Machine Vision Review

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Outline

Lecture 01: Matlab
Lecture 02: Within Image Processing
Lecture 03: Image Filter Design
Lecture 04: Statistical Implications
Lecture 05: The Correlation Coefficient
Lecture 06: Pixel Statistics & Template Matching
Lecture 07: Through Image Processing
Lecture 08: The Discrete Fourier Transform
Lecture 09: Convolution via the DFT
Lecture 10: Fast Object Tracking
Lecture 11: Peaks, Valleys, and Ridges

Discussion

Introduction to Matlab

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Using Matlab

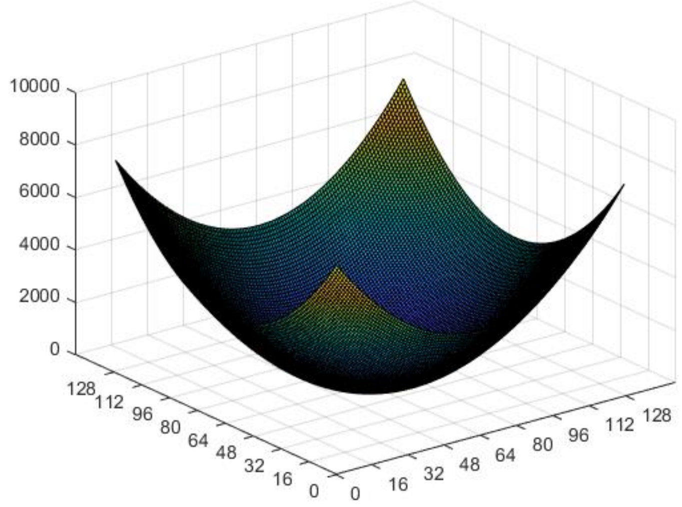
Some students were using Matlab for the first time.

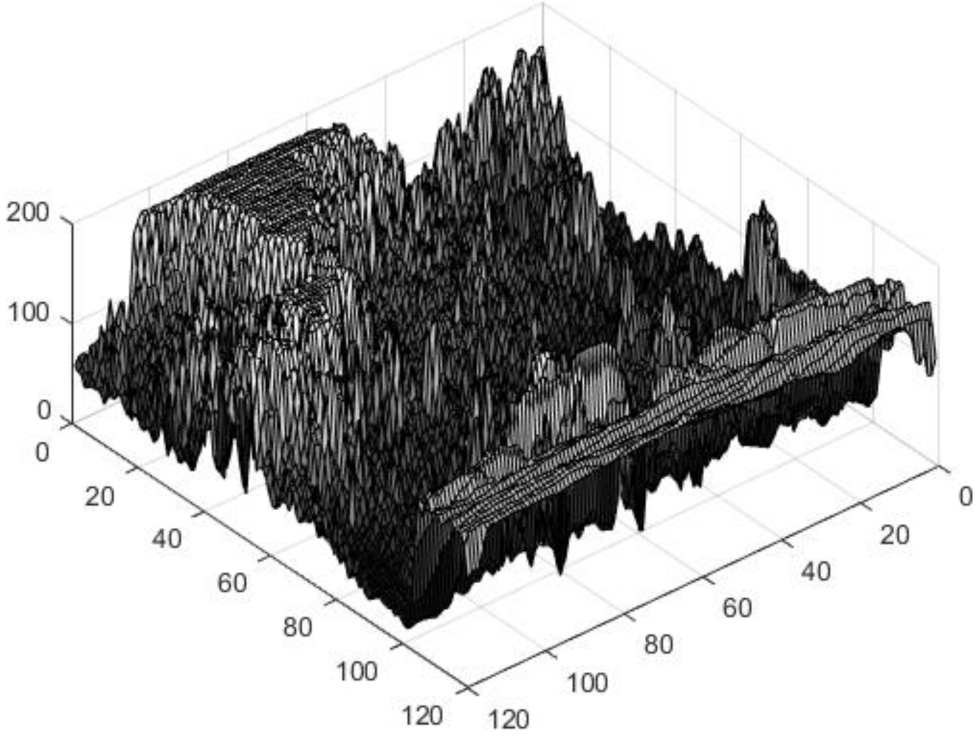
Installing Matlab

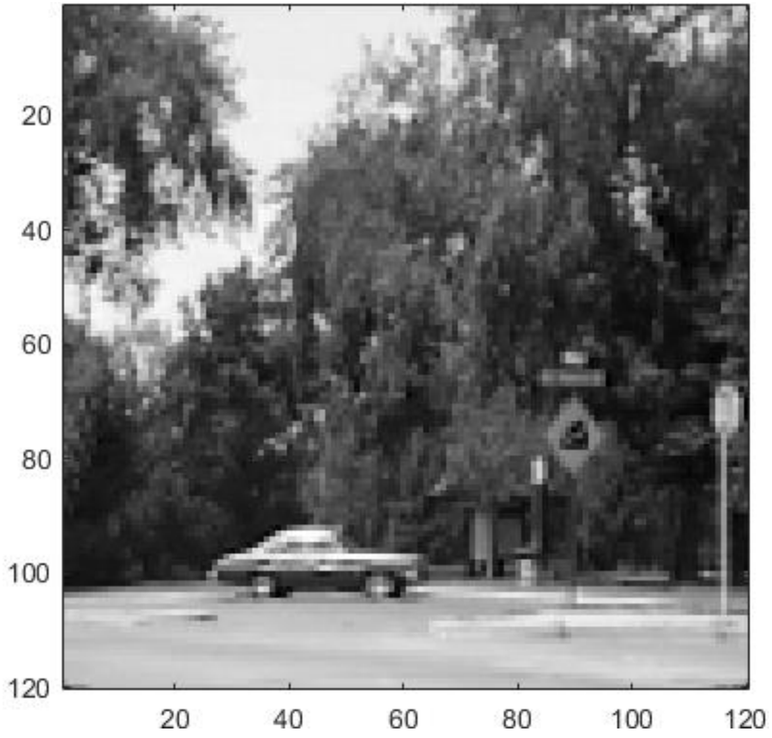
Using Matlab

Saving/Loading from/into Matlab

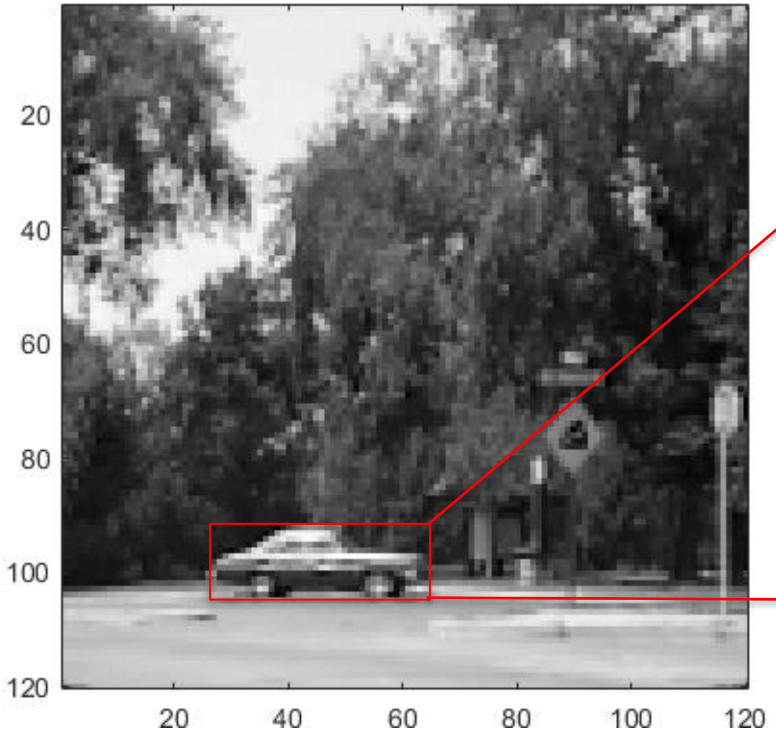
Functions in Matlab

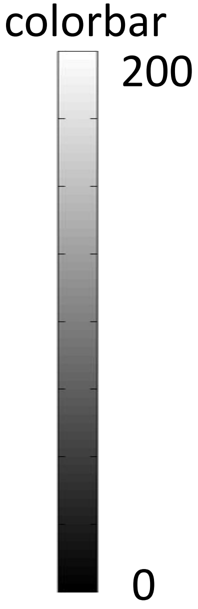






Using Matlab





38	37	31	32	28	26	25	27	24	13	35	126	174	194	200	200	200	199	197	194	180	132	94	45	35	41	43	62	39	33	50	70	42	45	44	43	45	38
33	29	30	33	32	26	28	21	16	86	157	171	186	168	170	181	193	196	188	162	161	141	77	66	53	38	43	51	39	40	61	74	60	59	54	47	32	26
35	27	36	36	31	25	14	43	127	172	165	143	139	92	91	99	131	119	92	83	96	123	122	66	72	55	36	49	38	48	61	68	53	47	40	40	24	32
30	26	23	15	26	56	116	174	183	147	122	129	106	121	126	136	157	149	138	135	137	145	162	126	82	95	88	84	73	66	68	63	46	32	31	40	27	27
21	38	98	152	178	179	173	180	184	182	179	185	190	189	180	182	180	185	186	182	192	197	197	198	198	198	197	197	197	193	189	183	169	154	141	127	84	50
60	135	144	124	114	109	101	90	58	59	116	123	105	112	114	116	133	147	128	92	116	122	101	84	83	98	101	105	133	139	136	130	133	132	125	91	68	60
89	95	94	98	97	97	93	94	124	142	139	124	123	111	121	123	122	109	131	172	177	150	133	125	128	131	111	79	75	69	72	77	82	72	73	87	58	63
124	41	47	49	49	50	28	16	31	36	31	45	51	51	49	49	52	47	31	22	27	40	46	46	40	58	40	19	55	64	44	29	57	83	135	123	45	43
93	34	38	40	40	38	28	60	104	95	45	31	45	44	43	44	44	44	46	45	45	46	45	44	44	45	34	66	99	113	74	37	29	35	84	100	51	43
105	84	69	61	45	42	37	71	90	102	68	31	40	40	39	38	40	40	42	40	39	39	40	40	39	38	32	83	99	113	90	74	22	24	56	69	78	92
119	122	120	123	126	127	108	107	175	159	61	18	24	17	39	46	50	45	47	51	56	51	32	30	29	27	43	88	134	170	140	84	15	46	135	154	178	169
146	137	137	139	99	99	89	57	94	86	27	9	21	75	121	122	124	122	122	121	128	135	135	133	124	117	99	68	96	119	74	17	15	70	94	91	100	107

```
car=fxy(93:104,27:64);
figure;
imagesc(car)
axis image, colormap(gray), axis off
print(gcf,'-dtiffn','-r100',['car'])

filename = 'cardata.xlsx';
writematrix(car,filename,'Sheet',1,'Range','A1')
```

Save into an excel spreadsheet!

Within Image Processing

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Convolution Kernel

Weights

w →

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3x3

$$\sum w_i = 1$$

There are different sizes and weightings to perform different functions.

weights

w_1	w_2	w_3	w_4	w_5
w_6	w_7	w_8	w_9	w_{10}
w_{11}	w_{12}	w_{13}	w_{14}	w_{15}
w_{16}	w_{17}	w_{18}	w_{19}	w_{20}
w_{21}	w_{22}	w_{23}	w_{24}	w_{25}

5x5 average

1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

/25

4 neighbor

0	0	0	0	0
0	0	1/5	0	0
0	1/5	1/5	1/5	0
0	0	1/5	0	0
0	0	0	0	0

Binomial

0	0	0	0	0
0	1/16	1/8	1/16	0
0	1/8	1/4	1/8	0
0	1/16	1/8	1/16	0
0	0	0	0	0

Gaussian

1	20	55	20	1
20	403	1097	403	20
55	1097	2981	1097	55
20	403	1097	403	20
1	20	55	20	1

/9365

Smoothing.

Convolution Kernel

Weights

$$w \longrightarrow \underset{3\text{x}3}{\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}} \qquad \sum w_i = 0$$

There are different sizes and weightings to perform different functions.

w_1	w_2	w_3	w_4	w_5
w_6	w_7	w_8	w_9	w_{10}
w_{11}	w_{12}	w_{13}	w_{14}	w_{15}
w_{16}	w_{17}	w_{18}	w_{19}	w_{20}
w_{21}	w_{22}	w_{23}	w_{24}	w_{25}

weights

0	0	0	0	0
0	-1	-1	-1	0
0	0	0	0	0
0	1	1	1	0
0	0	0	0	0

Gradient

0	0	0	0	0
0	-1	0	1	0
0	-2	0	2	0
0	-1	0	1	0
0	0	0	0	0

Sobel

0	0	0	0	0
0	0	1	0	0
0	1	-4	1	0
0	0	1	0	0
0	0	0	0	0

Laplacian

0	0	0	0	0
0	0	-1	-1	0
0	1	0	-1	0
0	1	1	0	0
0	0	0	0	0

Oblique Gradient

Sharpening.

Image Smoothing

Weights w

$$w = \frac{1}{5}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Pixel Values p

38	37	31	32	28	26	25	27	24	13	35	126	174	194	200	200	200	199	197	194	180	132	94	45	35	41	43	62	39	33	50	70	42	45	44	43	45	38
33	29	30	33	32	26	28	21	16	86	157	171	186	168	170	181	193	196	188	162	161	141	77	66	53	38	43	51	39	40	61	74	60	59	54	47	32	26
35	27	36	36	31	25	14	43	127	172	165	143	139	92	91	99	131	119	92	83	96	123	122	66	72	55	36	49	38	48	61	68	53	47	40	40	24	32
30	26	23	15	26	56	116	174	183	147	122	129	106	121	126	136	157	149	138	135	137	145	162	126	82	95	88	84	73	66	68	63	46	32	31	40	27	27
21	38	98	152	178	179	173	180	184	182	179	185	190	189	180	182	180	185	186	182	192	197	197	198	198	198	197	197	197	193	189	183	169	154	141	127	84	50
60	135	144	124	114	109	101	90	58	59	116	123	105	112	114	116	133	147	128	92	116	122	101	84	83	98	101	105	133	139	136	130	133	132	125	91	68	60
89	95	94	98	97	97	93	94	124	142	139	124	123	111	121	123	122	109	131	172	177	150	133	125	128	131	111	79	75	69	72	77	82	72	73	87	58	63
124	41	47	49	49	50	28	16	31	36	31	45	51	51	49	49	52	47	31	22	27	40	46	46	40	58	40	19	55	64	44	29	57	83	135	123	45	43
93	34	38	40	40	38	28	60	104	95	45	31	45	44	43	44	44	44	46	45	45	46	45	44	44	45	34	66	99	113	74	37	29	35	84	100	51	43
105	84	69	61	45	42	37	71	90	102	68	31	40	40	39	38	40	40	42	40	39	39	40	40	39	38	32	83	99	113	90	74	22	24	56	69	78	92
119	122	120	123	126	127	108	107	175	159	61	18	24	17	39	46	50	45	47	51	56	51	32	30	29	27	43	88	134	170	140	84	15	46	135	154	178	169
146	137	137	139	99	99	89	57	94	86	27	9	21	75	121	122	124	122	122	121	128	135	135	133	124	117	99	68	96	119	74	17	15	70	94	91	100	107

200

0

New image

Within Image Processing

0	1/5	0
1/5	1/5	1/5
0	1/5	0

4 neighbor

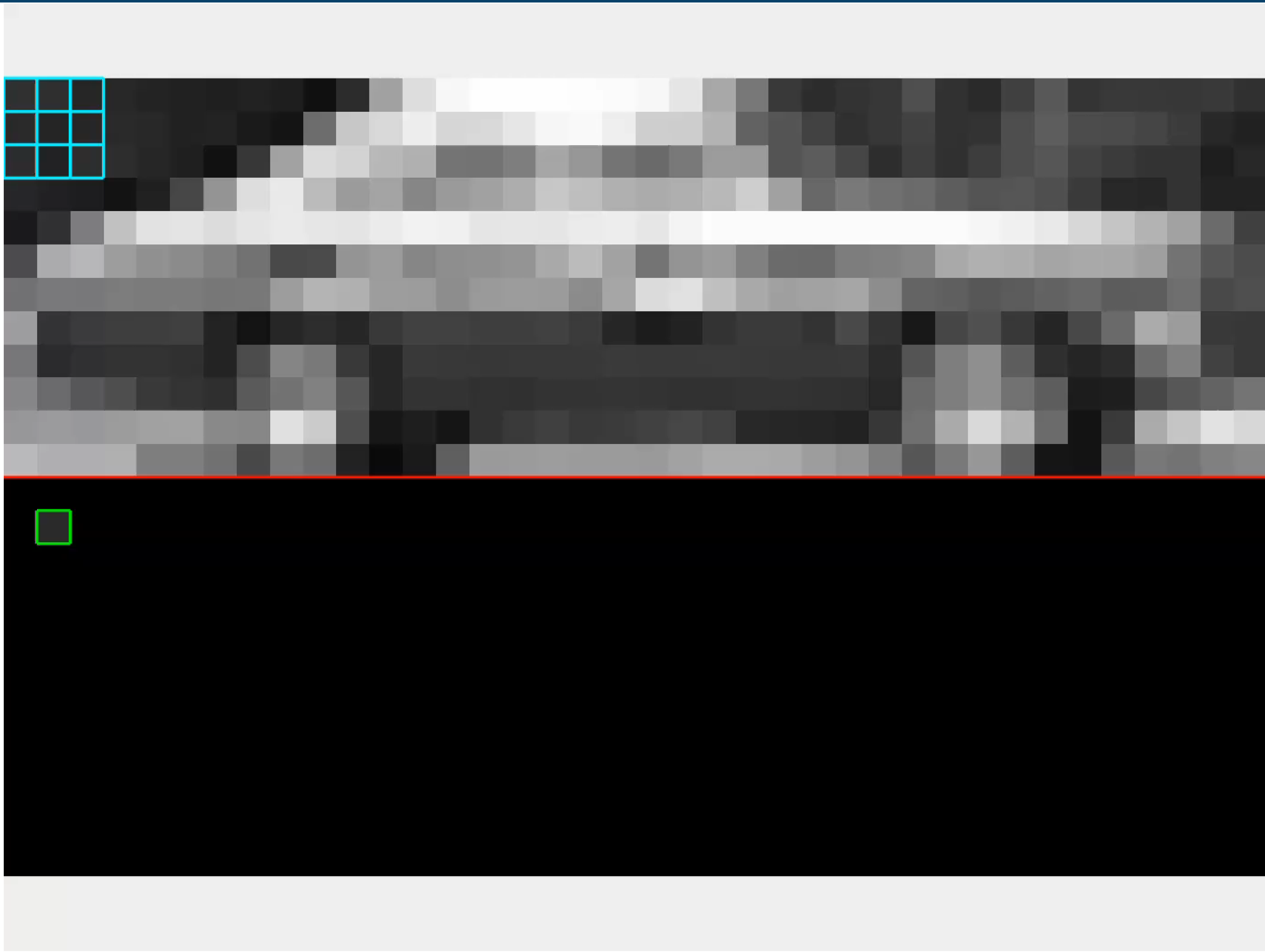

Image Filter Design

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Smoothing Filters

The zero mean Gaussian distribution with common variance σ^2 is

$$g(x,y)=e^{-\frac{x^2+y^2}{2\sigma^2}}.$$

(Note that we have neglected the normalizing constant.)

0.000335462627903

We can form a 5×5 Gaussian filter with σ^2=0.5.

Calculate the unnormalized weights.

.0003	.0067	.0183	.0067	.0003
.0067	.1353	.3679	.1353	.0067
.0183	.3679	1.0000	.3679	.0183
.0067	.1353	.3679	.1353	.0067
.0003	.0067	.0183	.0067	.0003

Divide all the values by the corners value.

Round to the nearest integer.

Divide by the sum of the integers.

1	20	55	20	1
20	403	1097	403	20
55	1097	2981	1097	55
20	403	1097	403	20
1	20	55	20	1

/9365

Image Smoothing

Gaussian

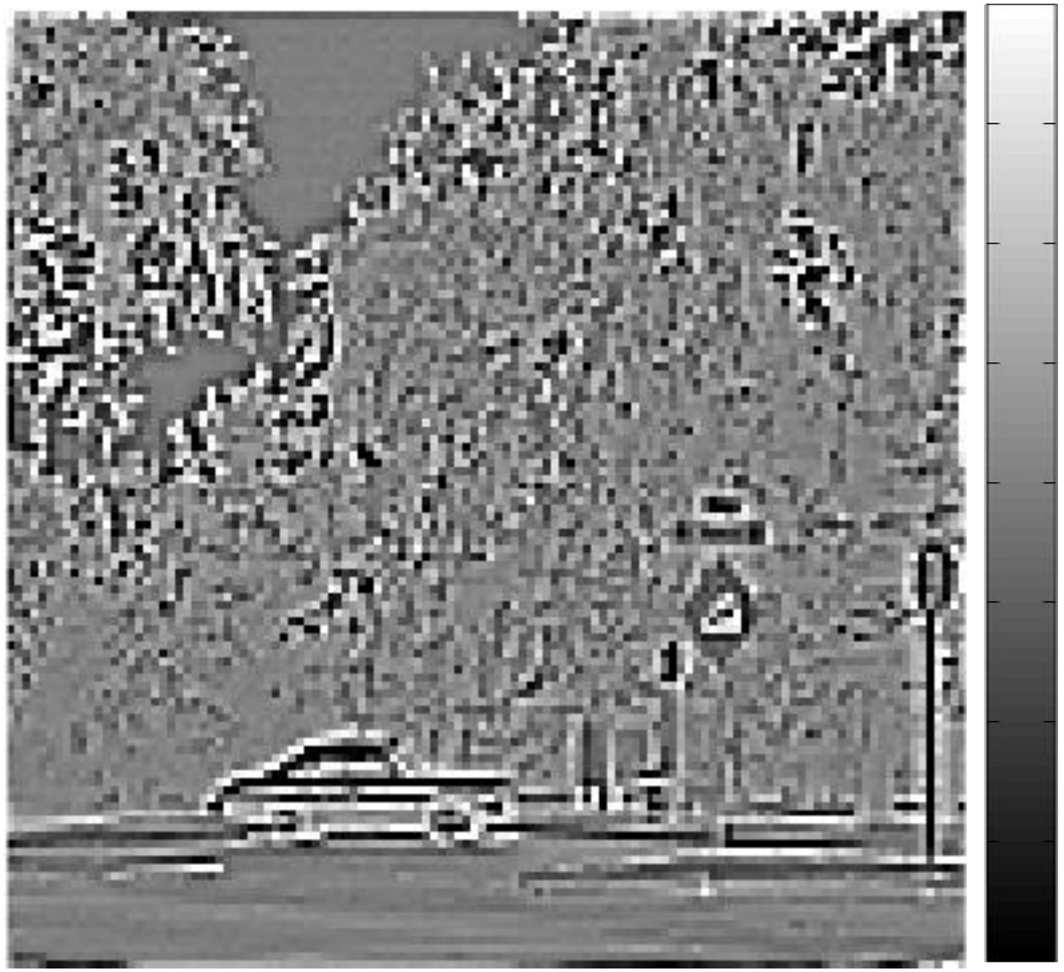
1	20	55	20	1
20	403	1097	403	20
55	1097	2981	1097	55
20	403	1097	403	20
1	20	55	20	1

/9365

Apply to whole image and examine the difference

Original

Smoothed



Difference

Sharpening Filters

The discrete version of the first derivatives $\frac{\partial}{\partial x} f(x,y)$ and $\frac{\partial}{\partial y} f(x,y)$

are $D_x = f(x,y) - f(x-1,y)$ and $D_y = f(x,y) - f(x,y-1)$ which in terms of kernels are

-1	1

and

-1
1

but may also be expressed as

-1	1
-1	1

and

-1	-1
1	1

or even with larger kernels.

Prewitt's 3×3 derivative kernels are

-1	-1	-1
0	0	0
1	1	1

and

-1	0	1
-1	0	1
-1	0	1

.

Derivative at center pixel.

Roberts cross gradient at 45° are

-1	0
0	1

and

0	-1
1	0

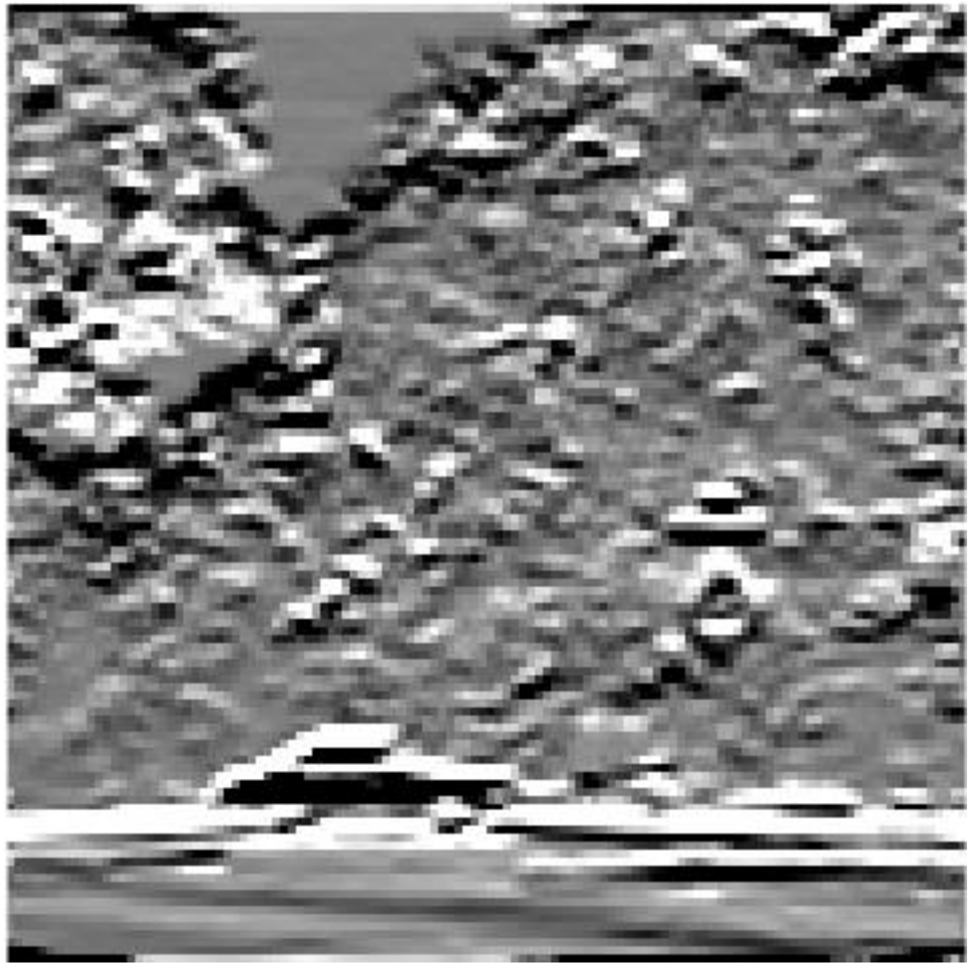
.

Sharpening Filters

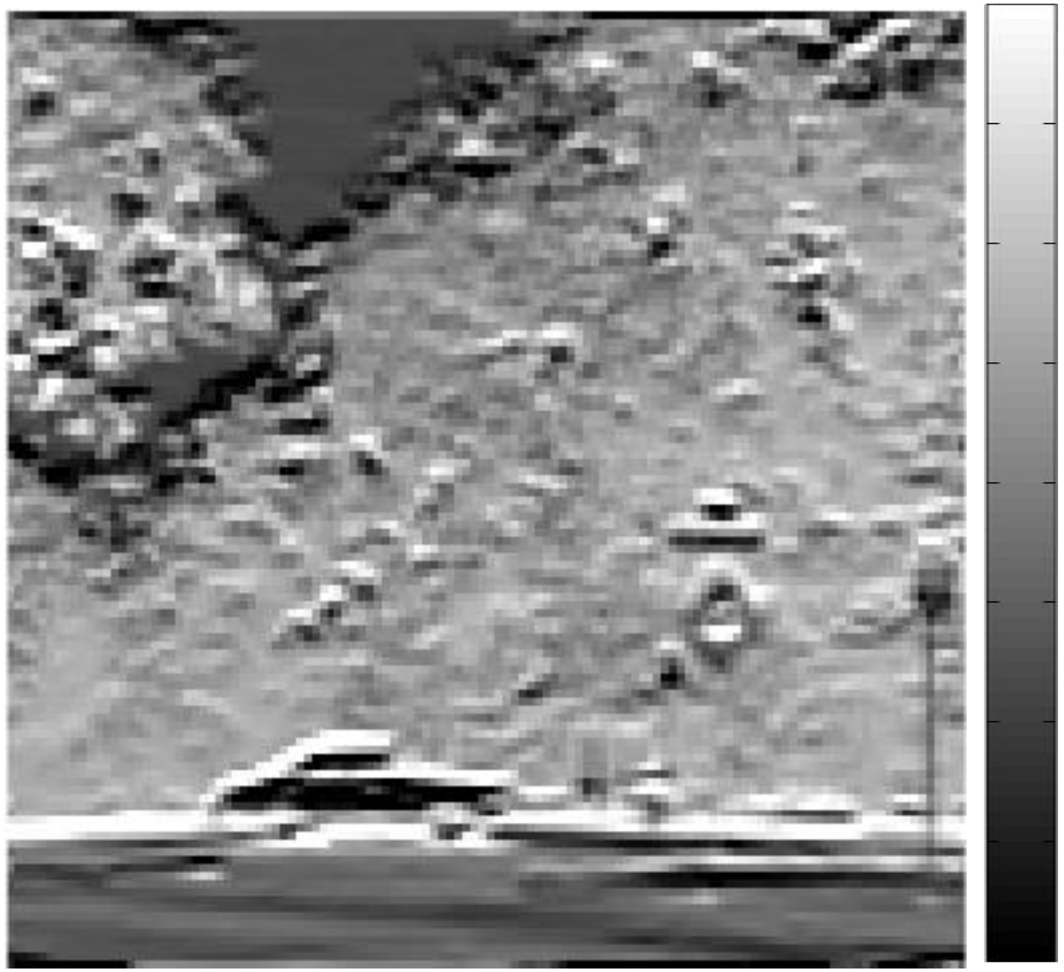
Derivative at center pixel.

-1	-1	-1
0	0	0
1	1	1

Original



U-D Gradient



Difference

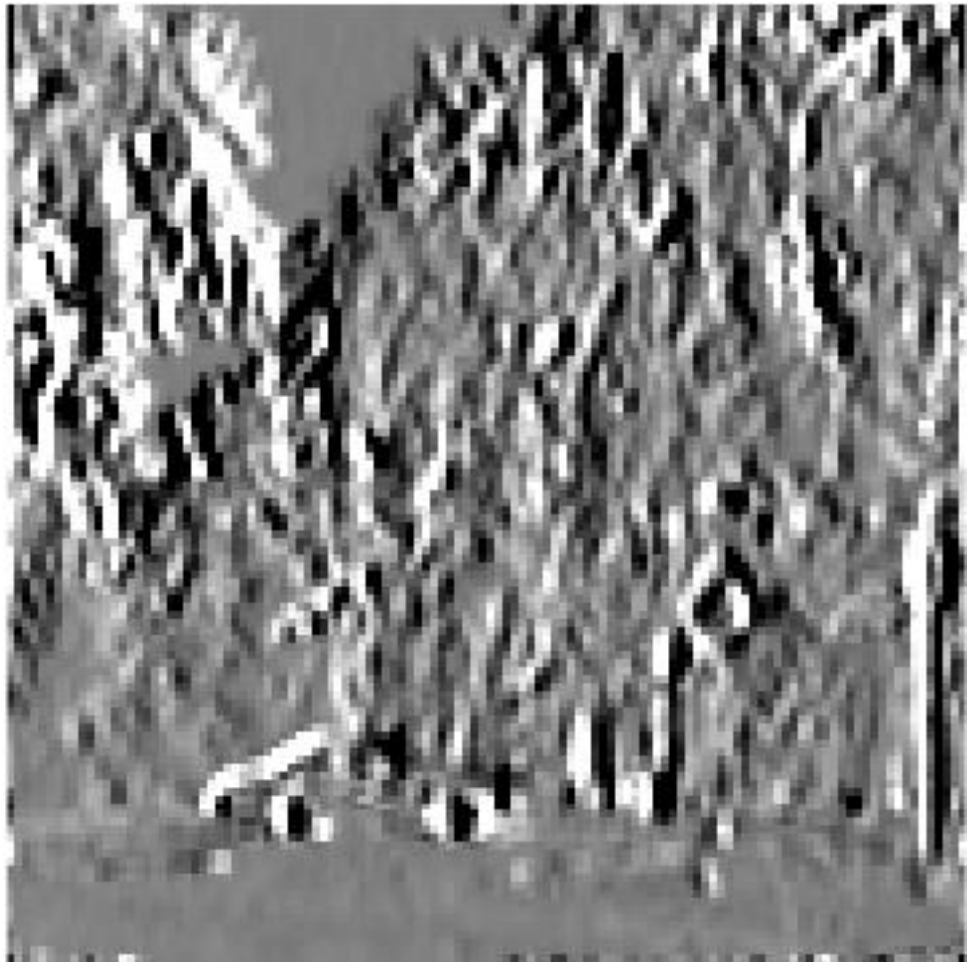
Sharpening Filters

Derivative at center pixel.

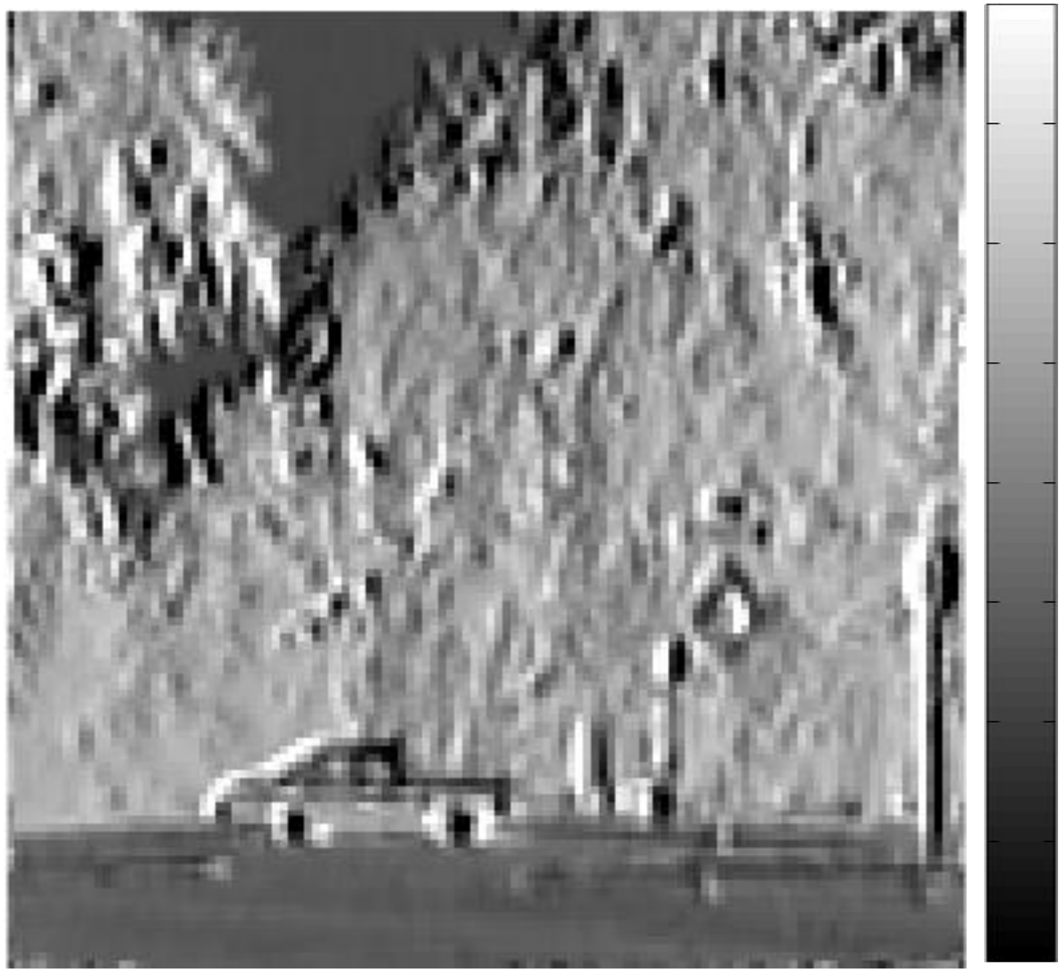
-1	0	1
-1	0	1
-1	0	1

200,
100,100

Original



L-R Gradient



Difference

0,
-100,-300

Hybrid Filters

We can boost the edges in an image with a “high boost” filter.

=

+

Original

Smoothed

High

This can be thought of as subtracting the low fom the original image.

Hybrid Filters

We add back part of the high to the original to boost the high.

=2

-

High Boost

Original

Low

HighBoost=A*(Original)-(Low)

HighBoost =255(HighBoost) /max(max(HighBoost)); %renormalize

Statistical Implications

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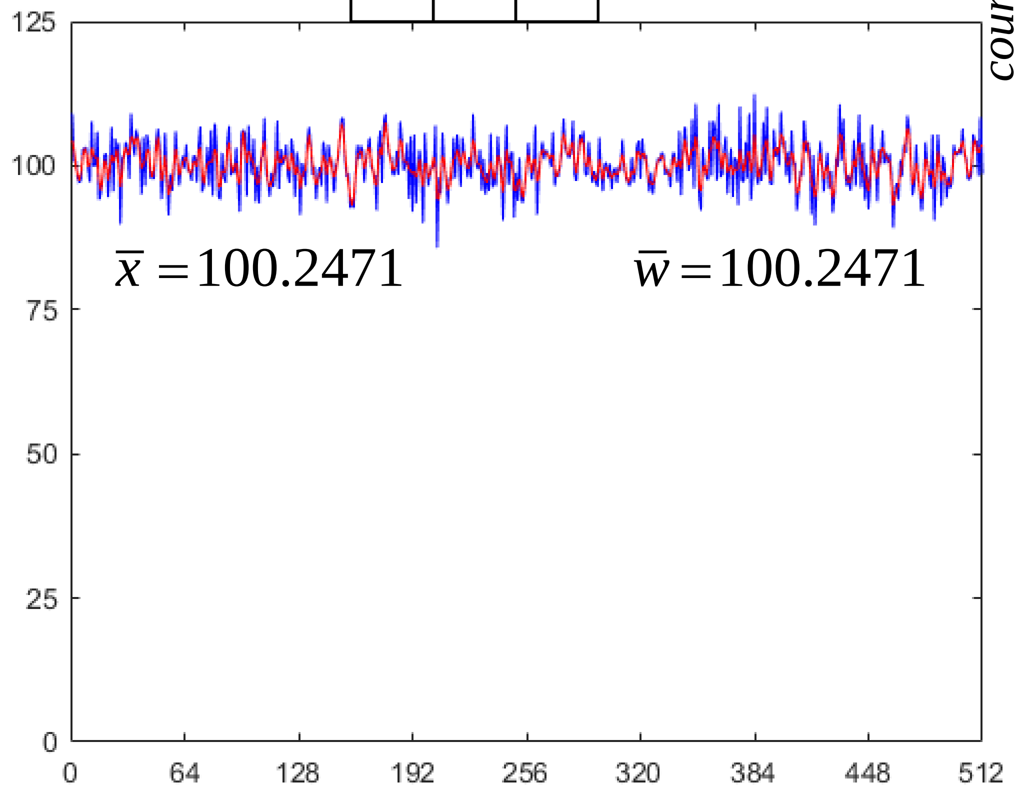
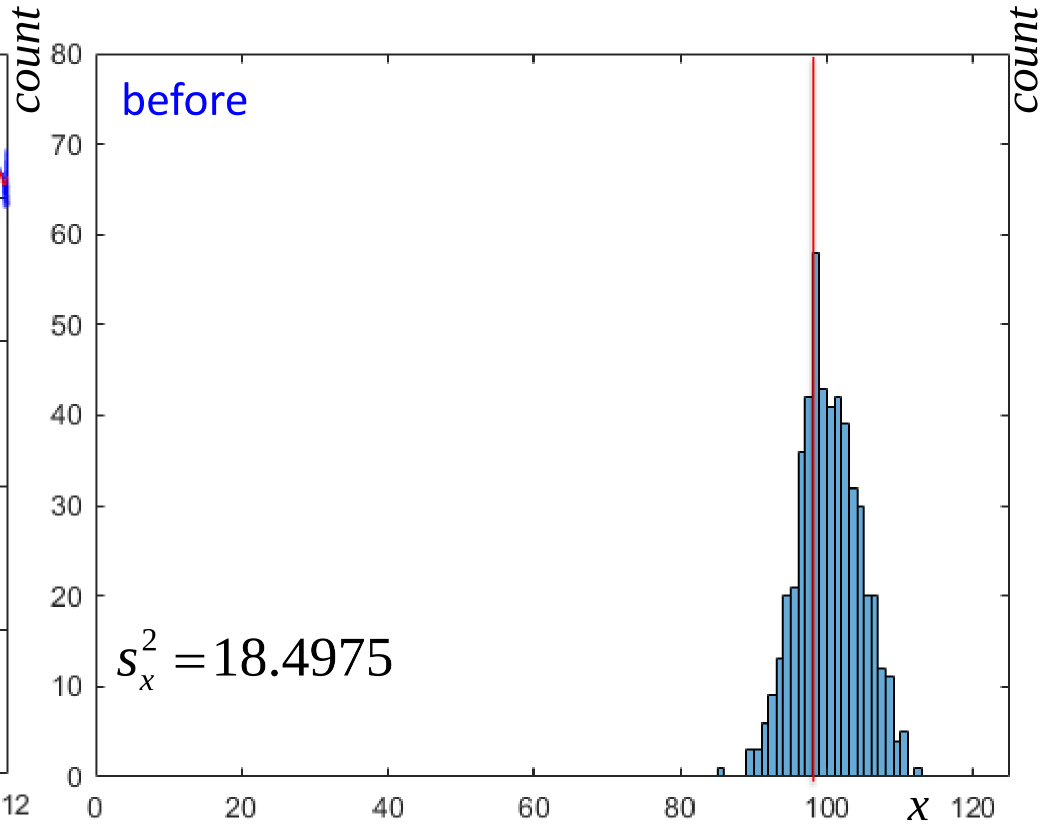
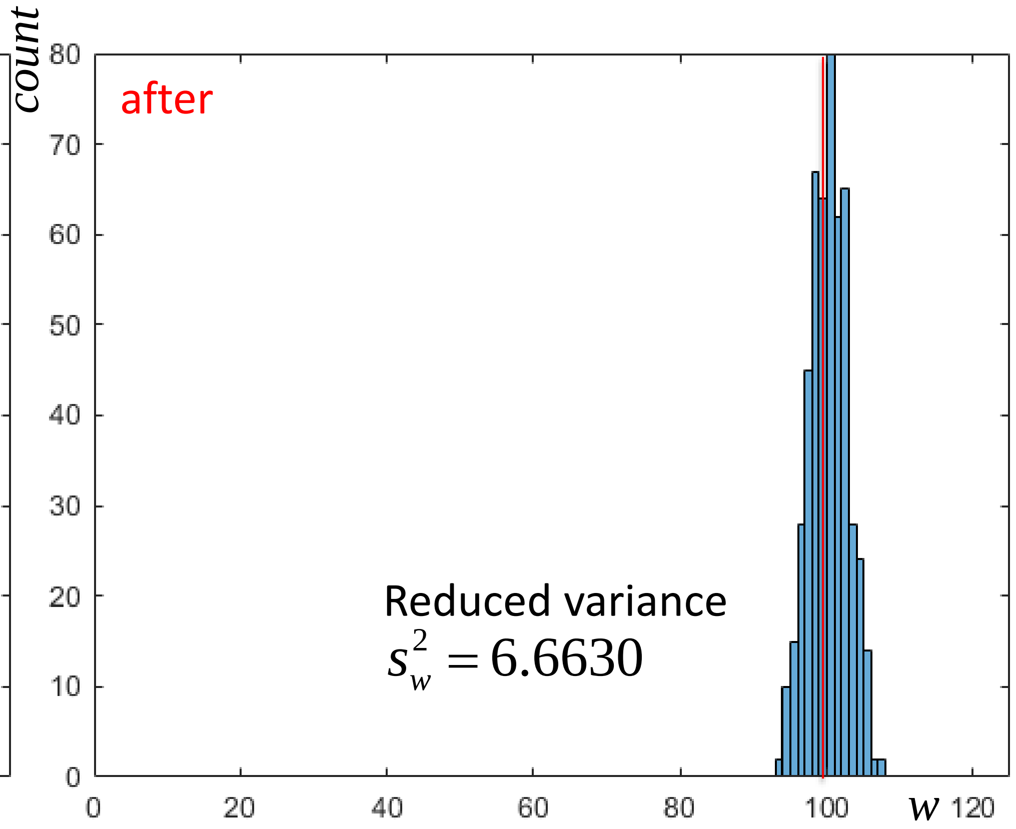
Introduction

If we were to smooth the time series (row of pixels) with say a 3 point binomial kernel, then what is the effect? Weighted average.

1/4	1/2	1/4



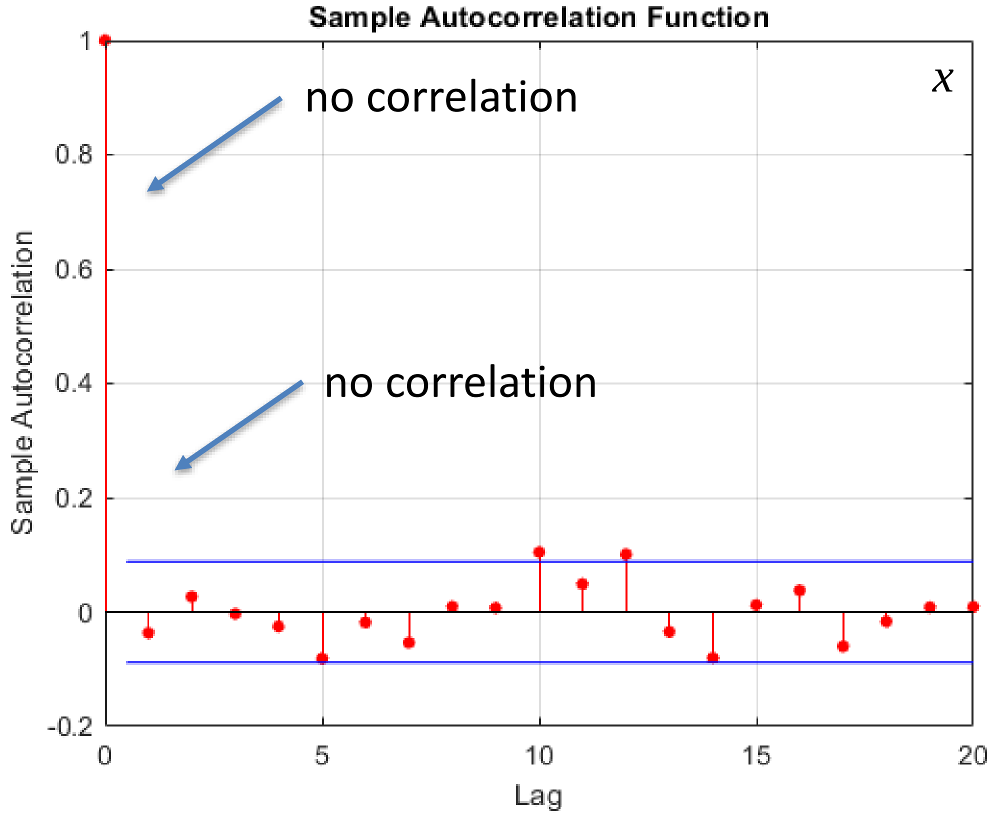


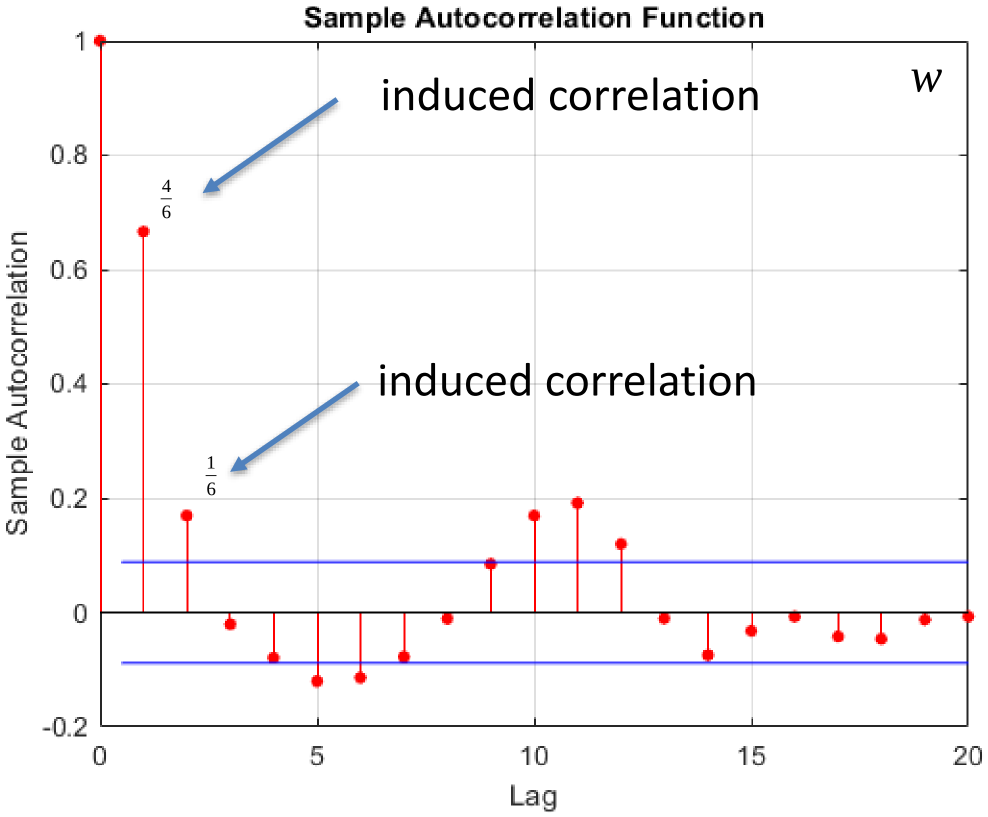


Reduced variance but induced temporal correlation.

Introduction

We can calculate the temporal autocorrelation and see induced correlation.





Make a copy of time series shifted by lag L with wrap around, then calculate The correlation between the two time series.

L	$corx$	$corw$
0	1.0000	1.0000
1	-0.0382	**0.6707**
2	0.0344	**0.1757**
3	-0.0049	-0.0162
4	-0.0210	-0.0768
5	-0.0791	-0.1179
6	-0.0147	-0.1125
7	-0.0577	-0.0785
8	0.0072	-0.0151
9	0.0022	0.0800
10	0.1061	0.1681
11	0.0519	0.1997
12	0.1088	0.1384
13	-0.0220	0.0073
14	-0.0805	-0.0661
15	0.0098	-0.0304
16	0.0428	-0.0082
17	-0.0609	-0.0446
18	-0.0187	-0.0516
19	0.0025	-0.0211
20	0.0056	-0.0209

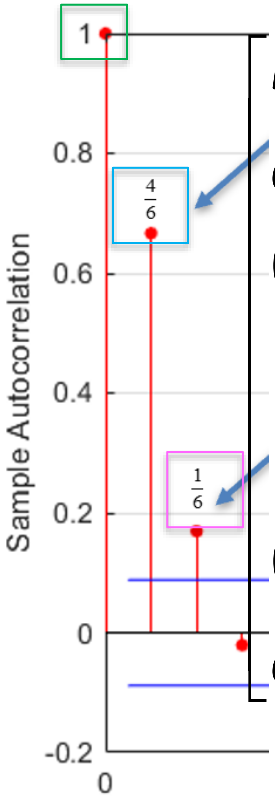
$\frac{4}{6}$ $\frac{1}{6}$

Let’s examine how and why does this happen?
This also happens in a 2D image!

Time Series Statistics

This slide assumes σ^2=1.

Because there is a two step correlation between pixels in the time series (row), we can determine the weights in the kernel used for convolution.



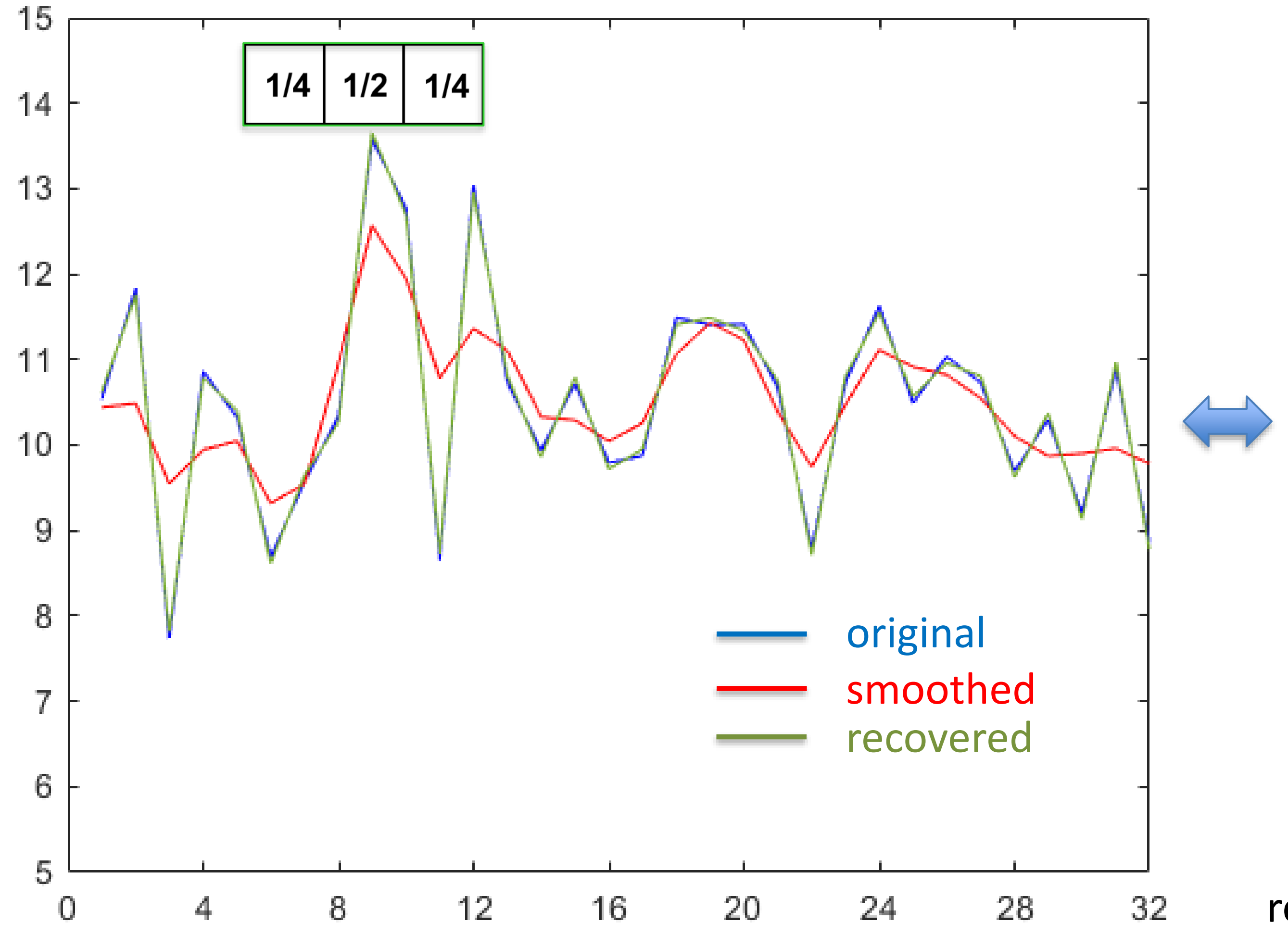
$$\underbrace{\begin{bmatrix} b & a & 0 & \cdots & \cdots & 0 & a \\ a & b & a & 0 & \cdots & \cdots & 0 \\ 0 & a & b & a & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & 0 & a & b & a & 0 \\ 0 & \vdots & \vdots & 0 & a & b & a \\ a & 0 & 0 & \cdots & 0 & a & b \end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix} b & a & 0 & \cdots & \cdots & 0 & a \\ a & b & a & 0 & \cdots & \cdots & 0 \\ 0 & a & b & a & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & 0 & a & b & a & 0 \\ 0 & \vdots & \vdots & 0 & a & b & a \\ a & 0 & 0 & \cdots & 0 & a & b \end{bmatrix}}_{A'} = \underbrace{\begin{bmatrix} b^2+2a^2 & 2ab & a^2 & 0 & \cdots & 0 & a^2 \\ 2ab & b^2+2a^2 & 2ab & a^2 & \cdots & \cdots & 0 \\ a^2 & 2ab & b^2+2a^2 & 2ab & a^2 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & a^2 & 2ab & b^2+2a^2 & 2ab & a^2 \\ a^2 & \vdots & \vdots & a^2 & 2ab & b^2+2a^2 & 2ab \\ 2ab & a^2 & 0 & \cdots & a^2 & 2ab & b^2+2a^2 \end{bmatrix}}_{AA'=cor(w)}$$

$\boxed{b^2+2a^2=1}$ $\boxed{2ab=\frac{2}{3}}$ $\boxed{a^2=\frac{1}{6}}$ $\longrightarrow$ $a=\frac{1}{\sqrt{6}}, b=\frac{\sqrt{6}}{3}$ mult by $\frac{\sqrt{6}}{4}$ for $a=\frac{1}{4}$, $b=\frac{1}{2}$.

Form $B=A^{-1}$ and deconvolve!

Time Series Statistics

Unsmoothing a time series with a deconvolution filter can be written as a matrix multiplication.



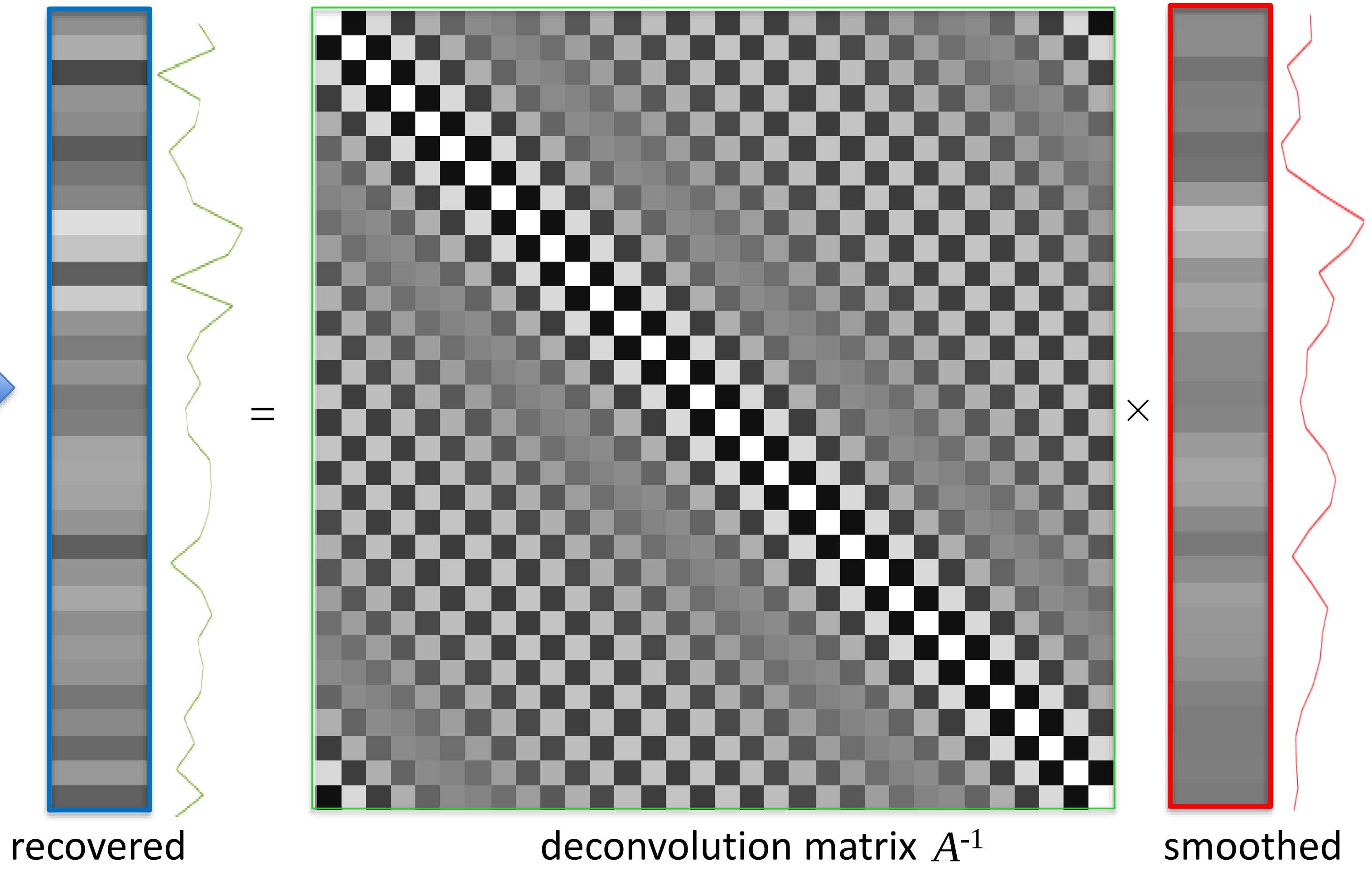


Image Statistics

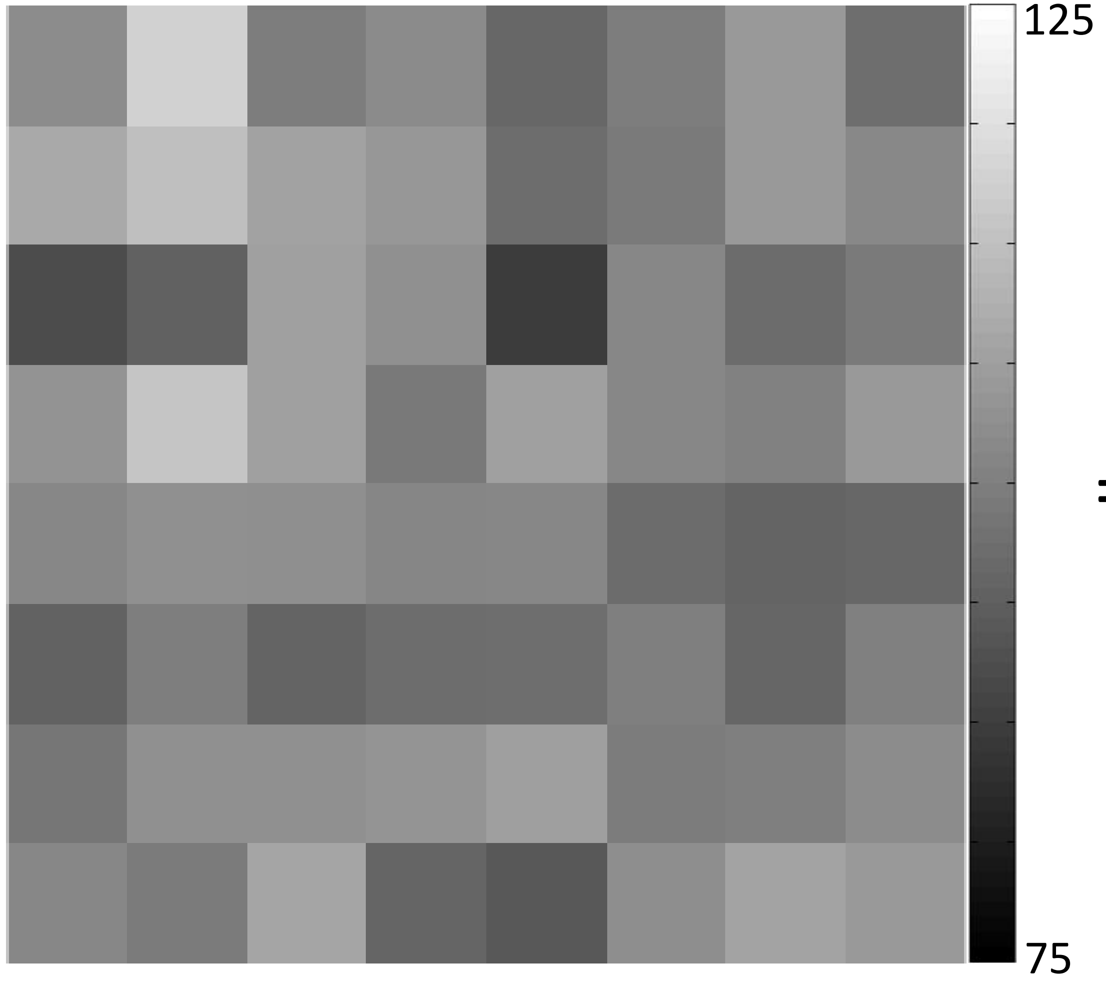
Imagine we have the below 8×8 noisy image and smooth with kernel.

0	1/8	0
1/8	1/2	1/8
0	1/8	0

Kernel

convolution

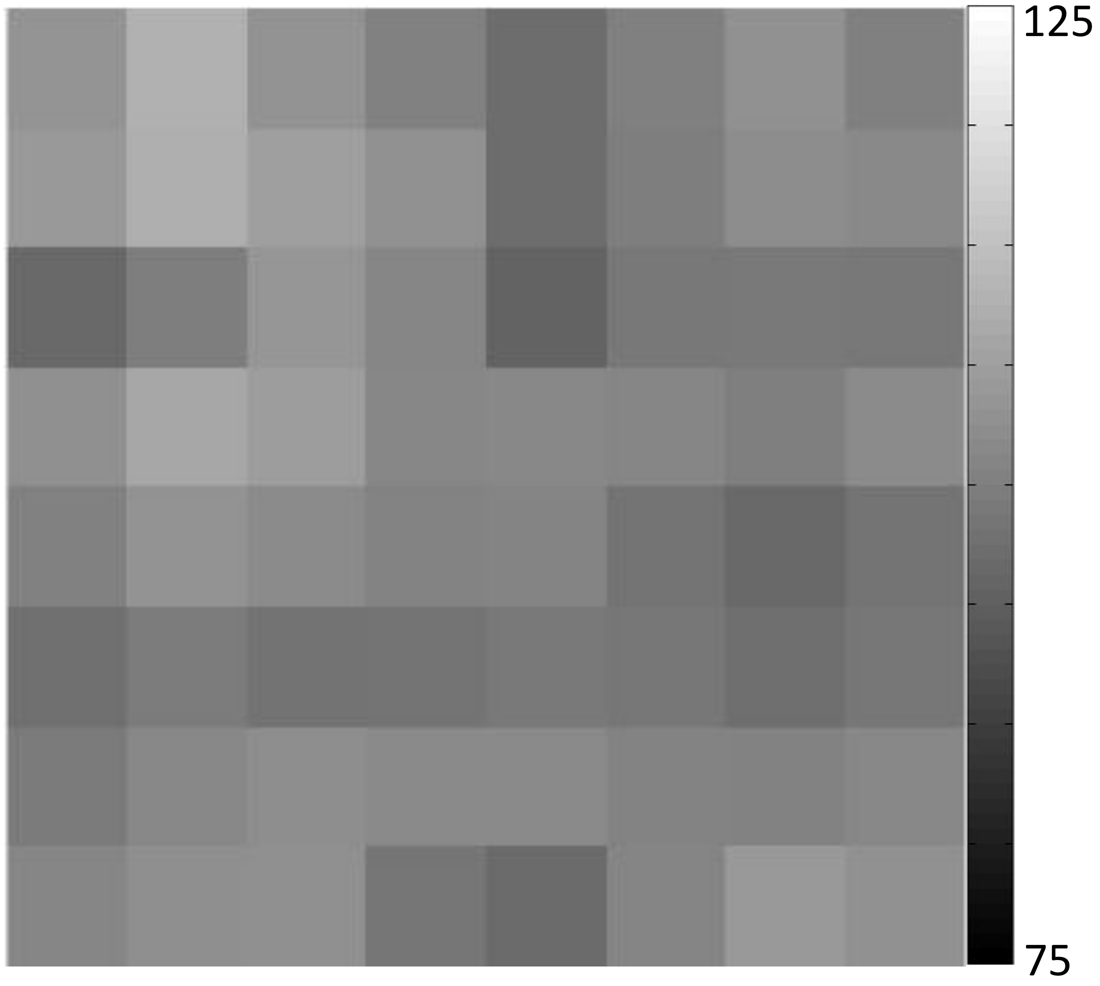
$*$



Input Image

$\bar{I} = 100.9576 \qquad s_I^2 = 28.4930$

=



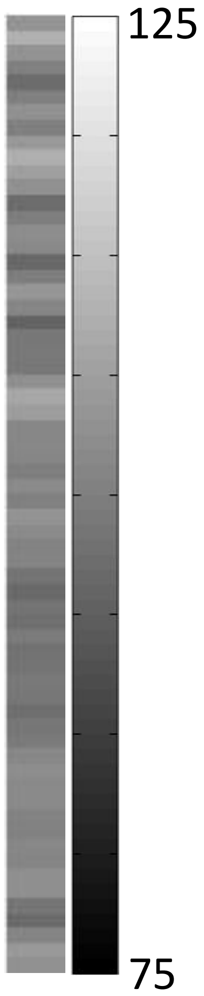
Output Image

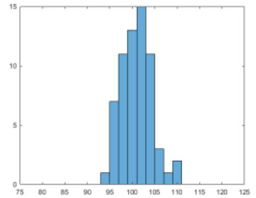
$\bar{O} = 100.9576 \qquad s_O^2 = 10.4009$

Image Statistics

0	1/8	0
1/8	1/2	1/8
0	1/8	0

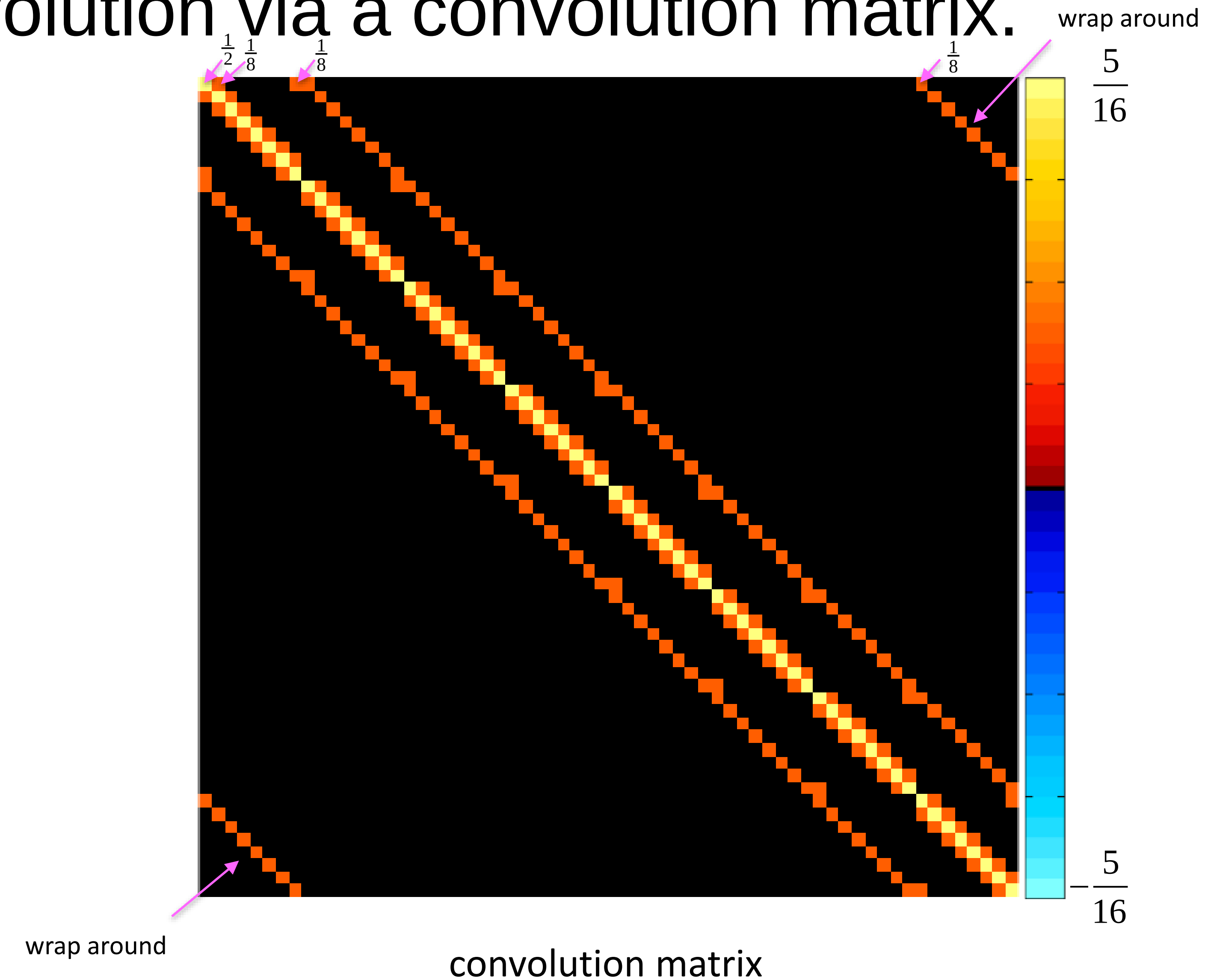
Then perform convolution via a convolution matrix.





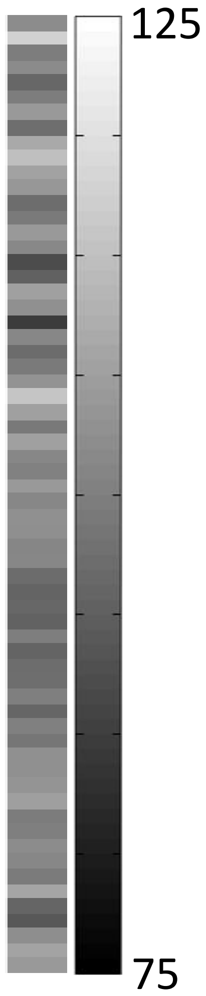
smoothed

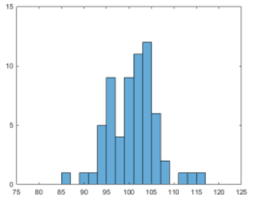
=



convolution matrix

$\times$



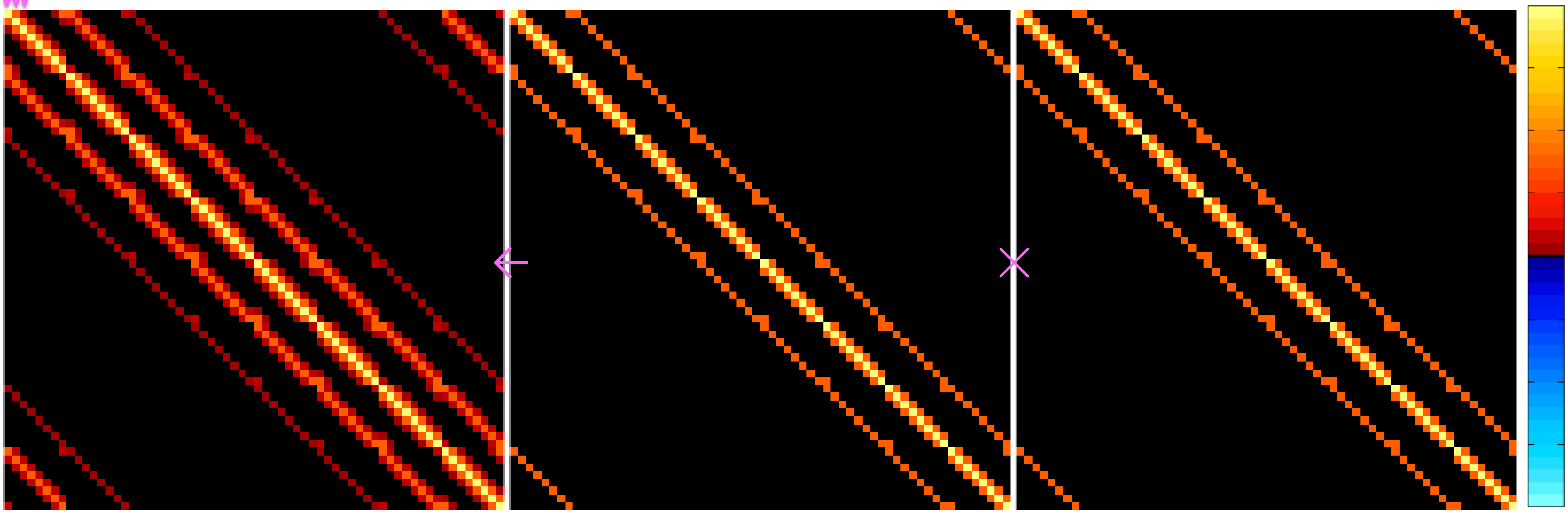


original

Image Statistics

0	1/8	0
1/8	1/2	1/8
0	1/8	0

We can calculate theoretically what the induced correlation is.

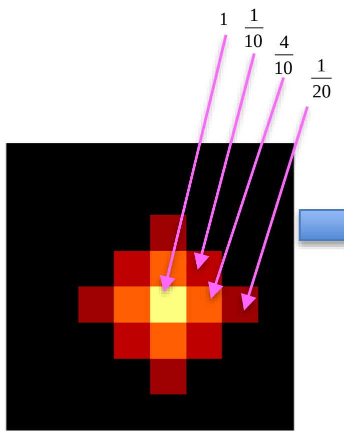


R correlation $\leftarrow$ A convolution matrix $\times$ A' convolution matrix’

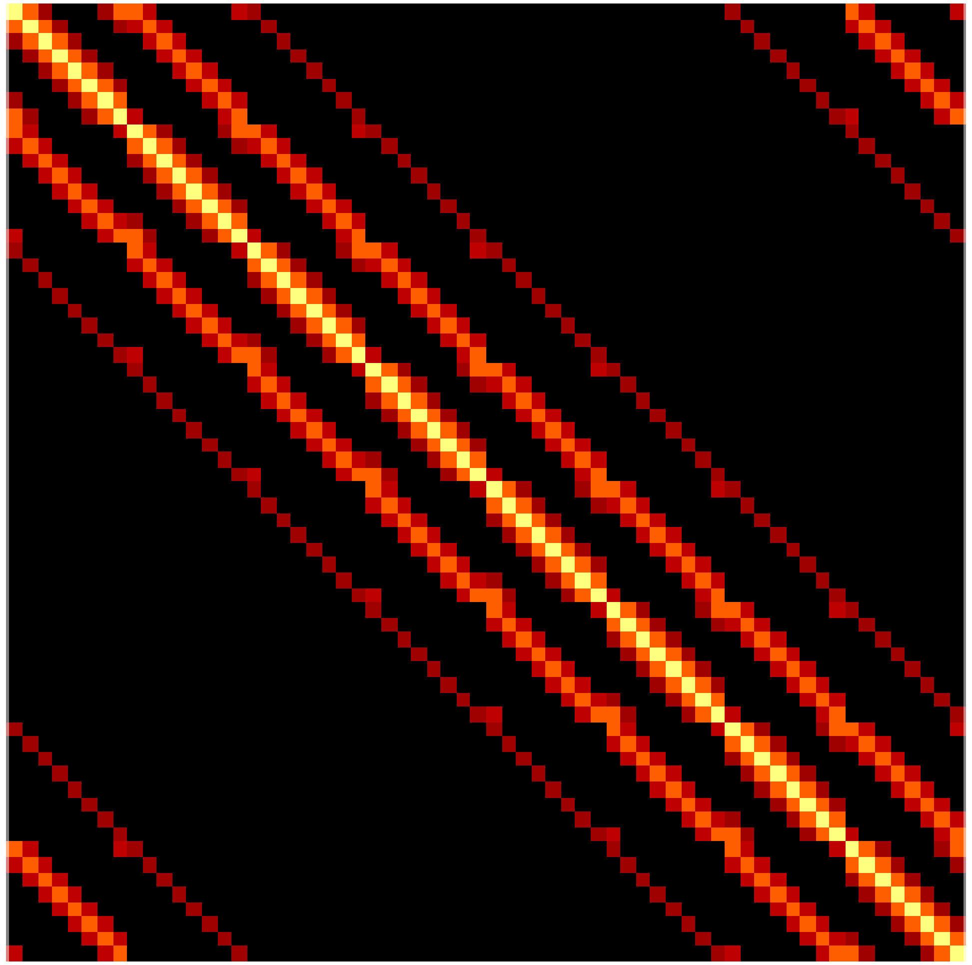
Image Statistics

0	1/8	0
1/8	1/2	1/8
0	1/8	0

If we had the induced correlation, we could calculate convolution matrix.

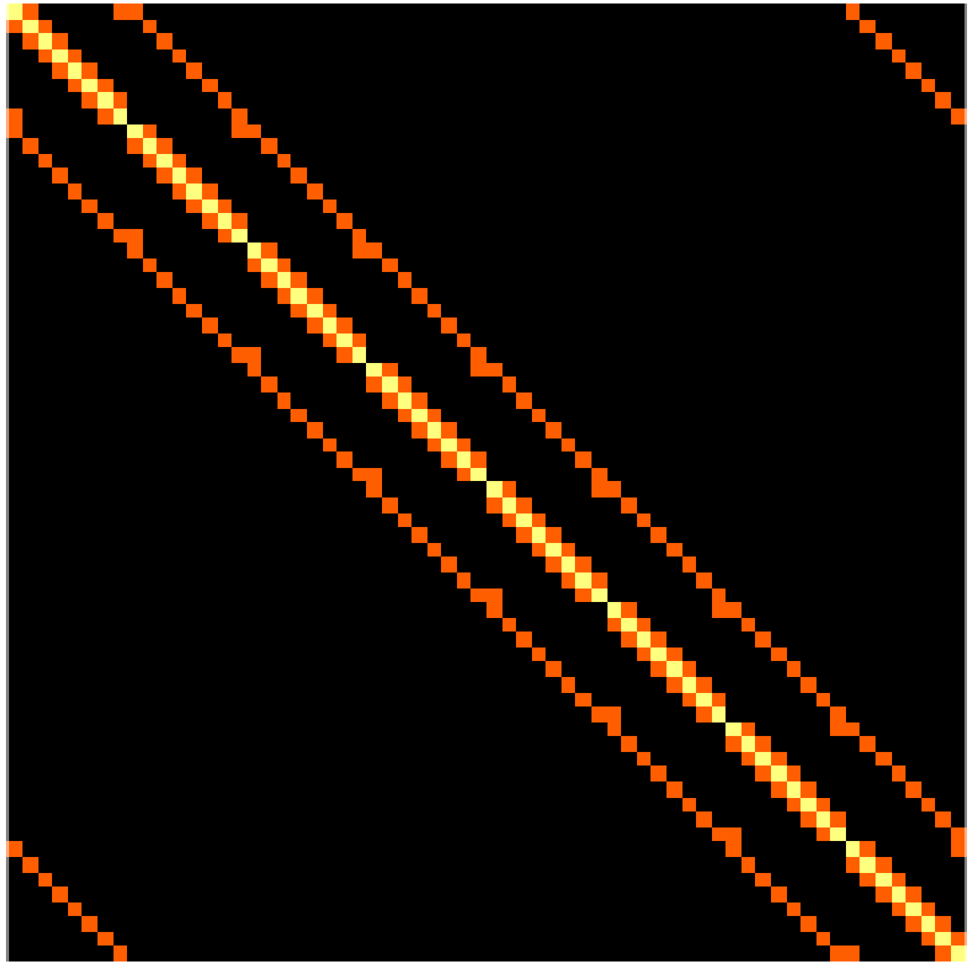


Correlation between (5,5) pixel and all others. Reshaped 37th row of R.



R

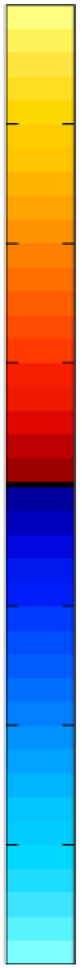
correlation



A

convolution matrix

1,5/16

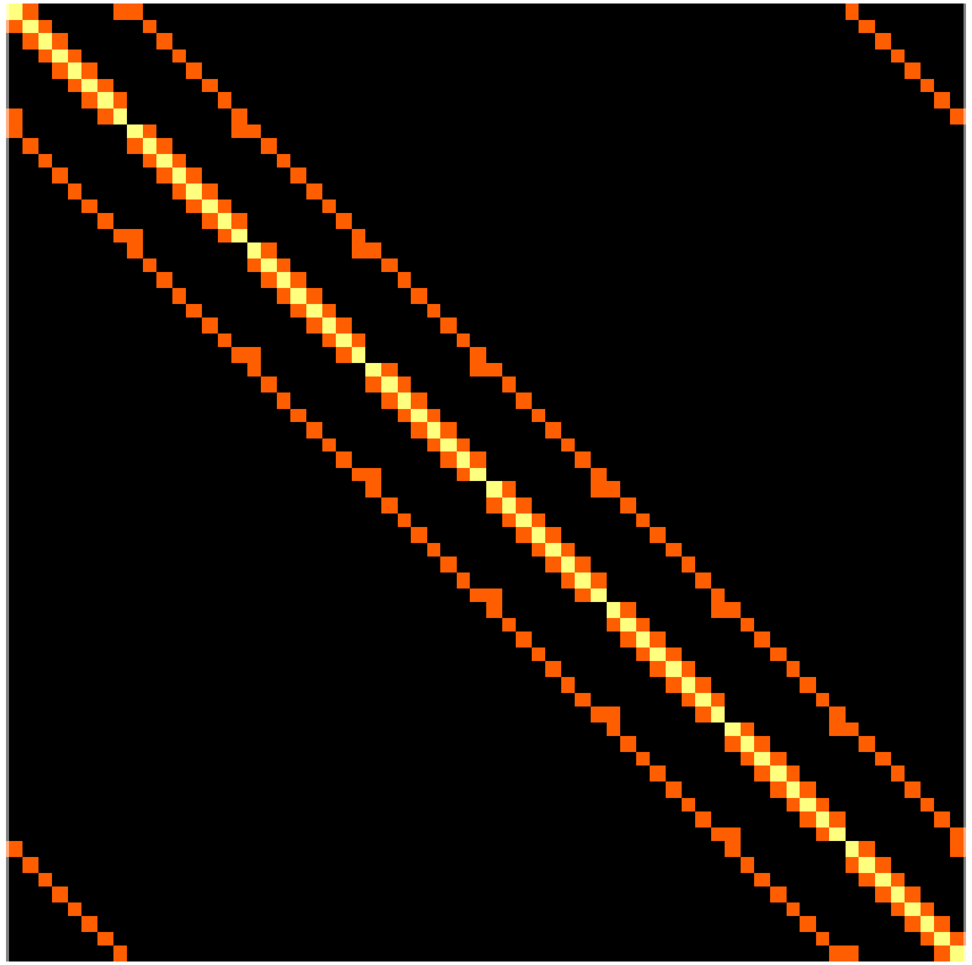


-1,-5/16

Image Statistics

0	1/8	0
1/8	1/2	1/8
0	1/8	0

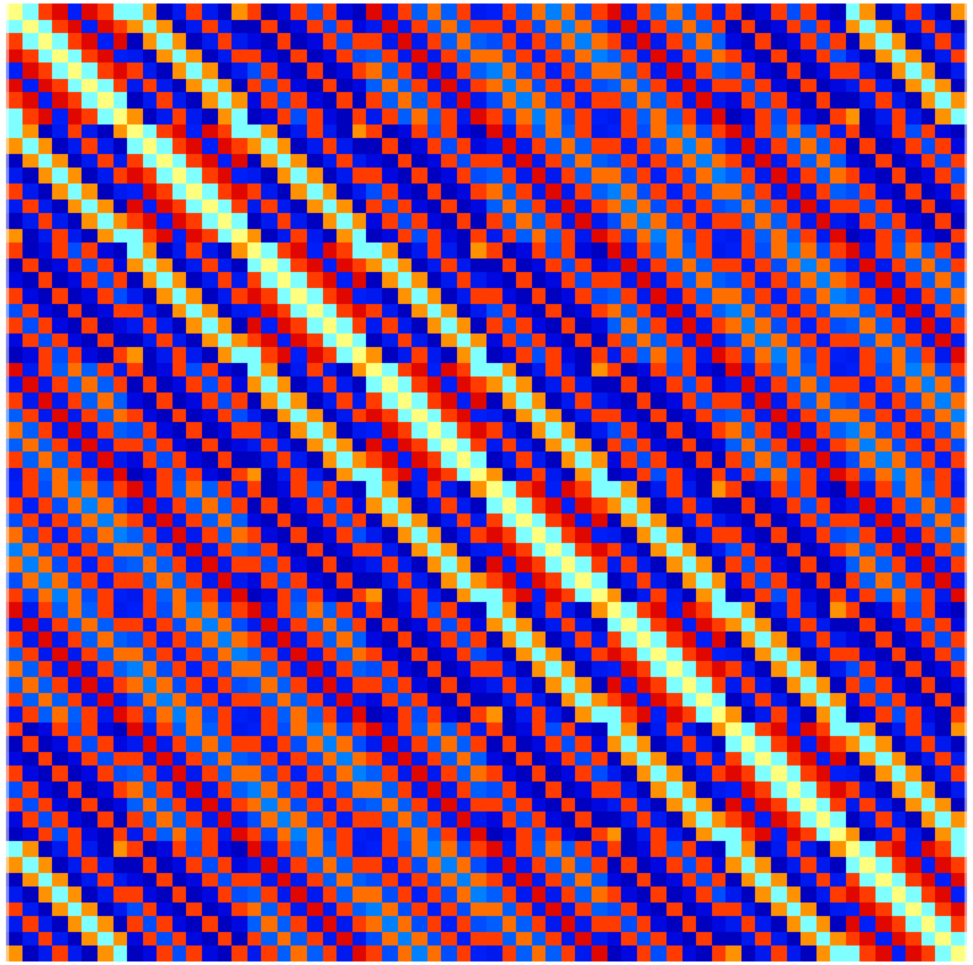
Calculate inverse of convolution matrix for a deconvolution matrix.



-1

A

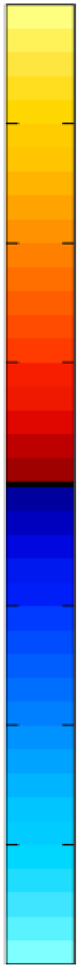
convolution matrix



A^{-1}

deconvolution matrix

1

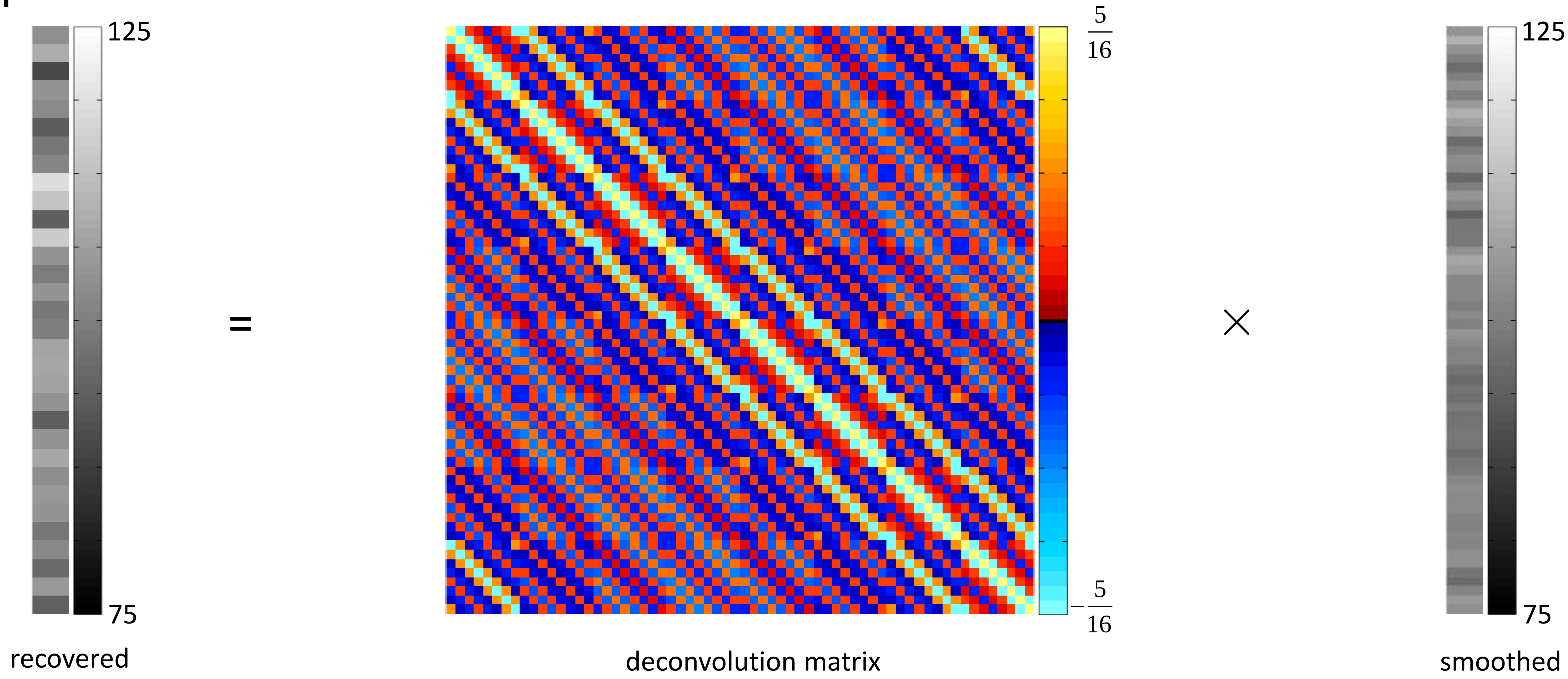


-1

Image Statistics

0	1/8	0
1/8	1/2	1/8
0	1/8	0

Then perform deconvolution via a deconvolution matrix.



The Correlation Coefficient

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The Correlation Distribution

From the variances s_1^2 , s_2^2 , and covariance s_{12} we can perform a transformation of variable to obtain the correlation coefficient $r = \dfrac{s_{12}}{s_1 s_2}$.

It has been shown that the alternative hypothesis ($\rho \neq 0$) PDF is

$$f(r) = \frac{n-2}{\sqrt{2\pi}} \frac{\Gamma(n-1)}{\Gamma\left(n-\frac{1}{2}\right)} \frac{(1-\rho^2)^{\frac{n-1}{2}}(1-r^2)^{\frac{n-4}{2}}}{(1-\rho r)^{n-\frac{3}{2}}} \quad {}_2F_1\left(\tfrac{1}{2},\tfrac{1}{2},n-\tfrac{1}{2},\tfrac{1}{2}(1+\rho r)\right)$$

F is the hypergeometric function

which under the null hypothesis ($\rho=0$) becomes

$$f(r \mid H_0) = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\frac{n-2}{2}\right)}(1-r^2)^{\frac{n-4}{2}} .$$

https://en.wikipedia.org/wiki/Pearson_correlation_coefficient

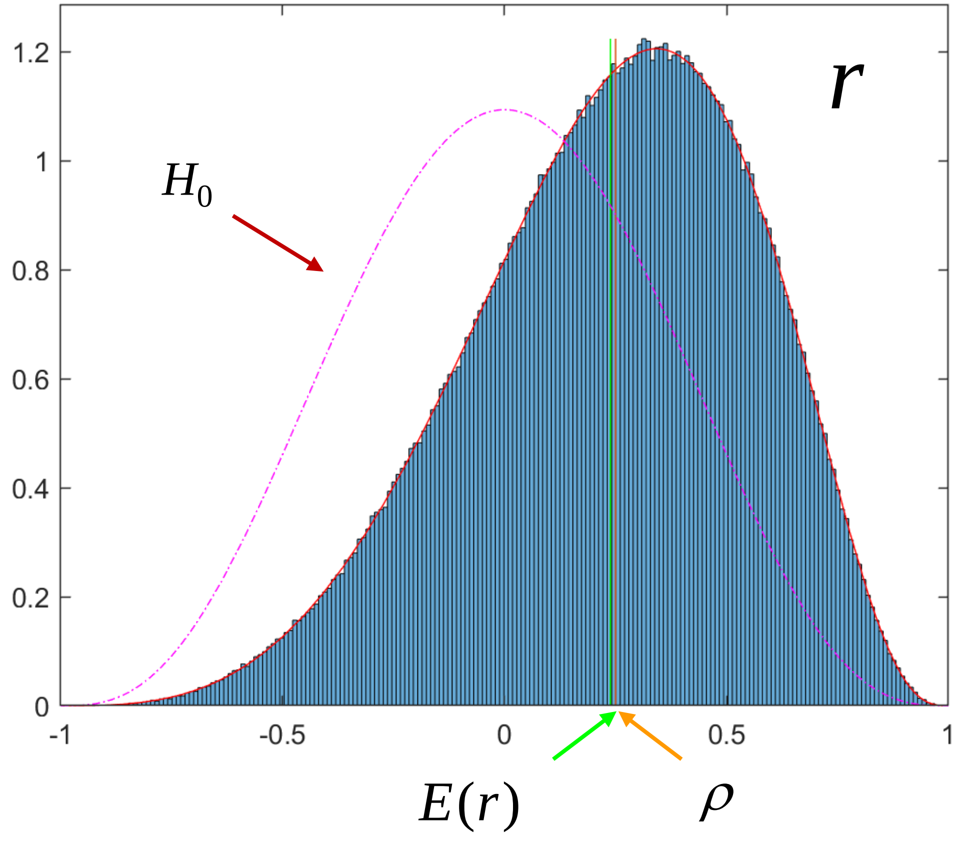
Hotelling: New Light on the Correlation Coefficient and its Transforms. JRSS-B, 15(2), 193-232, 1953.

The Correlation Distribution

Example: Using the same $S_{(1)},\ldots,S_{(L)}$ calculated $r_{(1)},\ldots,r_{(L)}$.

$$\underset{2\times 2}{\Sigma} = \begin{pmatrix} 4 & 2 \\ 2 & 16 \end{pmatrix}$$

$$\rho = 0.25$$



$$E(r) \approx 0.2383$$

$$\bar{r}_{adj} = 0.2453$$

Of note is that $E(r)$ is biased.

$$E(r) = \int_{-1}^{1} r f(r)\, dr$$

$$f(r) = \frac{n-2}{\sqrt{2\pi}} \frac{\Gamma(n-1)}{\Gamma(n-\frac{1}{2})} \frac{(1-\rho^2)^{\frac{n-1}{2}}(1-r^2)^{\frac{n-4}{2}}}{(1-\rho r)^{n-\frac{3}{2}}} \; {}_2F_1\left(\tfrac{1}{2},\tfrac{1}{2},n-\tfrac{1}{2},\tfrac{1}{2}(1+\rho r)\right)$$

$$E(r) = \rho + (1-\rho^2)\left(-\frac{\rho}{2n} - \frac{\rho - 9\rho^3}{8n^2} + \frac{\rho + 42\rho^3 - 75\rho^5}{16n^3} + \cdots\right)$$

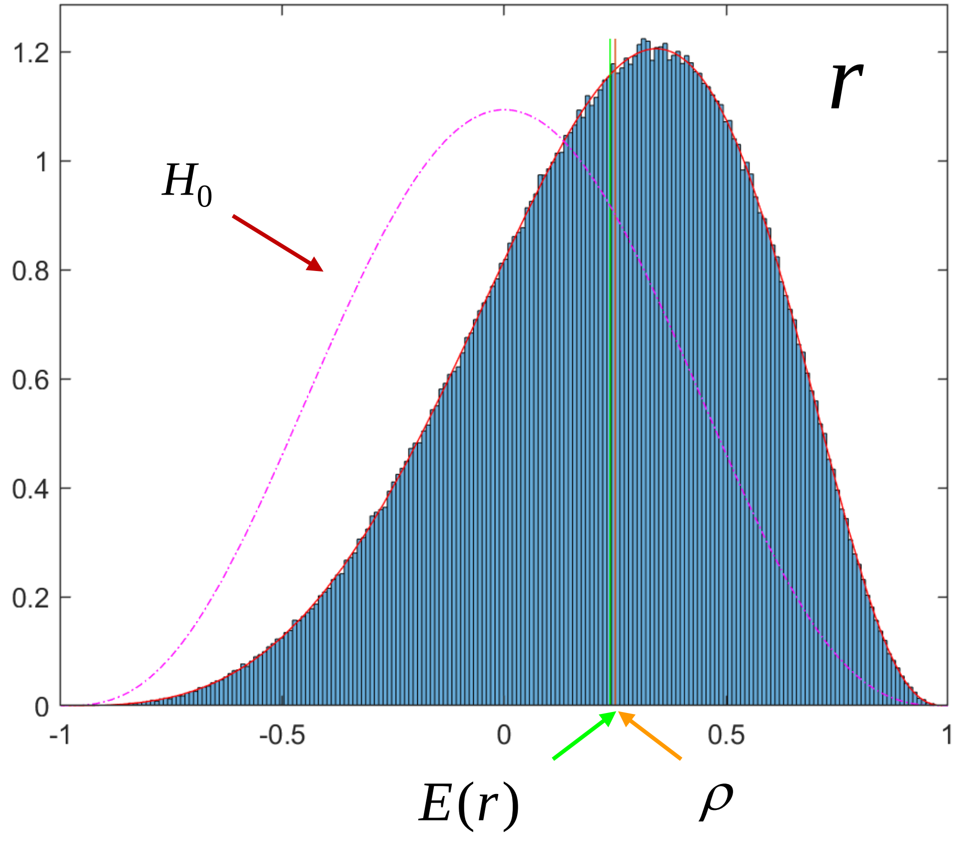
$$E(r) \approx \rho - \frac{\rho(1-\rho^2)}{2n} \quad \rightarrow \quad r_{adj} \approx r\left[1 + \frac{1-r^2}{2n}\right]$$

The Correlation Distribution

Example: Using the same $S_{(1)},\ldots,S_{(L)}$ calculated $r_{(1)},\ldots,r_{(L)}$.

$$\underset{2\times 2}{\Sigma}=\begin{pmatrix}4 & 2\\ 2 & 16\end{pmatrix}$$

$$\rho=0.25$$



$$E(r)\approx 0.2383$$

$$\bar{r}_{adj}=0.2453$$

Of note is that $E(r)$ is biased.

$$E(r)=\int_{-1}^{1} rf(r)\,dr$$

$$f(r)=\frac{n-2}{\sqrt{2\pi}}\frac{\Gamma(n-1)}{\Gamma(n-\frac{1}{2})}\frac{(1-\rho^2)^{\frac{n-1}{2}}(1-r^2)^{\frac{n-4}{2}}}{(1-\rho r)^{n-\frac{3}{2}}}\;{}_2F_1\left(\tfrac{1}{2},\tfrac{1}{2},n-\tfrac{1}{2},\tfrac{1}{2}(1+\rho r)\right)$$

$$E(r)=\rho+(1-\rho^2)\left(-\frac{\rho}{2n}-\frac{\rho-9\rho^3}{8n^2}+\frac{\rho+42\rho^3-75\rho^5}{16n^3}+\cdots\right)$$

$$E(r)\approx\rho-\frac{\rho(1-\rho^2)}{2n} \qquad \rightarrow \qquad r_{adj}\approx r\left[1+\frac{1-r^2}{2n}\right]$$

Pixel Statistics & Template Matching

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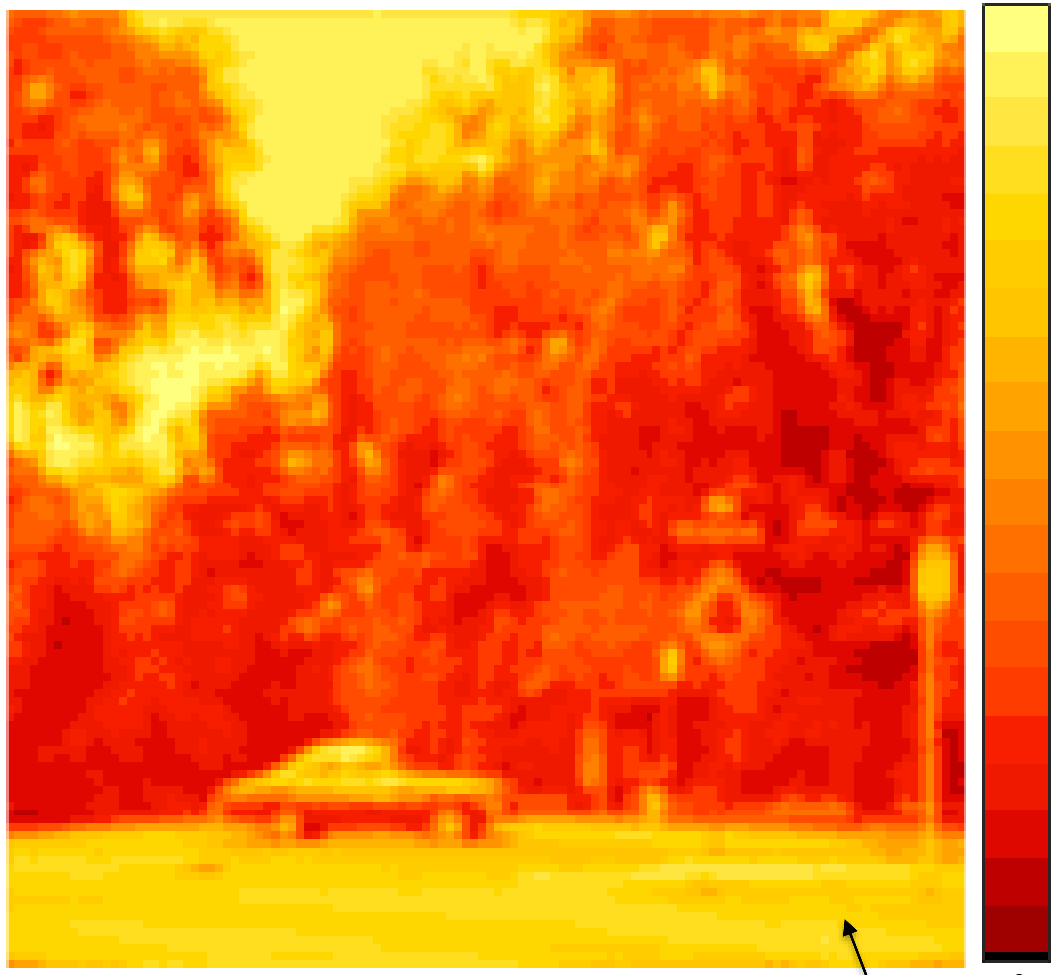
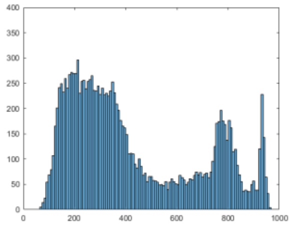
Pixel Statistics

We can use convolution to compute a local sum image.





$\sum x_i$

statistic image

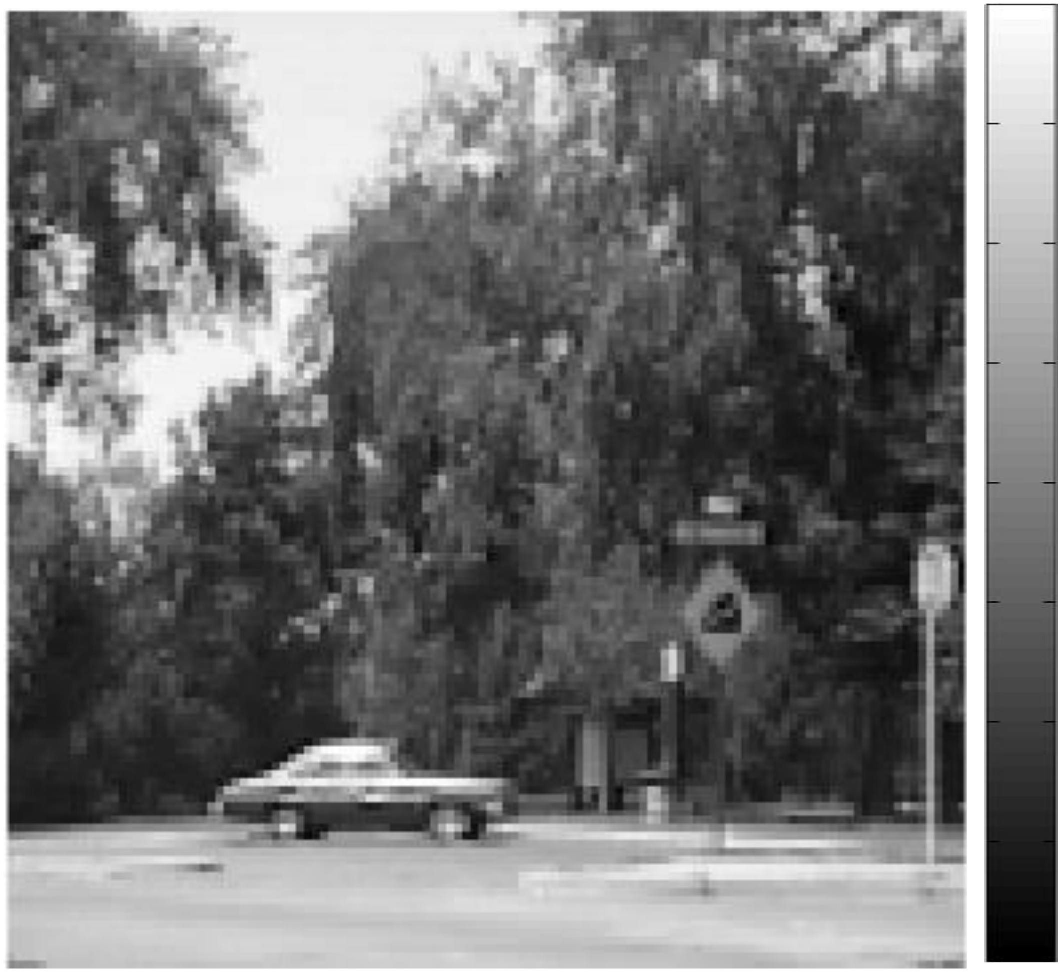
=

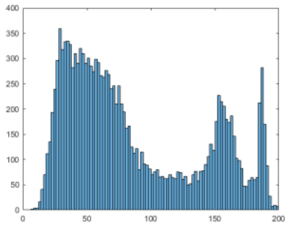
0	**1**	**0**
1	**1**	**1**
0	**1**	**0**

Kernel

$*$

convolution

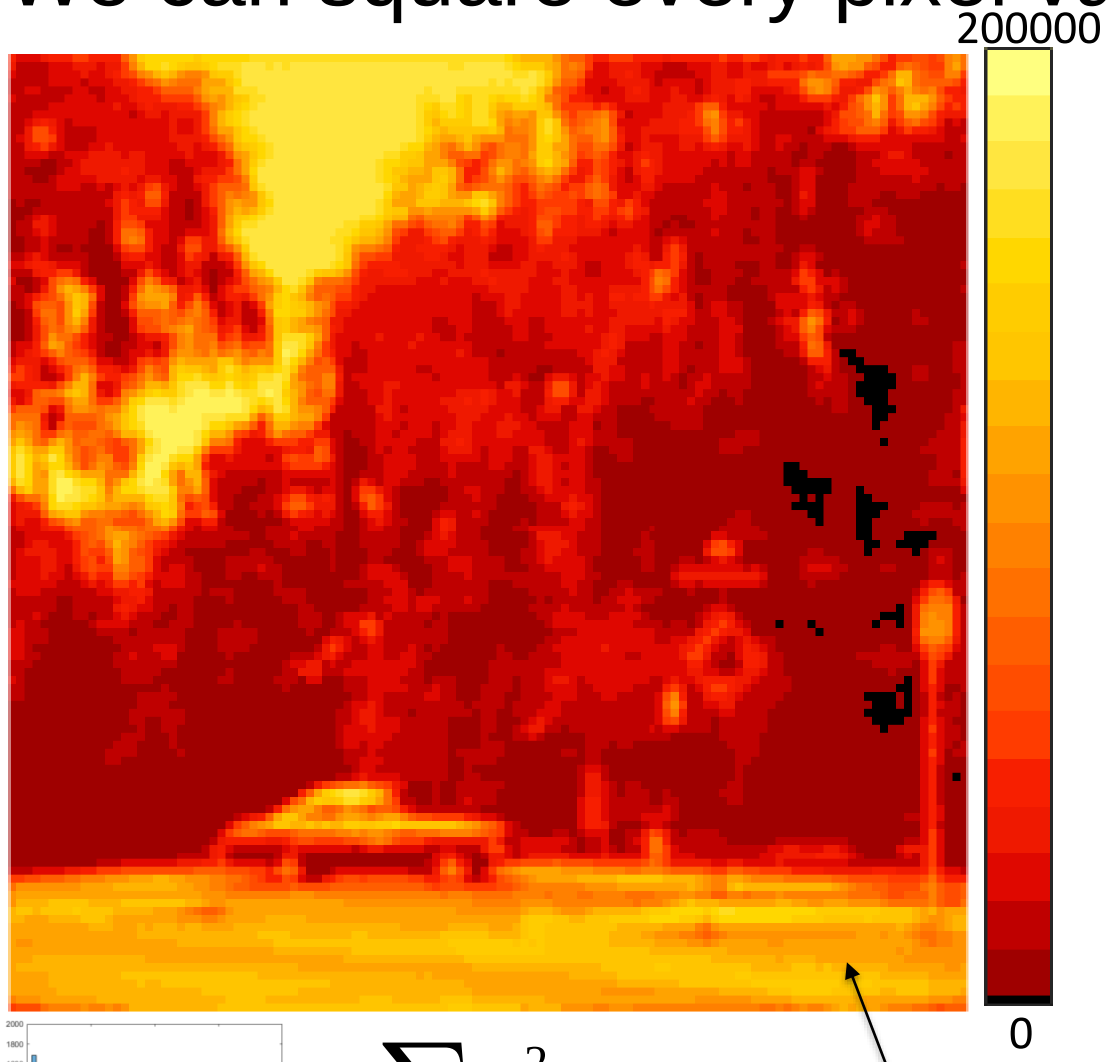


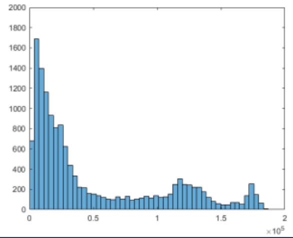


x_i

Pixel Statistics

We can square every pixel value and compute a sum of sqaures.





$\sum x_i^2$

statistic image

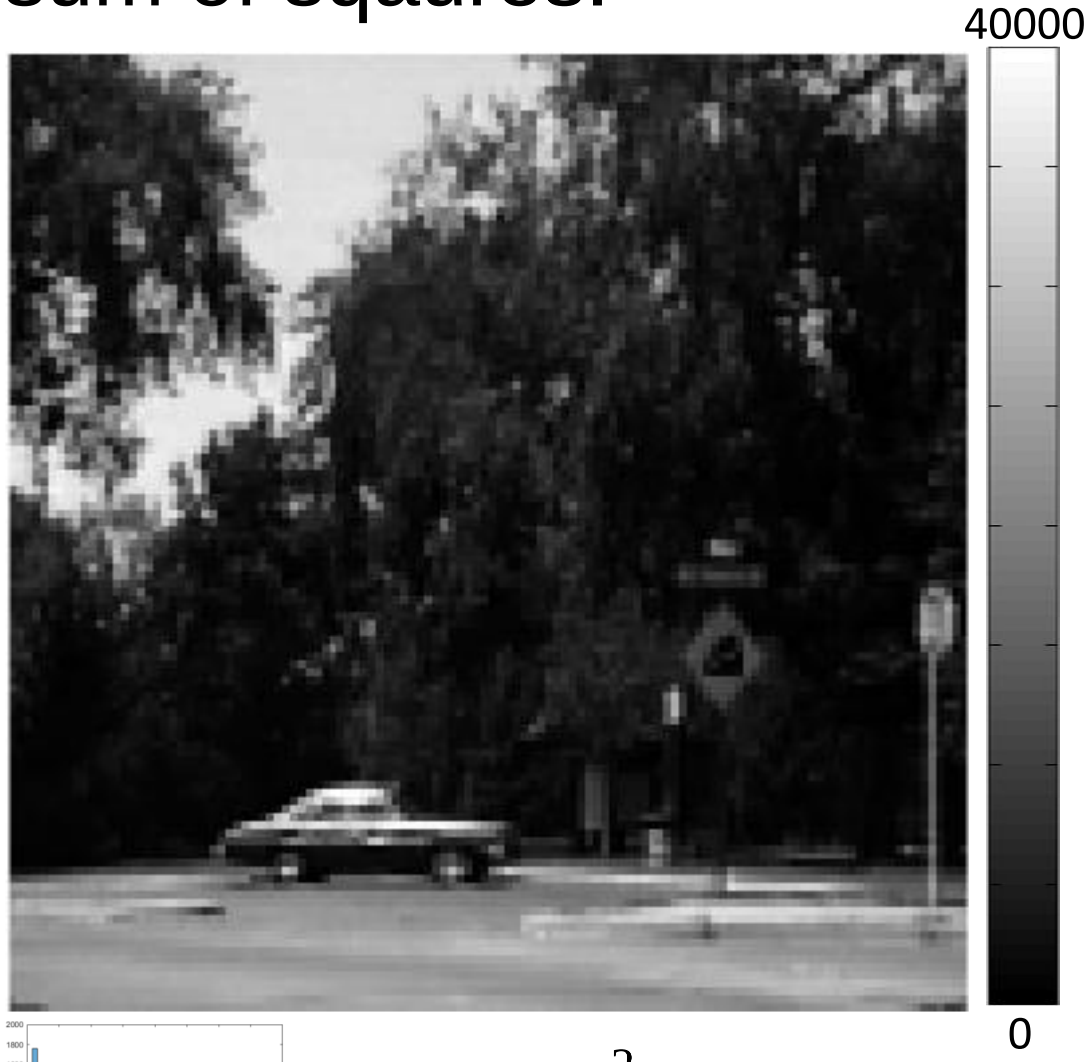
=

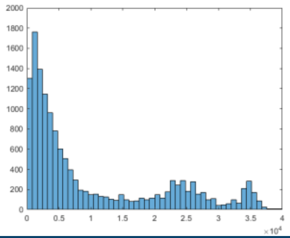
0	1	0
1	1	1
0	1	0

Kernel

$*$

convolution



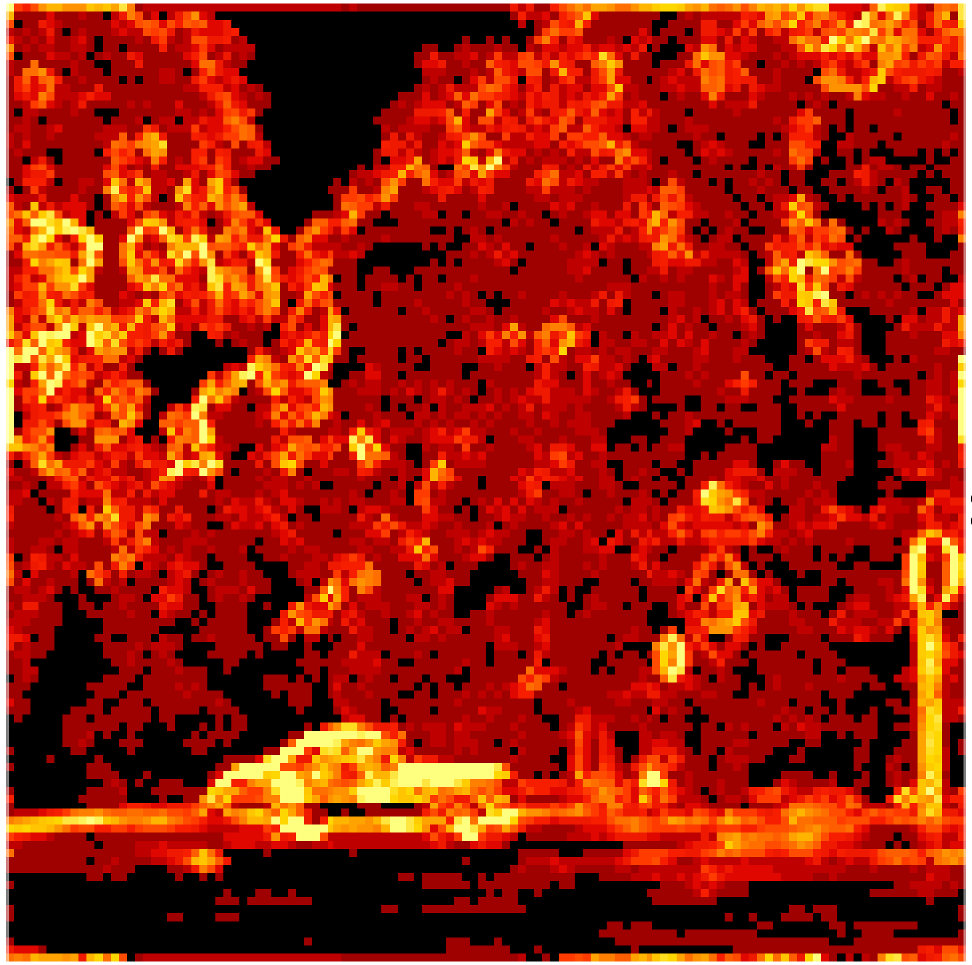


x_i^2

Pixel Statistics

$$s^2 = \frac{\sum x_i^2 - \left(\sum x_i\right)^2 / n}{n-1}$$

We can use sum and sum of squares to compute usual the local variance.



=

-

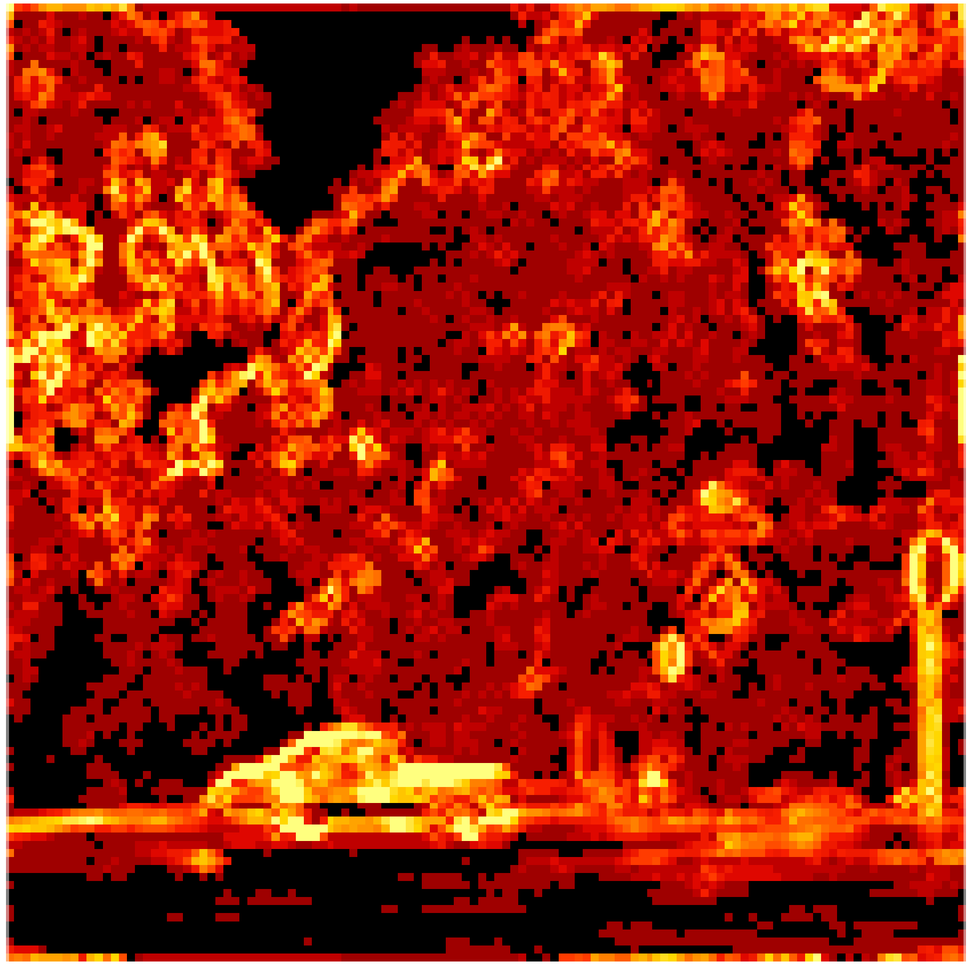
2

/ n

n-1

Pixel Statistics

The variance is low in homogeneous and high in heterogeneous areas.



Variance

Original

Mean

Perhaps we could smooth more in low variance areas?

Template Matching

We want to perform computations as fast as possible.

For our object template we can pre-compute

$\sum o_i$ $\sum o_i^2$

Then as we loop through our image, compute

$\sum p_i$ $\sum p_i^2$ $\sum o_i p_i$

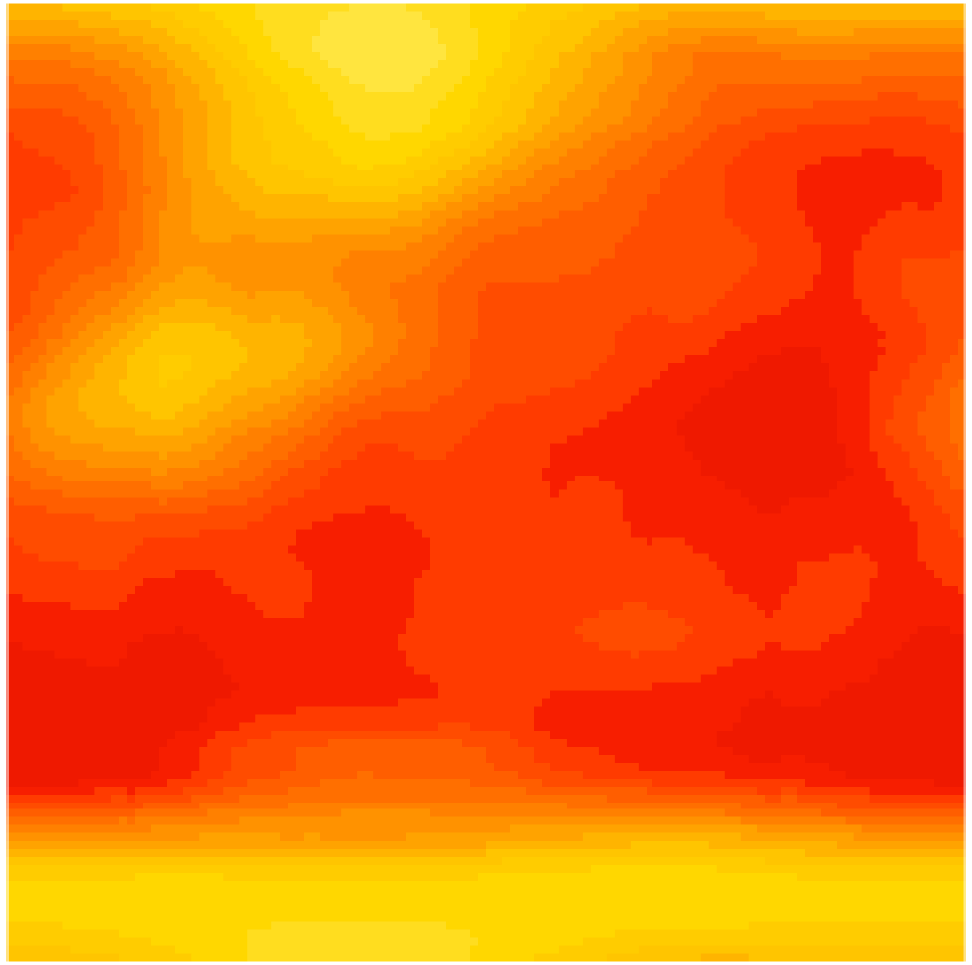
$$r = \frac{\sum p_i o_i - \frac{1}{n}\left(\sum p_i\right)\left(\sum o_i\right)}{\sqrt{\sum p_i^2 - \frac{1}{n}\left(\sum p_i\right)^2}\sqrt{\sum o_i^2 - \frac{1}{n}\left(\sum o_i\right)^2}}$$

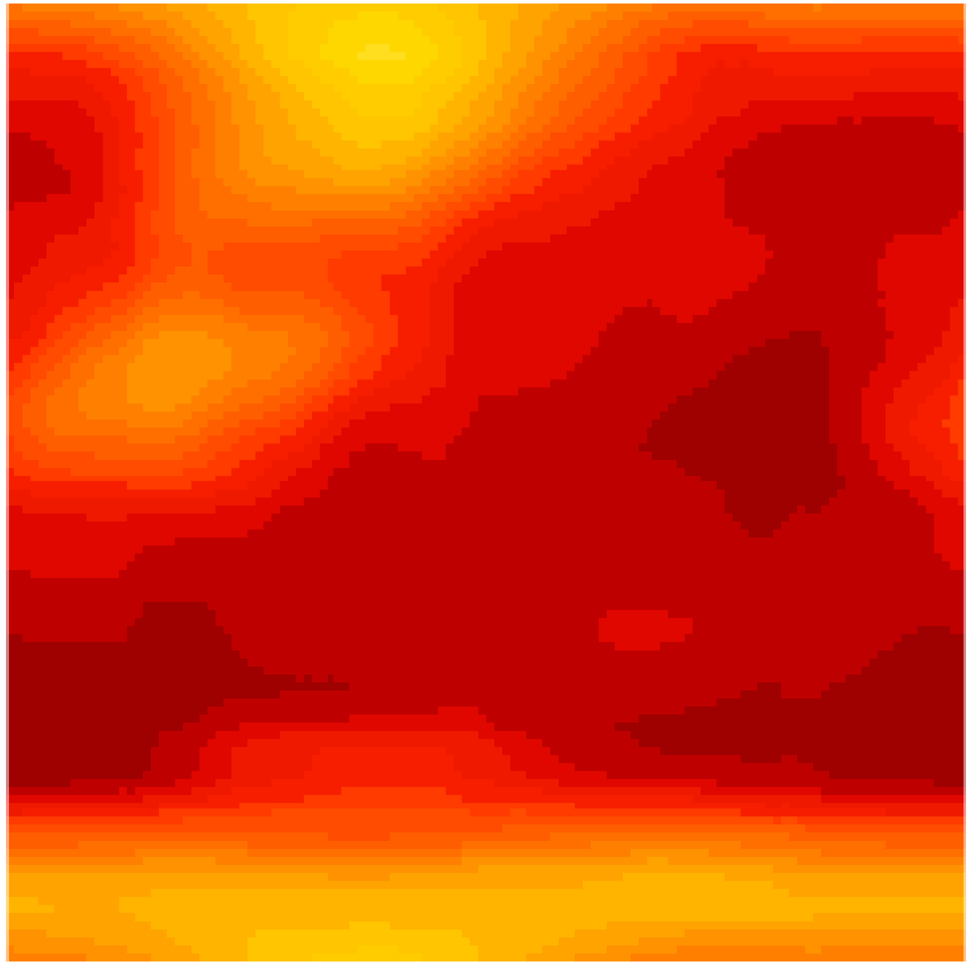
Template Matching

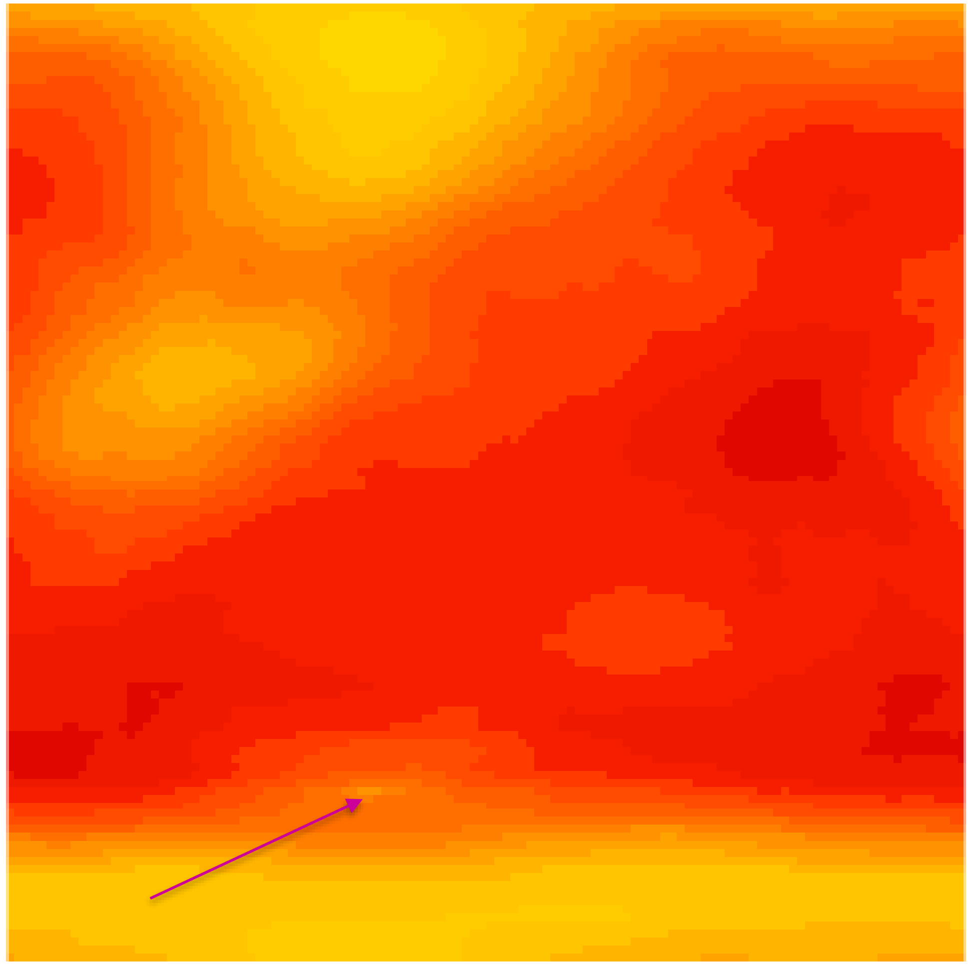
$$\sum o_i = 40120$$

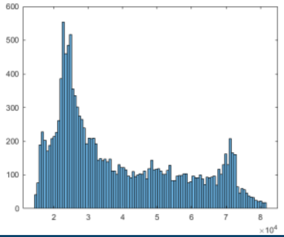
$$\sum o_i^2 = 4781080$$

Here are our statistic images.

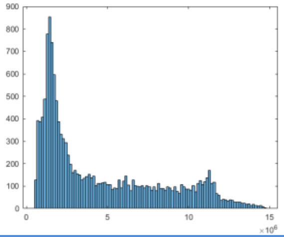




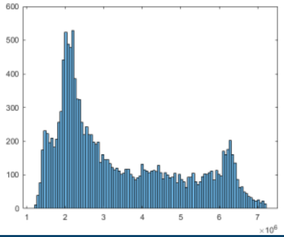




$$\sum p_i$$



$$\sum p_i^2$$



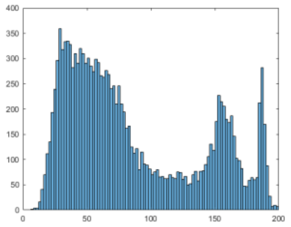
$$\sum o_i p_i$$

Template Matching

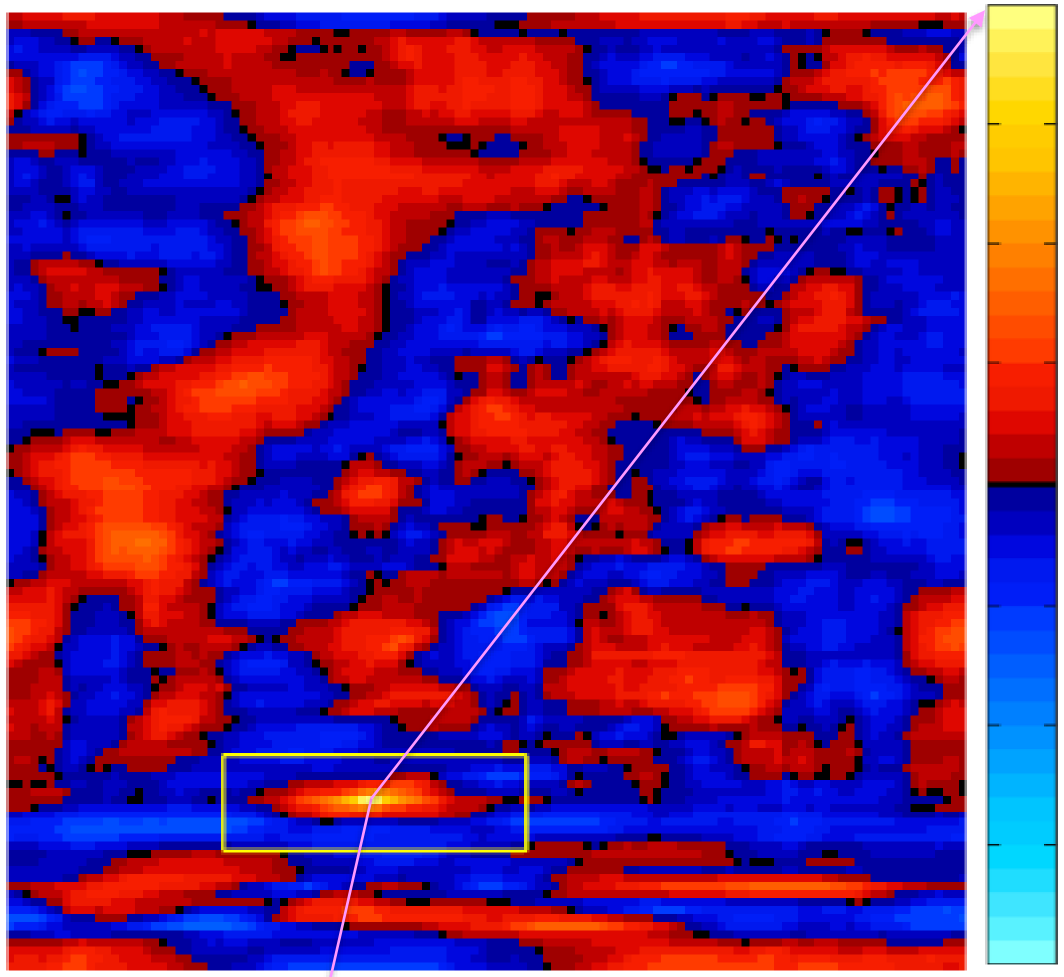
The correlation process of our object template with our scene is:

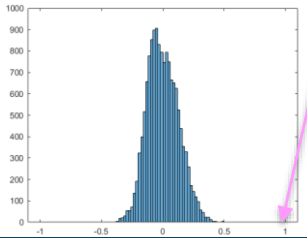
★

correlation



=





Through Image Processing

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Videos in Matlab

Original Images

General Properties:
Name: 'Cat.mp4'
Path: 'C:MATH4931
Duration: 24.8686
CurrentTime: 24.8686
NumFrames: 373

Video Properties:
Width: 1280
Height: 720
FrameRate: 15.0282
BitsPerPixel: 24
VideoFormat: 'RGB24'

Play Video

Videos in Matlab

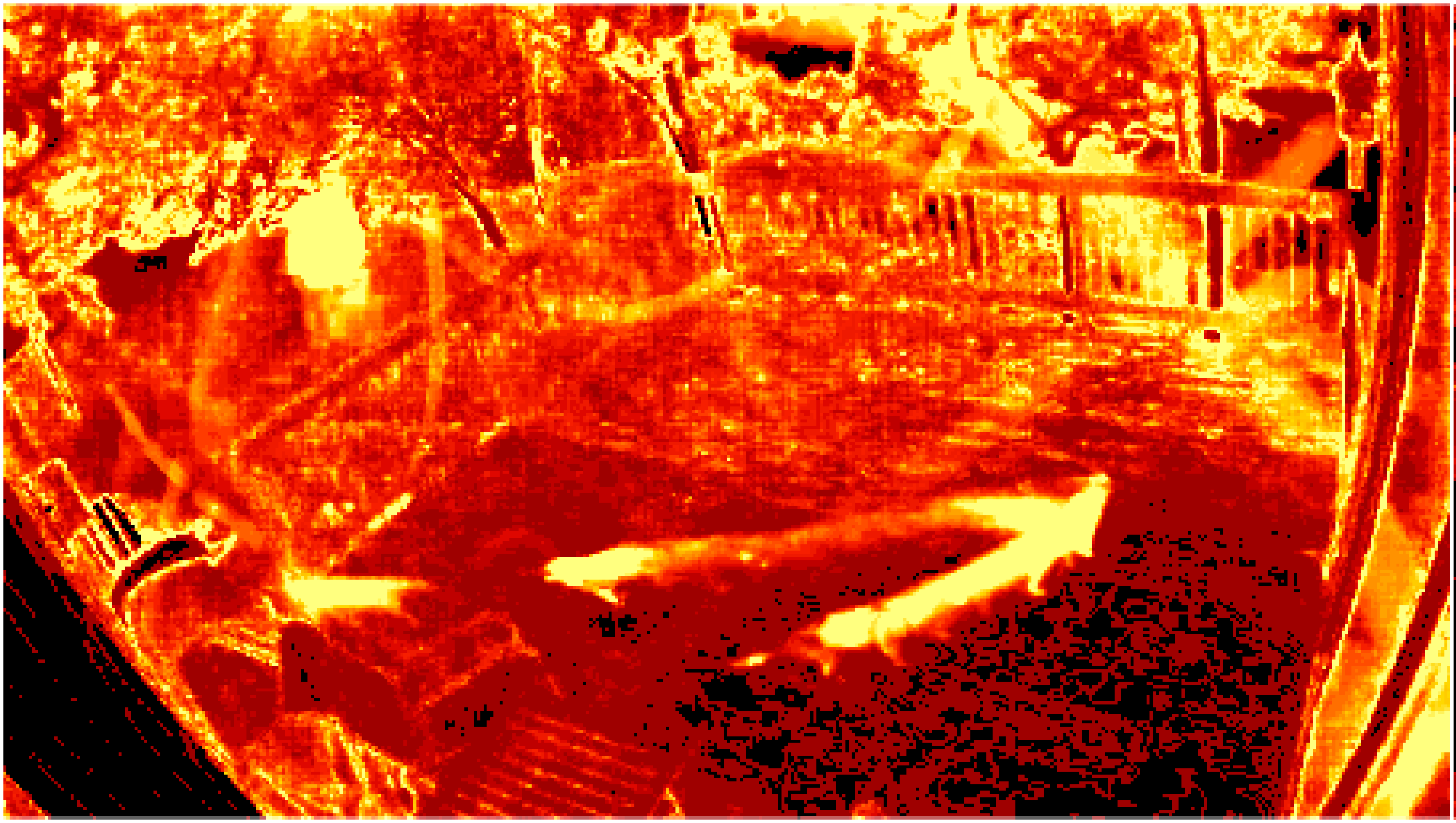


Image Mean

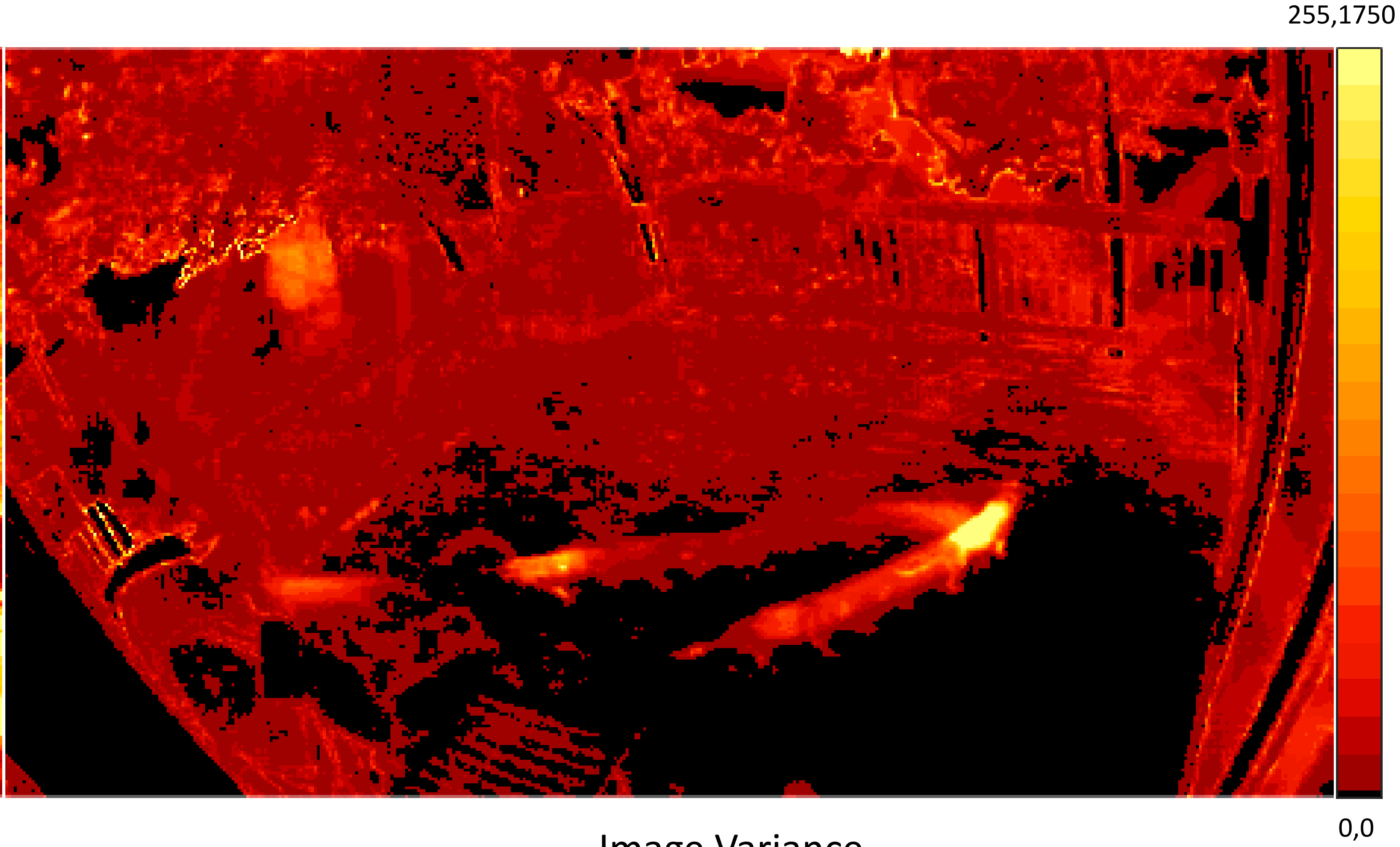


Image Variance

Pixel Temporal Convolution

To apply pixel temporal convolution we need to store as many images as the size of our kernel!

$\frac{1}{16}\times$

t=98

$\frac{3}{16}\times$

t=99

$\frac{5}{16}\times$

t=100

$\frac{7}{16}\times$

t=101

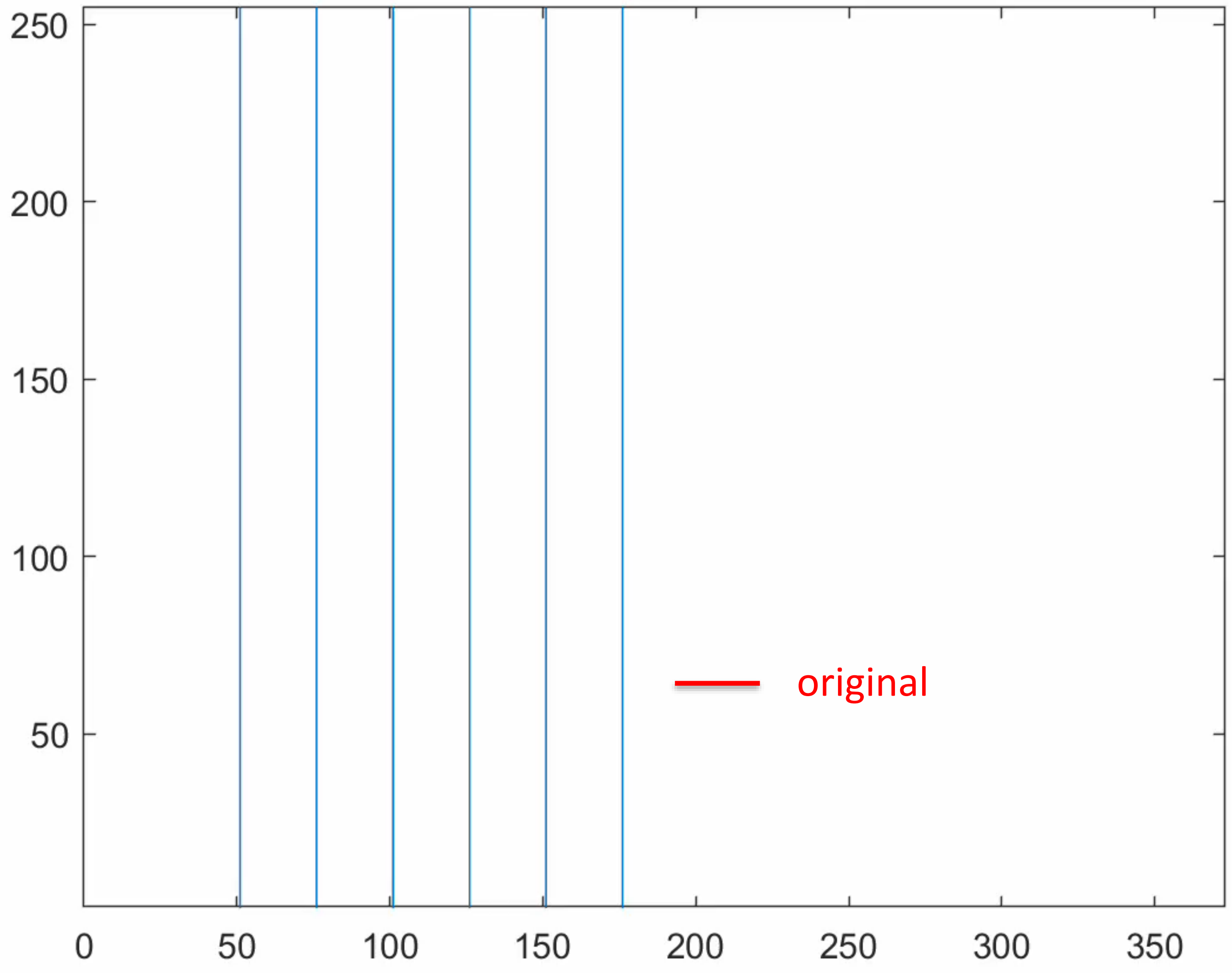
$=$

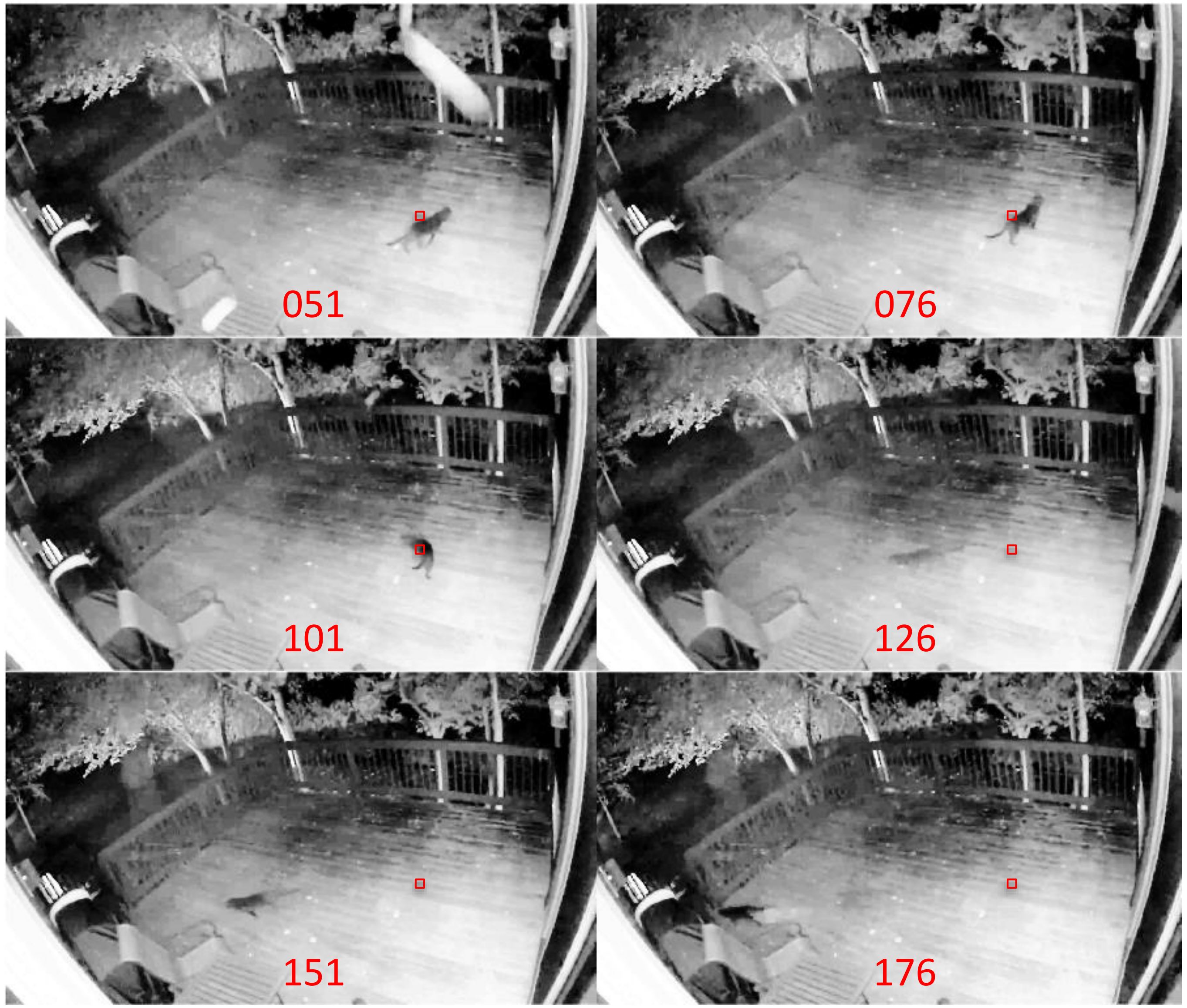
t=101

1/16	3/16	5/16	7/16

Pixel Temporal Convolution

View the time series of a particular pixel.





The Discrete Fourier Transform

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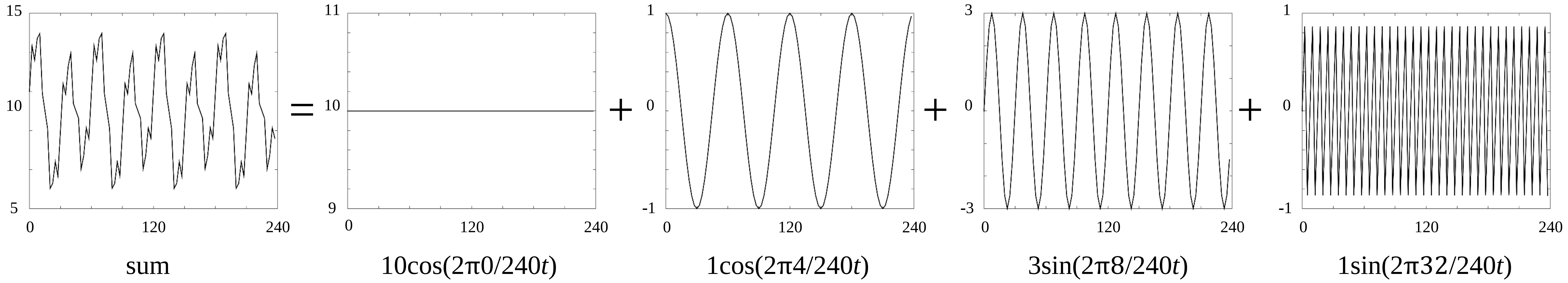
1D Discrete Fourier Transform

Example: Let's sample the continuous time series (1D function)

$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + \cos\left(2\pi\frac{4}{240}t\right) + 3\sin\left(2\pi\frac{8}{240}t\right) + \sin\left(2\pi\frac{32}{240}t\right)$$

at $t=1\Delta t, 2\Delta t, 3\Delta t, \ldots, n\Delta t$, where $n=96$ and $\Delta t=2.5\text{s}$ for a total time of 240s.



1D Discrete Fourier Transform

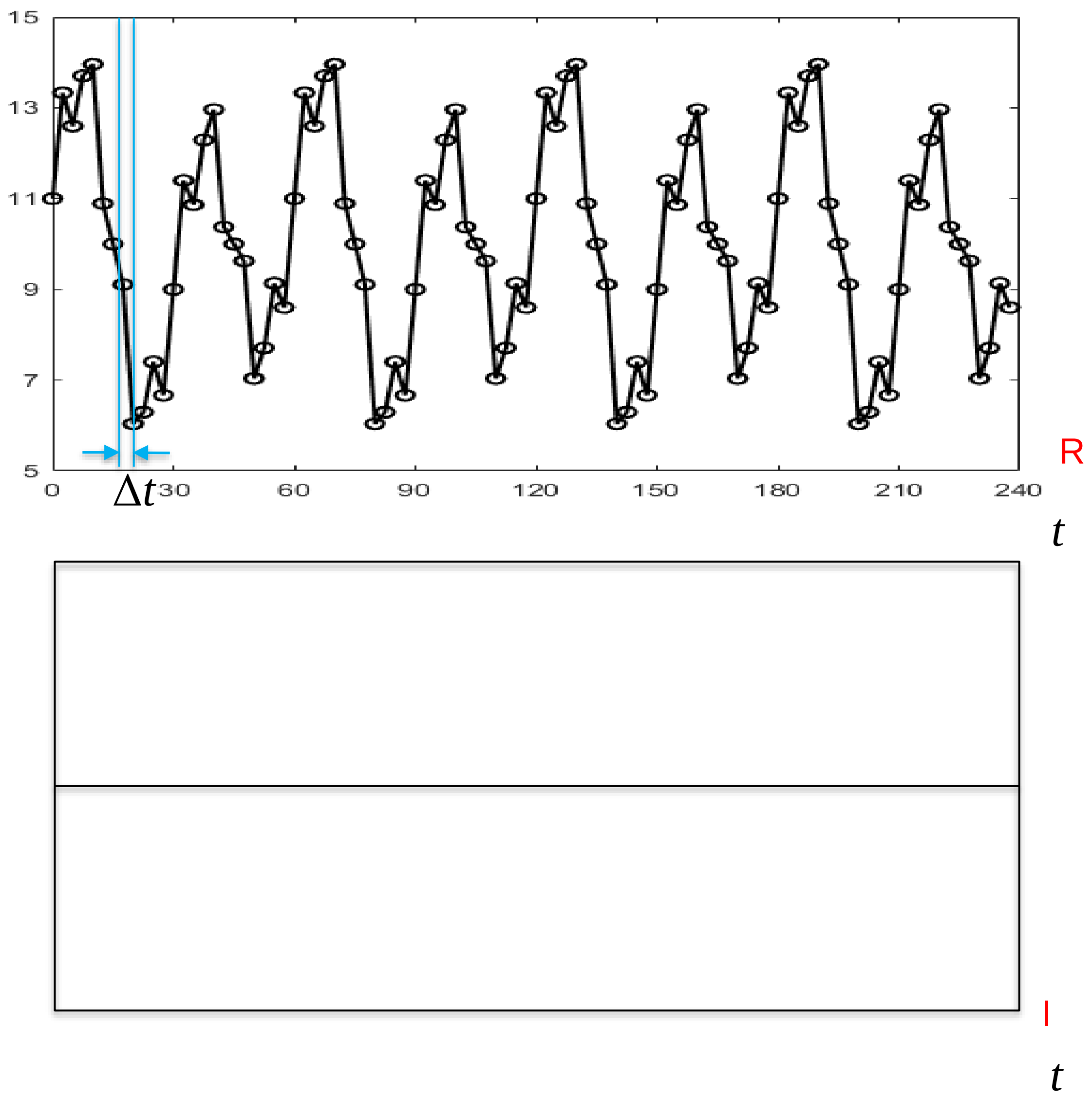
$n=96$

$\Delta t=2.5\text{s}$

$$\Delta\nu=\frac{1}{n\Delta t}=0.042Hz$$

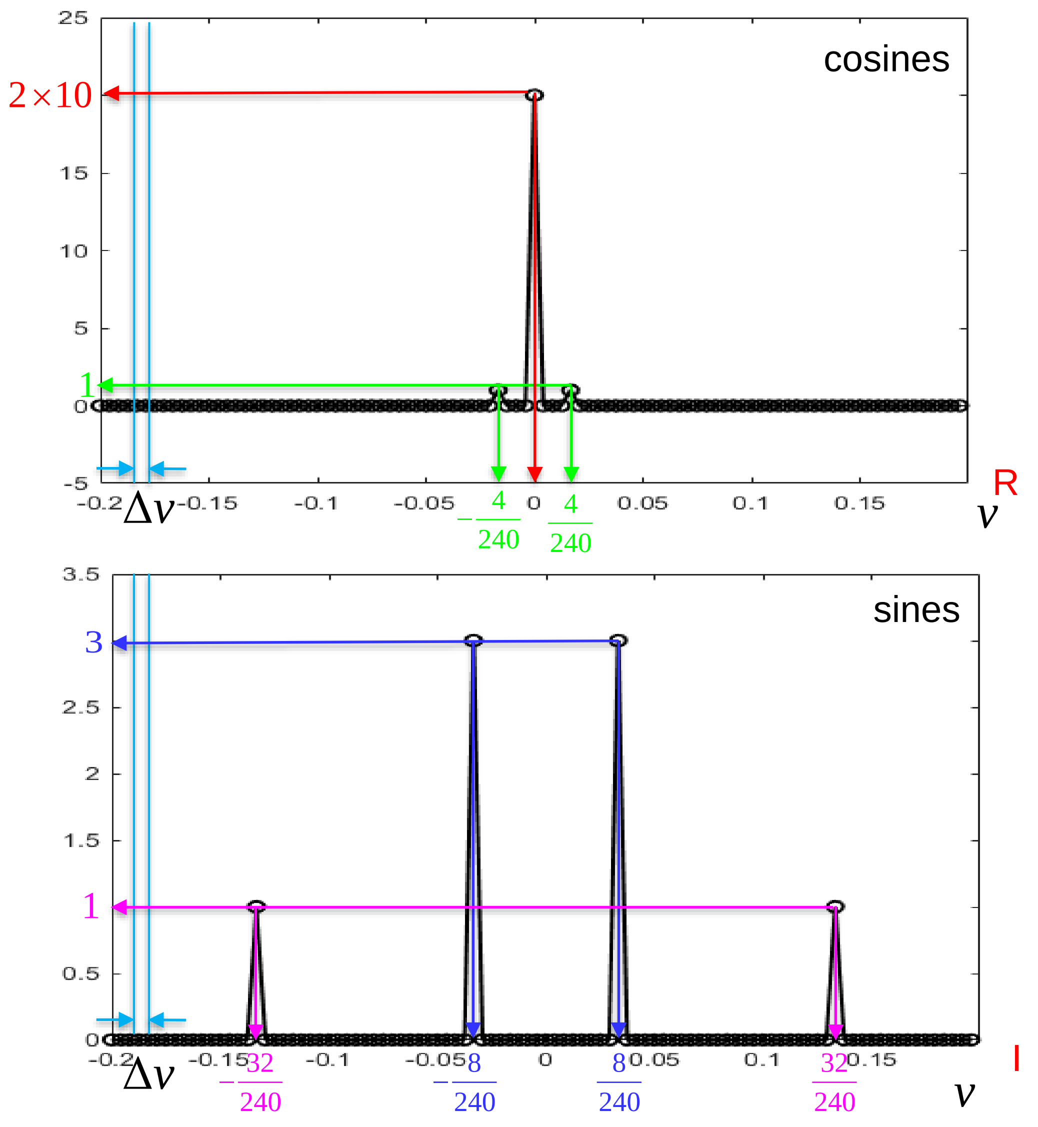
$$\nu_{\max}=\frac{n}{2}\Delta\nu=20Hz$$

Time Series



$\mathcal{F}$

Amplitude Coefficients

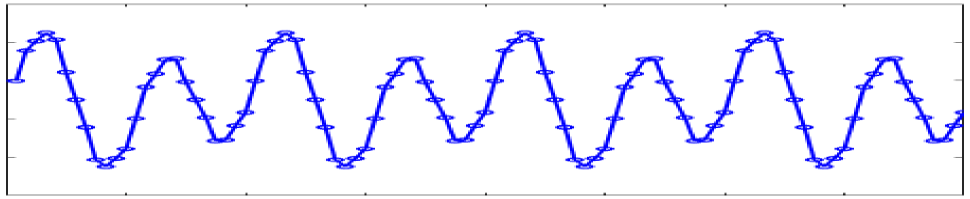


$$y(t)=10\cos\left(2\pi\frac{0}{240}t\right)+1\cos\left(2\pi\frac{4}{240}t\right)+3\sin\left(2\pi\frac{8}{240}t\right)+1\sin\left(2\pi\frac{32}{240}t\right)$$

*coefficients divided by $n/2$ for display

Time Series Convolution via the DFT

When we performed convolution of a time series with a kernel, we moved the kernel, multiplied, and created a new time point. This was repeated to create an entirely new time series.



smoothed

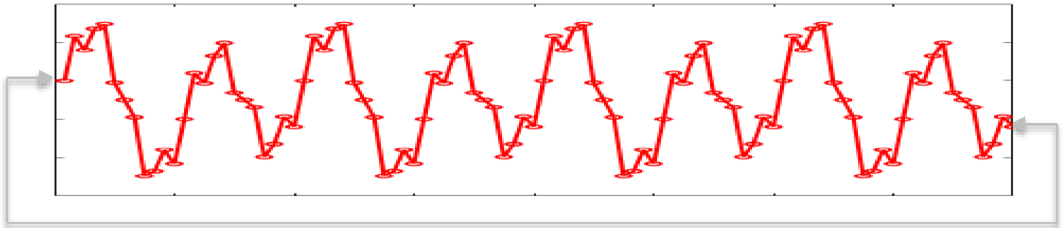
=

1/4	1/2	1/4

kernel

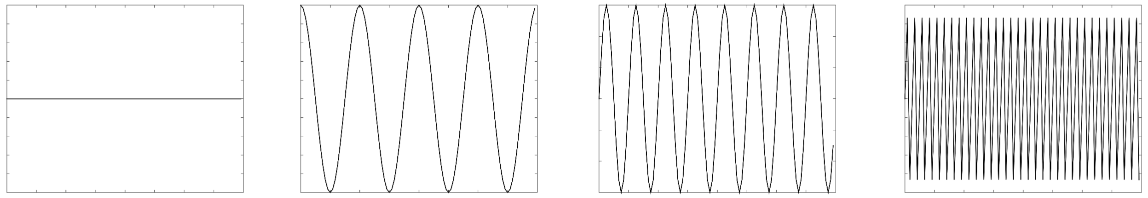
$*$

convolution



original

wrap

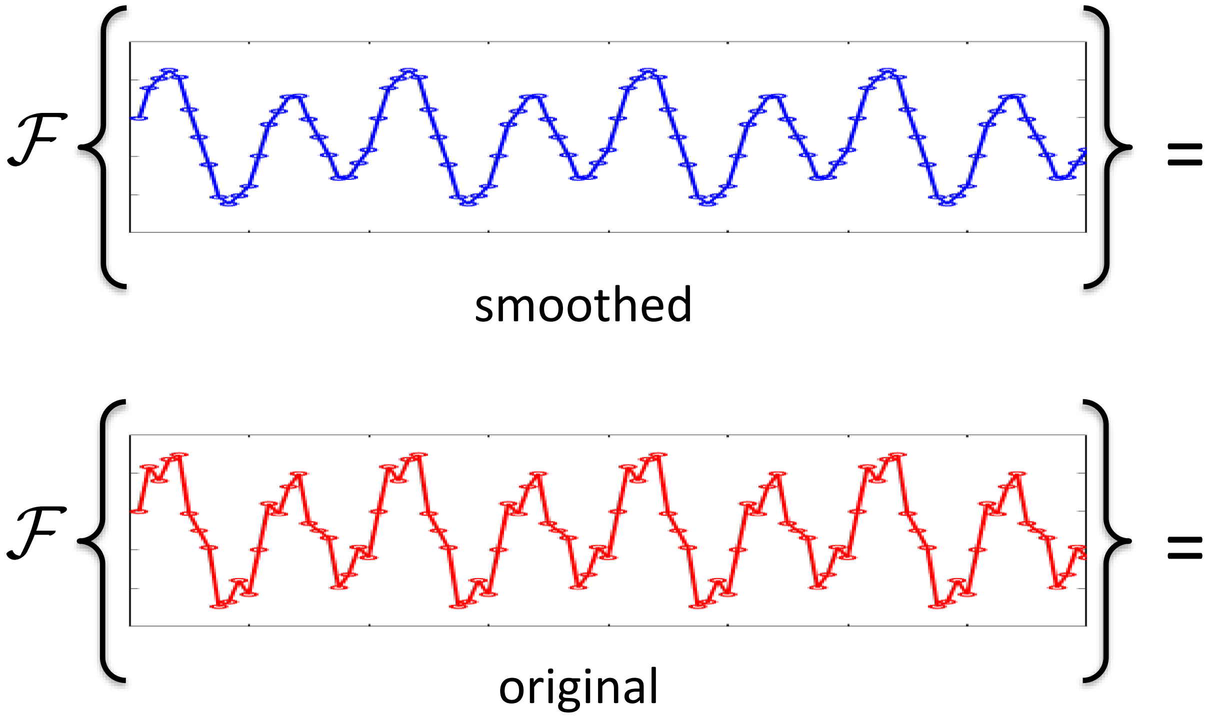


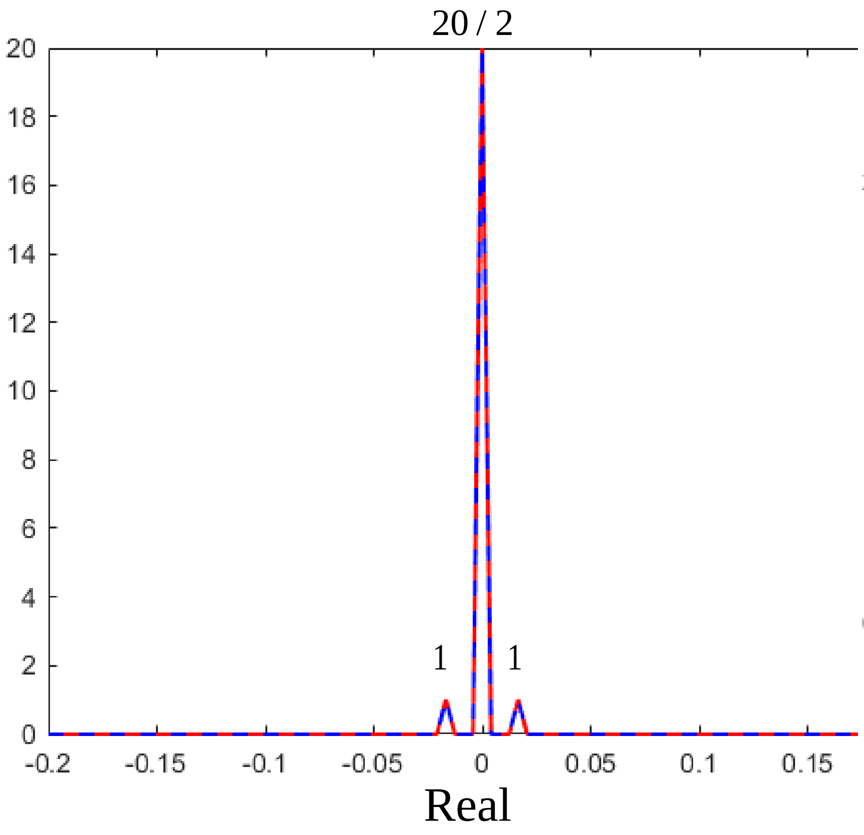
$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + 1\cos\left(2\pi\frac{4}{240}t\right) + 3\sin\left(2\pi\frac{8}{240}t\right) + 1\sin\left(2\pi\frac{32}{240}t\right)$$

Time Series Convolution via the DFT

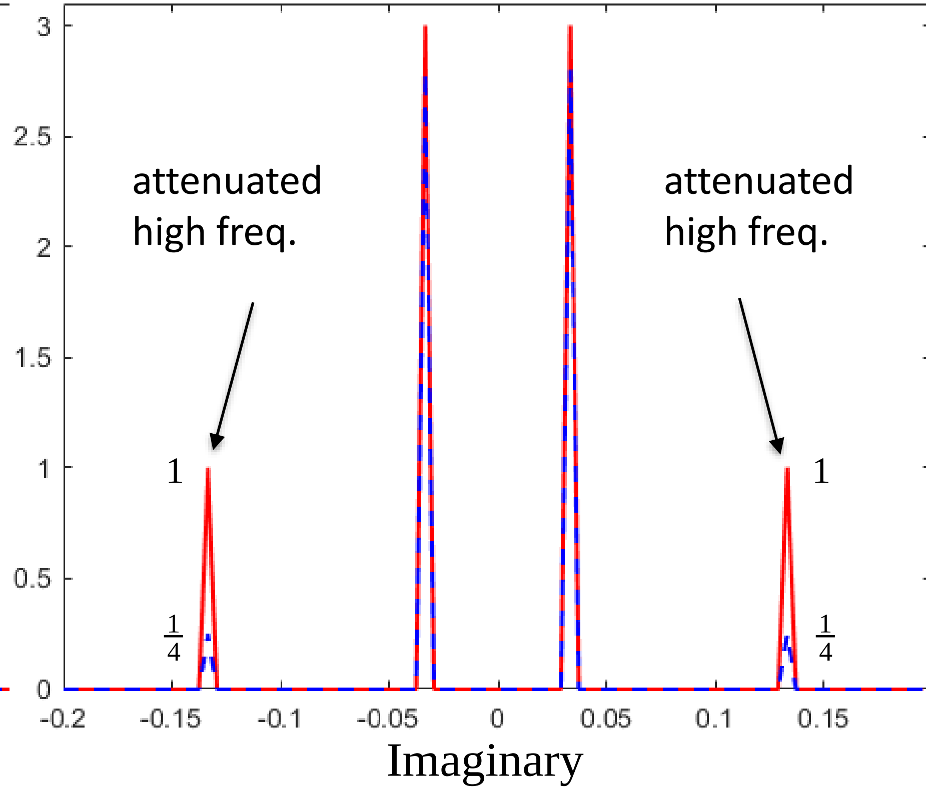
We can take the Fourier transform of the smoothed time series and compare it to the Fourier transform of the original input time series to see how the high frequency amplitude coefficients were attenuated.

1/4	1/2	1/4





$\sin\left(2\pi \frac{32}{240} t\right)$



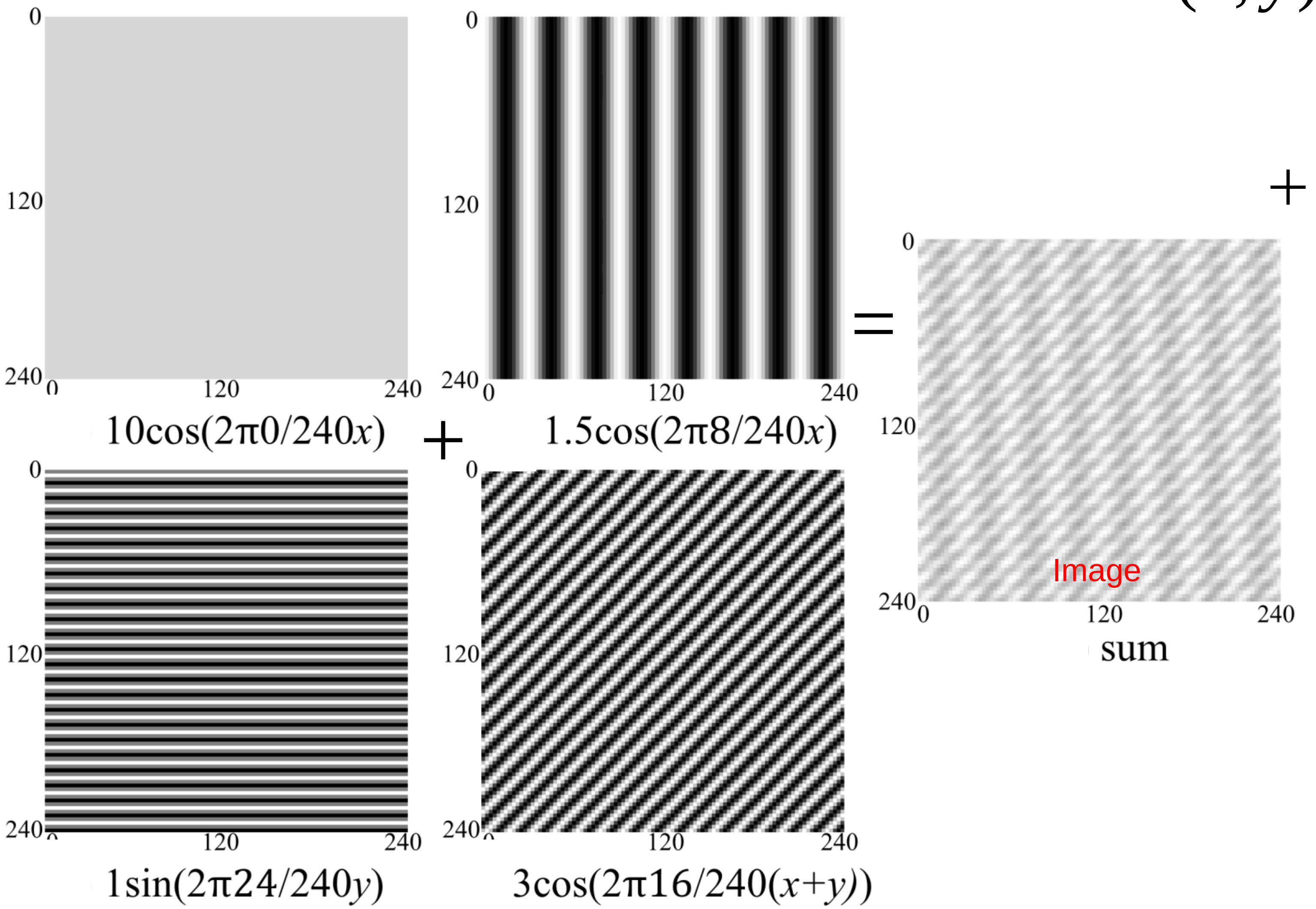
$$y(t) = 10\cos\left(2\pi \frac{0}{240} t\right) + 1\cos\left(2\pi \frac{4}{240} t\right) + 3\sin\left(2\pi \frac{8}{240} t\right) + 1\sin\left(2\pi \frac{32}{240} t\right)$$

*coefficients divided by $n/2$ for display

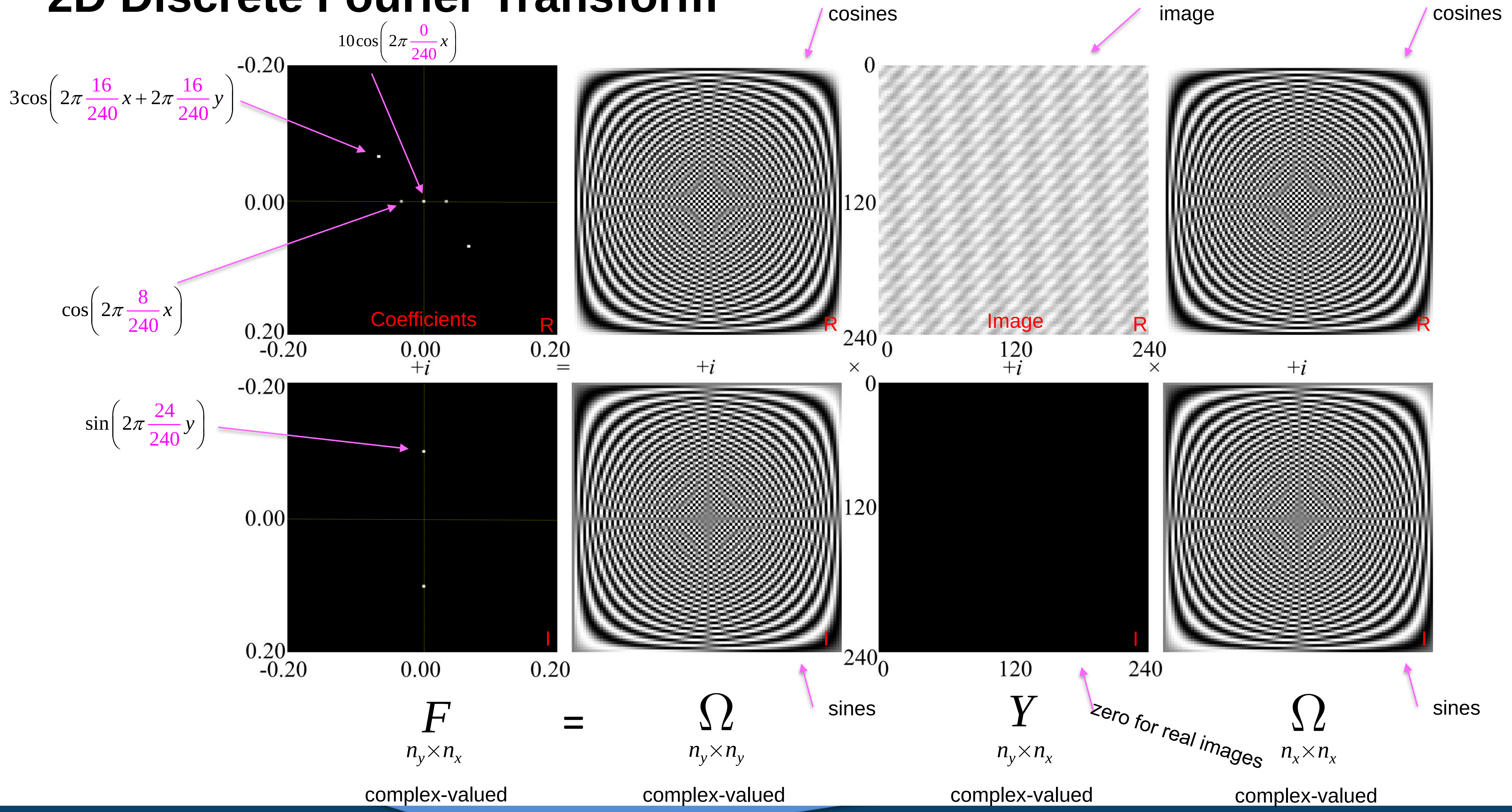
2D Discrete Fourier Transform

Example: Let's sample the continuous image scene (2D function)

$$Y(x,y) = 10\cos\left(2\pi\frac{0}{240}x\right) + \frac{3}{2}\cos\left(2\pi\frac{8}{240}x\right) + \sin\left(2\pi\frac{24}{240}y\right) + \cos\left(2\pi\left(\frac{16}{240}x + \frac{16}{240}y\right)\right)$$



2D Discrete Fourier Transform



Convolution via the DFT

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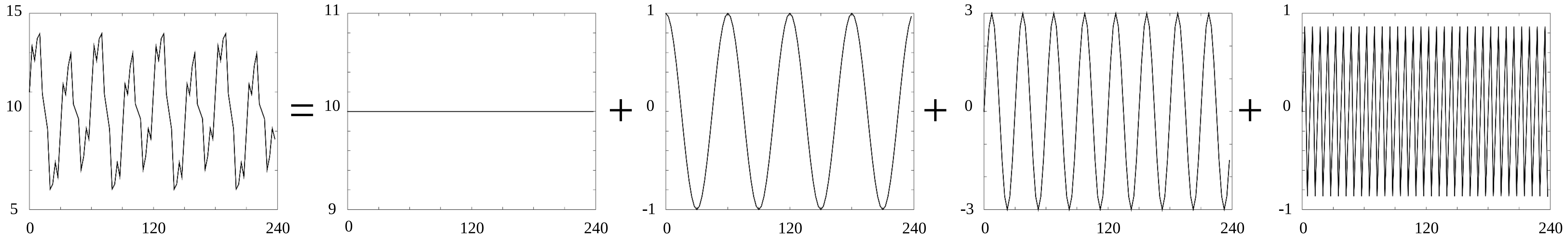
Time Series Convolution via the DFT

Example: Let's sample the continuous time series (1D function)

$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + \cos\left(2\pi\frac{4}{240}t\right) + 3\sin\left(2\pi\frac{8}{240}t\right) + \sin\left(2\pi\frac{32}{240}t\right)$$

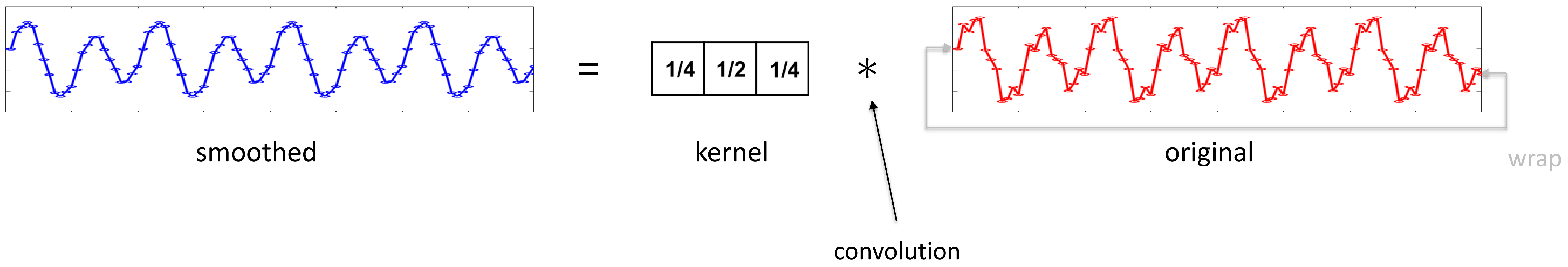
at $t=1\Delta t, 2\Delta t, 3\Delta t, \ldots, n\Delta t$, where $n=96$ and $\Delta t=2.5\text{s}$ for a total time of 240s.

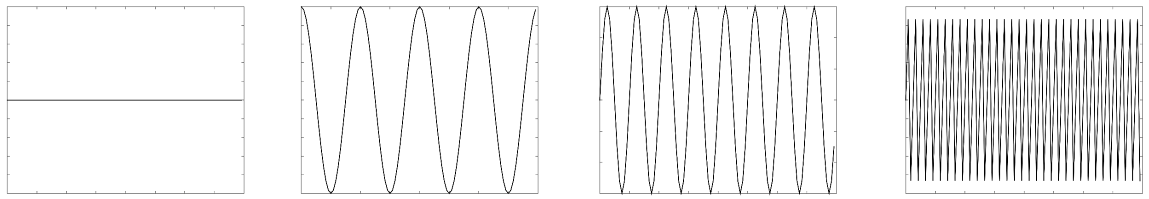


sum = $10\cos(2\pi 0/240t)$ + $1\cos(2\pi 4/240t)$ + $3\sin(2\pi 8/240t)$ + $1\sin(2\pi 32/240t)$

Time Series Convolution via the DFT

When we performed convolution of a time series with a kernel, we moved the kernel, multiplied, and created a new time point. This was repeated to create an entirely new time series.



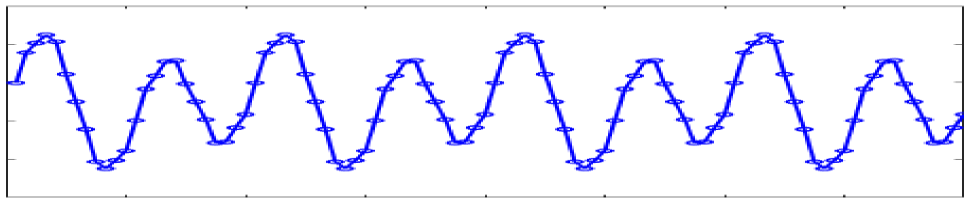


$$y(t)=10\cos\left(2\pi\frac{0}{240}t\right)+1\cos\left(2\pi\frac{4}{240}t\right)+3\sin\left(2\pi\frac{8}{240}t\right)+1\sin\left(2\pi\frac{32}{240}t\right)$$

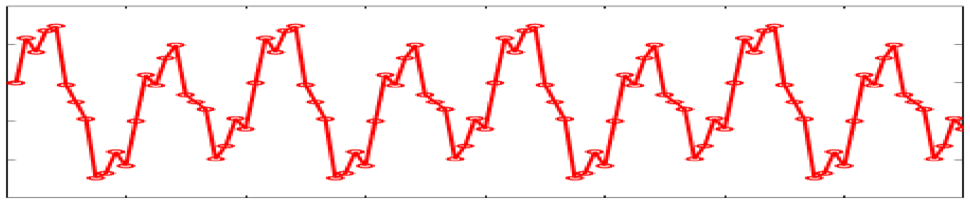
Time Series Convolution via the DFT

We can take the Fourier transform of the smoothed time series and compare it to the Fourier transform of the original input time series to see how the high frequency amplitude coefficients were attenuated.

1/4	**1/2**	**1/4**

$\mathcal{F}\Bigg\{$  $\Bigg\} =$

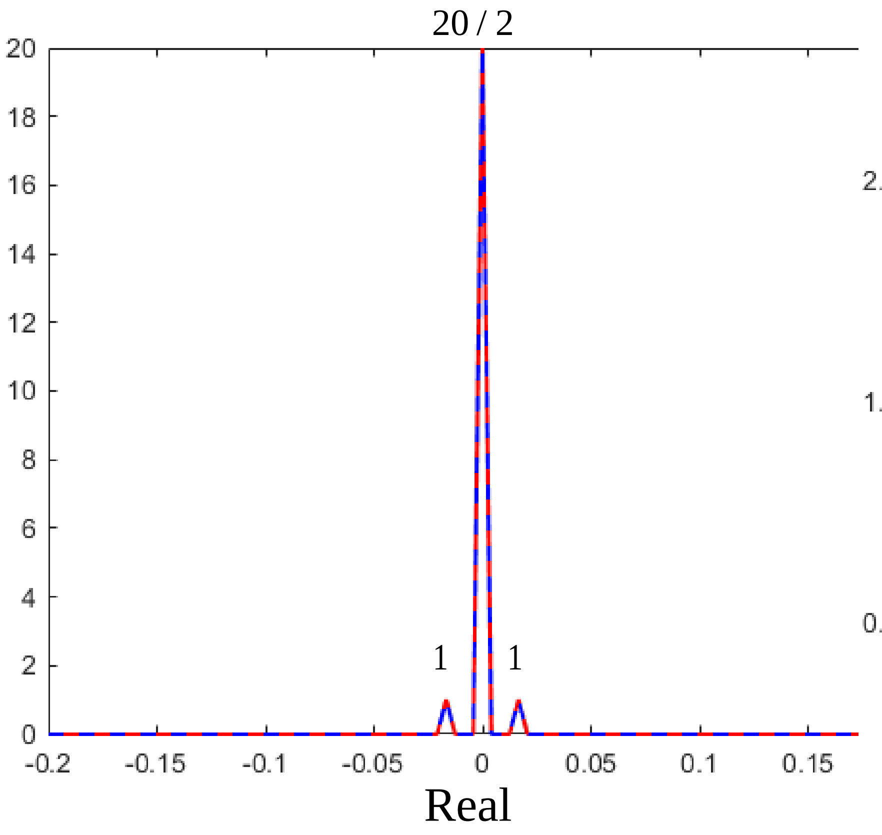
smoothed

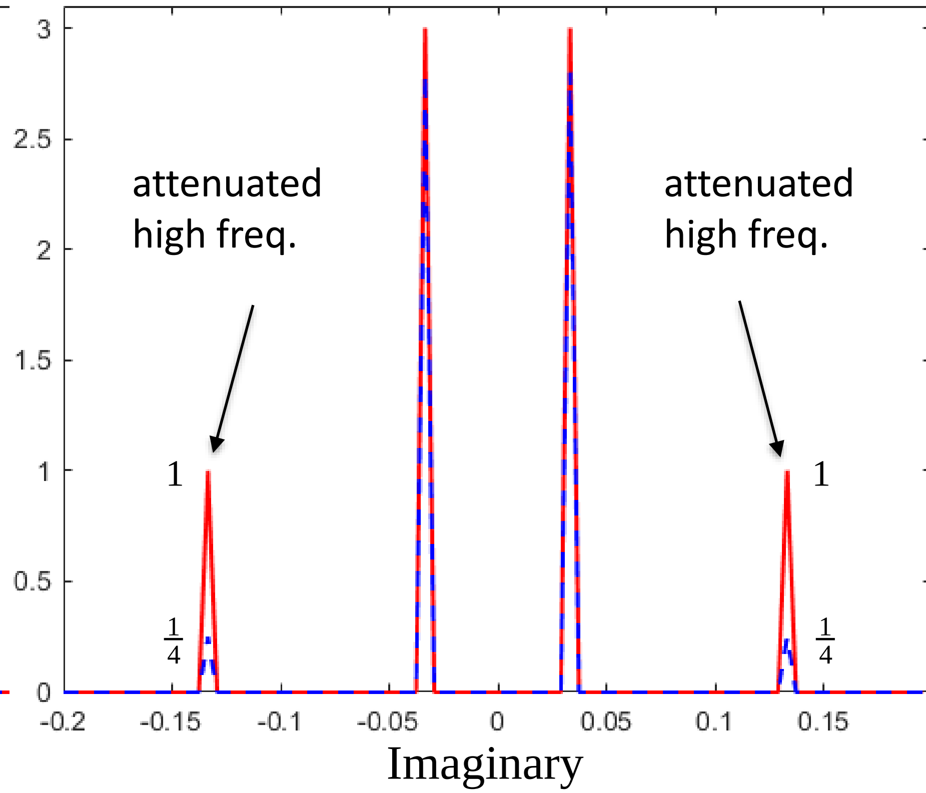
$\mathcal{F}\Bigg\{$  $\Bigg\} =$

original

$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + 1\cos\left(2\pi\frac{4}{240}t\right) + 3\sin\left(2\pi\frac{8}{240}t\right) + 1\sin\left(2\pi\frac{32}{240}t\right)$$

$$\sin\left(2\pi\frac{32}{240}t\right)$$



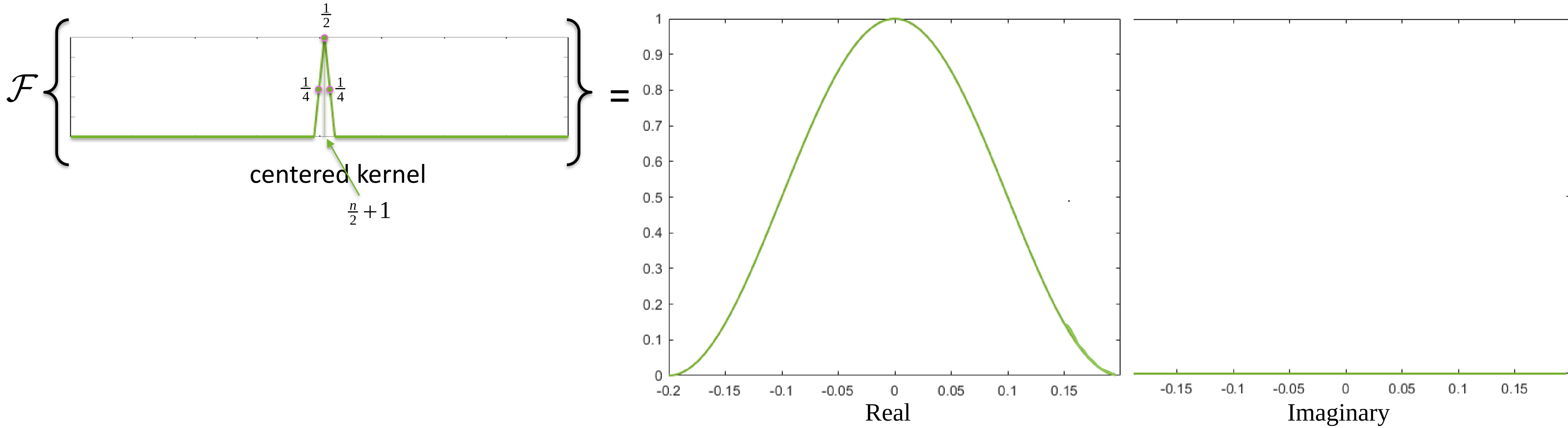


*coefficients divided by $n/2$ for display

Time Series Convolution via the DFT

To demonstrate the convolution theorem, we take the discrete Forward Fourier transform of the centered kernel …

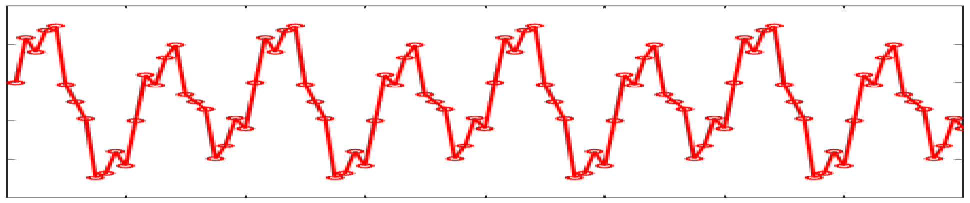
1/4	1/2	1/4



Time Series Convolution via the DFT

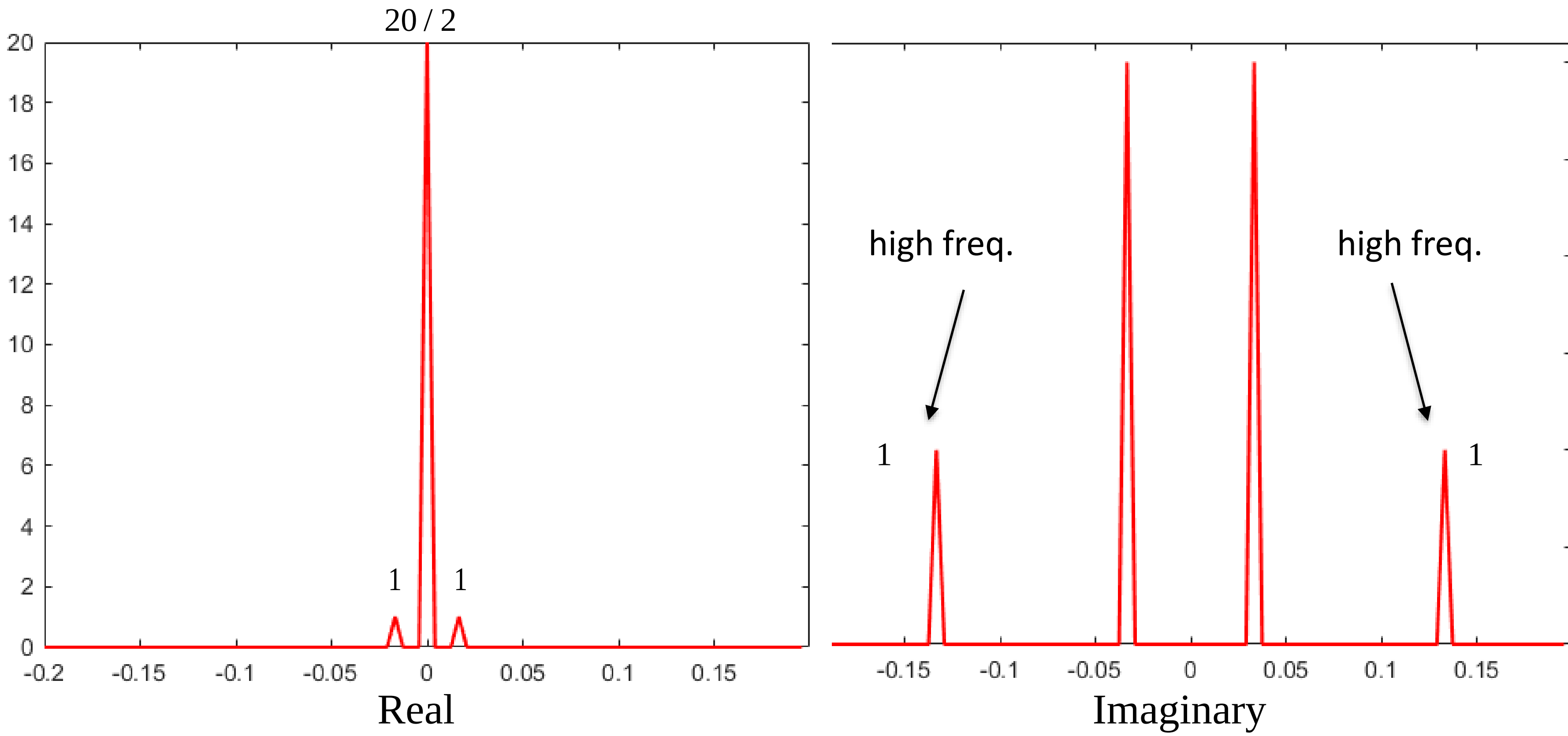
… and the forward discrete Fourier transform of the original input time series …

$\mathcal{F}\Bigg\{$



original

$\Bigg\} =$



$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + 1\cos\left(2\pi\frac{4}{240}t\right) + 3\sin\left(2\pi\frac{8}{240}t\right) + 1\sin\left(2\pi\frac{32}{240}t\right)$$

*coefficients divided by $n/2$ for display

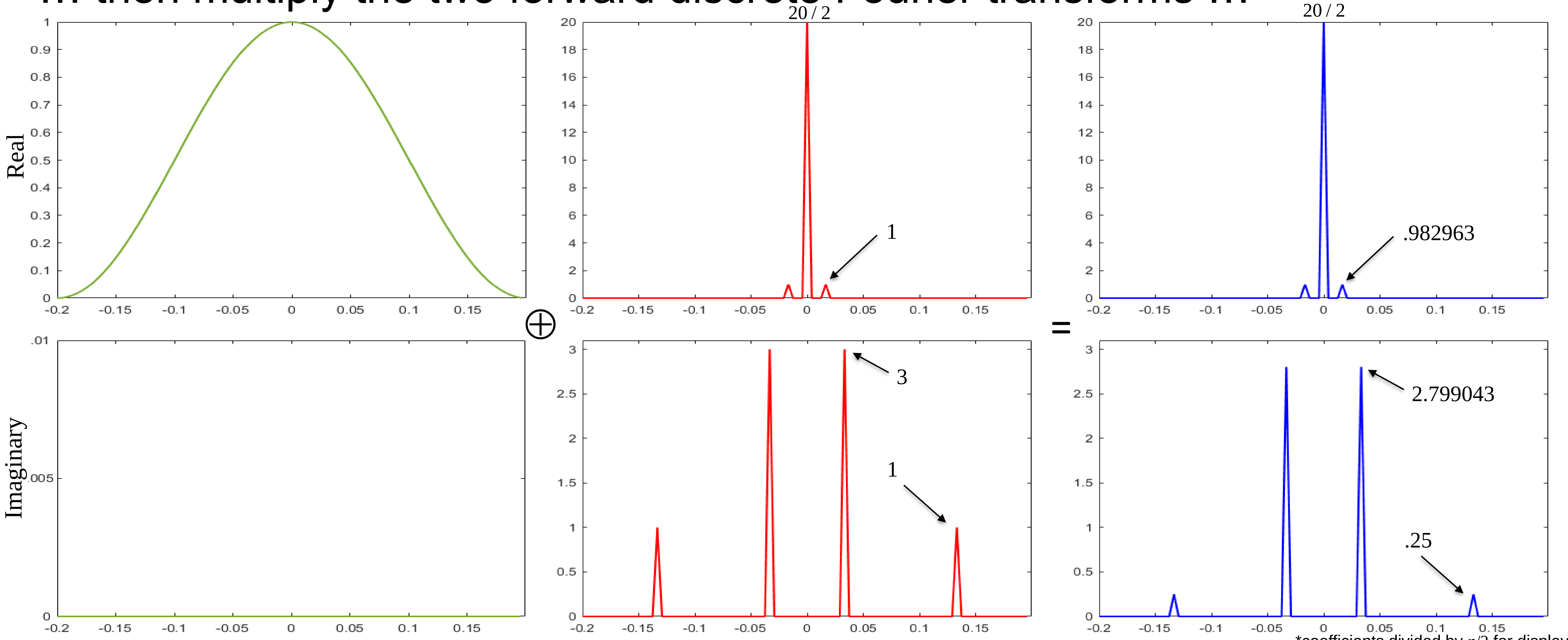
Time Series Convolution via the DFT

$$y(t)=10\cos\left(2\pi\frac{0}{240}t\right)+1\cos\left(2\pi\frac{4}{240}t\right)+3\sin\left(2\pi\frac{8}{240}t\right)+1\sin\left(2\pi\frac{32}{240}t\right)$$

$\oplus$ = direct product, element-wise complex multiplication

".*" in Matlab

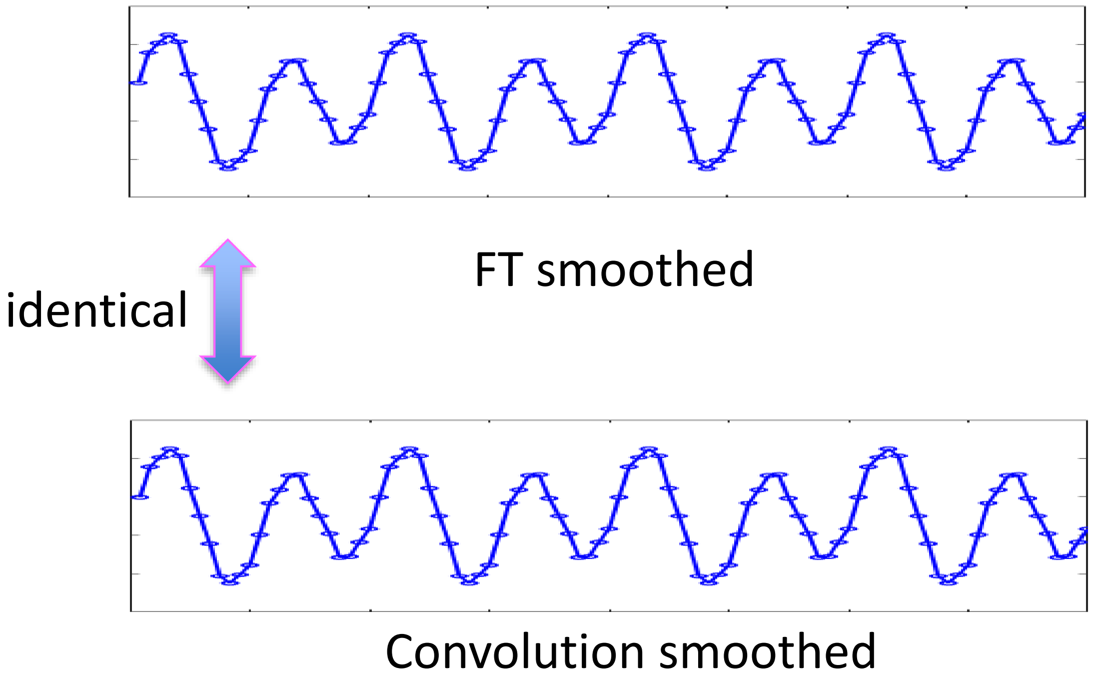
… then multiply the two forward discrete Fourier transforms …



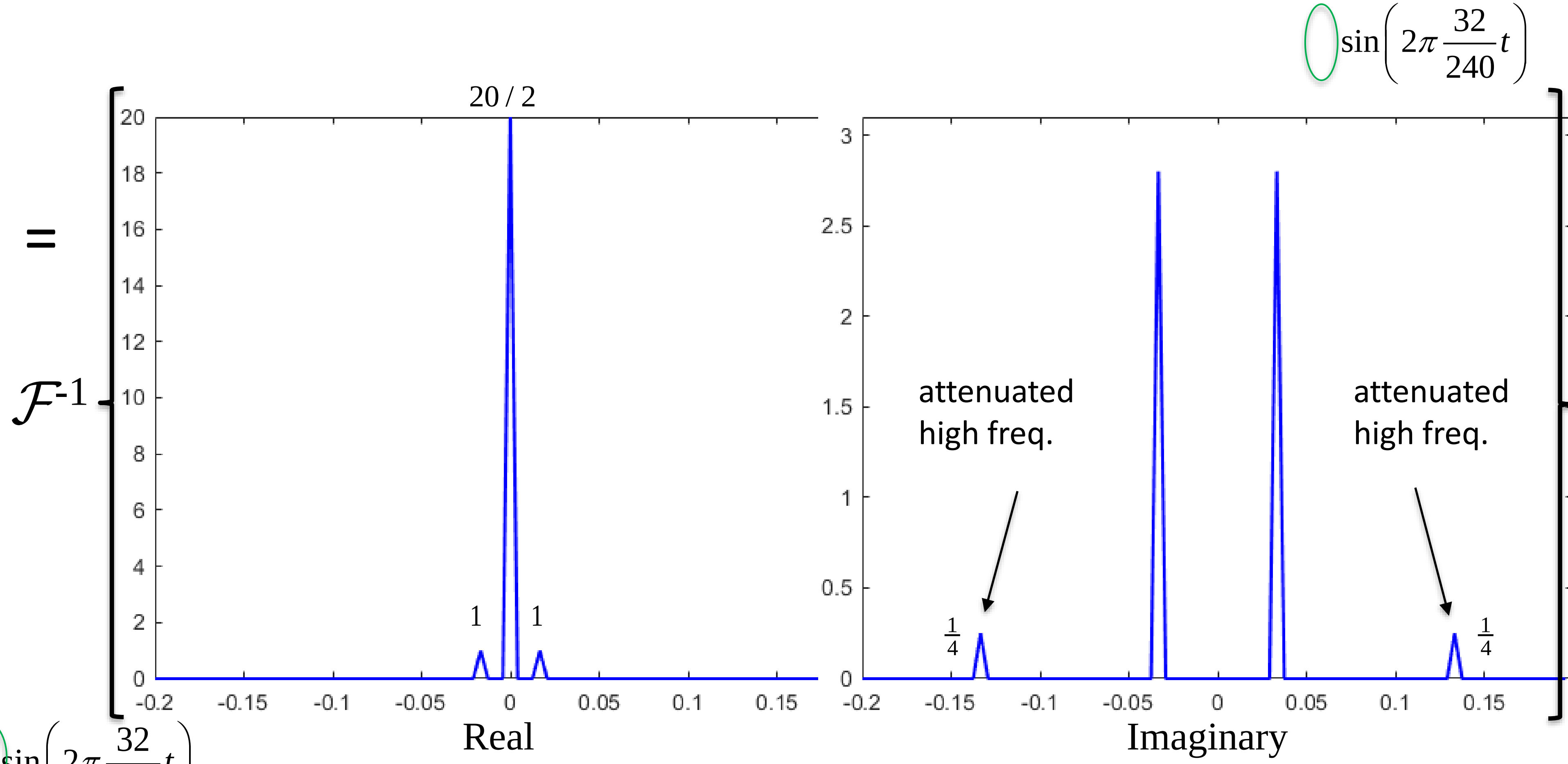
*coefficients divided by $n/2$ for display

Time Series Convolution via the DFT

… and inverse discrete Fourier transform to obtain the smoothed time series.



=

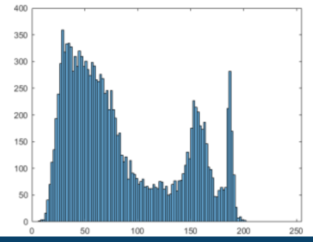


$$y(t) = 10\cos\left(2\pi\frac{0}{240}t\right) + .98\cos\left(2\pi\frac{4}{240}t\right) + 2.8\sin\left(2\pi\frac{8}{240}t\right) + .25\sin\left(2\pi\frac{32}{240}t\right)$$

*coefficients divided by $n/2$ for display

Image Convolution Example

We can use convolution in image space to compute a local weighted mean.



$\overline{x}_5$

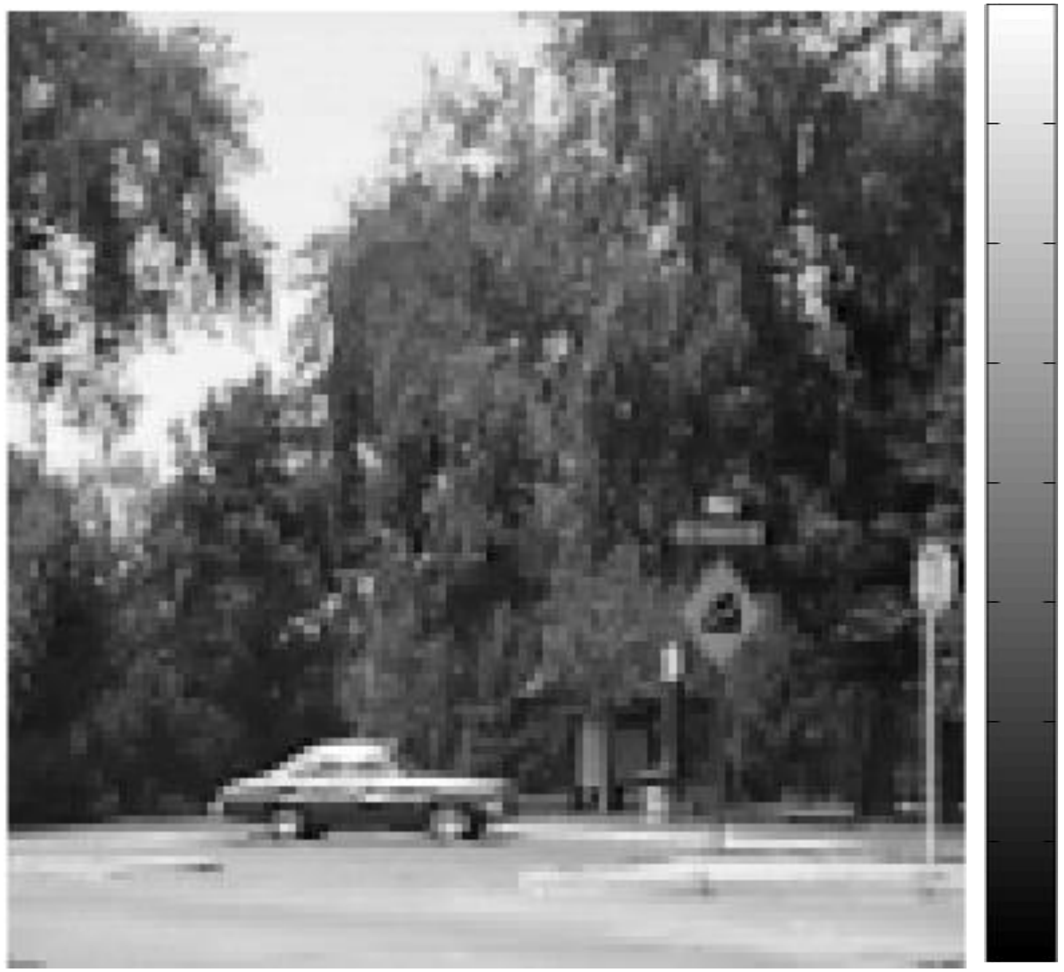
=

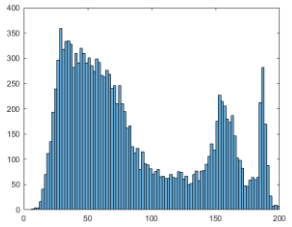
0	1/5	0
1/5	1/5	1/5
0	1/5	0

Kernel

$*$

convolution



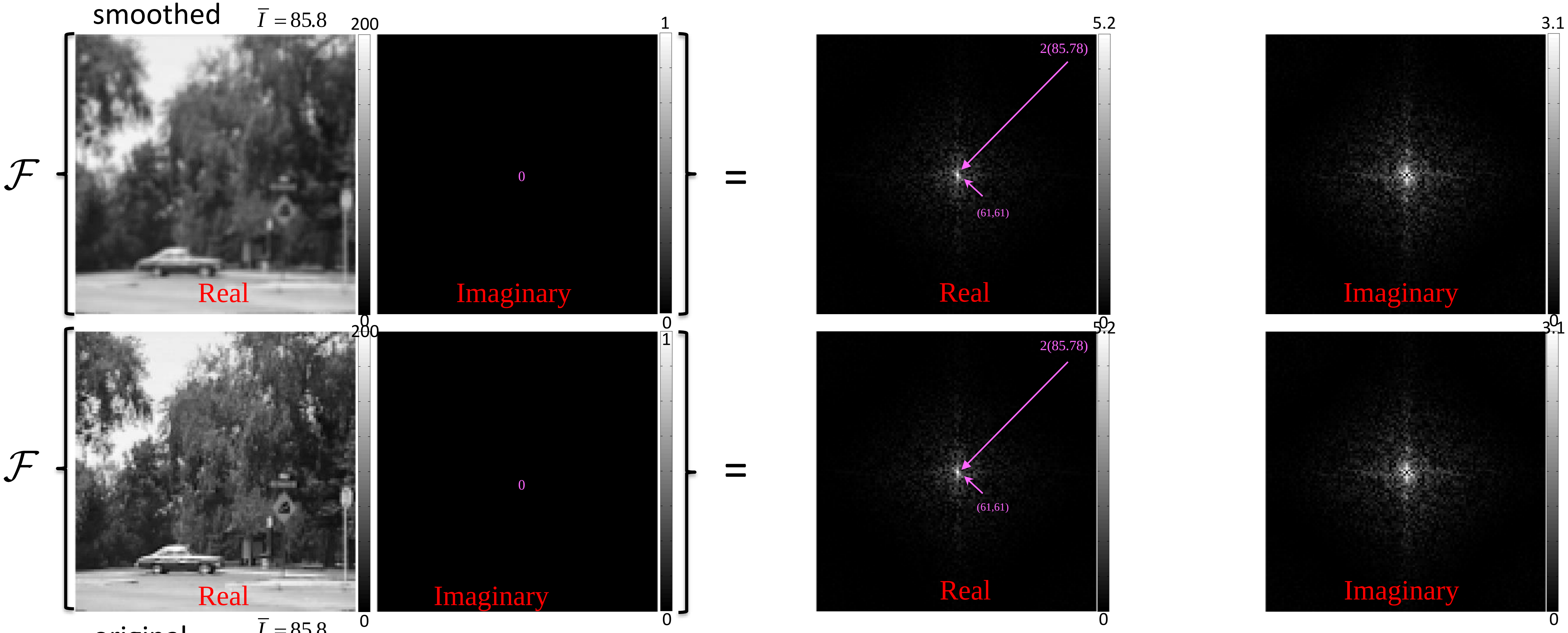


x_i

Image Convolution Example

120×120 image

We can Fourier transform of the smoothed image and compare it to that of the original input image to see high frequency coefficient attenuation.



*coefficients divided by $n_x n_y/2$ & log(abs(f)+1) for display

Image Convolution Example

⊕ = direct product, element
-wise complex multiplication

… then multiply the two forward discrete Fourier transform together …

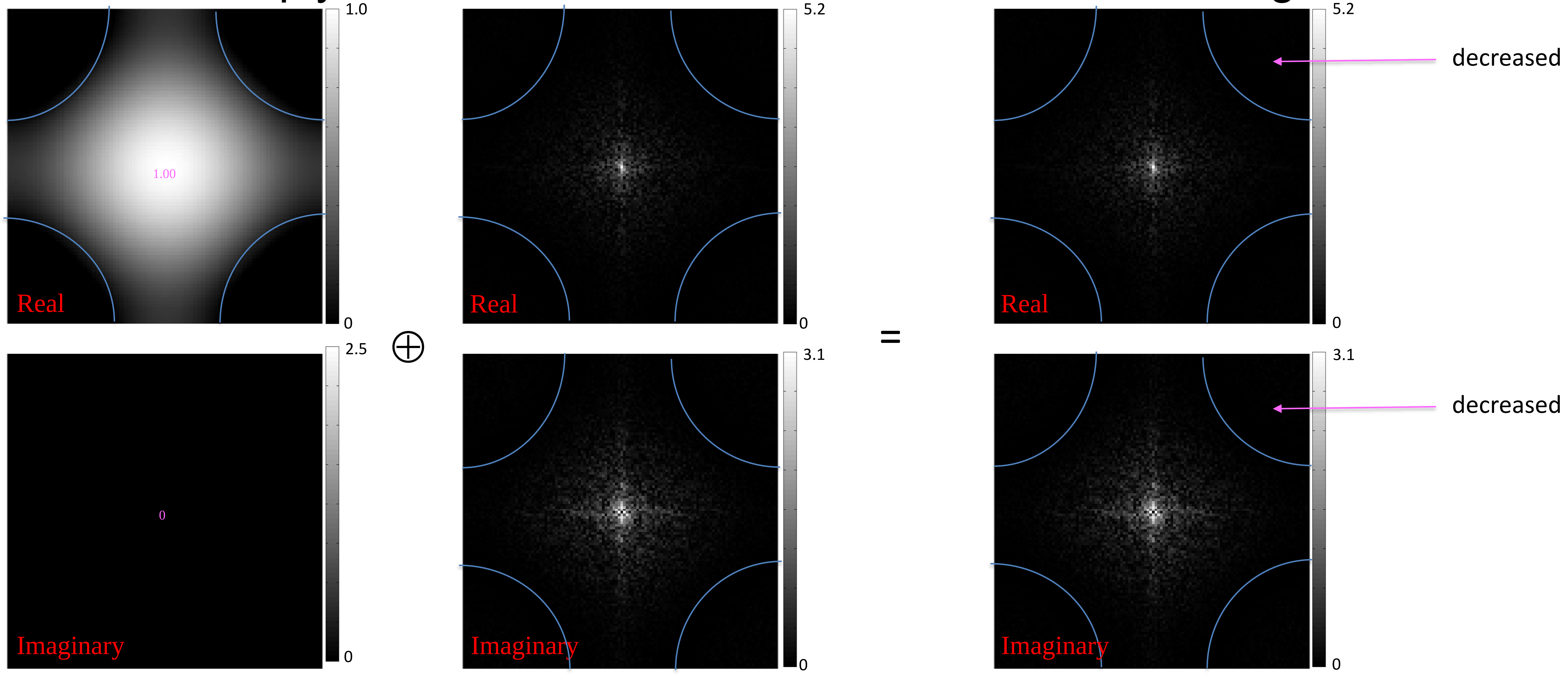
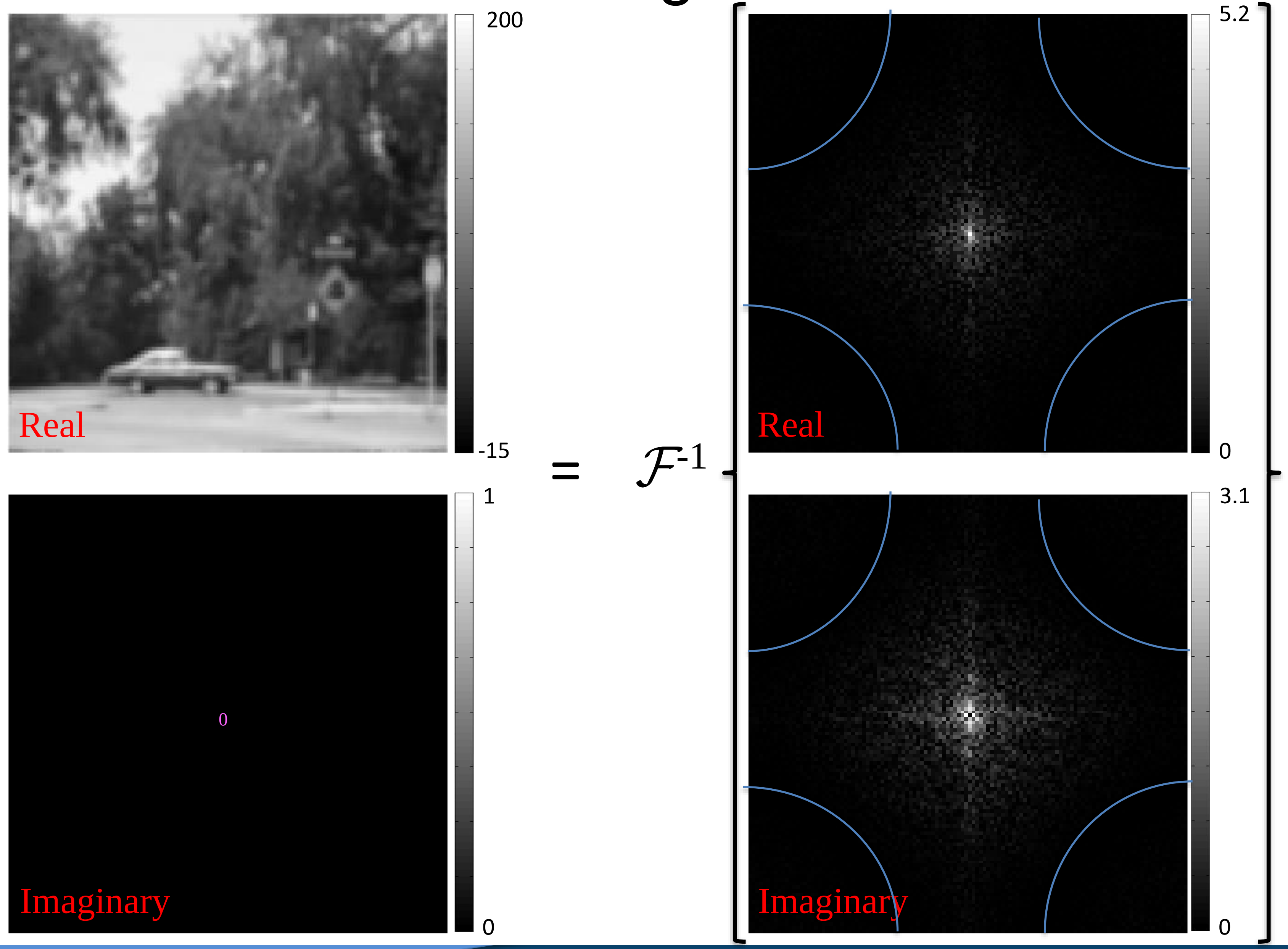


Image Convolution Example

… and inverse discrete Fourier transform to get our smoothed image.

This process yields the same result as convolution of the image with a kernel in image assuming wrap around!



Fast Object Tracking

Dr. Daniel B. Rowe
Professor of Computational Statistics
Department of Mathematical and Statistical Sciences
Marquette University

Template Matching via DFT

$$s^2 = \frac{\sum x_i^2 - \left(\sum x_i\right)^2 / n}{n-1}$$

We are going to use the DFT property to track cars with template matching.

For our object template we can pre-compute

$\sum o_i \qquad \sum o_i^2$

pre-compute once

compute each frame for all neighborhoods

Then using the DFT compute

$\sum p_i \qquad \sum p_i^2 \qquad \sum o_i p_i$

$$r = \frac{\sum p_i o_i - \frac{1}{n}\left(\sum p_i\right)\left(\sum o_i\right)}{\sqrt{\sum p_i^2 - \frac{1}{n}\left(\sum p_i\right)^2}\sqrt{\sum o_i^2 - \frac{1}{n}\left(\sum o_i\right)^2}}$$

Template Matching via DFT

We now have all the pieces that we need.

$\sum o_i = 10358226$

$\sum o_i^2 = 1929989924$

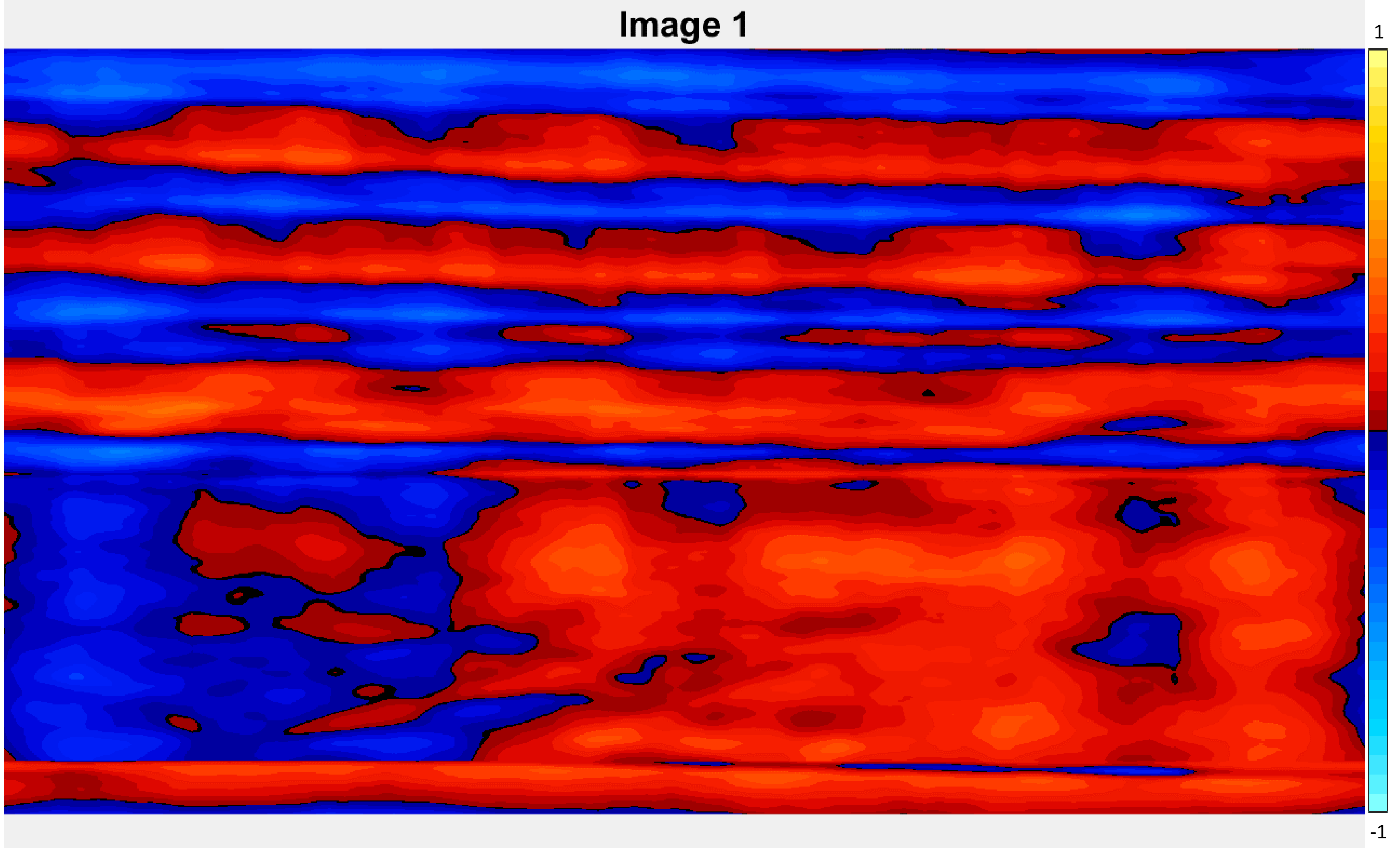
$\sum p_i$

$\sum p_i^2$

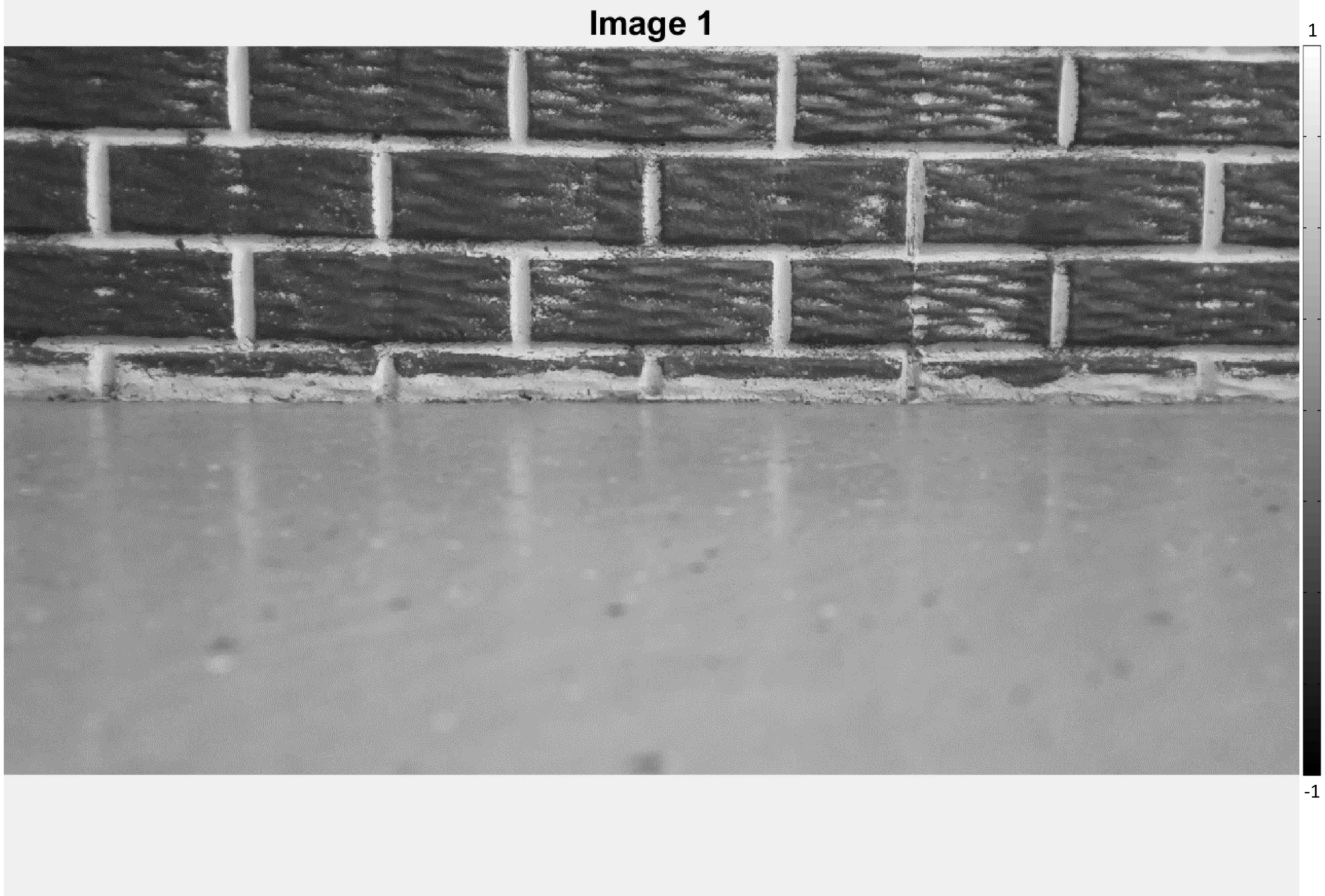
$\sum o_i p_i$

$$r = \frac{\sum p_i o_i - \frac{1}{n}\left(\sum p_i\right)\left(\sum o_i\right)}{\sqrt{\sum p_i^2 - \frac{1}{n}\left(\sum p_i\right)^2}\sqrt{\sum o_i^2 - \frac{1}{n}\left(\sum o_i\right)^2}}$$

Template Matching via DFT



Template Matching via DFT



Peaks, Valleys, and Ridges

Dr. Daniel B. Rowe
Professor of Computational Statistics
Department of Mathematical and Statistical Sciences
Marquette University

Introduction

A recent Marquette University graduate just landed their first job to develop an image recognition system to place eggs in an egg carton.

Image Critical Points

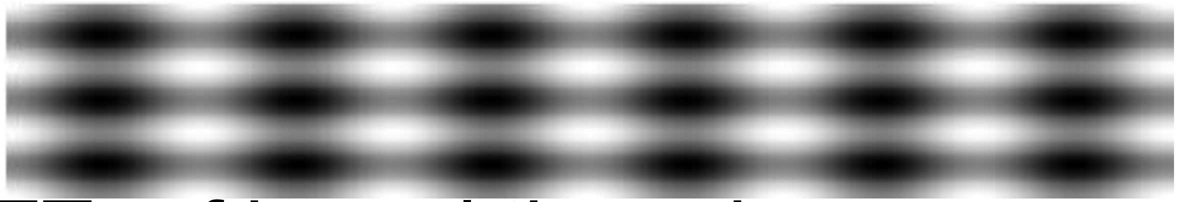
In the same way that we can find critical points for a function, we can find critical points in an image.

We can calculate all of the necessary discrete derivatives using kernels.

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}/6 \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}/6 \qquad \begin{bmatrix} 1 & -2 & 1 \\ 2 & -4 & 2 \\ 1 & -2 & 1 \end{bmatrix}/16 \qquad \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 1 & 2 & 1 \end{bmatrix}/16 \qquad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}/16$$

$$f_x = \frac{df(x,y)}{dx} \qquad f_y = \frac{df(x,y)}{dy} \qquad f_{xx} = \frac{d^2 f(x,y)}{dx^2} \qquad f_{yy} = \frac{d^2 f(x,y)}{dy^2} \qquad f_{xy} = \frac{d^2 f(x,y)}{dxy}$$

Image Critical Points

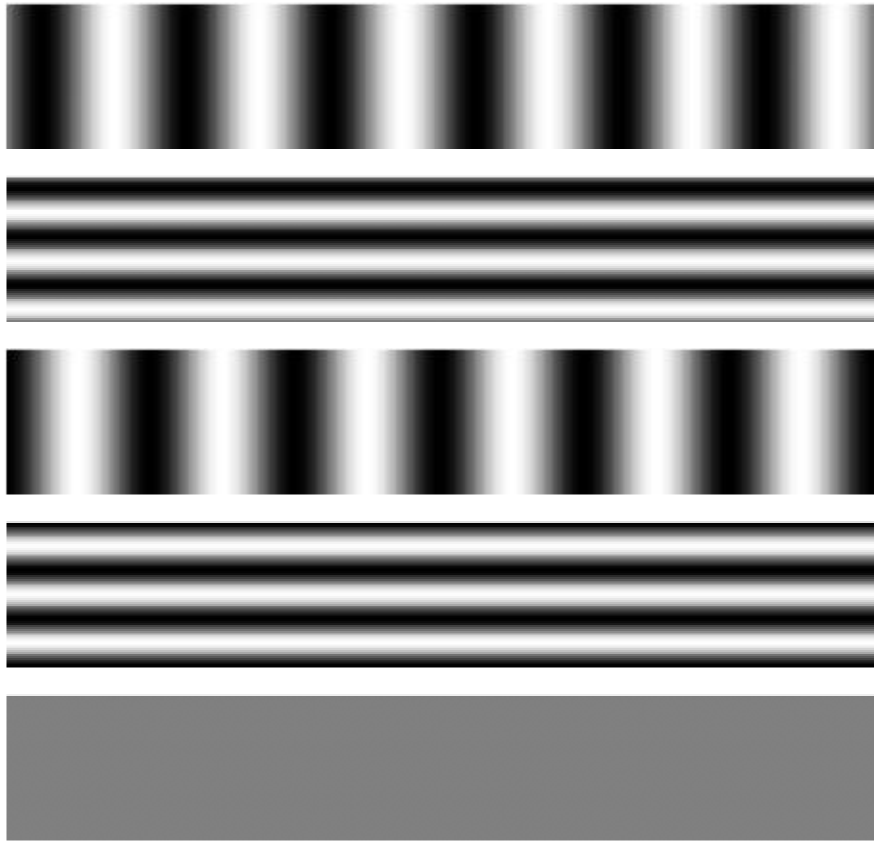
$I =$ 

Returning to the graduate's new project, the DFTs of kernel times image

Derivative images using kernels.

$$\mathcal{F}\{K\}\mathcal{F}\{I\}$$

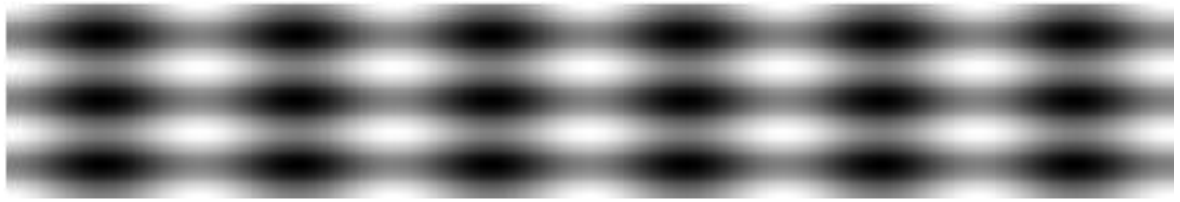
$I_x(x,y)$, $I_y(x,y)$, $I_{xx}(x,y)$, $I_{yy}(x,y)$, $I_{xy}(x,y)$



$\mathcal{F}^{-1}$ $\mathcal{F}^{-1}$ $\mathcal{F}^{-1}$ $\mathcal{F}^{-1}$ $\mathcal{F}^{-1}$

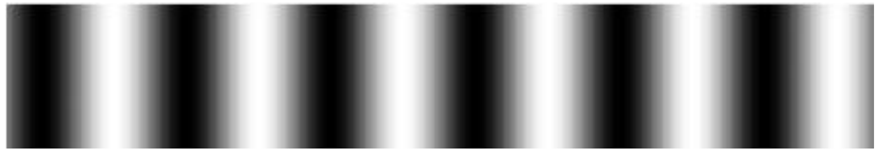
$\longleftarrow$

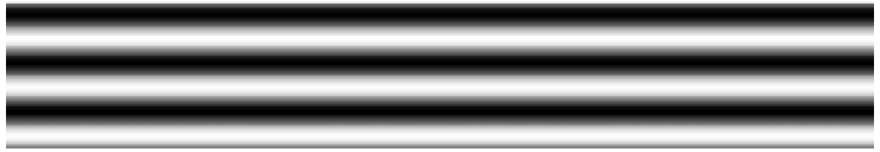
Image Critical Points

$I=$ 

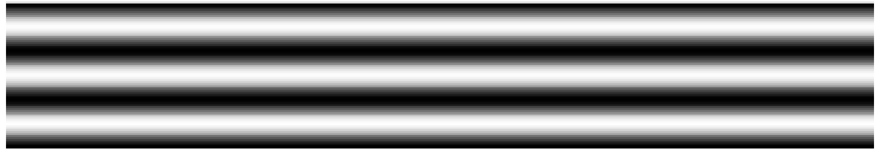
Continuing with the graduate's new project,

Derivative images using kernels.

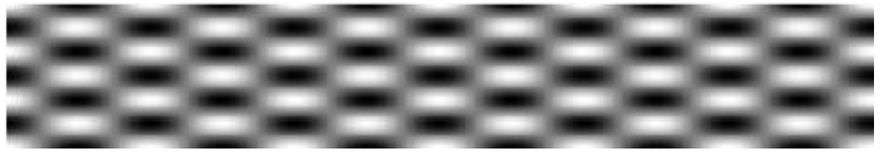
$I_x(x,y)$ 

$I_y(x,y)$ 

$I_{xx}(x,y)$ 

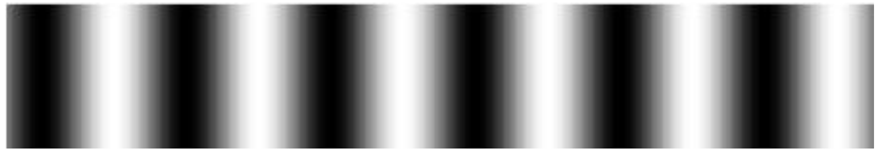
$I_{yy}(x,y)$ 

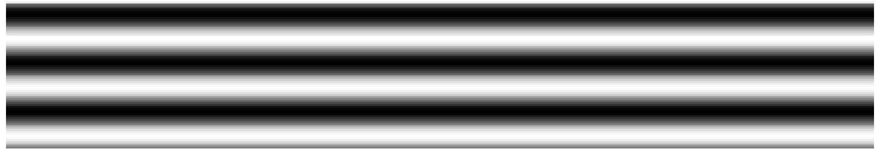
$I_{xy}(x,y)$ 

$I_D(x,y)$ 

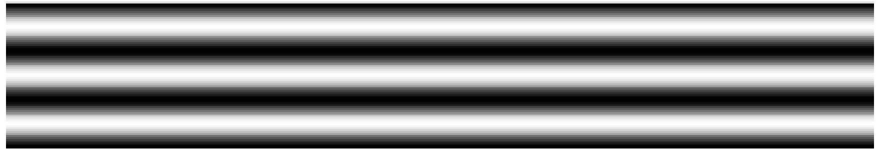
compare

Derivative images using Calculus.

$f_x(x,y)$ 

$f_y(x,y)$ 

$f_{xx}(x,y)$ 

$f_{yy}(x,y)$ 

$f_{xy}(x,y)$ 

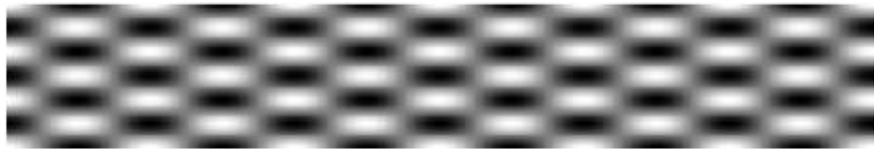
$f_D(x,y)$ 

Image Critical Points

This process can be applied to other images

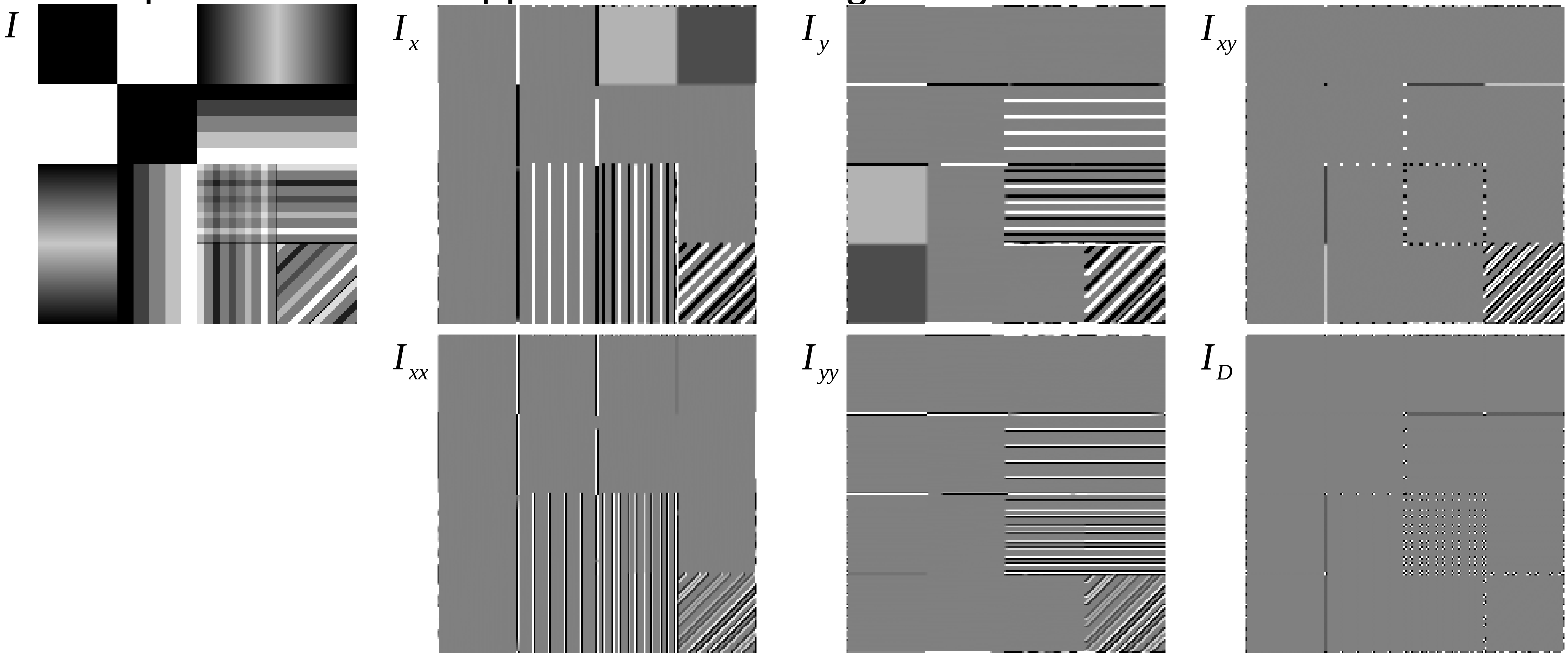
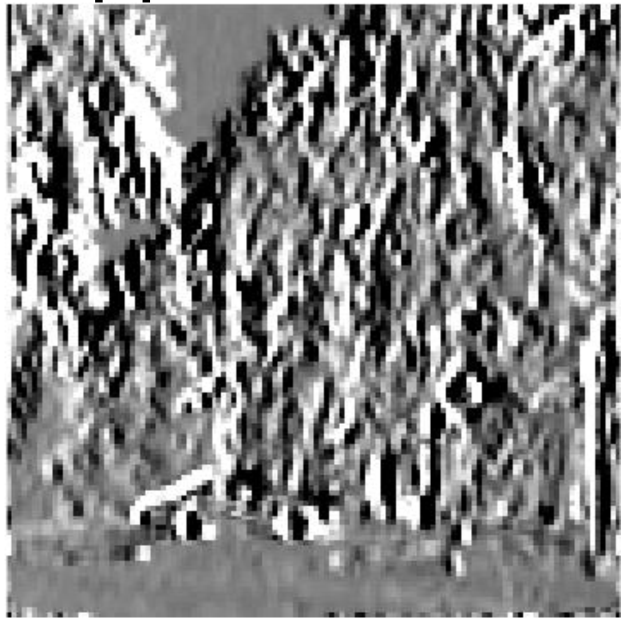


Image Critical Points

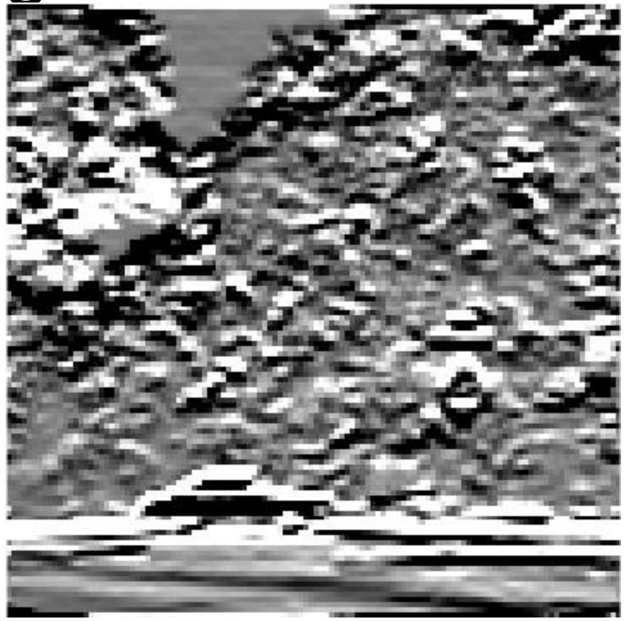
This process can be applied to other images

I

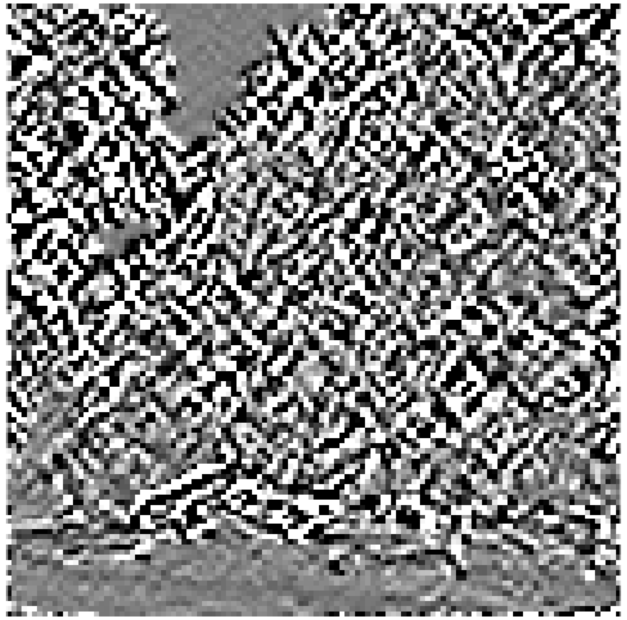
I_x



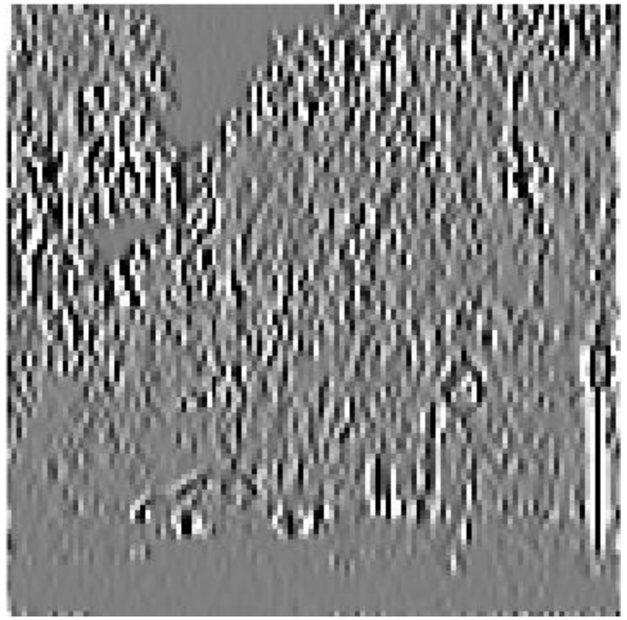
I_y



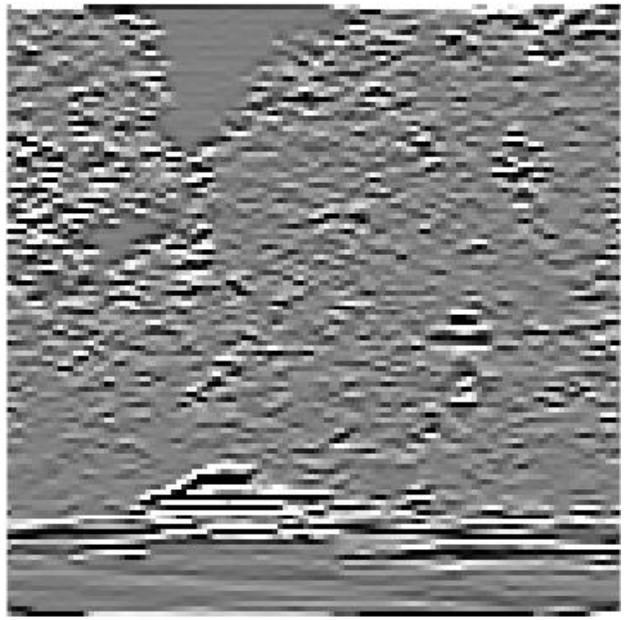
I_{xy}



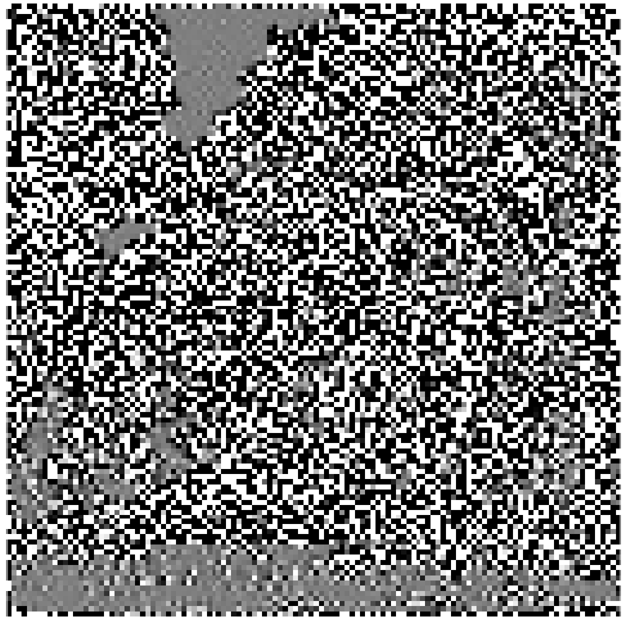
I_{xx}



I_{yy}



I_D



Discussion

We’ve taken a long journey through many topics.

We are able to load and process images in Matlab.

Convolution via the DFT can accomplish: image smoothing
image sharpening
image frequency decomposition
image template matching
image cartography

In this course you have learned a foundation for a lifetime of imaging.

Discussion

Thank You!