# Discrete Probability Mass Functions

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# Outline

## Probability Rules

## Binomial PMF

## Expectation of a RV

## Discussion

## Homework

# Probability Rules

## Properties

For a bivariate probability mass function (PMF) of $X$ and $Y$, $P(X,Y)$

1. $0 \leq P(X,Y) \leq 1$

2. $\sum_X \sum_Y P(X,Y) = 1$

3. $P(X) = \sum_Y P(X,Y)$

can swap $X$ and $Y$

4. $P(X,Y) = P(Y|X)P(X)$

$$P(Y|X) = \frac{P(X,Y)}{P(X)}$$

can swap $X$ and $Y$

5. $$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

can swap $X$ and $Y$

# Probability Rules

| Birth Gender | |
|---|---|
| $x$=0 | Female |
| $x$=1 | Male |

| Shirt Size | |
|---|---|
| $y$=0 | Small |
| $y$=1 | Medium |
| $y$=2 | Large |
| $y$=3 | X-Large |

## Example: Bivariate PMF

Consider two random variables of Marquette Undergraduate students, birth gender ($X$) and shirt size ($Y$).

| $P(X=x,Y=y)$ | | Shirt Size ($Y$) | | | |
|---|---|---|---|---|---|
| | | $y$=0 | $y$=1 | $y$=2 | $y$=3 |
| Birth Gender ($X$) | $x$=0 | 0.10 | 0.20 | 0.15 | 0.05 |
| | $x$=1 | 0.05 | 0.15 | 0.20 | 0.10 |

The probability of a randomly selected Marquette Undergraduate student being a male ($X$=1) that wears a large shirt ($Y$=2) is 0.20.

$$P(X=1,Y=2)=0.20$$

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1. $0 \le P(X,Y) \le 1$

   $0 \le P(X=x,Y=y) \le 1$

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2. $\sum_X \sum_Y P(X,Y)=1$

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$$\sum_{x=0}^{1} \sum_{y=0}^{3} P(X=x,Y=y)=1$$

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| | $x$=1 | 0.05 | 0.15 | 0.20 | 0.10 |

2. $\sum_X \sum_Y P(X,Y)=1$

$$\sum_{x=0}^{1} \sum_{y=0}^{3} P(X=x,Y=y)=1$$

$$\sum_{y=0}^{3} P(X=0,Y=y)+\sum_{y=0}^{3} P(X=1,Y=y)=1$$

# Probability Rules

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3. $P(Y)=\sum_X P(X,Y)$

$$P(Y=y)=\sum_{x=0}^{1} P(X=x,Y=y)$$

# Probability Rules

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3. $P(Y)=\sum_X P(X,Y)$

$$P(Y=y)=\sum_{x=0}^{1} P(X=x,Y=y)$$

$$P(Y=0)=\sum_{x=0}^{1} P(X=x,Y=0)$$

$$P(Y=0)=0.10+0.05=0.15$$

| Shirt Size | |
|---|---|
| $P(y=0)$ | 0.15 |
| $P(y=1)$ | 0.35 |
| $P(y=2)$ | 0.35 |
| $P(y=3)$ | 0.15 |

| Birth Gender | |
|---|---|
| $P(x=0)$ | 0.50 |
| $P(x=1)$ | 0.50 |

# Probability Rules

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|---|---|
| $x$=0 | Female |
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4. $$P(X\,|\,Y)=\frac{P(X,Y)}{P(Y)}$$

| Shirt Size | |
|---|---|
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| $P(x=0)$ | 0.50 |
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4. $$P(X\,|\,Y)=\frac{P(X,Y)}{P(Y)}$$

$$P(X=x\,|\,Y=y)=\frac{P(X=x,Y=y)}{P(Y=y)}$$

| Shirt Size | |
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| $P(y=0)$ | 0.15 |
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4. $$P(X|Y)=\frac{P(X,Y)}{P(Y)}$$

$$P(X=1|Y=2)=\frac{P(X=1,Y=2)}{P(Y=2)}$$

$$P(X=1|Y=2)=\frac{0.20}{0.35}=0.57$$

| Shirt Size | |
|---|---|
| $P(y=0)$ | 0.15 |
| $P(y=1)$ | 0.35 |
| $P(y=2)$ | 0.35 |
| $P(y=3)$ | 0.15 |

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|---|---|
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# Probability Rules

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5. $$P(Y|X)=\frac{P(X|Y)P(Y)}{P(X)}$$

$$P(Y=2|X=1)=\frac{P(X=1|Y=2)P(Y=2)}{P(X=1)}$$

| Shirt Size | |
|---|---|
| $P(y=0)$ | 0.15 |
| $P(y=1)$ | 0.35 |
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|---|---|
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| Birth Gender ($X$) | $x$=0 | 0.10 | 0.20 | 0.15 | 0.05 |
| | $x$=1 | 0.05 | 0.15 | 0.20 | 0.10 |

5. $$P(Y\mid X)=\frac{P(X\mid Y)P(Y)}{P(X)}$$

$$P(Y=2\mid X=1)=\frac{P(X=1\mid Y=2)P(Y=2)}{P(X=1)}$$

$$P(Y=2\mid X=1)=\frac{(0.57)(.35)}{0.50}=0.40$$

| Shirt Size | |
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| $P(y=0)$ | 0.15 |
| $P(y=1)$ | 0.35 |
| $P(y=2)$ | 0.35 |
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| Birth Gender | |
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| $P(x=0)$ | 0.50 |
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$P(X=1\mid Y=2)=0.57$

# Binomial PMF

Let's assume we have two independent events $E_1$ and $E_2$.

We know that $P(E_1 \text{ and } E_2)=P(E_1)P(E_2)$.

More generally, if we have $n$ independent events $E_1,\ldots,E_n$.

We know that $P(E_1 \text{ and } E_2 \ldots \text{ and } E_n) =P(E_1)P(E_2)\ldots P(E_n)$.

# Binomial PMF

Let's assume we are flipping a coin twice.

$E_1$=Head on first flip, $E_2$=Tail on second flip.

The probability of heads on any given flip is $p = P(H)$.

The probability of tails (not heads) on any given flip is $q = (1\text{-}p)$.

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Then $P(HT)=P(H)P(T)$ $=p(1\text{-}p)$.

Similarly $P(TH) = P(T)P(H)$ $= (1\text{-}p)p$.

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Then $P(HT)=P(H)P(T)$ $=p(1\text{-}p)$.

Similarly $P(TH) = P(T)P(H)$ $= (1\text{-}p)p$.

Let $x$ = # of heads in two flips of a coin.

$P(x{=}1) = P(HT)+P(TH)$

$= p(1\text{-}p)+(1\text{-}p)p = 2p(1\text{-}p)$.

2 ways to get one $H$ and one $T$
2 ways to get $x$=1 heads

# Binomial PMF

Bi means two like bicycle

An experiment with only two outcomes is called a Binomial exp.

Call one outcome *Success* and the other *Failure*.

Each performance of expt. is called a trial and are independent.

$$P(X=x\,|\,p)=\underbrace{\frac{n!}{x!(n-x)!}}_{\text{num}(x\text{ successes and }n\text{-}x\text{ failures})}\underbrace{p^{x}(1-p)^{n-x}}_{P(x\text{ successes and }n\text{-}x\text{ failures})}\qquad x=0,\ldots,n$$

Prob of exactly $x$ successes

$$\binom{n}{x}=\frac{n!}{x!(n-x)!}$$

$n$ = number of trials or times we repeat the experiment.

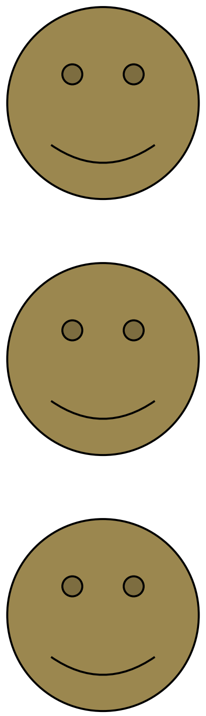
$x$ = the number of successes out of $n$ trials.

$p$ = the probability of success on an individual trial.

# Binomial PMF

Flip coin three times.

$p$=2/3 $x$= # of Heads $n(x)$= ways to get $x$ Heads



| $O$ | $P(O)$ |
|---|---|
| $HHH$ | $8/27$ |
| $HHT$ | $4/27$ |
| $HTH$ | $4/27$ |
| $HTT$ | $2/27$ |
| $THH$ | $4/27$ |
| $THT$ | $2/27$ |
| $TTH$ | $2/27$ |
| $TTT$ | $1/27$ |

$n$=3
$x$=1
$p$=2/3

| $x$ | $n(x)$ |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 3 |
| 3 | 1 |

| $x$ | $P(x)$ |
|---|---|
| 0 | $1/27$ |
| 1 | $6/27$ |
| 2 | $12/27$ |
| 3 | $8/27$ |

$$P(x\,|\,p)=\frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}$$

$$P(1)=\frac{3!}{1!(3-1)!}(2/3)^1(1-2/3)^{3-1}$$

$$P(1)=6/27$$

# Binomial PMF

The Binomial PMF can equivalently be represented as.

$$P(X_1 = x_1, X_2 = x_2 \mid p_1, p_2) = \frac{n!}{x_1! x_2!} p_1^{x_1} p_2^{x_2}$$

for the probability of exactly $x_1$ of outcome 1 and $x_2$ of outcome 2 in $n$ trials where the probability of outcome 1 is $p_1$ and of outcome 2 is $p_2$ where $x_1+x_2=n$ and $p_1+p_2=1$.

If we define vectors $x=(x_1,x_2)'$ and $p=(p_1,p_2)'$ then

$$P(x \mid p) = \frac{n!}{x_1! x_2!} p_1^{x_1} p_2^{x_2}$$

# Binomial PMF

A generalization of the binomial PMF is the Multinomial PMF where there are $K$ different possibilities with $p_k$ and $x_k$ being the probability and count for the $k^{\text{th}}$ category, $k=1,\ldots,K$.

$$P(X_1 = x_1,\ldots,X_K = x_K \mid p_1, p_2,\ldots, p_K) = \frac{n!}{x_1!x_2!\cdots x_K!} p_1^{x_1} p_2^{x_2} \ldots p_K^{x_K}$$

Define vectors $x=(x_1,x_{2,},\ldots,x_K)'$ and $p=(p_1,p_{2\,,},\ldots,p_K)'$ then

$$P(x \mid p) = \frac{n!}{\prod_{k=1}^{K} x_k!} \prod_{k=1}^{K} p_k^{x_k}$$

where $x_1+x_2\ldots+x_K=n$ and $p_1+p_2\ldots+p_K=1$.

# Expectation of RV

## PMF

$$E(Y)=\sum_{Y} YP(Y)$$

$$\text{var}(Y)=\sum_{Y}(Y-E(Y))^{2}P(Y)$$

## Conditional PMF

$$E(Y\,|\,X)=\sum_{Y} YP(Y\,|\,X)$$

$$\text{var}(Y\,|\,X)=\sum_{Y}(Y-E(Y\,|\,X))^{2}P(Y\,|\,X)$$

# Expectation of RV

## PMF

$$E(Y) = \sum_{y=0}^{3} yP(Y = y)$$
$$= 0(0.15) + 1(0.35) + 2(0.35) + 3(0.15)$$
$$= 1.50$$

| Shirt Size | |
|---|---|
| $P(y=0)$ | 0.15 |
| $P(y=1)$ | 0.35 |
| $P(y=2)$ | 0.35 |
| $P(y=3)$ | 0.15 |

$$\text{var}(Y) = \sum_{y=0}^{3} (y - E(Y))^2 P(Y = y)$$

$$\text{var}(Y) = (0-1.5)^2(0.15) + (1-1.5)^2(0.35)$$
$$+(2-1.5)^2(0.35) + (3-1.5)^2(0.15)$$
$$= 0.85$$

# Expectation of RV

# Conditional PMF

$$E(Y|X=1)=\sum_{y=0}^{3} yP(Y=y|X=1)$$
$$=0(0.10)+1(0.30)+2(0.4)+3(0.20)$$
$$=1.70$$

$$\text{var}(Y|X=1)=\sum_{y=0}^{3}(y-E(Y=y|X=1))^2 P(Y=y|X=1)$$
$$=0(0-1.7)^2(0.10)+(1-1.7)^2(0.30)$$
$$+(2-1.7)^2(0.4)+(3-1.7)^2(0.20)$$
$$=0.52$$

| $P(Y=y\|X=x)$ | | Shirt Size ($Y$) | | | |
|---|---|---|---|---|---|
| | | $y$=0 | $y$=1 | $y$=2 | $y$=3 |
| Birth Gender ($X$) | $x$=0 | 0.20 | 0.40 | 0.30 | 0.10 |
| | $x$=1 | 0.10 | 0.30 | 0.40 | 0.20 |

# Discussion

**Questions?**

# Homework 2

1. Use Matlab to generate 1000 observations from the Birth Gender ($X$) and Shirt Size ($Y$) bivariate PMF.

| $P(X=x,Y=y)$ | | Shirt Size ($Y$) | | | |
|---|---|---|---|---|---|
| | | $y$=0 | $y$=1 | $y$=2 | $y$=3 |
| **Birth Gender ($X$)** | $x$=0 | 0.10 | 0.20 | 0.15 | 0.05 |
| | $x$=1 | 0.05 | 0.15 | 0.20 | 0.10 |

a. Fill in a table of counts.

| $n(X=x,Y=y)$ | | Shirt Size ($Y$) | | | |
|---|---|---|---|---|---|
| | | $y$=0 | $y$=1 | $y$=2 | $y$=3 |
| **Birth Gender ($X$)** | $x$=0 | | | | |
| | $x$=1 | | | | |

b. Estimate the bivariate PMF.

| $\hat{P}(X=x,Y=y)$ | | Shirt Size ($Y$) | | | |
|---|---|---|---|---|---|
| | | $y$=0 | $y$=1 | $y$=2 | $y$=3 |
| **Birth Gender ($X$)** | $x$=0 | | | | |
| | $x$=1 | | | | |

c. Estimate $P(Y=2|X=1)$.

d. Estimate $E(Y)$ and $E(Y|X=1)$, $y=0,1,2,3$.

# Homework 2

2. Use Matlab to generate 1000 observations from the Binomial PMF with $n=2$ and $p=2/3$.

a. Fill in a table of counts.

| $x$ | $x=0$ | $x=1$ | $x=2$ |
|---|---|---|---|
| $n(X=x)$ | | | |

b. Estimate the univariate PMF.

| $x$ | $x=0$ | $x=1$ | $x=2$ |
|---|---|---|---|
| $P(X=x)$ | | | |

c. Estimate $E(X)$.