# Chapter 10: Nonparametric Tests II Supplement

Dr. Daniel B. Rowe
Professor of Computational Statistics
Department of Mathematical and Statistical Sciences
Marquette University

# 10.3 Tests with Matched Samples – Sign Test

We learned the parametric matched difference hypothesis test,

$H_0$: $\mu_d \leq 0$ *vs*. $H_1$: $\mu_d > 0$ (prove greater than), $\mu_d = \mu_1 - \mu_2$

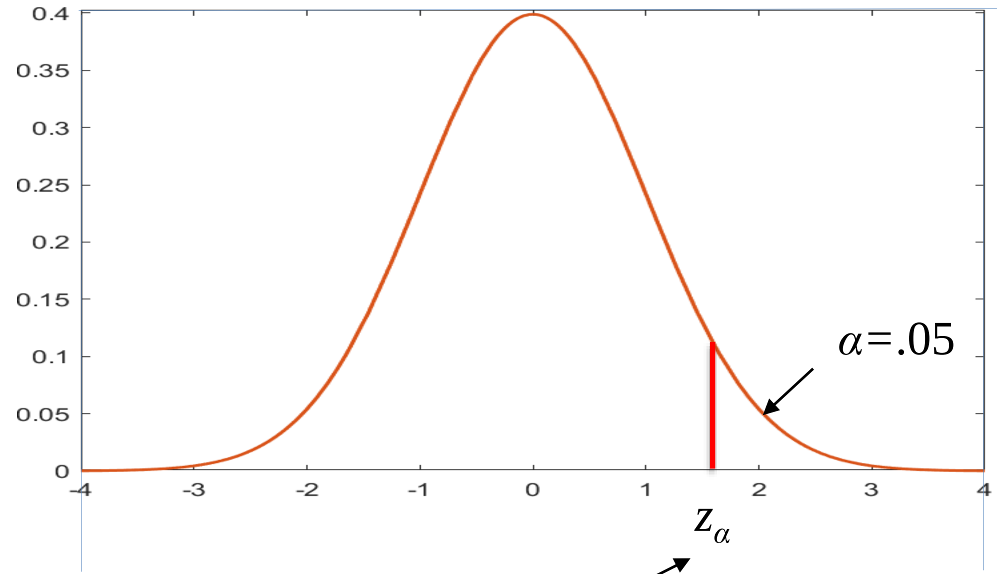
We know for the parametric test, we reject for “large" average differences

$\bar{X}_d = \frac{1}{n}\sum_{i=1}^{n} d_i$ or $z = \frac{\bar{X}_d}{s / \sqrt{n}}$ , (assuming $n$ large).

# 10.3 Tests with Matched Samples – Sign Test

$H_0$: $\mu_d \leq 0$ *vs*. $H_1$: $\mu_d > 0$ (prove greater than), $\mu_d = \mu_1 - \mu_2$

When we calculate $z = \dfrac{\bar{X}_d}{s/\sqrt{n}}$, we see where it lies




The $z$ value that has an area $\alpha$ larger than it.

We look up the $z$ value in the table.

$Z_\alpha$ = z value with area $\alpha$ less than
$z_\alpha$ = z value with area $\alpha$ greater than

$Z_\alpha = -1.645$
$z_\alpha = +1.645$

$z_\alpha = -Z_\alpha$

| $Z_i$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |

Table 1.

So we need to either multiply $Z_\alpha$=-1.645 by -1 to get 1.645 or turn it into $H_0$: $\mu_d = 0$ *vs*. $H_1$: $\mu_d < 0$ by using $-\bar{X}_d$ and $z = \dfrac{-\bar{X}_d}{s/\sqrt{n}}$, $\mu_d = \mu_2 - \mu_1$.

# 10.3 Tests with Matched Samples – Sign Test

The same thing occurs in nonparametric testing with the sign test.

$H_0: \delta=0$ *vs*. $H_1: \delta > 0$ (prove greater than), $\delta=MD_1-MD_2$

For the nonparametric sign test, we reject for a “large" number of differences greater than $0$.

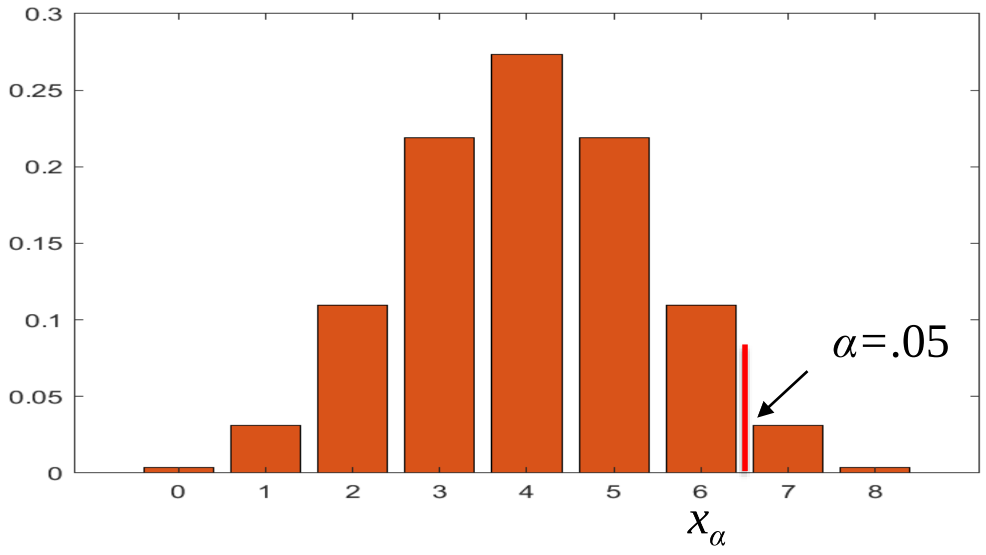
$x$ = (the number of differences > 0) .

# 10.3 Tests with Matched Samples – Sign Test

| x | P(X=x) | CumSum | CumSumR |
|---|---|---|---|
| 0 | 0.004 | 0.004 | 1.000 |
| 1 | 0.031 | 0.035 | 0.996 |
| 2 | 0.109 | 0.145 | 0.965 |
| 3 | 0.219 | 0.363 | 0.856 |
| 4 | 0.273 | 0.637 | 0.637 |
| 5 | 0.219 | 0.856 | 0.363 |
| 6 | 0.109 | 0.965 | 0.145 |
| 7 | 0.031 | 0.996 | 0.035 |
| 8 | 0.004 | 1.000 | 0.004 |

$H_0$: $\mu_d \leq 0$ *vs*. $H_1$: $\mu_d > 0$ (prove greater than), $\delta = MD_1 - MD_2$

When we calculate $x = \#\ d\text{'s} > 0$, we see where it lies




The $x$ value that has an area $\alpha$ larger than it.

We look up the x value in the table.

$X_\alpha$ = x value with area $\alpha$ less than $X_\alpha = 1$

$x_\alpha$ = x value with area $\alpha$ greater than $x_\alpha = 7$

$x_\alpha = n - X_\alpha$

| One-Sided Test $\alpha$ | .05 | .025 | .01 | .005 |
|---|---|---|---|---|
| $n$ | | | | |
| 8 | 1 | 0 | 0 | 0 |

Table 6.

So we need to either subtract $X_\alpha$=1 from $n$=8 to get 7 or turn it into $H_0$: $\mu_d = 0$ *vs*. $H_1$: $\mu_d < 0$

by using $x = \#\ d\text{'s} < 0$, $\delta = MD_2 - MD_1$.

# 10.5 Summary

**Sign Test** (one sample)

$x$ = number of observations $> MD_0$
$<$

**Mann-Whitney U Test**

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

$$U = \min(U_1, U_2)$$

**Sign Test** (two sample)

$x$ = number of differences $> 0$
$<$

**Wilcoxon Signed Rank Test**
(two sample)

$W = \min(W+, W-)$
$W+$ = sum of positive ranks
$W-$ = sum of negative ranks

**Kruskal-Wallis Test**
(three or more samples)

$$H = \left( \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} \right) - 3(N+1)$$

$R_j$ = sum of ranks for sample $j$.

# Questions?

# Homework 10

Read Chapter 10.

Problems # 6 (Sign Test), 7 (Wilcoxon Signed Rank Test),
8 (Kruskal-Wallis Test) the $n_1=n_2=n_3=n_4=5$ critical value is $7.377$.