Bayesian Non-Conjugate Priors

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Outline

Prior Information

Deterministic Integration

Stochastic Integration

Non-Conjugate Prior for Binomial RVs
(Parabolic prior for p)

Discussion

Homework

Prior Information

When we expect observations from a particular distribution, $f(x|\theta)$, it may be the case that the information that we have about the parameter(s) θ do not fit into the conjugate prior framework.
It is too constraining.

In such cases, we have non-conjugate prior distributions which when combined with the likelihood for the forthcoming observations does not necessarily form a “nice” $f(x,\theta)$ or “friendly” $f(\theta|x)$.

We may need to use advanced computational methods.

Prior Information

These advanced computational methods have the goal to compute the (marginal) posterior mean

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \theta f(\theta \mid x_1,\ldots,x_n) d\theta$$ ← Can't always be found with pencil and paper!

where

$$f(\theta \mid x_1,\ldots,x_n) = \frac{f(x_1,\ldots,x_n \mid \theta) f(\theta)}{f(x_1,\ldots,x_n)} .$$

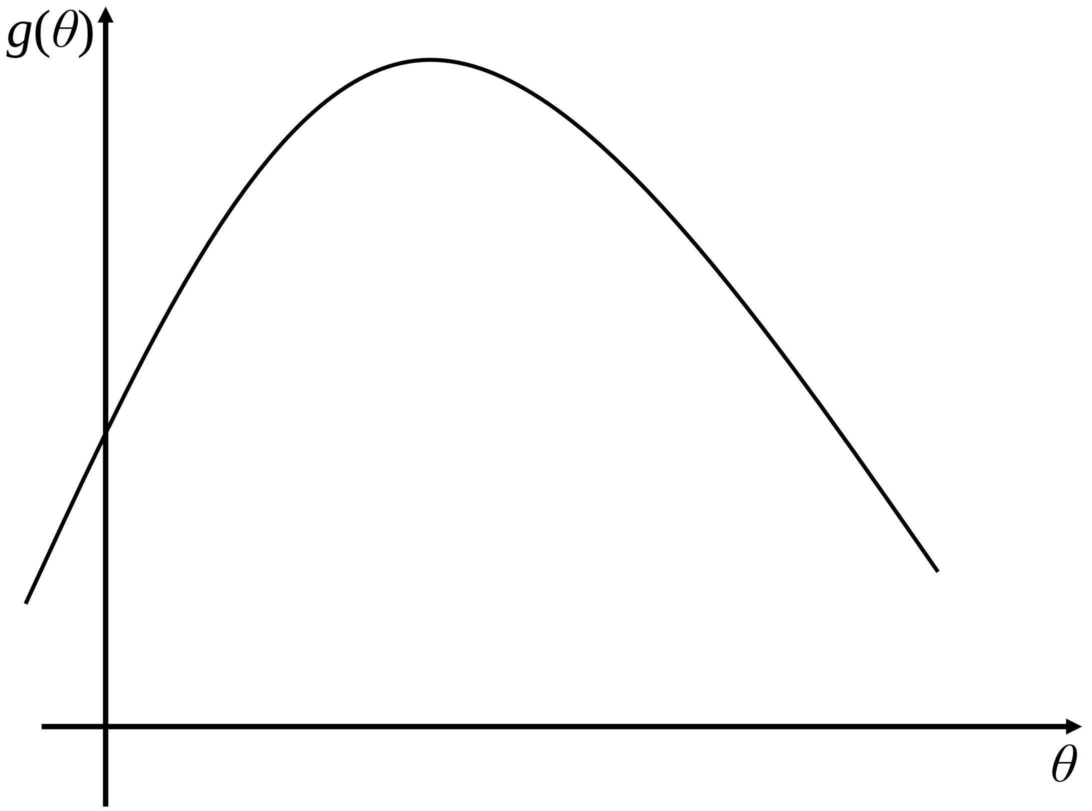
But to do this, don't forget that we also need

$$f(x_1,\ldots,x_n) = \int_\theta f(x_1,\ldots,x_n \mid \theta) f(\theta) d\theta .$$ ← Can't always be found with pencil and paper!

Deterministic Integration

goal

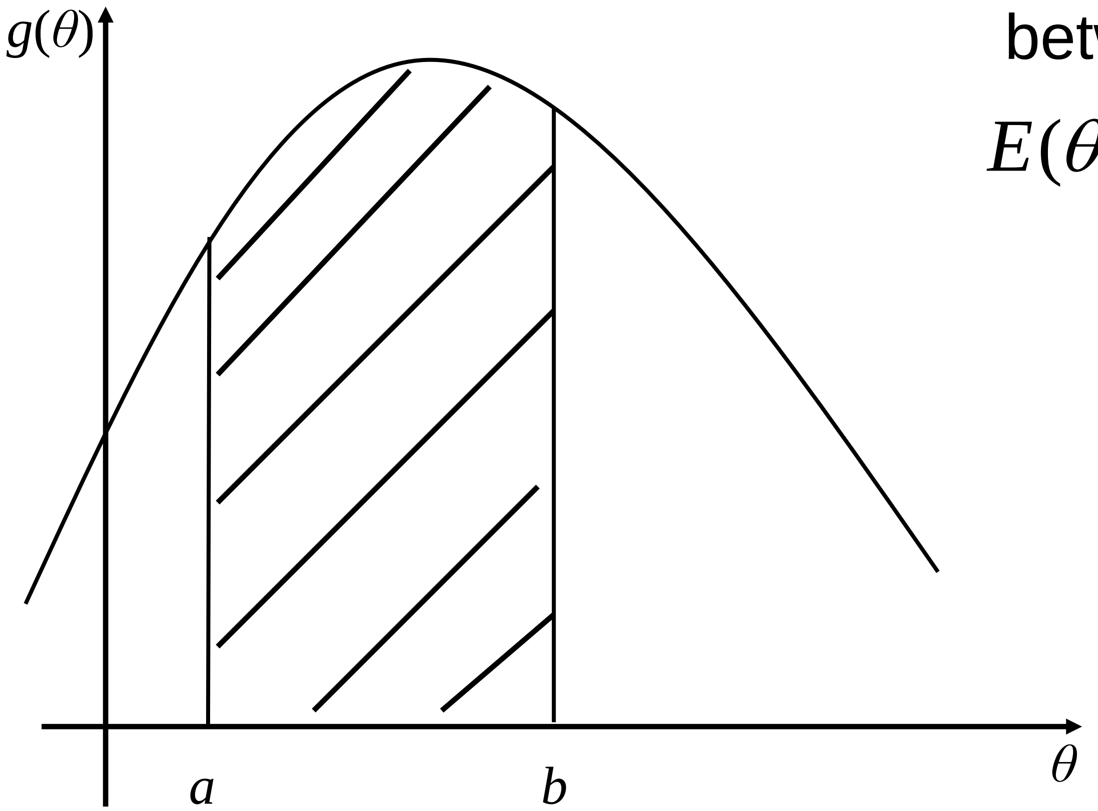
$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



Area under curve between a and b.

$$E(\theta \mid \cdot) = \int_\theta g(\theta) d\theta$$

think of as

$$g(\theta) = \theta f(\theta)$$

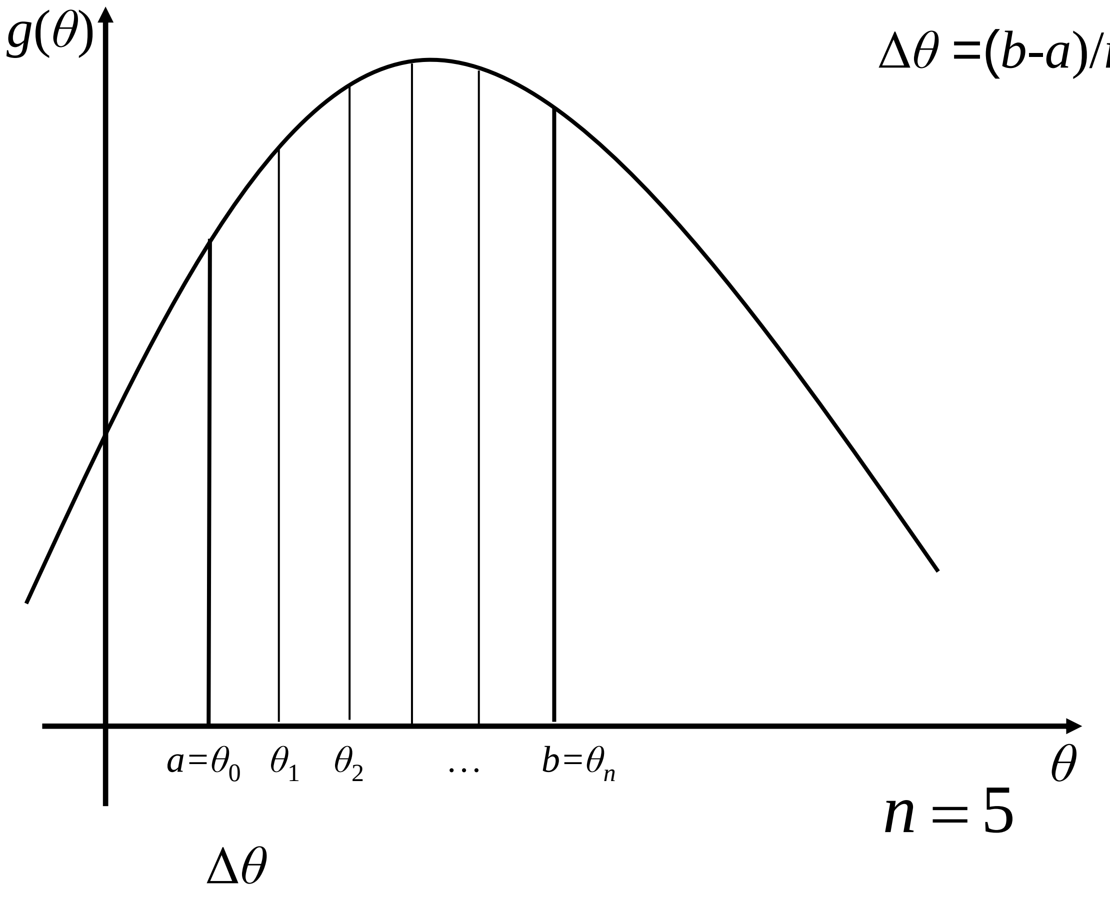
Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$

Divide into intervals: $\Delta\theta$ small

$\Delta\theta = (b-a)/n$ $\quad\Delta\theta = \theta_i - \theta_{i-1}$



think of as

$$g(\theta) = \theta f(\theta)$$

Deterministic Integration

goal

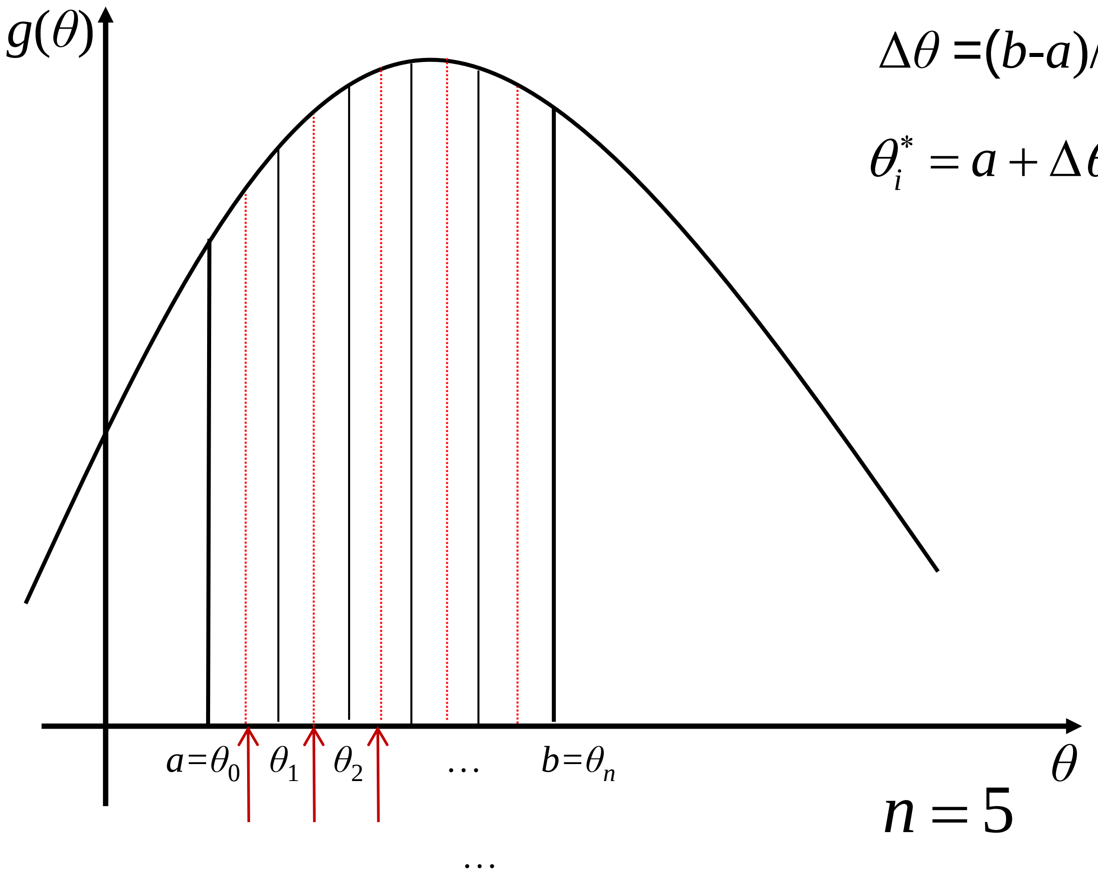
$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$

Divide into intervals: $\Delta\theta$ small

$\Delta\theta = (b\text{-}a)/n$ $\quad \Delta\theta = \theta_i - \theta_{i-1}$

$\theta_i^* = a + \Delta\theta/2 + (i-1)\Delta\theta$

$i = 1,\ldots,n$

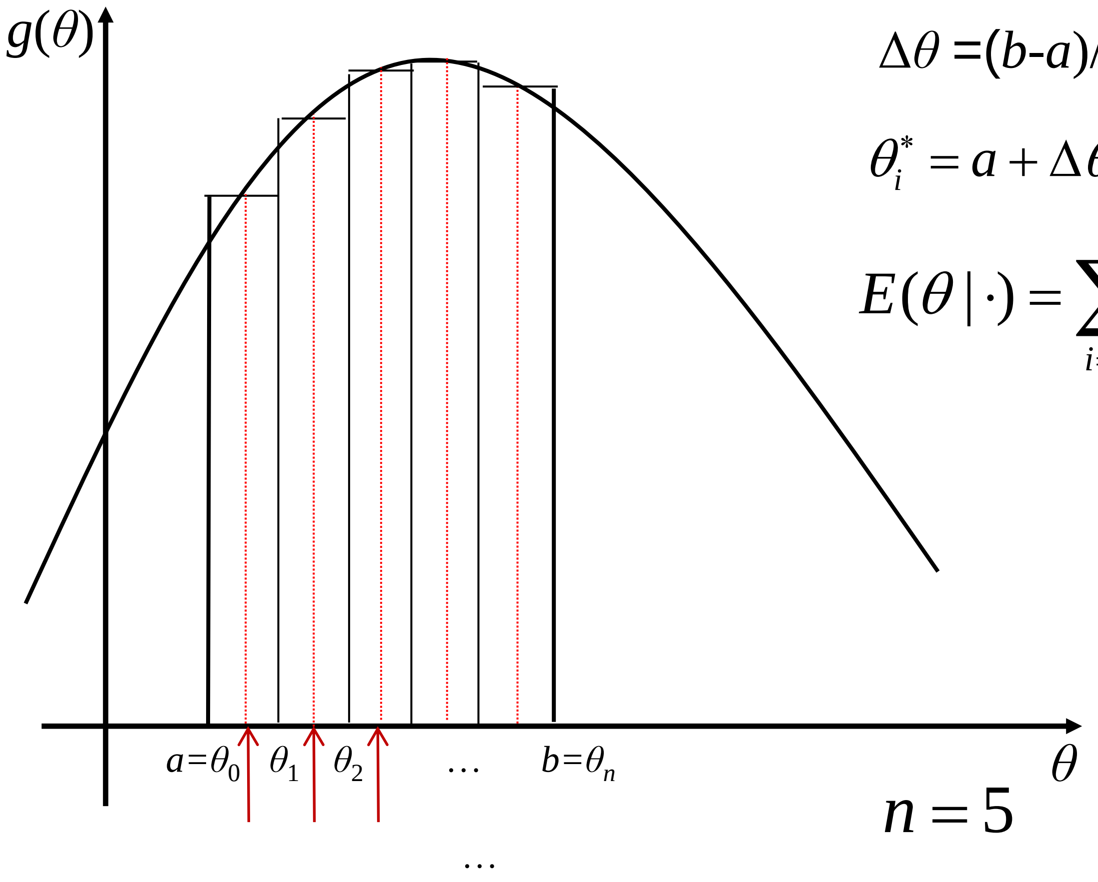


$n = 5$

Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



Divide into intervals: $\Delta\theta$ small

$\Delta\theta = (b-a)/n$ $\quad\quad \Delta\theta = \theta_i - \theta_{i-1}$

$$\theta_i^* = a + \Delta\theta/2 + (i-1)\Delta\theta$$

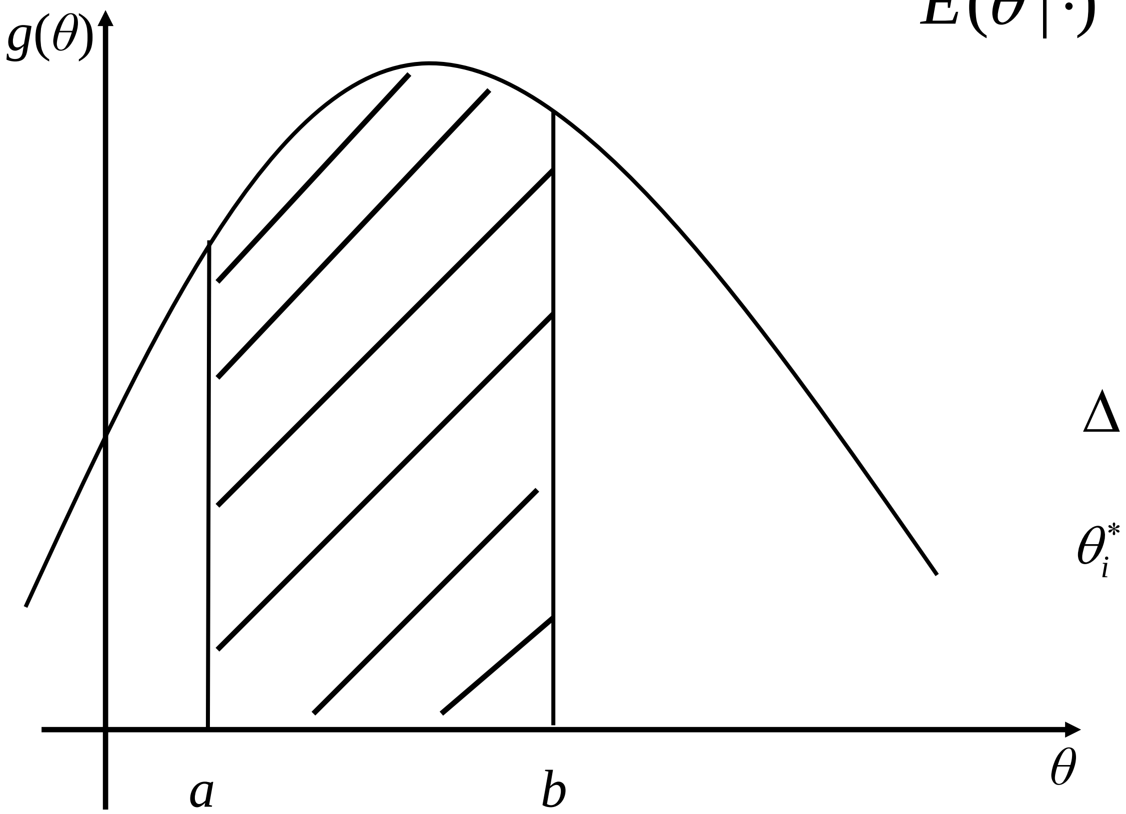
$$E(\theta \mid \cdot) = \sum_{i=1}^{n} g(\theta_i^*)\Delta\theta \quad\quad i = 1,\ldots,n$$

$n = 5$

Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



$$E(\theta \mid \cdot) = \lim_{\Delta\theta \to 0} \sum_{i=1}^{n} g(\theta_i^*)\Delta\theta$$

$$= \int_{\theta=a}^{b} g(\theta)d\theta$$

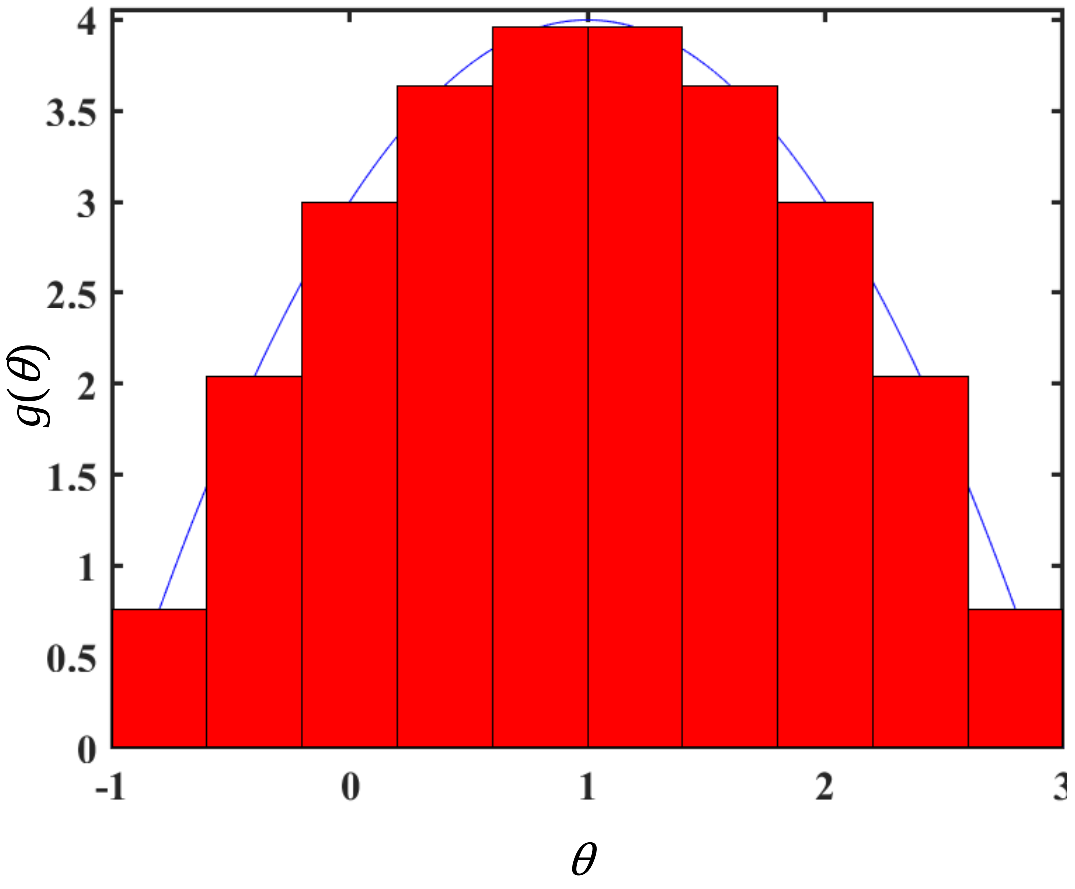
$$\Delta\theta = (b-a)/n$$

$$\theta_i^* = a + \Delta\theta/2 + (i-1)\Delta\theta$$

Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



$g(\theta) = 4-(\theta-1)^2$

← numerical

n=10, $\Delta\theta$=0.400

$$\hat{E}(\theta \mid \cdot) = \Delta\theta \sum_{i=1}^{10} g(\theta_i^*) = 10.7200$$

analytic

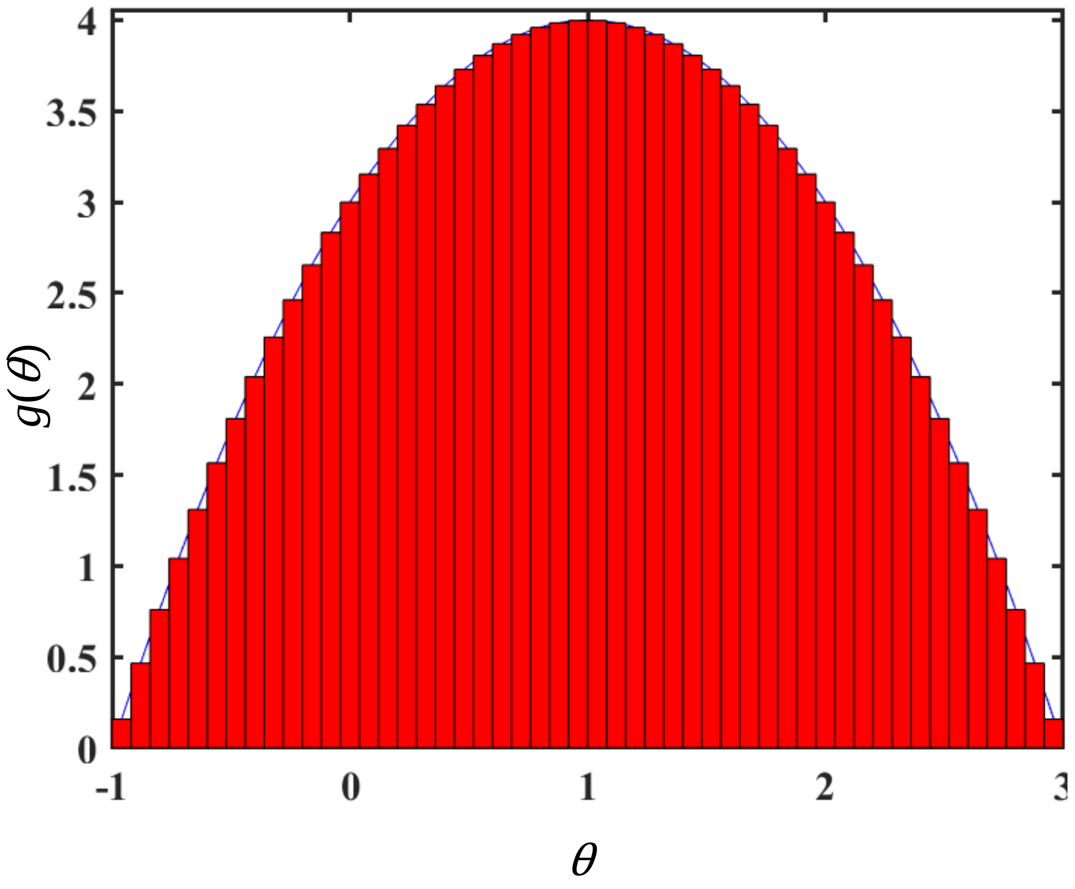
$$\int_{\theta=-1}^{3} [4-(\theta-1)^2] d\theta = 10.6667$$

```
% Numerical Integral
a=-1; b=3;
n=10;   dt=(b-a)/n;
tpts=(a+dt/2:dt:b)';
gpts=4-(tpts-1).^2;
Ethat=dt*sum(gpts)
```

Deterministic Integration

goal

$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



$$g(\theta) = 4 - (\theta - 1)^2$$

← numerical

n=50, $\Delta\theta$=0.080

$$\hat{E}(\theta \mid \cdot) = \Delta\theta \sum_{i=1}^{50} g(\theta_i^*) = 10.6688$$

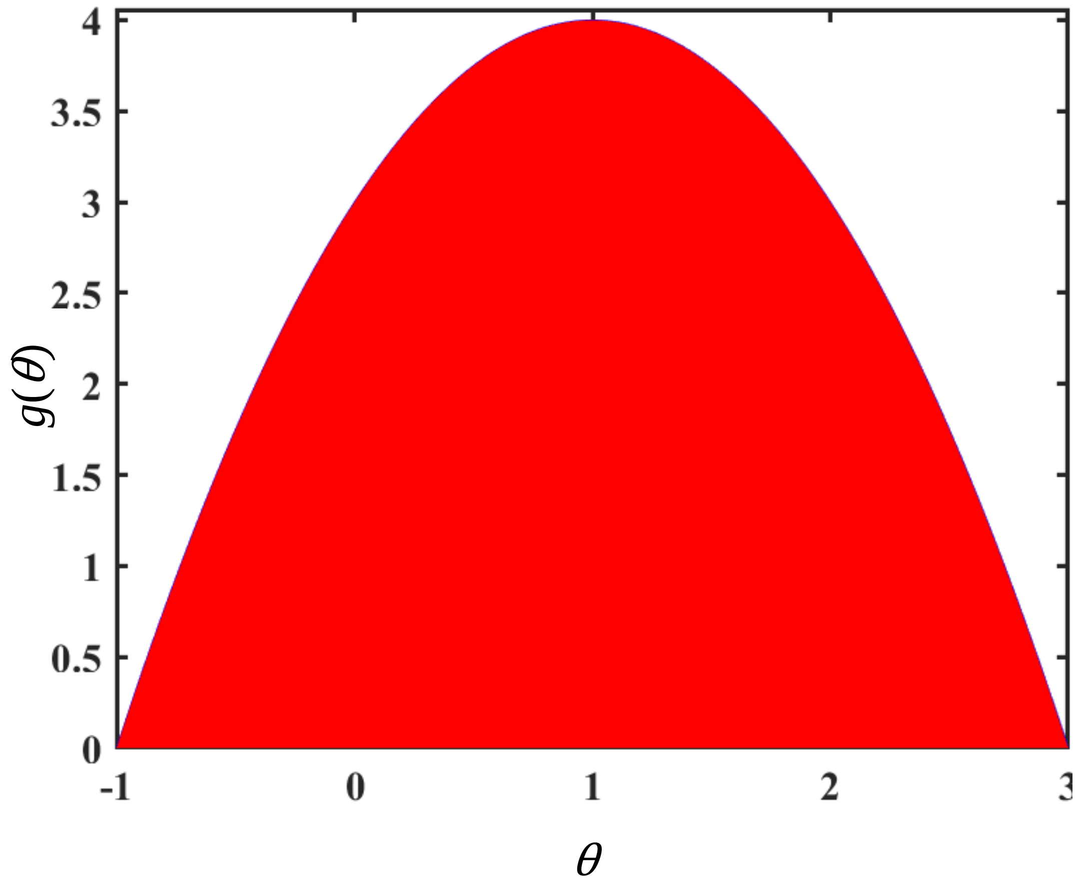
analytic

$$\int_{\theta=-1}^{3} [4 - (\theta - 1)^2] d\theta = 10.6667$$

```
% Numerical Integral
a=-1; b=3;
n=50;   dt=(b-a)/n;
tpts=(a+dt/2:dt:b)';
gpts=4-(tpts-1).^2;
Ethat=dt*sum(gpts)
```

Deterministic Integration

goal
$$E(\theta \mid x_1,\ldots,x_n) = \int_\theta \underbrace{\theta f(\theta \mid x_1,\ldots,x_n)}_{g(\theta)} d\theta$$



$$g(\theta) = 4 - (\theta - 1)^2$$

← numerical

n=1000, $\Delta\theta$=0.004

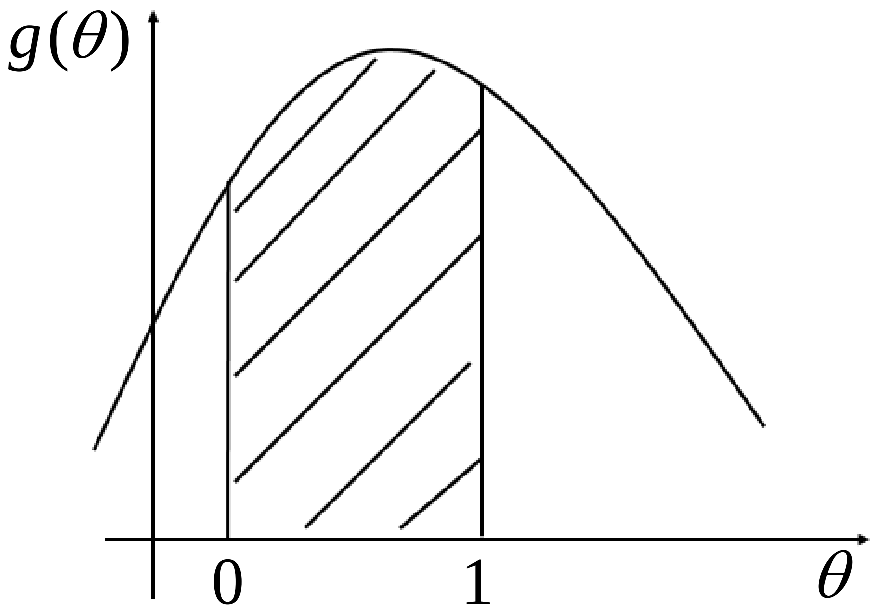
$$\hat{E}(\theta \mid \cdot) = \Delta\theta \sum_{i=1}^{1000} g(\theta_i^*) = 10.6667$$

analytic

$$\int_{\theta=-1}^{3} [4 - (\theta - 1)^2] d\theta = 10.6667$$

```
% Numerical Integral
a=-1; b=3;
n=1000; dt=(b-a)/n;
tpts=(a+dt/2:dt:b)';
gpts=4-(tpts-1).^2;
Ethat=dt*sum(gpts)
```

Stochastic Integration



We can also integrate a function $g(\theta)$ via random uniform numbers.

$$E(\theta|\cdot) = \int_0^1 g(u)du$$ ← changed symbol of integration

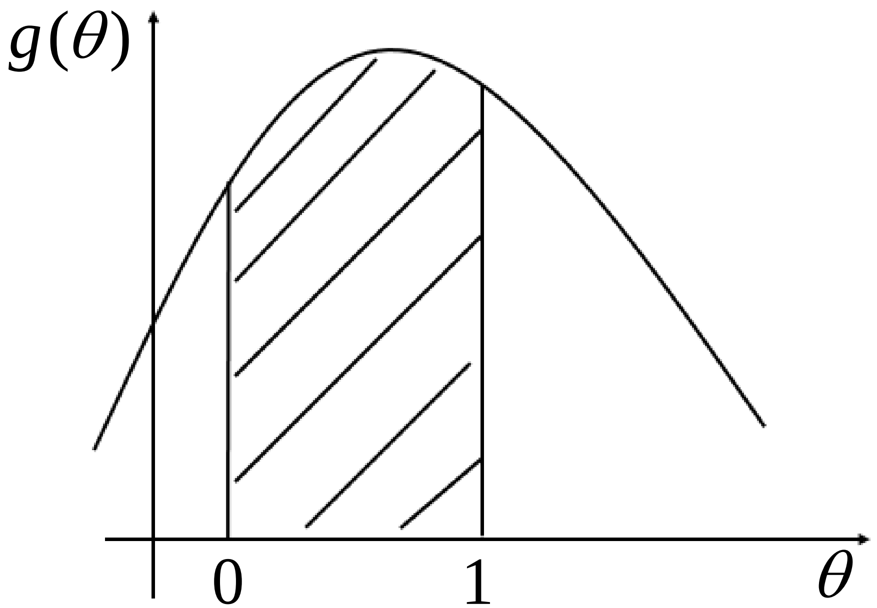
We can view this as there being a PDF $f(u)$ where u is uniformly distributed over $(0,1)$

$$f(u) = 1 \qquad 0 \le u \le 1$$

and we are calculating the expected value

$$E(\theta|\cdot) = E(g(u)) = \int_0^1 g(u)f(u)du \ .$$ ← key idea

Stochastic Integration



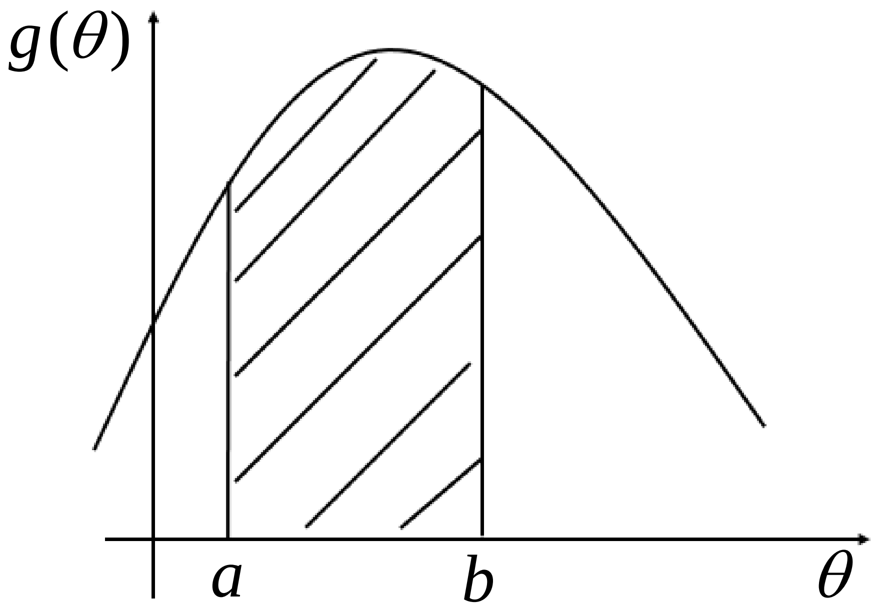
If we have a random sample $u_1,\ldots,u_n$ from $f(u)$, then we can calculate a sample version of

$$E(\theta\,|\,\cdot) = E(g(u)) = \int_0^1 g(u)\,f(u)\,du$$

with the iid sample $u_1,\ldots,u_n$, from $f(u)$.
The computed values $g(u_1),\ldots,g(u_n)$ are an iid sample with mean $E(g(u))$.

$$\frac{1}{n}\sum_{i=1}^{n} g(u_i) \to E[g(u)] = E(\theta\,|\,\cdot) \quad \text{as } n \to \infty$$

Stochastic Integration



If we want $E(\theta|\cdot)=\int_a^b g(\theta)d\theta$, then we can transform θ to u as u =(θ-a)/(b-a), with du =$d\theta$ /(b-a)

$$E(\theta|\cdot)=\int_a^b g(\theta)d\theta \qquad d\theta=(b-a)du$$

$$\theta=(b-a)u+a$$

Then our integral becomes

$$E(\theta|\cdot)=\int_0^1 g\left(a+(b-a)u\right)(b-a)du$$

$$h(u)=g\left(a+(b-a)u\right)(b-a)$$

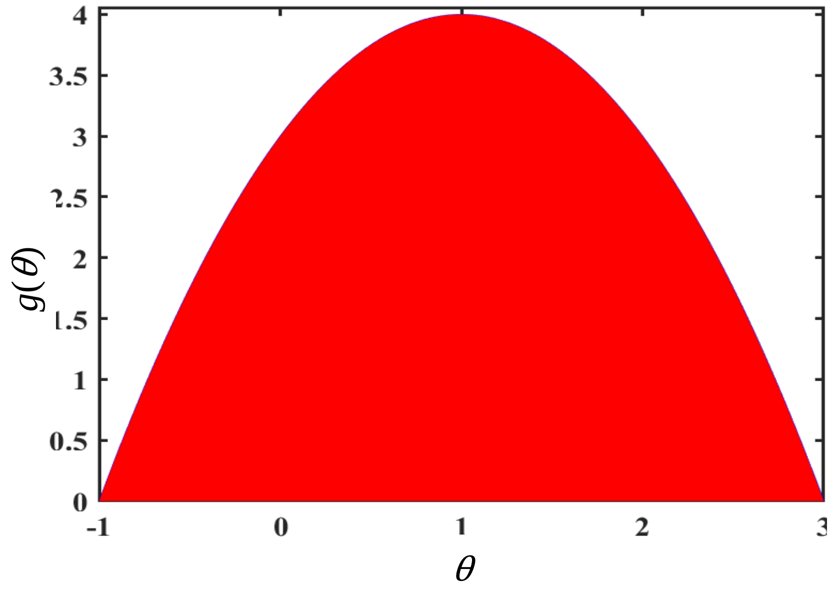
$$f(u)=1 \qquad 0\le u\le 1$$

which is the same as

$$E(\theta|\cdot)=E(h(u))=\int_0^1 h(u)f(u)du \, .$$

Stochastic Integration

Deterministic



$$g(\theta)=4-(\theta-1)^2$$

Let’s use this idea to evaluate the same integral.

$$(b-a)=4$$

Deterministic

$$E(\theta\,|\,\cdot)=\int_\theta g(\theta)d\theta$$

$$\hat{E}_n(\theta\,|\,\cdot)=\Delta\theta\sum_{i=1}^{n} g(\theta_i^*)$$

$$\hat{E}_{10}(\theta\,|\,\cdot)=10.7200$$

Stochastic

$$h(u)=g\big(a+(b-a)u\big)(b-a)$$

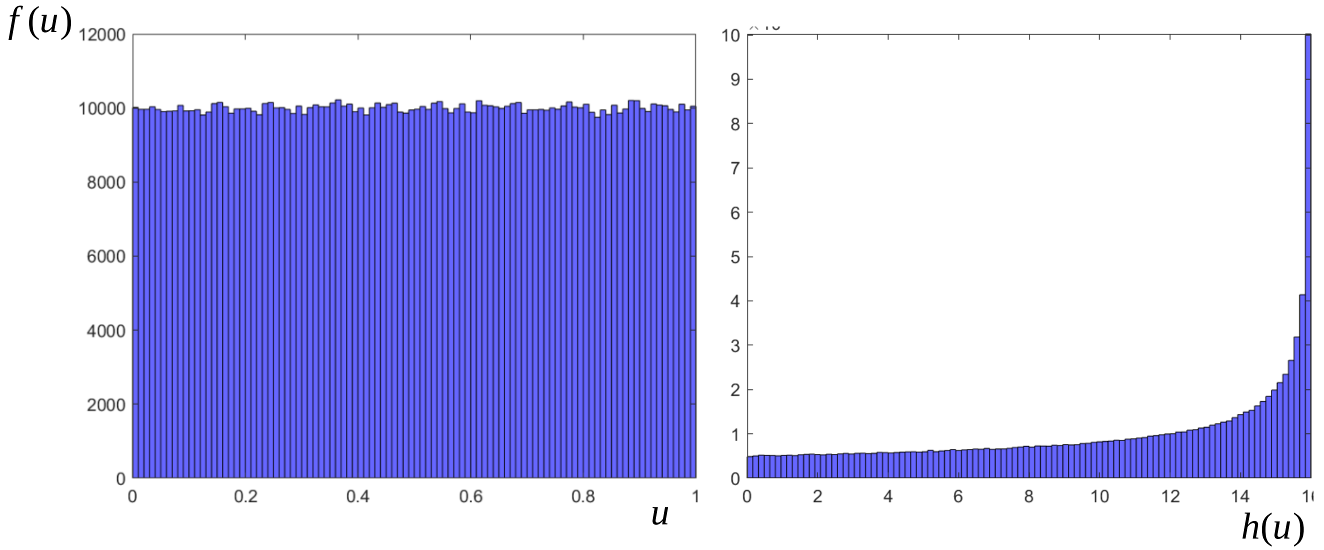
$$\frac{1}{n}\sum_{i=1}^{n}h(u_i)=10.6718$$

```
% stochastic integral
rng('default')
a=-1; b=3; n=10^6;
u=rand(n,1);
t=(a+(b-a)*u);
hu=(4-(t-1).^2)*(b-a);
Ethat=sum(hu)/n
```

Theoretical

$$\int_{x=-1}^{3}[4-(\theta-1)^2]d\theta=10.6667$$

Stochastic



Stochastic Integration

If we want $E(\theta|\cdot)=\int_0^\infty g(\theta)d\theta$, then we can transform θ to u as u=1/(θ +1), with du=–1/(θ +1)$^2d\theta$ = –$u^2d\theta$

$$E(\theta|\cdot)=\int_0^\infty g(\theta)d\theta$$

$$d\theta=-du/u^2$$

$$\theta=1/u-1$$

limits

$$\theta=0\rightarrow u=1$$

$$\theta=\infty\rightarrow u=0$$

Then our integral becomes

$$E(\theta|\cdot)=\int_0^1 g(1/u-1)\frac{du}{u^2}$$

$$u=1/(\theta+1)$$

$$h(u)=g(1/u-1)/u^2$$

$$E(\theta|\cdot)=\int_0^1 h(u)f(u)du$$

and we can use the same technique.

$$\bar{E}(\theta|\cdot)=\frac{1}{n}\sum_{i=1}^{n}h(u_i)$$

Stochastic Integration

logistic function

If we want $E(\theta|\cdot)=\int_{-\infty}^{\infty} g(\theta)d\theta$, then we can transform θ to u as $u=e^{\theta}/(1+e^{\theta})$, with $du=e^{\theta}/(1+e^{\theta})^2 d\theta=ud\theta/(1-u)$

$$E(\theta|\cdot)=\int_{-\infty}^{\infty} g(\theta)d\theta$$

$$d\theta = udu/(1-u)$$

$$\theta=\ln\left(\frac{u}{1-u}\right)$$

limits

$$\theta=-\infty \to u=0$$

$$\theta=+\infty \to u=1$$

Then our integral becomes

$$E(\theta|\cdot)=\int_{0}^{1} g\left(\ln\left(\frac{u}{1-u}\right)\right)\frac{du}{u(1-u)}$$

$$E(\theta|\cdot)=\int_{0}^{1} h(u)f(u)du$$

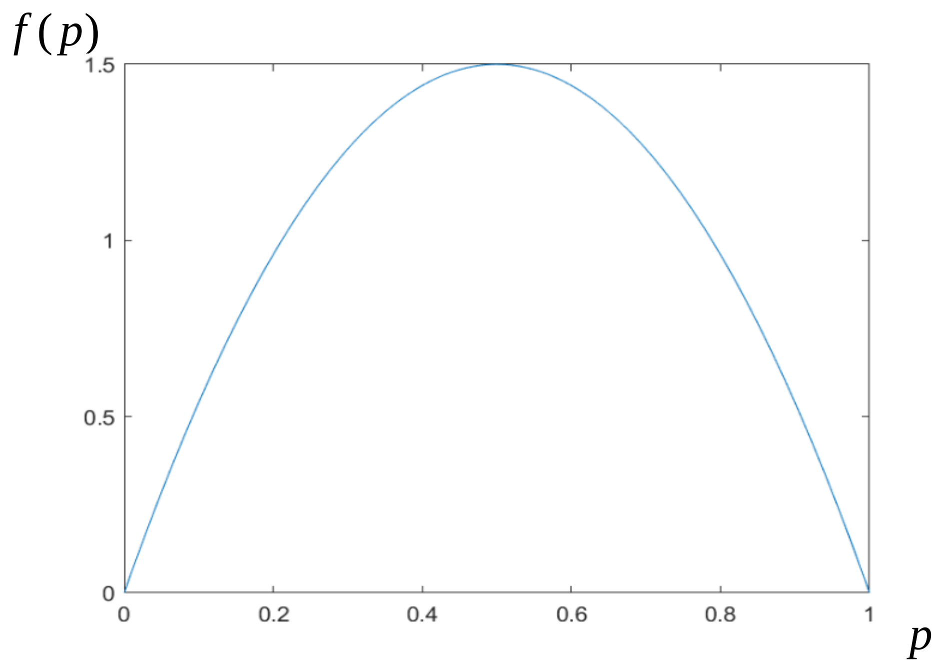
$$u=e^{\theta}/(1+e^{\theta})$$

$$h(u)=g\left(\ln\left(\frac{u}{1-u}\right)\right)\frac{1}{u(1-u)}$$

and we can use the same technique.

$$\bar{E}(\theta|\cdot)=\frac{1}{n}\sum_{i=1}^{n} h(u_i)$$

Non-Conjugate Prior For Binomial RVs



Binomial observation x:

Let's imagine that our prior information for p was consistent with the following parabolic prior for p,

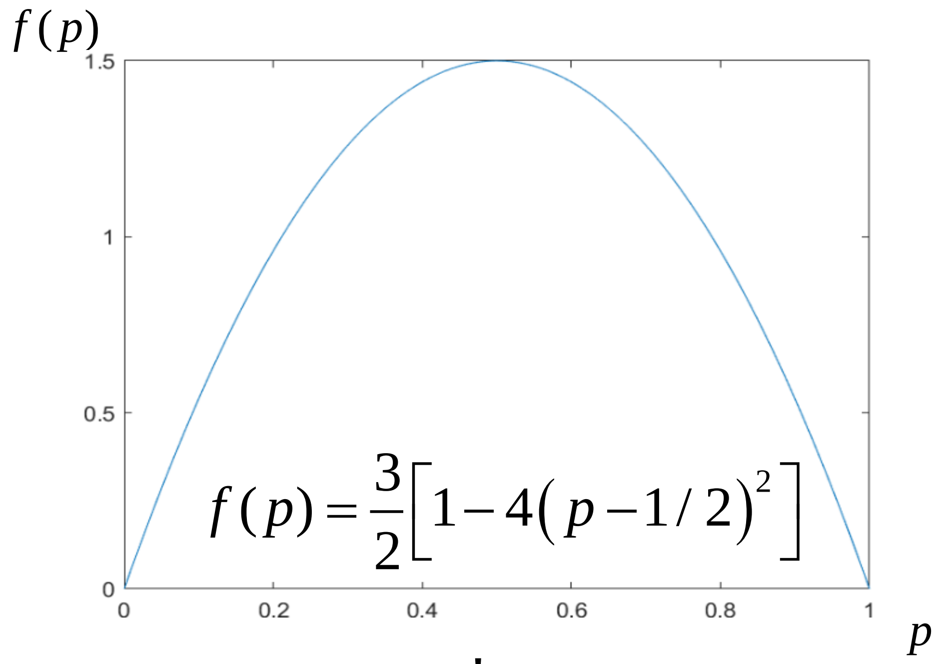
$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right] \qquad p\in[0,1]$$

to go along with the binomial likelihood for x

$$f(x\mid p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \qquad p\in[0,1]$$

$$x=0,1,\ldots,n$$

Non-Conjugate Prior For Binomial RVs



$$f(x\mid p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$p\in[0,1]$$

Binomial observation x:

The parabolic prior for p and binomial likelihood for x do not combine to form a "nice" joint distribution $f(x,p)$

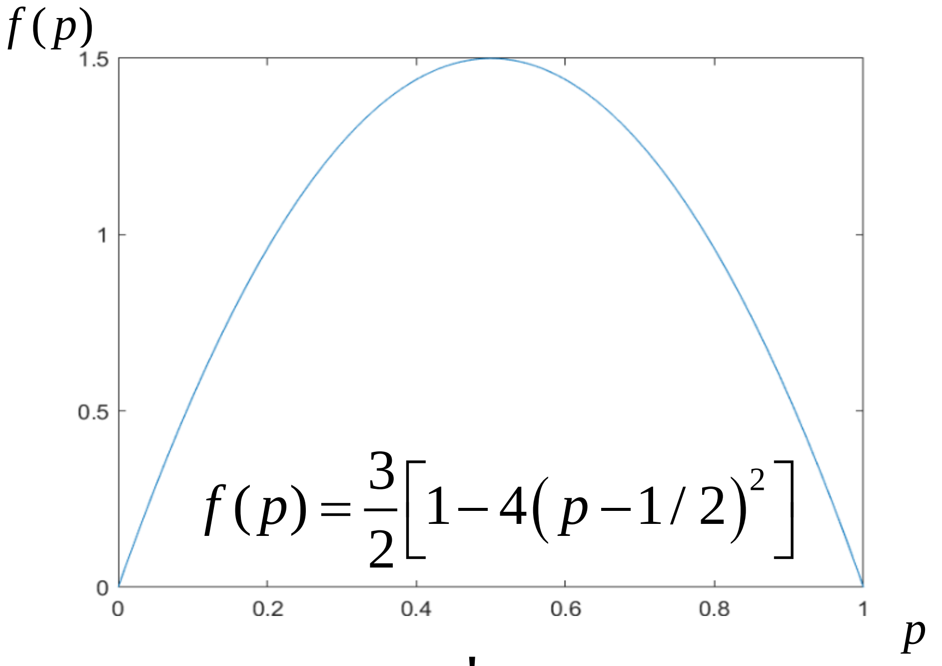
$$f(x,p)=\frac{3}{2}\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right]$$

or yield a "friendly" posterior distribution $f(p|x)$ for p.

So we need to resort to more advanced methods.

i.e. Try harder for pencil & paper integral, deterministic integration (rectangles), stochastic integration (random u's), MCMC.

Non-Conjugate Prior For Binomial RVs



$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

$$f(x\,|\,p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$p\in[0,1]$$

Binomial observation x:

The $f(p)$ and $f(x|p)$ do not combine to form a "nice" $f(x,p)$ or yield a "friendly" posterior $f(p|x)$.

It can be shown (homework problem) that

$$f(x)=6\frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

and thus

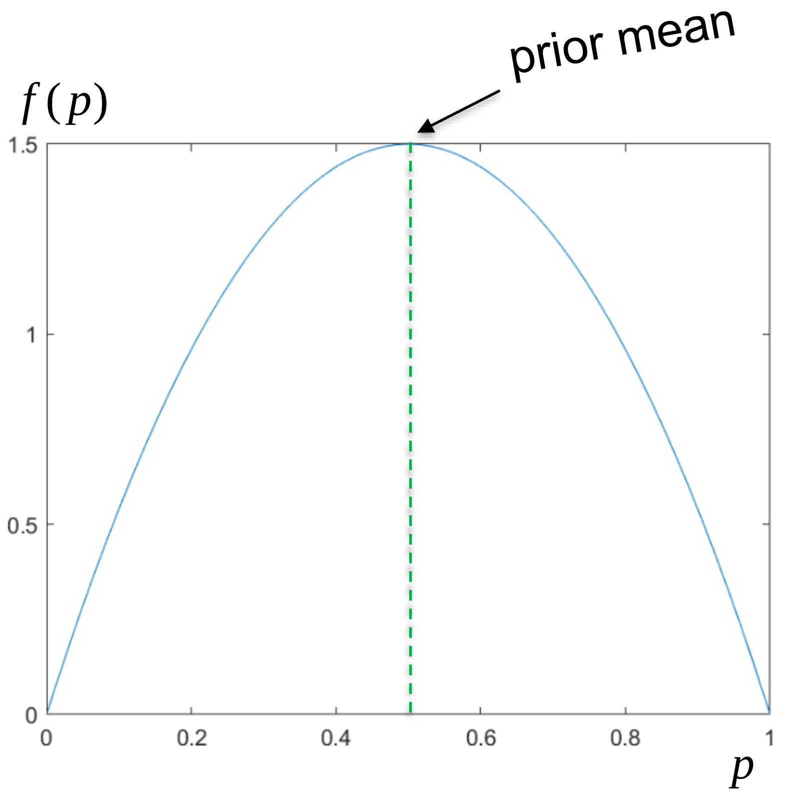
$$f(p\,|\,x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right].$$

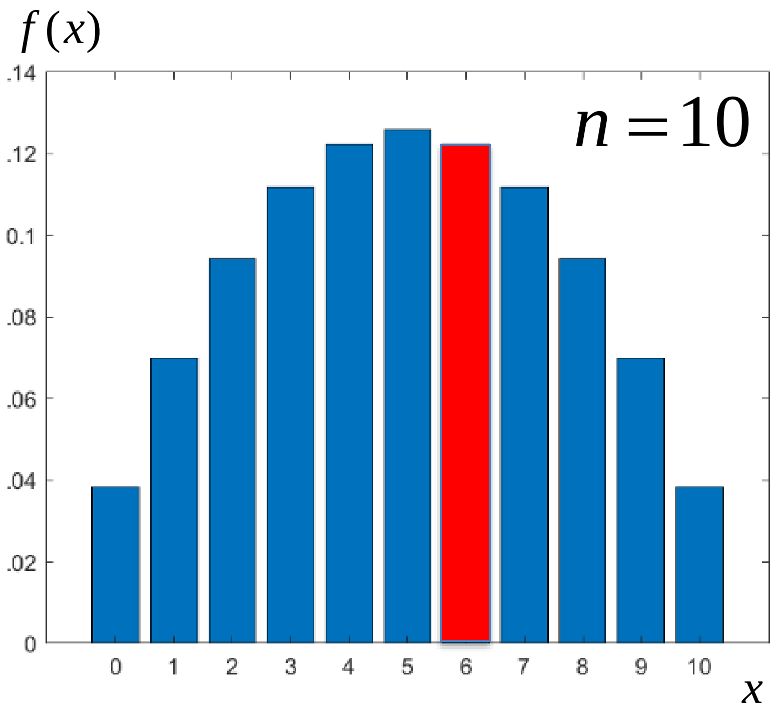
Non-Conjugate Prior For Binomial RVs

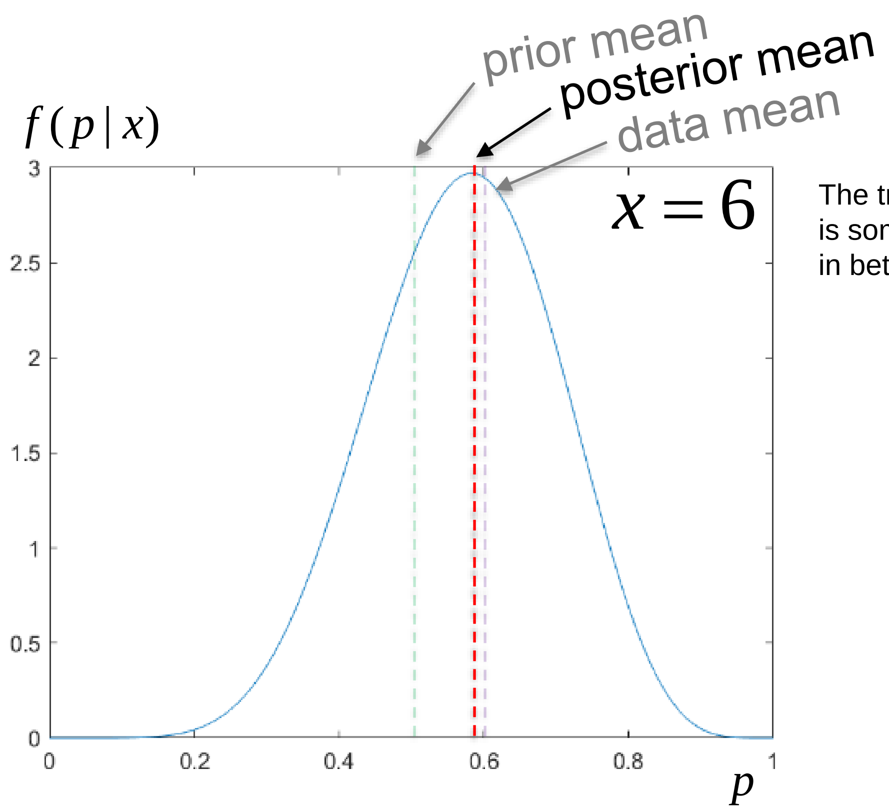
$$f(x\,|\,p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

Binomial observation x: $$f(p\,|\,x)=\frac{f(x\,|\,p)f(p)}{f(x)}$$

$$p\in[0,1]$$







The truth
is somewhere
in between!

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

$$f(x)=6\frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

$$f(p\,|\,x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right]$$

Non-Conjugate Prior For Binomial RVs

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

Binomial observation x:

From this posterior PDF for p,

$$f(p|x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right]$$

$$p\in[0,1]$$

$$n=10$$
$$x=6$$

we need to compute summary measures.

i.e. mode, mean, median, variance of $p|x$.

Similar to what we do when we have conjugate priors.

Non-Conjugate Prior For Binomial RVs

$$f(x\,|\,p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

Binomial observation x:

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

We can differentiate to maximize the posterior

$$f(p\,|\,x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right]$$

$$n=10$$
$$x=6$$

$$p\in[0,1]$$

Or we can we can try every value of p that is $\Delta p= 0.0001$ apart and select the one that yields the maximum.

$$\underset{p}{\text{ArgMax}}\; f(p\,|\,x,n)=0.5833$$

Non-Conjugate Prior For Binomial RVs

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

Binomial observation x:

To calculate the expected value $E(p|x)$ of

$$f(p|x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right],$$

$$p\in[0,1]$$

$$n=10$$
$$x=6$$

there are several techniques at our disposal

exact integration (calculus)

deterministic (*numerical analysis*)

and stochastic (*statistical simulation*).

Non-Conjugate Prior For Binomial RVs

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

Binomial observation x:

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right]$$

Stochastic and numerical integration.

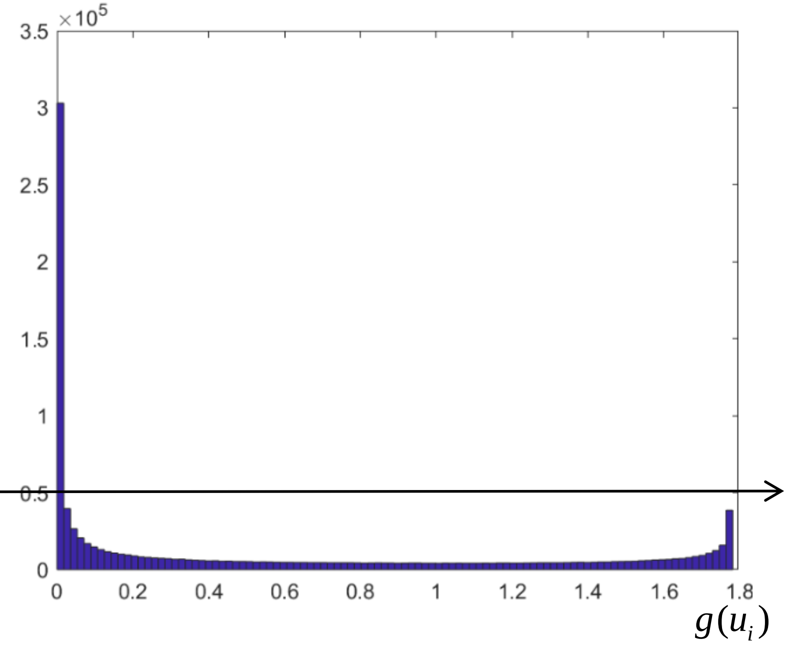
$$f(p|x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right]$$

$$p\in[0,1]$$

$$n=10$$
$$x=6$$

$$E_{10^6}(p|x,n)=0.5721$$

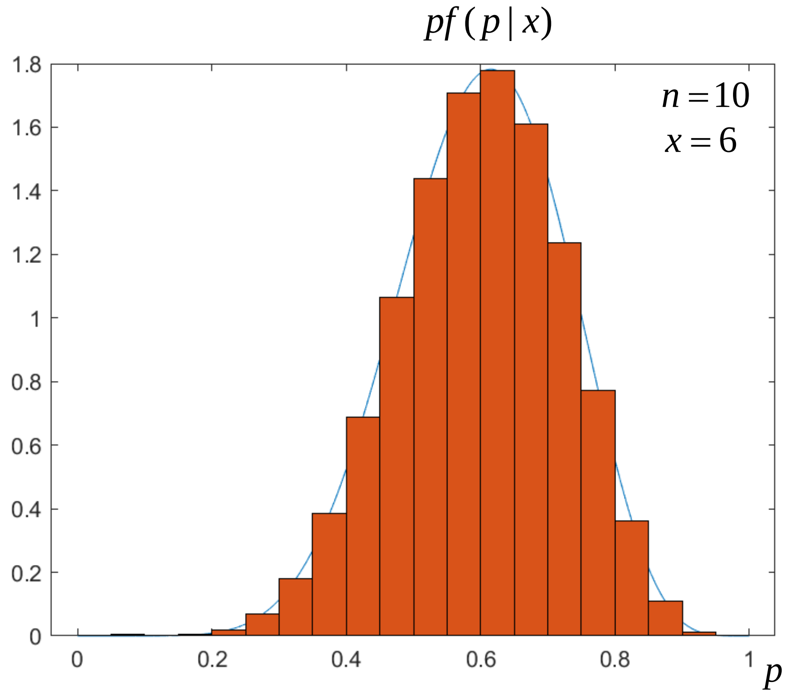
$n=10^6$ $\quad \frac{1}{n}\sum_{i=1}^{n}g(u_i)$



$$E_{20}(p|x,n)=0.5714$$

$\Delta p=0.05$ $\quad \Delta p\sum f(p_{mid}|x)$

same answer for 5 rectangles



Discussion

Life is more difficult when we have non-conjugate priors!

$$f(x_1,\ldots,x_n)=\int_\theta f(x_1,\ldots,x_n\,|\,\theta)\,f(\theta)\,d\theta$$

$$f(\theta\,|\,x_1,\ldots,x_n)=\frac{f(x_1,\ldots,x_n\,|\,\theta)\,f(\theta)}{f(x_1,\ldots,x_n)}$$

$$E(\theta\,|\,x_1,\ldots,x_n)=\int_\theta \theta\, f(\theta\,|\,x_1,\ldots,x_n)\,d\theta$$

For each n.

Deterministic and Stochastic integration requires $f(x_1,\ldots,x_n)$.

It may be extremely difficult to get. May need other methods!

Discussion

Questions?

Pencil and Paper Integration

$$E(\theta | x_1,...,x_n) = \int_\theta \theta f(\theta | x_1,...,x_n) d\theta$$

Deterministic Numerical Integration

$$\hat{E}(\theta | \cdot) = \Delta\theta \sum_{i=1}^{n} g(\theta_i^*)$$

$$\Delta\theta = (b-a)/n$$

$$\theta_i^* = a + \Delta\theta/2 + (i-1)\Delta\theta$$

Stochastic Simulation Integration

$$\bar{E}(\theta | \cdot) = \frac{1}{n}\sum_{i=1}^{n} h(u_i)$$

$a < \theta < b$	$0 < \theta < \infty$	$-\infty < \theta < \infty$
$u = (\theta - a)/(b-a)$	$u = 1/(\theta+1)$	$u = e^\theta / (1+e^\theta)$
$h(u) = g\left(a + (b-a)u\right)(b-a)$	$h(u) = g(1/u - 1)/u^2$	$h(u) = g\left(\ln\left(\frac{u}{1-u}\right)\right)\frac{1}{u(1-u)}$

Homework 9

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$
$$p\in[0,1]$$

1. With the parabolic prior for p (prove integrates to 1)

$$f(p)=\frac{3}{2}\left[1-4\left(p-1/2\right)^2\right] \qquad p\in[0,1]$$

and binomial likelihood for x,

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x} \qquad x=0,1,\ldots,n$$

a) prove that the marginal PDF for x is

$$f(x)=6\frac{(x+1)(n-x+1)}{(n+3)(n+2)(n+1)}$$

← Want at least a page of calculus.

Hint: This is a Beta PDF integral.

b) and that the posterior for $p|x$ is

$$f(p|x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right].$$

Homework 9

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$p\in[0,1]$$

2. With the posterior PDF for p

$$f(p|x)=\frac{1}{4}\frac{(n+3)!}{(x+1)!(n-x+1)!}p^x(1-p)^{n-x}\left[1-4\left(p-1/2\right)^2\right] , \qquad x=0,1,\ldots,n \quad .$$

a) Calculate the mode and prove or disprove that it is $0.583\bar{3}$. n=10 and x=6

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Use deterministic AND stochastic methods.

$f'(p|x)=0$?

b) Calculate the mean and prove or disprove that it is 0.5714.

Use both deterministic AND stochastic integration.

$\int_{p=0}^{1} pf(p|x)dp$?

Homework 9

$$f(x\,|\,p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$p\in[0,1]$$

3*.Use the following non-conjugate prior PDF for p

$$f(p)=\frac{\pi}{2}\cos[\pi(p-1/2)] \qquad p\in[0,1]$$

to combine with the binomial likelihood for $x|p$.

a) Try pencil & paper options to prove $f(p)$ integrates to 1.

b) Try pencil & paper options to calculate $f(x)$ and $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil and paper, calculate with deterministic and stochastic methods.

c) Try pencil & paper options to calculate the mode of $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil & paper, calculate with deterministic and stochastic methods.

d) Try pencil & paper options to calculate the mean of $f(p|x)$. $n=10$ and $x=6$

If you can't do pencil & paper, calculate with deterministic and stochastic methods.

* For students in 5790.

Homework 9

$$f(x|p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$p\in[0,1]$$

4**. Make up your own ***fun*** non-conjugate prior PDF for p

$$f(p)=?????\qquad p\in[0,1]$$

to combine with the binomial likelihood for $x|p$.

a) Try pencil & paper options to calculate $f(x)$ and hence $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

b) Try pencil & paper options to calculate the mode of $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

c) Try pencil & paper options to calculate the mean of $f(p|x)$.

If you can't do pencil and paper, calculate computationally. $n=10$ and $x=6$

** For students that have had MSSC 6010 and 6020.

Homework 9

$$f(x \mid p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

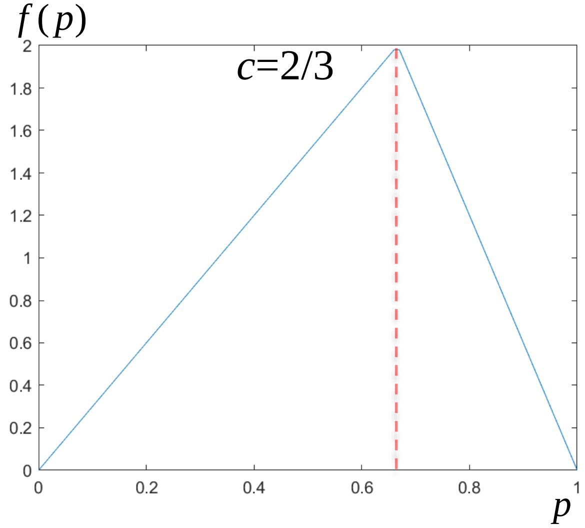
$$p \in [0,1]$$

Potential *Fun* Non-Conjugate Priors

Triangular

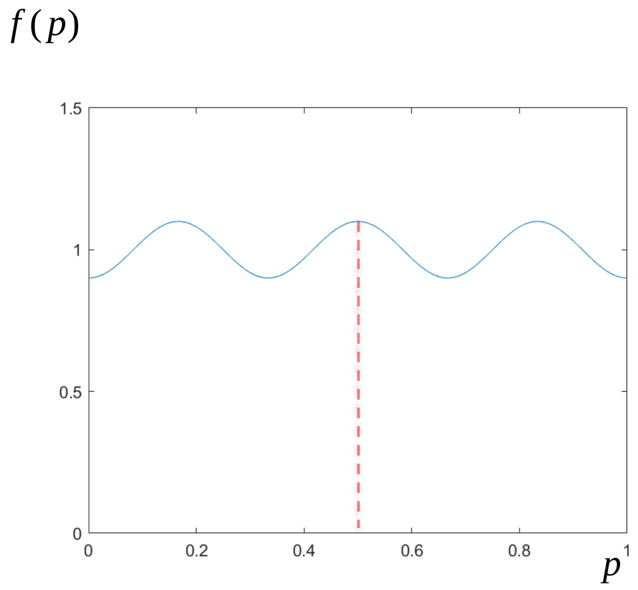
$$f(p) = \begin{cases} \frac{2p}{c}, & 0 \le p \le c \\ \frac{2(1-p)}{1-c}, & c \le p \le 1 \end{cases}$$

c is mode



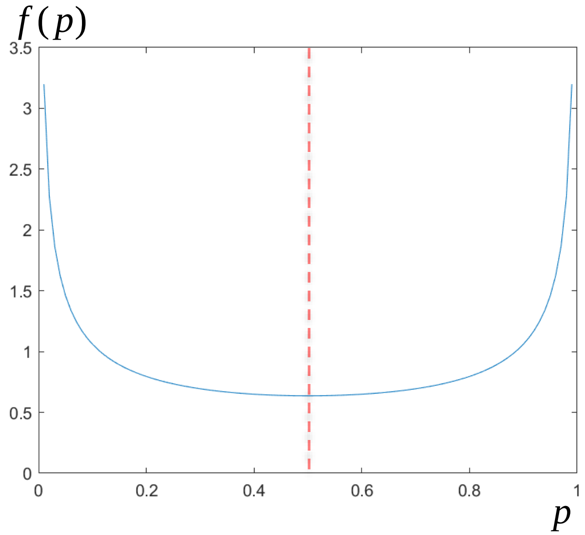
Roller Coaster

$$f(p) = \cos(6\pi p + \pi) + 1$$



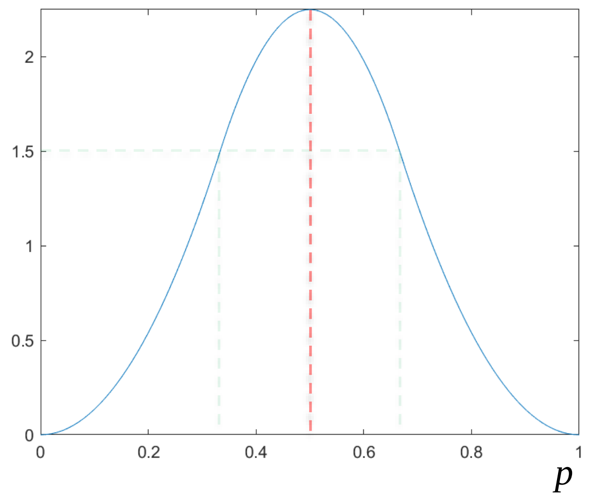
Arcsine U Shaped

$$f(p) = \frac{1}{\pi\sqrt{p(1-p)}}$$



3-Piece Quadratic

$$f(p) = \begin{cases} \frac{27}{2} p^2 & 0 \le p \le \frac{1}{3} \\ -27(p-\frac{1}{2})^2 + \frac{9}{4} & \frac{1}{3} \le p \le \frac{2}{3} \\ \frac{27}{2}(p-1)^2 & \frac{2}{3} \le p \le 1 \end{cases}$$



Homework 9

$$f(x \mid p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$p \in [0,1]$$

5**. Is there a difference between using conjugate vs non-conjugate priors?

a) Are the two posterior PDFs and CDFs different? Make plots.

b) Are the posterior means and variances different?

c) Are the 95th percentiles different?

** For students that have had MSSC 6010 and 6020.