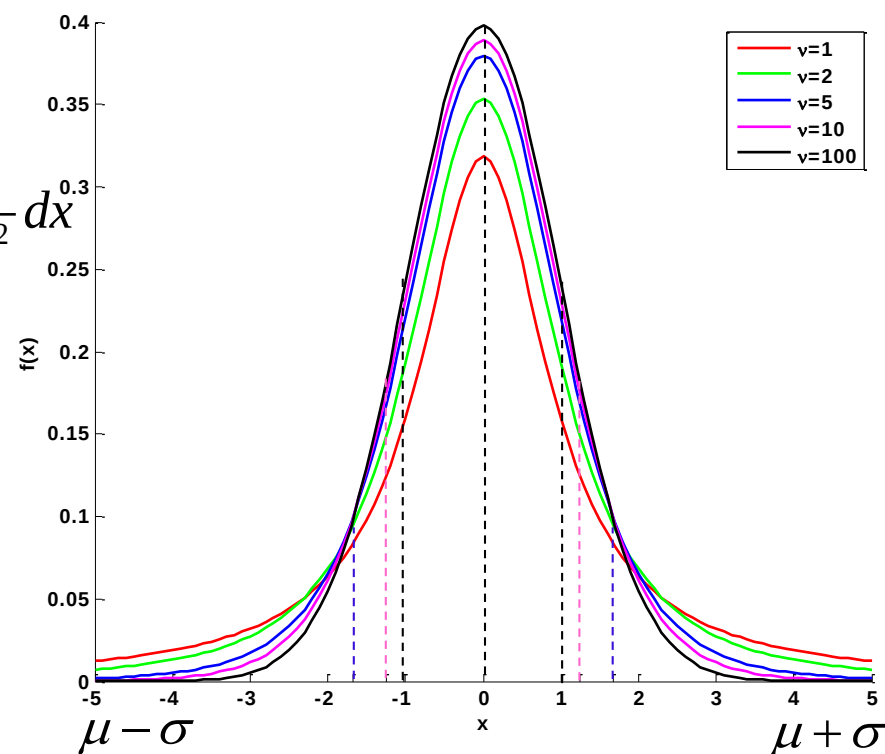
## Student-t:

that

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x | \theta) dx$$

$$= \int_{-\infty}^{\infty} (x - \mu)^2 \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}} dx$$

$$= \frac{\nu}{\nu - 2}$$



# Continuous Distributions

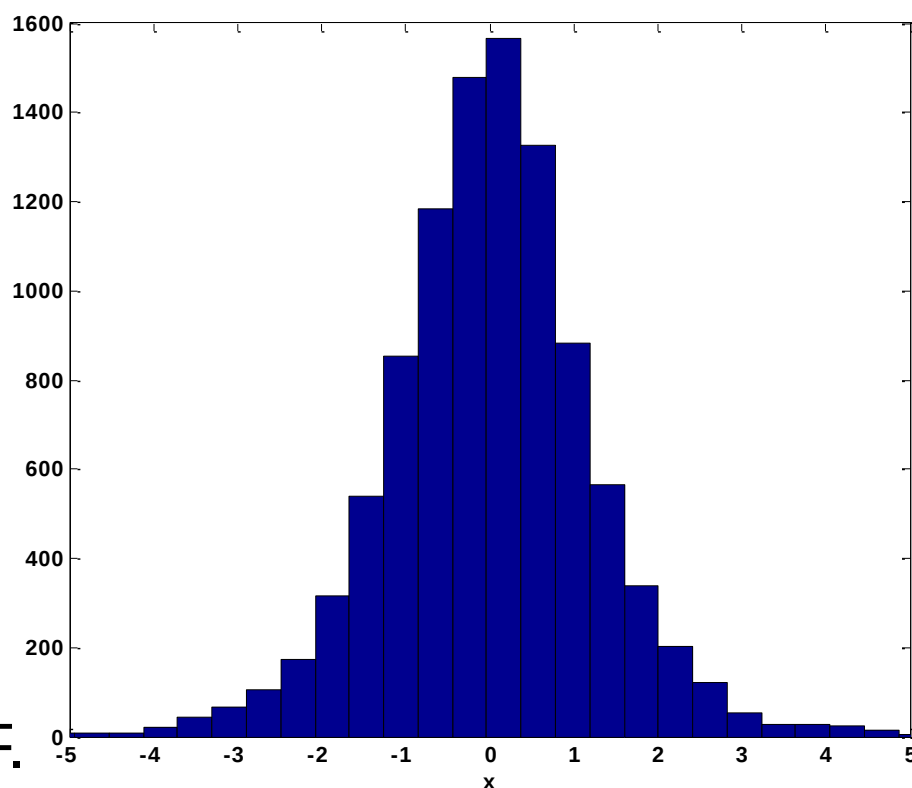
## Student-t:

```
nu=5;,num=10^4;
x=trnd(nu,num,1);
mean(x)
var(x)
hist(x,50), xlim([-5 5])
```

|            | True   | Simulated |
|------------|--------|-----------|
| $\mu$      | 0      | -0.0062   |
| $\sigma^2$ | 1.6667 | 1.6318    |

Can also find and plot ECDF.

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

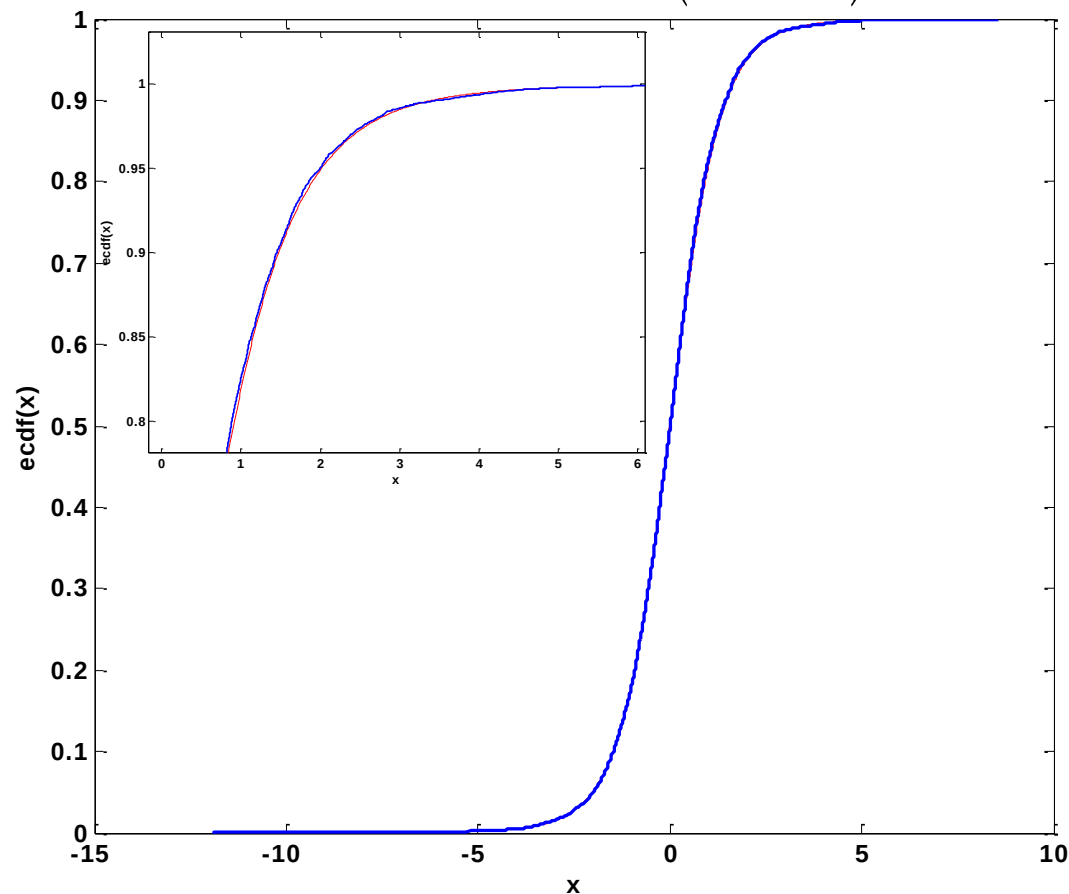


# Continuous Distributions

Student-t:

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

```
nu=5;  
y=tcdf((-5:.01:5),nu);  
plot((-5:.01:5),y, 'r')  
hold on  
[F,xx]=ecdf(x);  
stairs(xx,F,'LineWidth',2)
```



# Continuous Distributions

## Student-t:

The Student-t Distribution, can be generalized to have location and scale parameters, so that

$x \sim t(\nu, \delta, \tau)$  if

$$f(x | \nu, \delta, \tau) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{(\tau^2)^{-1/2}}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu} \left(\frac{x - \delta}{\tau}\right)^2\right)^{(\nu+1)/2}}, \quad x \in \mathbb{R}$$

where  $\nu = 1, 2, \dots$  .

# Continuous Distributions

## Cauchy (Lorentzian):

A random variable  $x$  has a continuous (standard) Cauchy distribution,  $x \sim \text{Cauchy}$  if

$$f(x | \nu) = \frac{1}{\pi} \frac{1}{1 + x^2}, \text{ where, } x \in \mathbb{R}.$$

Cauchy distribution is a special case of the Student-t distribution with  $\nu=1$ .

$$f(x | \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$

# Continuous Distributions

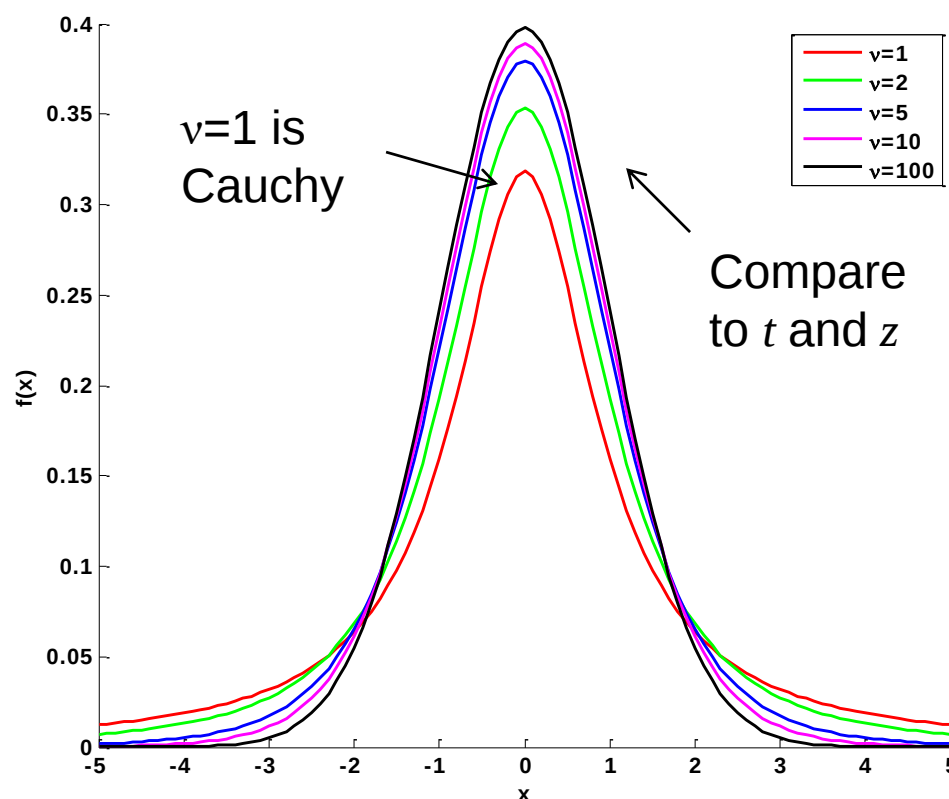
## Student-t (Cauchy):

```

x=(-5:.1:5)';
nu=[1,2,5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tpdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])

```

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$



# Continuous Distributions

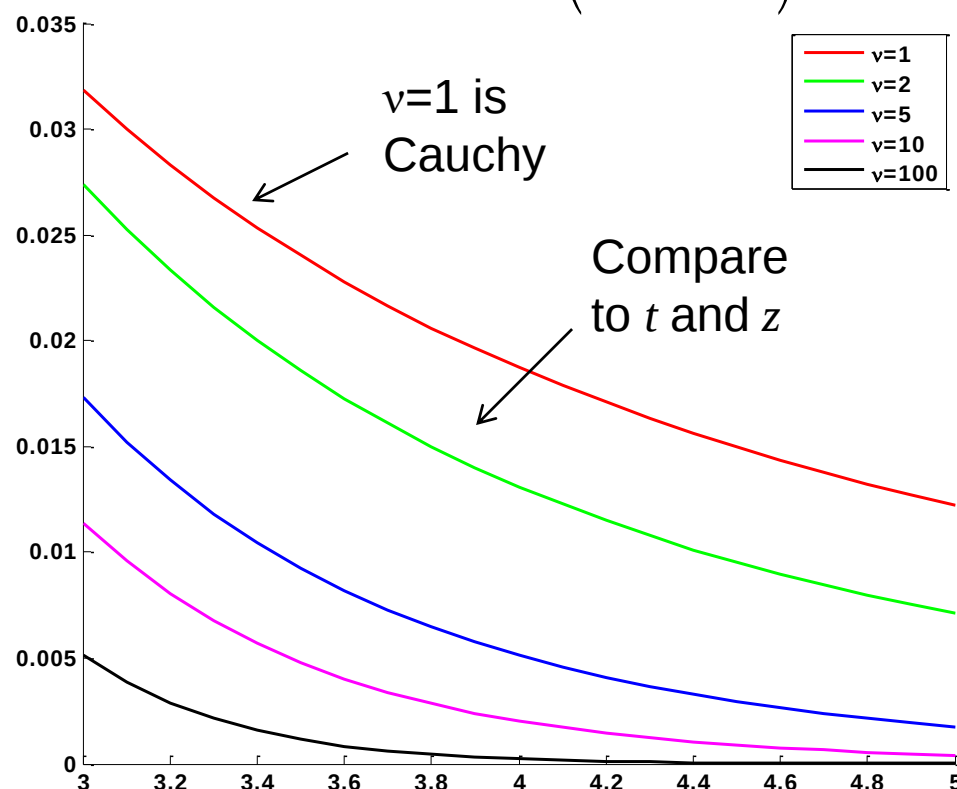
## Student-t (Cauchy):

```

x=(-5:.1:5)';
nu=[1,2,5,10,100];
figure(1)
hold on
for count=1:length(nu)
    y = tpdf(x,nu(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([-5 5])

```

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu}x^2\right)^{(\nu+1)/2}}$$



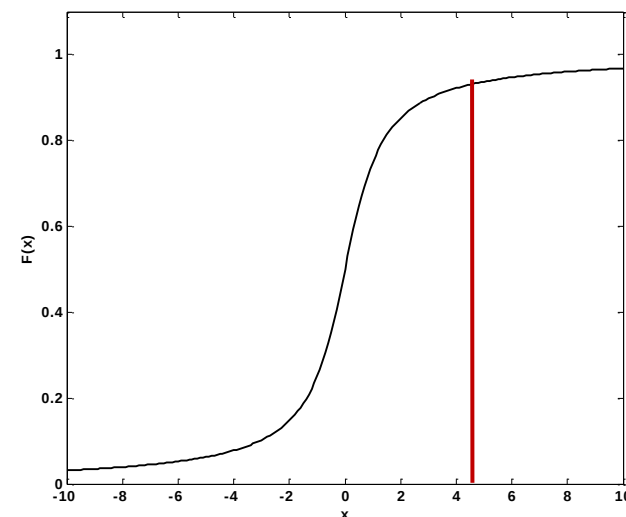
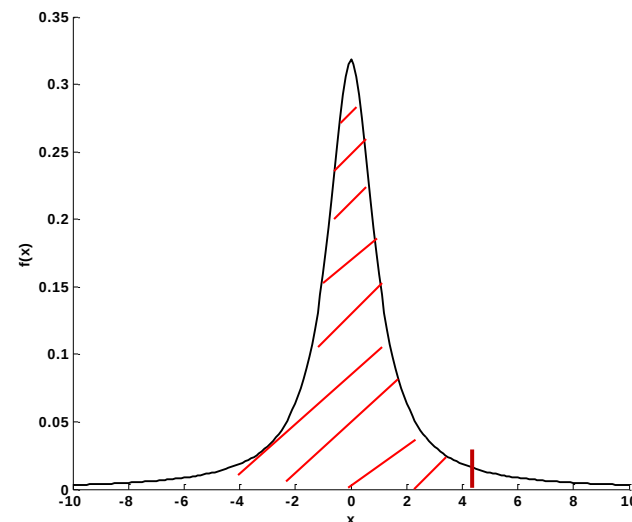
# Continuous Distributions

## Cauchy:

The CDF of the continuous Cauchy distribution is

$$F(x|\theta) = \int_{t=-\infty}^x f(t|\theta)dt$$

$$\begin{aligned} F(x) &= \int_{t=-\infty}^x \frac{1}{\pi} \frac{1}{1+t^2} dt \\ &= \frac{1}{2} + \frac{1}{\pi} \arctan(x) \end{aligned}$$

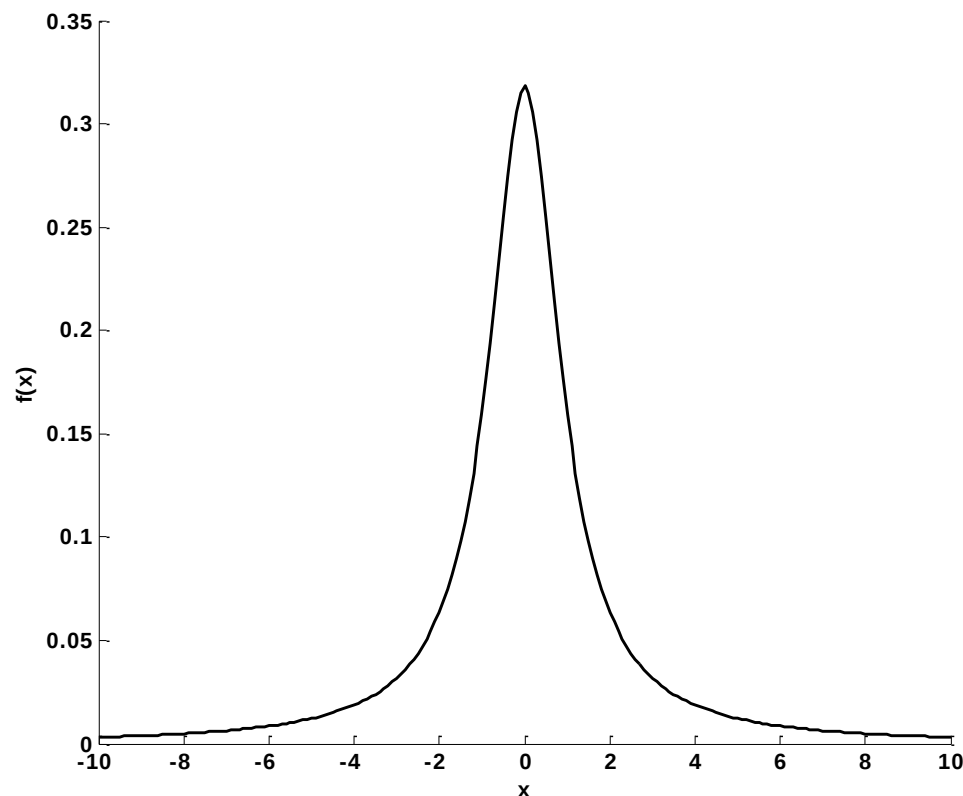


# Continuous Distributions

## Cauchy:

It can be shown that

$$\begin{aligned}\mu &= \int_{-\infty}^{\infty} xf(x|\theta)dx \\ &= \int_{-\infty}^{\infty} x \frac{1}{\pi} \frac{1}{1+x^2} dx \\ &= \text{not defined}\end{aligned}$$



# Continuous Distributions

## Cauchy:

It can be shown that

median

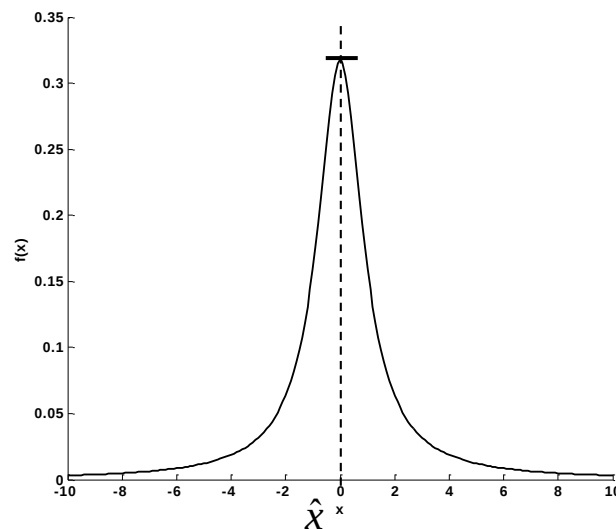
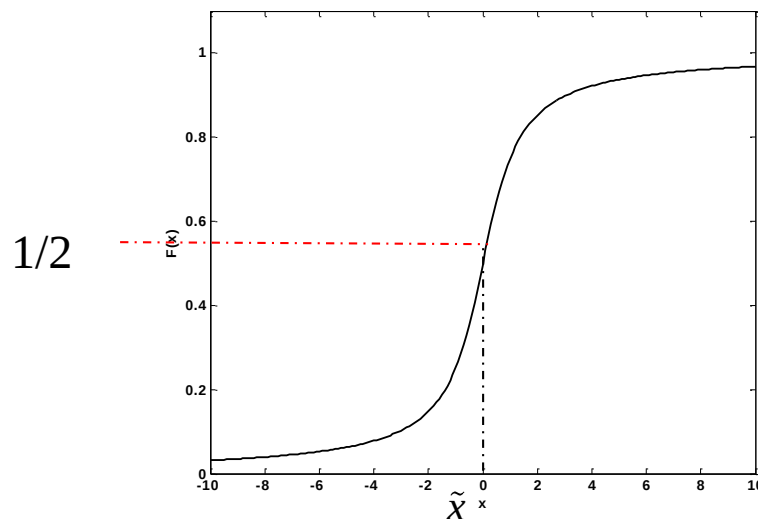
$$\int_{x=-\infty}^{\tilde{x}} f(x|\theta) dx = \frac{1}{2}$$

$$\tilde{x} = 0$$

mode

$$\left. \frac{\partial}{\partial x} f(x|\theta) \right|_{\hat{x}} = 0$$

$$\hat{x} = 0$$

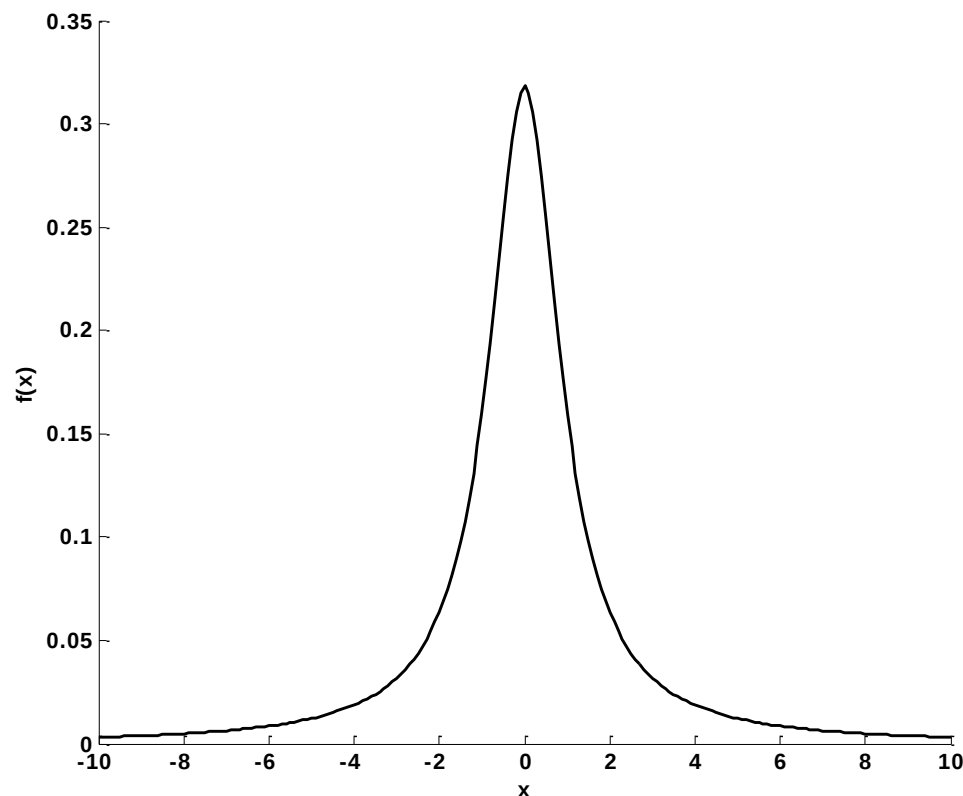


# Continuous Distributions

## Cauchy:

that

$$\begin{aligned}\sigma^2 &= \int_{-\infty}^{\infty} (x - \mu)^2 f(x|\theta) dx \\ &= \int_{-\infty}^{\infty} (x - \mu)^2 \frac{1}{\pi} \frac{1}{1 + x^2} dx \\ &= \text{not defined}\end{aligned}$$



# Continuous Distributions

Cauchy:

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

```
nu=1;,num=10^4;
```

```
x=trnd(nu,num,1);
```

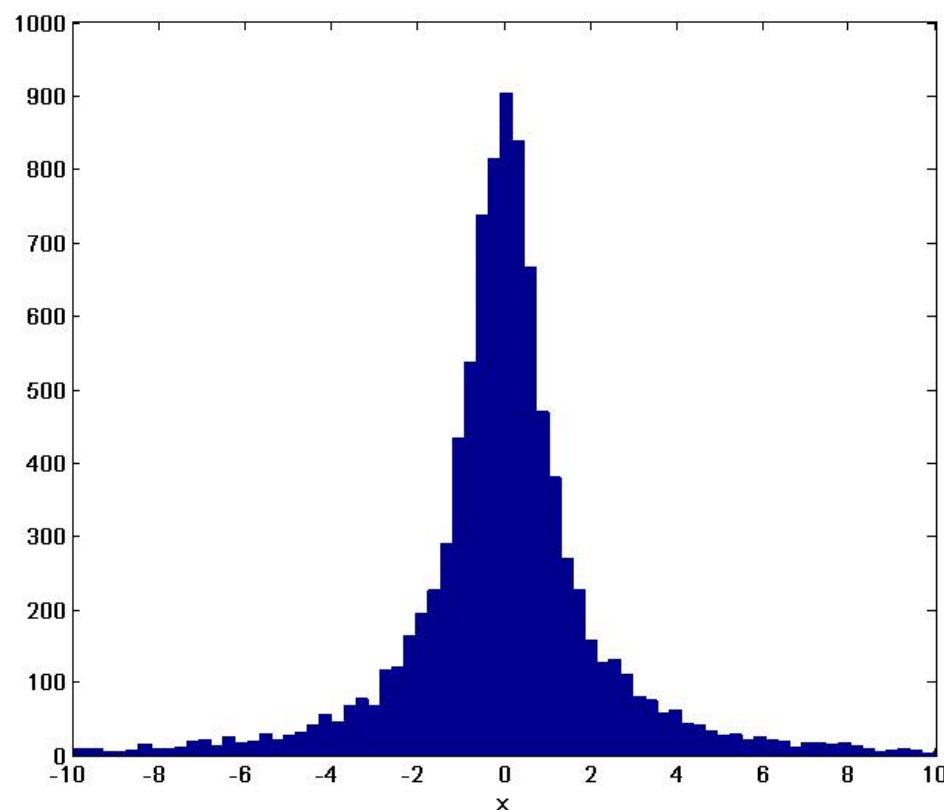
```
mean(x)
```

```
var(x)
```

```
hist(x,10^5), xlim([-10 10])
```

|            | True | Simulated            |
|------------|------|----------------------|
| $\mu$      | -    | 4.2813               |
| $\sigma^2$ | -    | $9.3156 \times 10^4$ |

Can also find and plot ECD



# Continuous Distributions

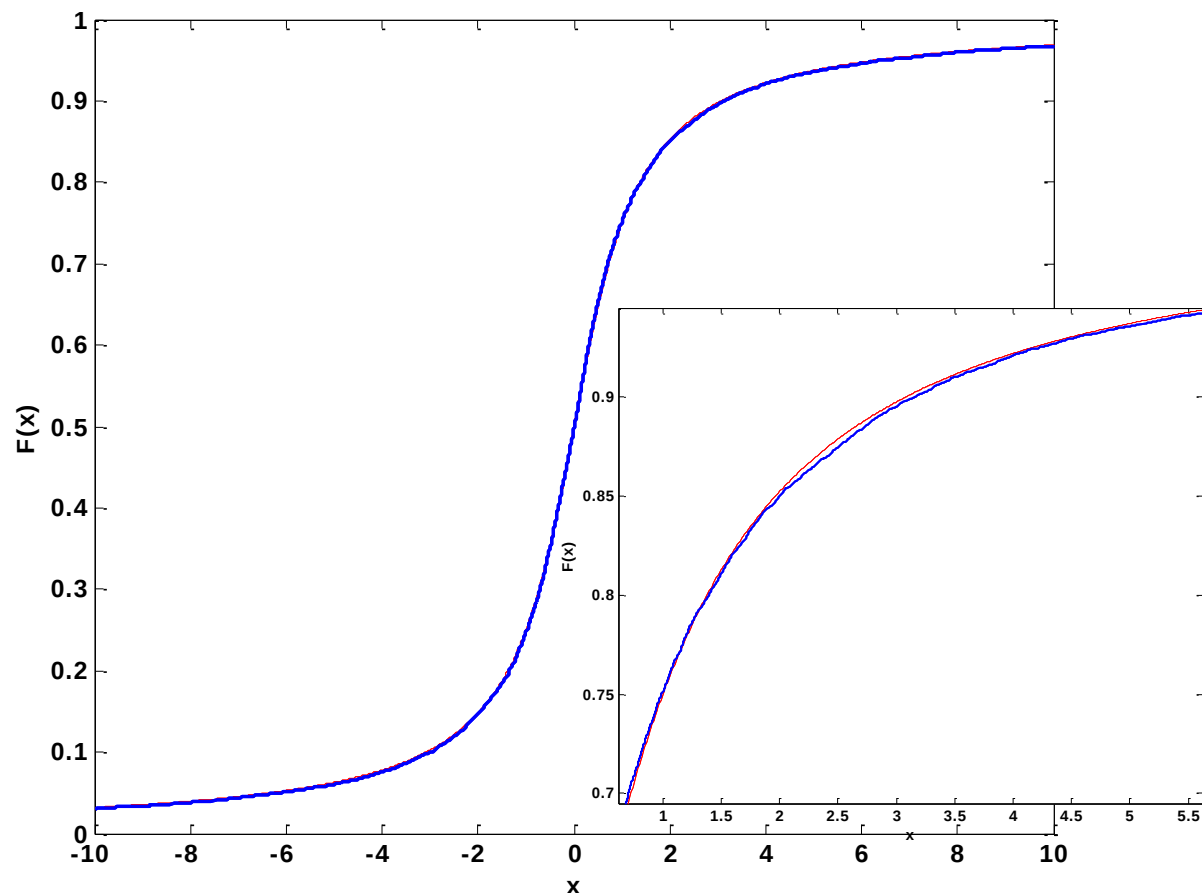
## Cauchy:

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

```

nu=1;
y=tcdf((-10:.01:10),nu);
plot((-10:.01:10),y, 'r')
hold on
[F,xx]=ecdf(x);
stairs(xx,F,'LineWidth',2)
axis([-10 10 0 1])

```



# Continuous Distributions

F:

A random variable  $x$  has a continuous F distribution,  $x \sim F(\nu_1, \nu_2)$  if

$$f(x | \nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}, \quad x > 0$$

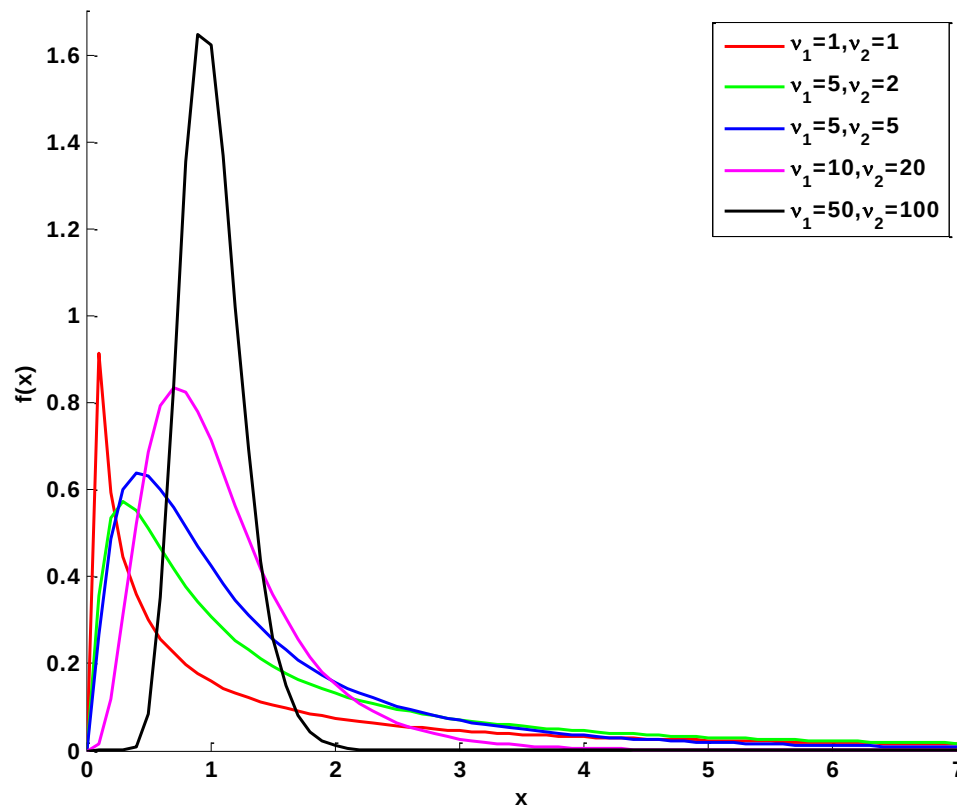
where  $\nu_1, \nu_2 = 1, 2, \dots$ .

# Continuous Distributions

F:

```
x=(0:.1:15)';
nu1=[1,5,5,10,50];, nu2=[1,2, 5,20,100];
figure(1)
hold on
for count=1:length(nu1)
    y = fpdf(x,nu1(count),nu2(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([0 7]), ylim([0 1.7])
```

$$f(x | \nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}$$

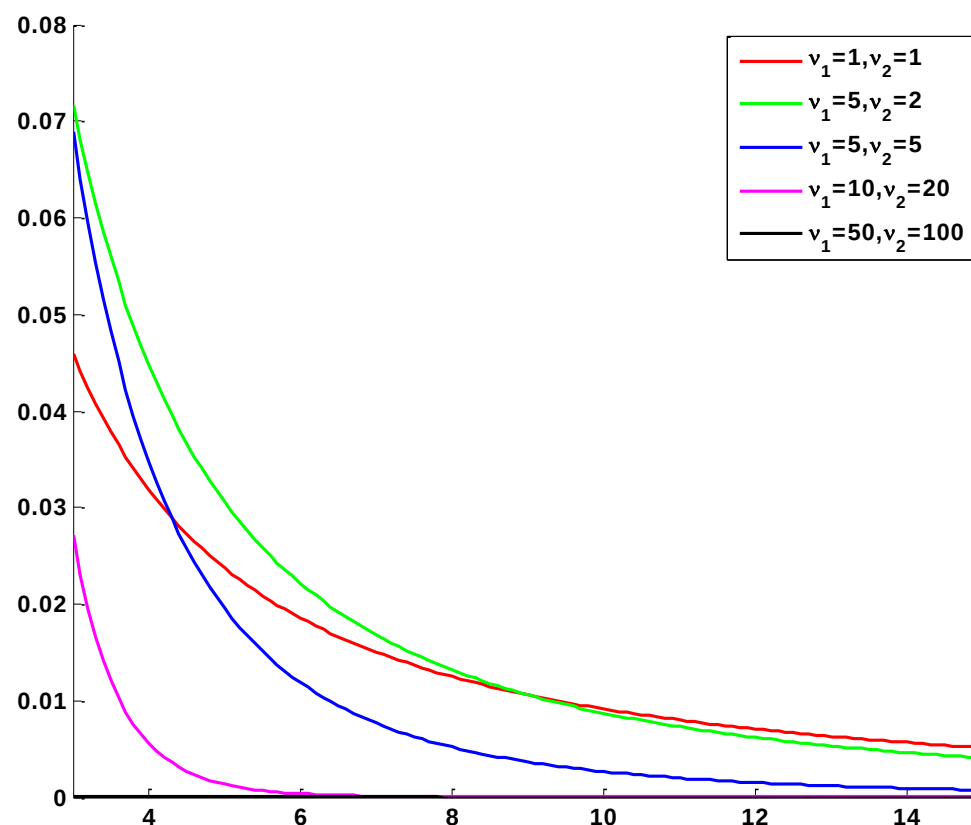


# Continuous Distributions

F:

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x=(0:.1:15)';
nu1=[1,5,5,10,50];, nu2=[1,2, 5,20,100];
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    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([3 15]),ylim([0 .08])
```

$$f(x | \nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}$$



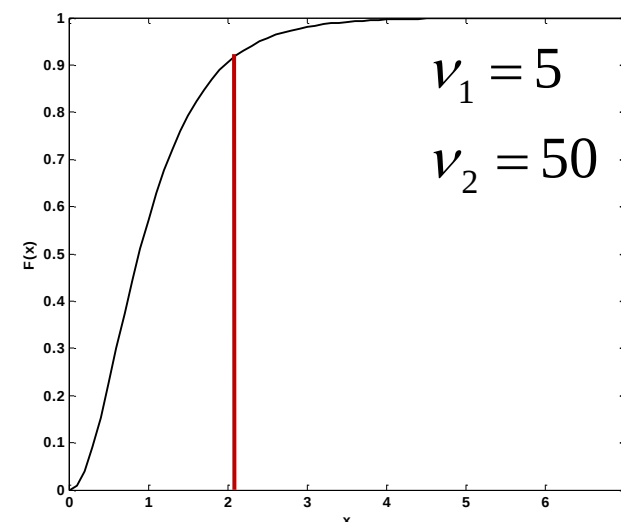
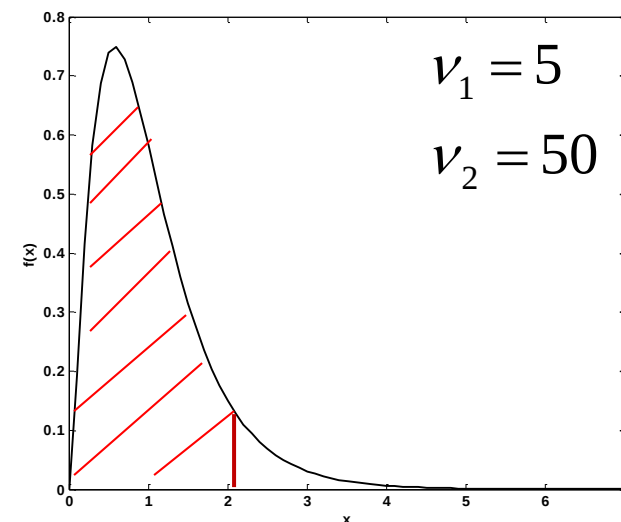
# Continuous Distributions

**F:**

The CDF of the continuous F distribution is

$$F(x|\theta) = \int_{t=-\infty}^x f(t|\theta)dt$$

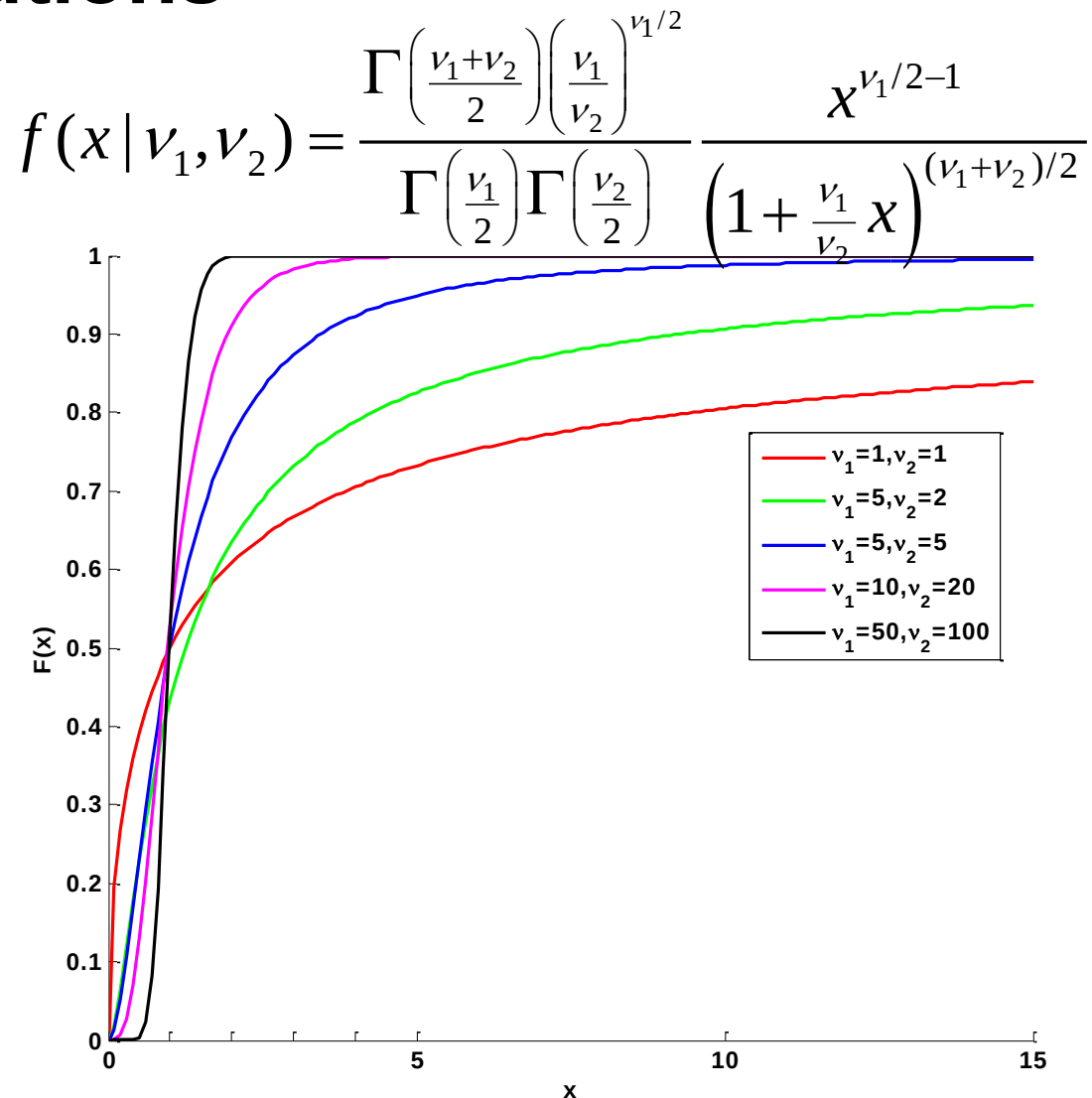
$$F(x|\nu_1, \nu_2) = \int_{t=0}^x \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)} \frac{t^{\nu_1/2-1}}{\left(1 + \frac{\nu_1}{\nu_2}t\right)^{(\nu_1+\nu_2)/2}} dt$$



# Continuous Distributions

F:

```
x=(0:.1:15)';
nu1=[1,5,5,10,50];, nu2=[1,2,5,20,100];
figure(1)
hold on
for count=1:length(nu1)
    y = fcdf(x,nu1(count),nu2(count));
    if count==1
        plot(x,y,'r','LineWidth',2)
    elseif count==2
        plot(x,y,'g','LineWidth',2)
    elseif count==3
        plot(x,y,'b','LineWidth',2)
    elseif count==4
        plot(x,y,'m','LineWidth',2)
    elseif count==5
        plot(x,y,'k','LineWidth',2)
    end
end
xlim([0 15])
```



# Continuous Distributions

F:

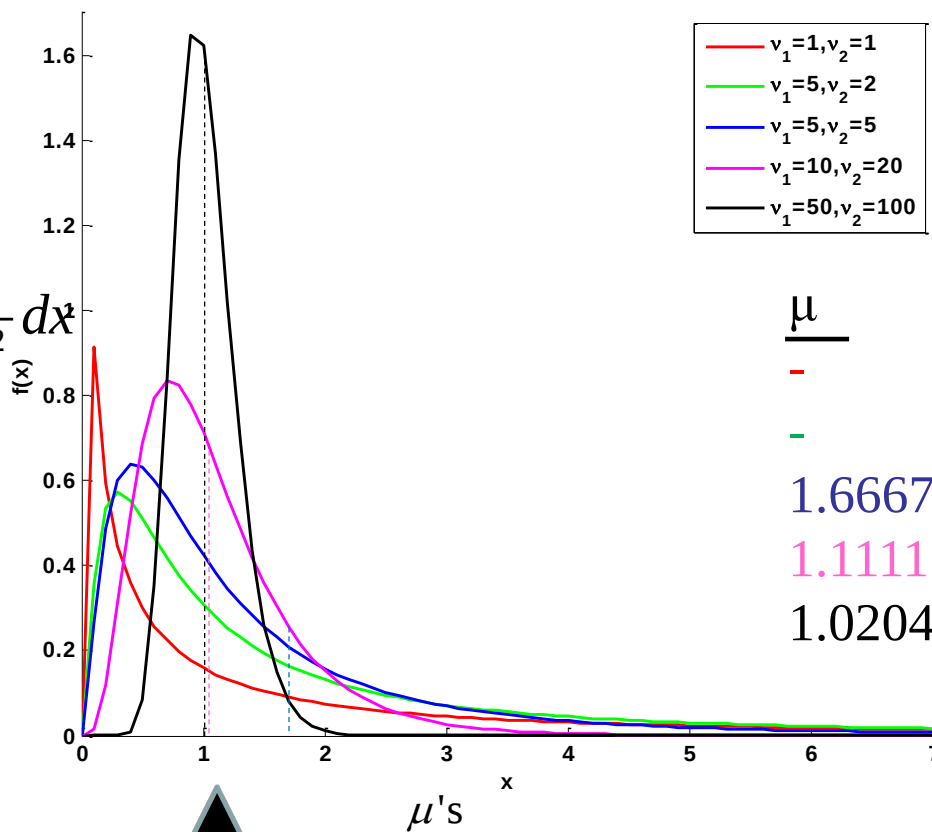
$$f(x | \nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}$$

It can be shown that

$$\mu = \int_{-\infty}^{\infty} x f(x | \theta) dx$$

$$= \int_{-\infty}^{\infty} x \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{\nu\pi}} \frac{1}{\left(1 + \frac{1}{\nu} x^2\right)^{(\nu+1)/2}} dx$$

$$= \frac{\nu_2}{\nu_2 - 2} \quad \nu_2 > 2$$



# Continuous Distributions

## F:

It can be shown that

median

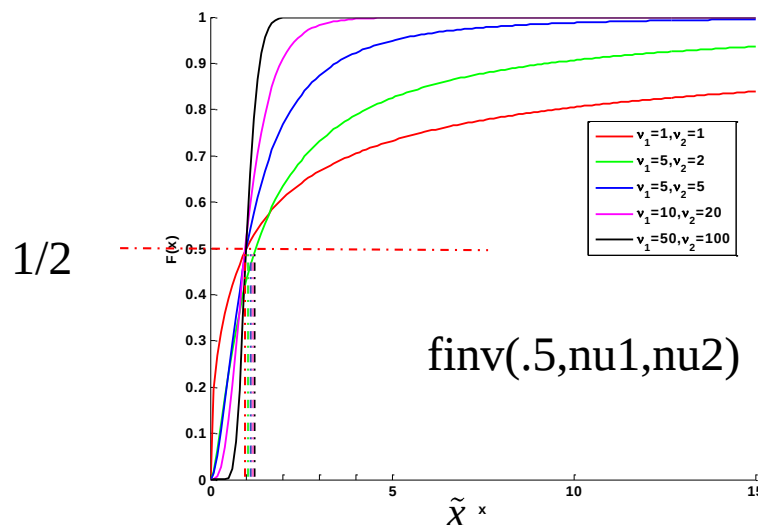
$$\int_{x=-\infty}^{\tilde{x}} f(x|\theta) dx = \frac{1}{2}$$

$\tilde{x}$  = no closed form

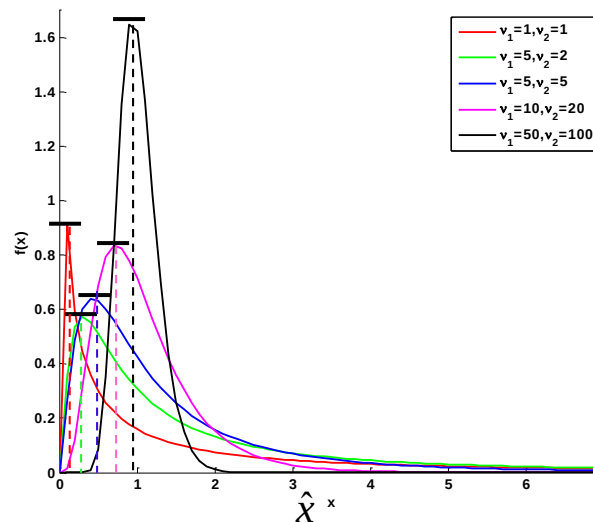
mode

$$\left. \frac{\partial}{\partial x} f(x|\theta) \right|_{\hat{x}} = 0$$

$$\hat{x} = \frac{\nu_1 - 2}{\nu_1} \frac{\nu_2}{\nu_2 + 2} \quad \nu_1 > 2$$



| $\tilde{x}$ |
|-------------|
| 1.0000      |
| 1.2519      |
| 1.0000      |
| 0.9663      |
| 0.9933      |



| $\hat{x}$ |
|-----------|
| -         |
| .3000     |
| .4286     |
| .7273     |
| .9412     |

# Continuous Distributions

## F:

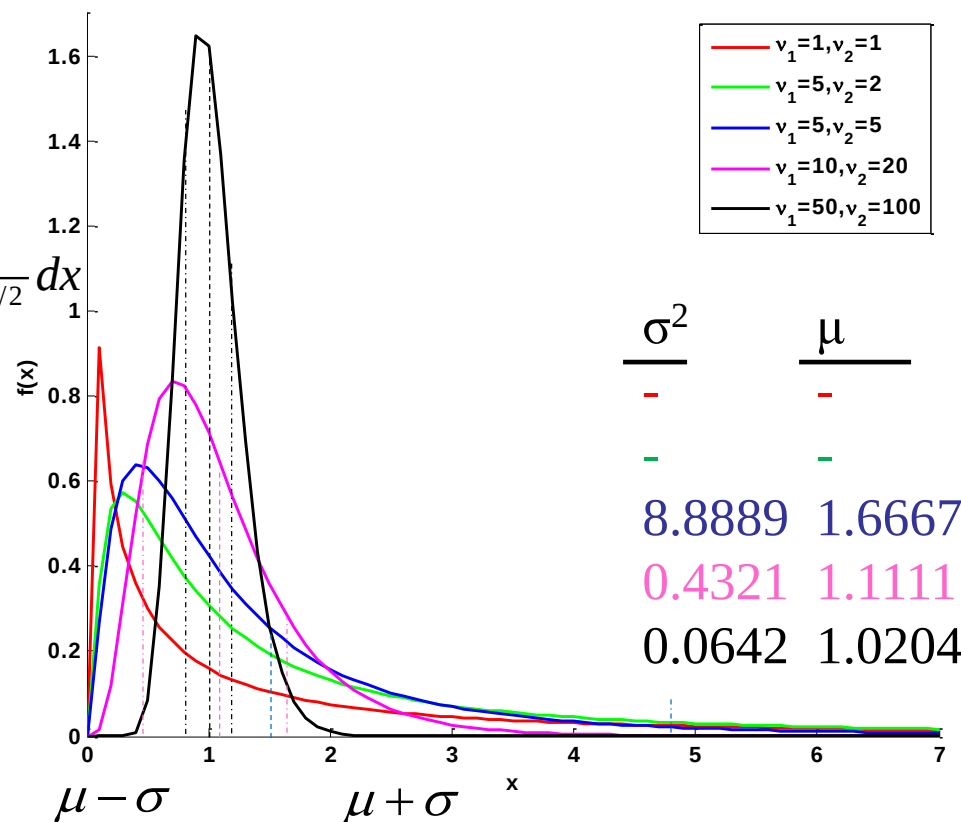
that

$$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x | \theta) dx$$

$$= \int_{-\infty}^{\infty} (x - \mu)^2 \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2-1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}} dx$$

$$= \frac{2\nu_2^2(\nu_1 + \nu_2 - 2)}{\nu_1(\nu_2 - 2)^2(\nu_2 - 4)}$$

$$\nu_2 > 4$$



# Continuous Distributions

F:

$$f(x | \nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}$$

```
nu1=5; nu2=50; num=10^4;
```

```
x=frnd(nu1, nu2, num, 1);
```

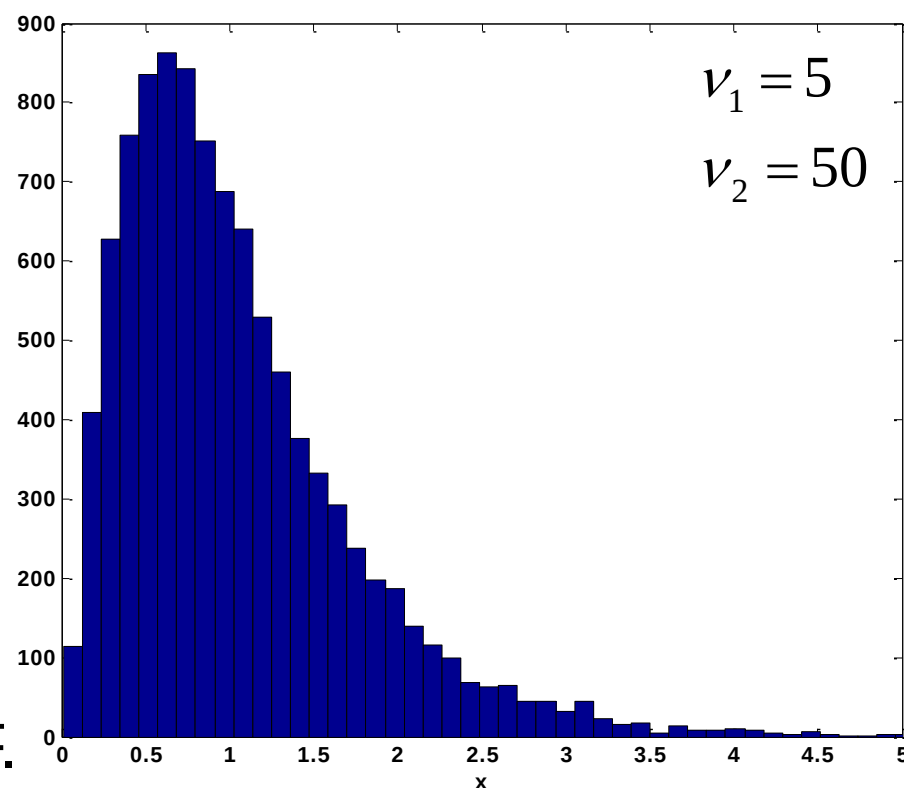
```
mean(x)
```

```
var(x)
```

```
hist(x, 50), xlim([0 5])
```

```
[mu, sigma2] = fstat(nu1, nu2)
```

|            | True   | Simulated |
|------------|--------|-----------|
| $\mu$      | 1.0417 | 1.0373    |
| $\sigma^2$ | 0.5001 | 0.4859    |



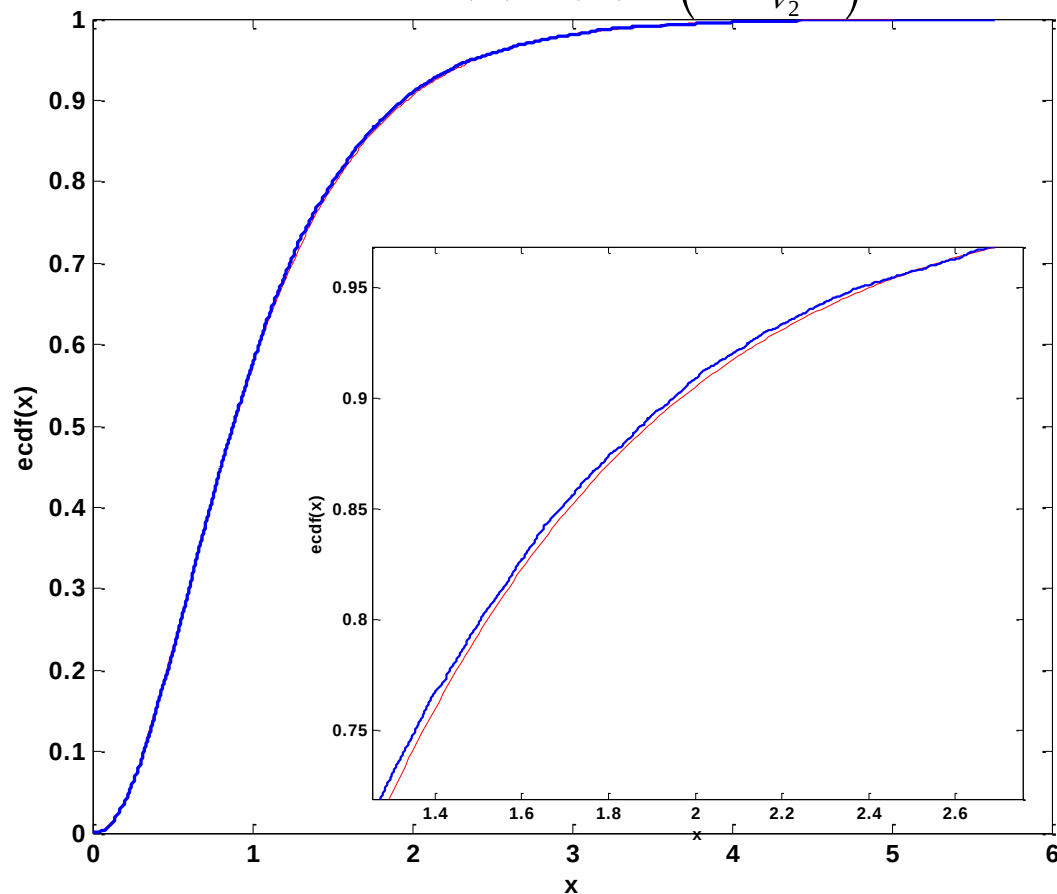
Can also find and plot ECDF.

# Continuous Distributions

F:

```
nu1=5;;nu2=50;;num=10^4;
y=fcdff(0:.01:5,nu1,nu2);
plot((0:.01:5),y, 'r')
hold on
[F,xx]=ecdf(x);
stairs(xx,F,'LineWidth',2)
```

$$f(x|\nu_1, \nu_2) = \frac{\Gamma\left(\frac{\nu_1 + \nu_2}{2}\right) \left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\Gamma\left(\frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2}\right)} \frac{x^{\nu_1/2 - 1}}{\left(1 + \frac{\nu_1}{\nu_2} x\right)^{(\nu_1 + \nu_2)/2}}$$



# Homework 5:

- 1) Let  $x \sim t(5)$ , using pencil and paper, find  $P(\mu - \sigma \leq x \leq \mu + \sigma)$  .
- 2) Show by pencil and paper that the variance of the Student- $t$  distribution is  $\sigma^2 = \frac{\nu}{\nu - 2}$  .
- 3) Numerically integrate Student- $t$  pdf with rectangles to find the 99<sup>th</sup> percentile. i.e. find  $x_0$  such that  $P(x \leq x_0) = .99$  for  $\nu = 5$  .

## Homework 5:

- 4) Generate  $10^6$  Student- $t$   $\nu = 5$  random variables.  
Empirically determine the 99<sup>th</sup> percentile.  
i.e. Find the  $.99 \cdot 10^6$  largest value  $x_0$ .  
such that  $P(x \leq x_0) \approx .99$  .  
Compare values to 3) and to value from  $t$ -table.

# Homework 5:

- 5) Make a histogram of the random variables in 4) with at least 50 bins.
- 6) Make a empirical CDF from the values in 4).

## Homework 5:

$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

7) Let  $x \sim \text{Cauchy}$  distributed. Derive the distribution of  $y = \sigma x + \mu$ .

8) Numerically integrate the Cauchy PDF with rectangles to find the 99<sup>th</sup> percentile. i.e. find  $x_0$  such that  $P(x \leq x_0) \approx .99$ . Compare to the exact percentile.

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan(x)$$

9) Generate  $10^6$  Cauchy random variates.

- Make a histogram, 50 bins.
- Compute sample mean and variance.
- Multiply each random variate by 2 and add 5.
- Make a histogram.
- Compute sample mean and variance.

# Homework 5:

10) Generate  $10^6$  Cauchy random variables.

Empirically determine the 50<sup>th</sup> and 99<sup>th</sup> percentiles.

i.e. Find the  $.50 \cdot 10^6$  and  $.99 \cdot 10^6$  largest values

$x_0$  such that  $P(x \leq x_0) \approx .5$  and  $P(x \leq x_0) \approx .99$ .

Compare values to value 8.

## Homework 5:

- 11) Write a Matlab program to numerically differentiate the  $F$  distribution and estimate the mode.  $\nu_1 = 5, \nu_2 = 50$   
 $\Delta x = 1/100$
- 12) Generate  $10^6$   $F$  distributed random variables.  $\nu_1 = 5$   
 $\nu_2 = 50$   
Empirically determine the 50<sup>th</sup> and 99<sup>th</sup> percentiles.  
i.e. Find the  $.50 \cdot 10^6$  and  $.99 \cdot 10^6$  largest values  
 $x_0$  such that  $P(x \leq x_0) \approx .5$  and  $P(x \leq x_0) \approx .99$ .  
Compare first value to 11 and second value to table.