Chapter 3 Quantifying the Extent of Disease

$$Point\ Prevalence\ (PP) = \frac{Number\ of\ persons\ with\ disease}{Number\ of\ persons\ examined\ at\ baseline}$$

$$Cumulative\ Incidence\ (CI) = \frac{Number\ of\ persons\ who\ develop\ disease\ during\ specified\ period}{Number\ of\ persons\ at\ risk\ at\ baseline}$$

$$Incidence\ Rate\ (IR) = \frac{Number\ of\ persons\ who\ develop\ disease\ during\ specified\ period}{Sum\ of\ lengths\ of\ time\ during\ which\ persons\ are\ disease\ free}$$

$$Risk\ Difference\ (RD) = PP_{exposed} - PP_{unexposed} = CI_{exposed} - CI_{unexposed} = IR_{exposed} - IR_{unexposed}$$

$$Population\ Attributable\ Risk\ (PAR) = \frac{PP_{overall} - PP_{unexposed}}{PP_{overall}} = \frac{CI_{overall} - CI_{exposed}}{CI_{overall}} = \frac{IR_{overall} - IR_{exposed}}{IR_{overall}}$$

$$Relative\ Risk\ (RR) = \frac{PP_{exposed}}{PP_{unexposed}} = \frac{CI_{exposed}}{CI_{unexposed}} = \frac{IR_{exposed}}{IR_{unexposed}}$$

$$Odds\ Ratio\ (OR) = \frac{PP_{exposed} / (1 - PP_{exposed})}{PP_{unexposed} / (1 - PP_{unexposed})} = \frac{CI_{exposed} / (1 - CI_{exposed})}{CI_{unexposed} / (1 - CI_{unexposed})}$$

Chapter 4 Summarizing Data Collected in the Sample

Mean: $\bar{X} = \frac{\sum X}{n}$

Standard Deviation: $s = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}} = \sqrt{\frac{\sum X^2 - \frac{1}{n}\left(\sum X\right)^2}{n-1}}$

Median: Middle Value in Ordered Set (Q_2)

First Quartile: Q_1 = Value holding 25% below

Third Quartile: Q_3 = Value holding 75% below

Interquartile Range: $IQR = Q_3 - Q_1$

Criteria for Outliers: Values below $Q_1 - .5\ IQR$ or above $Q_3 + 1.5\ IQR$

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Chapter 5 The Role of Probability

$Basic\ Probability = P(Characteristic) = (Number\ of\ persons\ with\ characteristic) / N$

$Sensitivity = P(screen\ positive \mid disease) = a/(a+c)$

$\text{Specificity} = P(screen\ negative \mid disease\ \ free) = d/(b+d)$

$False\ Positive\ Fraction = P(screen\ positive \mid disease\ free) = b/(b+d)$

$False\ Negative\ Fraction = P(screen\ negative \mid disease) = c/(a+c)$

$Positive\ Predictive\ Value = P(disease \mid screen\ positive) = a/(a+b)$

$Negative\ Predictive\ Value = P(disease\ \ free \mid screen\ negative) = d/(c+d)$

$Independent\ Events{:}\ P(A \mid B) = P(A)\ or\ P(B \mid A) = P(B)$

$Binomial\ Distribution: P(x\ successes) = \dfrac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$Standard\ Normal\ Distribution: z = (x-\mu)/\sigma \quad (Table\ 1)$

$Percentiles\ of\ the\ Normal\ Distribution: X = \mu + z\sigma \quad (Table\ 1A)$

$Application\ of\ Central\ Limit\ Theorem: z = \dfrac{x-\mu}{\sigma/\sqrt{n}} \quad (Table\ 1)$

Chapter 6 Confidence Interval Estimates

Number of Groups: Parameter	Confidence Interval, $n<30$	Confidence Interval, $n\geq30$
One sample, continuous: CI for μ	$\bar{X} \pm t_{\frac{\alpha}{2},df} \dfrac{s}{\sqrt{n}}$	$\bar{X} \pm z_{\frac{\alpha}{2}} \dfrac{s}{\sqrt{n}}$
One sample, dichotomous: CI for p	(Not taught in this class.)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$
Two independent samples, continuous: CI for μ_1-μ_2	$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},df} S_P \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$ $S_P = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$ $df = n_1 + n_2 - 2$	$(\bar{X}_1 - \bar{X}_2) \pm z_{\frac{\alpha}{2}} S_P \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$ $S_P = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$
Two matched samples, continuous: CI for μ_d=μ_1-μ_2	$\bar{X}_d \pm t_{\frac{\alpha}{2},df} \dfrac{s_d}{\sqrt{n}}$, $df=n-1$	$\bar{X}_d \pm z_{\frac{\alpha}{2}} \dfrac{s_d}{\sqrt{n}}$
One sample, continuous: CI for μ	$\bar{X} \pm t_{\frac{\alpha}{2},df} \dfrac{s}{\sqrt{n}}$	$\bar{X} \pm z_{\frac{\alpha}{2}} \dfrac{s}{\sqrt{n}}$
One sample, dichotomous: CI for p	(Not taught in this class.)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

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Chapter 6 Confidence Interval Estimates

Number of Groups: Parameter	Confidence Interval, $n<30$	Confidence Interval, $n\geq 30$
Two independent samples, continuous: CI for μ_1-μ_2	$(\bar{X}_1-\bar{X}_2)\pm t_{\frac{\alpha}{2},df} S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_P=\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ $df=n_1+n_2-2$	$(\bar{X}_1-\bar{X}_2)\pm z_{\frac{\alpha}{2}} S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_P=\sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$
Two matched samples, continuous: CI for $\mu_d=\mu_1$-μ_2	$\bar{X}_d\pm t_{\frac{\alpha}{2},df}\frac{s_d}{\sqrt{n}}$, $df=n-1$	$\bar{X}_d\pm z_{\frac{\alpha}{2}}\frac{s_d}{\sqrt{n}}$
Two independent samples, dichotomous: CI for $RD=(p_1$-$p_2)$	(Not taught in this class.)	$(\hat{p}_1-\hat{p}_2)\pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
CI for $ln(RR)=ln(p_1/p_2)$	(Not taught in this class.)	$\ln(RR)\pm z_{\frac{\alpha}{2}}\sqrt{\frac{(n_1-X_1)/X_1}{n_1}+\frac{(n_2-X_2)/X_2}{n_2}}$
CI for $RR=p_1/p_2$	(Not taught in this class.)	$\exp(Lower\ Limit), \exp(Upper\ \ Limit)$
CI for $ln(OR)=ln([p_1/(1-p_1)]/[p_2/(1-p_2)])$	(Not taught in this class.)	$ln(OR)\pm z_{\frac{\alpha}{2}}\sqrt{\frac{1}{X_1}+\frac{1}{n_1-X_1}+\frac{1}{X_2}+\frac{1}{n_2-X_2}}$
CI for $OR=[p_1/(1-p_1)]/[p_2/(1-p_2)]$	(Not taught in this class.)	$\exp(Lower\ Limit), \exp(Upper\ \ Limit)$

Chapter 7 Hypothesis Testing Procedures

Number of Groups: Parameter	Test Statistic, $n<30$	Test Statistic, $n\geq 30$
One sample, continuous: μ	$t=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$, $df=n-1$ (Technically assume normal data.)	$z=\frac{\bar{X}-\mu_0}{s/\sqrt{n}}$
One sample, dichotomous: p	Binomial Test (Not taught in this class.)	$z=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$
One Sample, Categorical and Ordinal: $p_1,\ldots,p_k$	Multinomial Test (Not taught in this class.)	$\chi^2=\sum\frac{(O-E)^2}{E}$, $df=k-1$

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Number of Groups, Outcome: Parameter	Test Statistic, $n<30$	Test Statistic, $n\geq30$
Two independent samples, continuous: $\mu_1 = \mu_2$	$t = \frac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$, $df=n_1+n_2-2$ (Technically assumes populations are normal, independent within and between populations, and population variances equal.)	$z = \frac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$, $S_P = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$
Two matched samples, continuous: μ_d	$t = \frac{\bar{X}_d}{s_d/\sqrt{n}}$, $df=n-1$ (Technically assumes data is normal, pairs independent of each other, variances equal between populations.)	$z = \frac{\bar{X}_d}{s_d/\sqrt{n}}$, $\bar{X}_d = \frac{1}{n}\sum_{i=1}^{n} d_i$, $s_d = \sqrt{\frac{\Sigma d^2-(\Sigma d)^2/n}{n-1}}$
Two independent samples, dichotomous: $p_1=p_2$	Multinomial Test (Not taught in this class.)	$z = \frac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$, $\hat{p}_1 = \frac{X_1}{n_1}$, $\hat{p}_2 = \frac{X_2}{n_2}$, $\hat{p} = \frac{X_1+X_2}{n_1+n_2}$
More than two samples, continuous: $\mu_1 = \ldots = \mu_k$	Same as large sample. (Technically assumes populations are normal, independent within and between populations, and population variances equal.)	$F = \frac{MSB}{MSE}$, $df_1=k-1$, $df_2=N-k$ $MSB = \frac{1}{k-1}\sum_{j=1}^{k} n_j(\bar{X}_j-\bar{X})^2$ $MSE = \frac{1}{N-k}\sum_{j=1}^{k}\sum_{i=1}^{n_j} n_j(X_i-\bar{X}_j)^2$
Two or more samples, Categorical and Ordinal: $p_{11},\ldots,p_{rc}$	Multinomial Test (Not taught in this class.)	$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$, $df=(r-1)(c-1)$

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Chapter 9 Multivariable Models

Correlation Coefficient: r	$r = \frac{\text{cov}(x,y)}{\sqrt{s_x^2 s_y^2}}$ $\text{cov}(x,y) = \frac{1}{n-1}\left[\sum XY - \frac{1}{n}\left(\sum Y\right)\left(\sum X\right)\right]$ $s_x^2 = \frac{1}{n-1}\left[\sum X^2 - \frac{1}{n}\left(\sum X\right)^2\right]$, $s_y^2 = \frac{1}{n-1}\left[\sum Y^2 - \frac{1}{n}\left(\sum Y\right)^2\right]$
Linear Regression: $\hat{y} = b_0 + b_1 x$	$b_1 = r\frac{s_y}{s_x}$, $b_0 = \bar{Y} - b_1\bar{X}$
Logistic Regression: $\hat{p} = \frac{1}{1+e^{-b_0-b_1x_1-\ldots-b_px_p}}$	$ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = b_0 + b_1x_1 + \ldots + b_px_p$ $\hat{OR} = e^{\hat{\beta}_1\Delta_1+\ldots+\hat{\beta}_p\Delta_p}$

Chapter 10 Nonparametric Tests

Sign Test: $MD=MD_0$ (One Sample)	x = number of observations > MD_0 If value<MD_0, –. If value=MD_0, 0. If value>MD_0, +.
Mann-Whitney U Test: $MD_1=MD_2$ (not-Paired)	$U_1 = n_1n_2 + \frac{n_1(n_1+1)}{2} - R_1$ $U_2 = n_1n_2 + \frac{n_2(n_2+1)}{2} - R_2$ $U = \min(U_1, U_2)$
Sign Test: $\delta=0$ (Paired)	If difference<0, –. If difference=0, 0. If difference>0, +. If H_1: $\delta>0$, x = number of observations > 0. If H_1: $\delta<0$, x = number of observations < 0. Use min(–,+) and Table 6.
Wilcoxon Signed Rank Test: $\delta=0$ (Paired)	$W = \min(W+, W-)$ $W+$ = sum of positive ranks $W-$ = sum of negative ranks
Kruskal-Wallis Test: $MD_1=\ldots=MD_k$ (ANOVA)	$H = \left(\frac{12}{N(N+1)}\sum_{j=1}^{k}\frac{R_j^2}{n_j}\right) - 3(N+1)$

Chapter 11 Survival Analysis

Actuarial Life Table:	N_t = # *event free during interval t* D_t = # *who die in interval t* C_t = # *censored in interval t* N_{t^*} = *avg. # at risk in interval t*, $N_{t^*}=N_t-C_t/2$ q_t = *prop. die in interval t*, $q_t=D_t/N_{t^*}$ p_t = *prop. survive in interval t*, $p_t=1-q_t$ S_t = *prop. survive past interval t*
Kaplan-Meier Life Table:	$S_{t+1}=S_t\frac{N_t-D_t}{N_t}$ $SE(S_t)=S_t\sqrt{\sum\frac{D_t}{N_t(N_t-D_t)}}$
Chi-Square Test: k=# groups $\sum_{t=1}^{T}O_{ij}$ =Obs. deaths in Group j $\sum_{t=1}^{T}E_{ij}$ =Exp. deaths in Group j	$\chi^2=\sum_{j=1}^{2}\frac{\left(\sum_{t=1}^{T}O_{ij}-\sum_{t=1}^{T}E_{ij}\right)^2}{\sum_{t=1}^{T}E_{ij}}$, $df=k-1$
Cox Proportional Hazards Model:	$h(t)=h_0(t)\exp(b_1x_1+b_2x_2+\ldots+b_px_p)$

Table 1: Standard Normal z

Table entries represent $P(Z < Z_i)$
e.g., $P(Z < -1.96) = 0.0250$, $P(Z < 1.96) = 0.9750$

Z_i	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table entries represent $P(Z < Z_i)$
e.g., $P(Z < -1.96) = 0.0250$, $P(Z < 1.96) = 0.9750$

Z_i	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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Table 2: t Distribution

Table entries represent values from t distribution with upper-tail area equal to α.
e.g., $P(t_{df} > t) = \alpha$, e.g., $P(t_6 > 1.943) = 0.05$

Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test α	.20	.10	.05	.02	.01
One-Sided Test α	.10	.05	.025	.01	.005
df					
1	3.078	6.314	12.71	31.82	63.66
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

Table entries represent values from t distribution with upper-tail area equal to α.
e.g., $P(t_{df} > t) = \alpha$, e.g., $P(t_6 > 1.943) = 0.05$

Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test α	.20	.10	.05	.02	.01
One-Sided Test α	.10	.05	.025	.01	.005
df					
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690
46	1.300	1.679	2.013	2.410	2.687
47	1.300	1.678	2.012	2.408	2.685
48	1.299	1.677	2.011	2.407	2.682
49	1.299	1.677	2.010	2.405	2.680
50	1.299	1.676	2.009	2.403	2.678
51	1.298	1.675	2.008	2.402	2.676
52	1.298	1.675	2.007	2.400	2.674
53	1.298	1.674	2.006	2.399	2.672
54	1.297	1.674	2.005	2.397	2.670
55	1.297	1.673	2.004	2.396	2.668
56	1.297	1.673	2.003	2.395	2.667
57	1.297	1.672	2.002	2.394	2.665
58	1.296	1.672	2.002	2.392	2.663
59	1.296	1.671	2.001	2.391	2.662
60	1.296	1.671	2.000	2.390	2.660

Table entries represent values from t distribution with upper-tail area equal to α.
e.g., $P(t_{df} > t) = \alpha$, e.g., $P(t_6 > 1.943) = 0.05$

Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test α	.20	.10	.05	.02	.01
One-Sided Test α	.10	.05	.025	.01	.005
df					
61	1.296	1.670	2.000	2.389	2.659
62	1.295	1.670	1.999	2.388	2.657
63	1.295	1.669	1.998	2.387	2.656
64	1.295	1.669	1.998	2.386	2.655
65	1.295	1.669	1.997	2.385	2.654
66	1.295	1.668	1.997	2.384	2.652
67	1.294	1.668	1.996	2.383	2.651
68	1.294	1.668	1.995	2.382	2.650
69	1.294	1.667	1.995	2.382	2.649
70	1.294	1.667	1.994	2.381	2.648
71	1.294	1.667	1.994	2.380	2.647
72	1.293	1.666	1.993	2.379	2.646
73	1.293	1.666	1.993	2.379	2.645
74	1.293	1.666	1.993	2.378	2.644
75	1.293	1.665	1.992	2.377	2.643
∞	1.282	1.645	1.960	2.326	2.576

Table 3: χ^2 Distribution

Table entries represent values from χ^2 distribution with upper tail area equal to α.
$P(\chi^2_{df} > \chi^2) = \alpha$, e.g., $P(\chi^2_3 > 7.81) = 0.05$

	α				
df	.10	.05	.025	.01	.005
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84
4	7.78	9.49	11.14	13.28	14.86
5	9.24	11.07	12.83	15.09	16.75
6	10.64	12.59	14.45	16.81	18.55
7	12.02	14.07	16.01	18.48	20.28
8	13.36	15.51	17.53	20.09	21.95
9	14.68	16.92	19.02	21.67	23.59
10	15.99	18.31	20.48	23.21	25.19
11	17.28	19.68	21.92	24.72	26.76
12	18.55	21.03	23.34	26.22	28.30
13	19.81	22.36	24.74	27.69	29.82
14	21.06	23.68	26.12	29.14	31.32
15	22.31	25.00	27.49	30.58	32.80
16	23.54	26.30	28.85	32.00	34.27
17	24.77	27.59	30.19	33.41	35.72
18	25.99	28.87	31.53	34.81	37.16
19	27.20	30.14	32.85	36.19	38.58
20	28.41	31.41	34.17	37.57	40.00
21	29.62	32.67	35.48	38.93	41.40
22	30.81	33.92	36.78	40.29	42.80
23	32.01	35.17	38.08	41.64	44.18
24	33.20	36.42	39.36	42.98	45.56
25	34.38	37.65	40.65	44.31	46.93
26	35.56	38.89	41.92	45.64	48.29
27	36.74	40.11	43.19	46.96	49.64
28	37.92	41.34	44.46	48.28	50.99
29	39.09	42.56	45.72	49.59	52.34
30	40.26	43.77	46.98	50.89	53.67
40	51.81	55.76	59.34	63.69	66.77
50	63.17	67.50	71.42	76.15	79.49
60	74.40	79.08	83.30	88.38	91.95
70	85.53	90.53	95.02	100.4	104.2
80	96.58	101.9	106.6	112.3	116.3
90	107.6	113.1	118.1	124.1	128.3
100	118.5	124.3	129.6	135.8	140.2

Table 4: F Distribution

$P(F_{df_1, df_2} > F) = 0.05$,
e.g., $P(F_{3,20} > 3.10) = 0.05$

	df_1													
df_2	1	2	3	4	5	6	7	8	9	10	20	30	40	50
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	248.0	250.1	251.1	251.8
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.45	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.28	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.23	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.19	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.16	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.12	2.04	1.99	1.97
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.10	2.01	1.96	1.94
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.07	1.98	1.94	1.91
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.05	1.96	1.91	1.88
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.03	1.94	1.89	1.86
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.01	1.92	1.87	1.84
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	1.99	1.90	1.85	1.82
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	1.97	1.88	1.84	1.81
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	1.96	1.87	1.82	1.79
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	1.94	1.85	1.81	1.77
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	1.93	1.84	1.79	1.76

$P(F_{df_1, df_2} > F) = 0.05$,
e.g., $P(F_{3,20} > 3.10) = 0.05$

df_2 \ df_1	1	2	3	4	5	6	7	8	9	10	20	30	40	50
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	1.92	1.83	1.78	1.75
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	1.91	1.82	1.77	1.74
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	1.90	1.81	1.76	1.72
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	1.89	1.80	1.75	1.71
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	1.88	1.79	1.74	1.70
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11	1.87	1.78	1.73	1.69
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.14	2.10	1.86	1.77	1.72	1.68
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	1.85	1.76	1.71	1.68
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08	1.85	1.75	1.70	1.67
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.84	1.74	1.69	1.66
41	4.08	3.23	2.83	2.60	2.44	2.33	2.24	2.17	2.12	2.07	1.83	1.74	1.69	1.65
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	1.83	1.73	1.68	1.65
43	4.07	3.21	2.82	2.59	2.43	2.32	2.23	2.16	2.11	2.06	1.82	1.72	1.67	1.64
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	1.81	1.72	1.67	1.63
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	1.81	1.71	1.66	1.63
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.15	2.09	2.04	1.80	1.71	1.65	1.62
47	4.05	3.20	2.80	2.57	2.41	2.30	2.21	2.14	2.09	2.04	1.80	1.70	1.65	1.61
48	4.04	3.19	2.80	2.57	2.41	2.29	2.21	2.14	2.08	2.03	1.79	1.70	1.64	1.61
49	4.04	3.19	2.79	2.56	2.40	2.29	2.20	2.13	2.08	2.03	1.79	1.69	1.64	1.60
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.78	1.69	1.63	1.60
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96	1.71	1.61	1.55	1.52
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.68	1.57	1.52	1.48

MATH 4740/MSSC 5740 Formula Sheet

Binomial Probabilities p=1/2

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0.250	0.125	0.063	0.031	0.016	0.008	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.500	0.375	0.250	0.156	0.094	0.055	0.031	0.018	0.010	0.005	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.250	0.375	0.375	0.313	0.234	0.164	0.109	0.070	0.044	0.027	0.016	0.010	0.006	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3		0.125	0.250	0.313	0.313	0.273	0.219	0.164	0.117	0.081	0.054	0.035	0.022	0.014	0.009	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000
4			0.063	0.156	0.234	0.273	0.273	0.246	0.205	0.161	0.121	0.087	0.061	0.042	0.028	0.018	0.012	0.007	0.005	0.003	0.002	0.001	0.001	0.000
5				0.031	0.094	0.164	0.219	0.246	0.246	0.226	0.193	0.157	0.122	0.092	0.067	0.047	0.033	0.022	0.015	0.010	0.006	0.004	0.003	0.002
6					0.016	0.055	0.109	0.164	0.205	0.226	0.226	0.209	0.183	0.153	0.122	0.094	0.071	0.052	0.037	0.026	0.018	0.012	0.008	0.005
7						0.008	0.031	0.070	0.117	0.161	0.193	0.209	0.209	0.196	0.175	0.148	0.121	0.096	0.074	0.055	0.041	0.029	0.021	0.014
8							0.004	0.018	0.044	0.081	0.121	0.157	0.183	0.196	0.196	0.185	0.167	0.144	0.120	0.097	0.076	0.058	0.044	0.032
9								0.002	0.010	0.027	0.054	0.087	0.122	0.153	0.175	0.185	0.185	0.176	0.160	0.140	0.119	0.097	0.078	0.061
10									0.001	0.005	0.016	0.035	0.061	0.092	0.122	0.148	0.167	0.176	0.176	0.168	0.154	0.136	0.117	0.097
11										0.000	0.003	0.010	0.022	0.042	0.067	0.094	0.121	0.144	0.160	0.168	0.168	0.161	0.149	0.133
12											0.000	0.002	0.006	0.014	0.028	0.047	0.071	0.096	0.120	0.140	0.154	0.161	0.161	0.155
13												0.000	0.001	0.003	0.009	0.018	0.033	0.052	0.074	0.097	0.119	0.136	0.149	0.155
14													0.000	0.000	0.002	0.005	0.012	0.022	0.037	0.055	0.076	0.097	0.117	0.133
15														0.000	0.000	0.001	0.003	0.007	0.015	0.026	0.041	0.058	0.078	0.097
16															0.000	0.000	0.001	0.002	0.005	0.010	0.018	0.029	0.044	0.061
17																0.000	0.000	0.000	0.001	0.003	0.006	0.012	0.021	0.032
18																	0.000	0.000	0.000	0.001	0.002	0.004	0.008	0.014
19																		0.000	0.000	0.000	0.000	0.001	0.003	0.005
20																			0.000	0.000	0.000	0.000	0.001	0.002
21																				0.000	0.000	0.000	0.000	0.000
22																					0.000	0.000	0.000	0.000
23																						0.000	0.000	0.000
24																							0.000	0.000
25																								0.000

Binomial Cumulative Probabilities p=1/2

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0.250	0.125	0.063	0.031	0.016	0.008	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.750	0.500	0.313	0.187	0.109	0.063	0.035	0.020	0.011	0.006	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	1.000	0.875	0.688	0.500	0.344	0.227	0.145	0.090	0.055	0.033	0.019	0.011	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3		1.000	0.938	0.813	0.656	0.500	0.363	0.254	0.172	0.113	0.073	0.046	0.029	0.018	0.011	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000
4			1.000	0.969	0.891	0.773	0.637	0.500	0.377	0.274	0.194	0.133	0.090	0.059	0.038	0.025	0.015	0.010	0.006	0.004	0.002	0.001	0.001	0.000
5				1.000	0.984	0.938	0.855	0.746	0.623	0.500	0.387	0.291	0.212	0.151	0.105	0.072	0.048	0.032	0.021	0.013	0.008	0.005	0.003	0.002
6					1.000	0.992	0.965	0.910	0.828	0.726	0.613	0.500	0.395	0.304	0.227	0.166	0.119	0.084	0.058	0.039	0.026	0.017	0.011	0.007
7						1.000	0.996	0.980	0.945	0.887	0.806	0.709	0.605	0.500	0.402	0.315	0.240	0.180	0.132	0.095	0.067	0.047	0.032	0.022
8							1.000	0.998	0.989	0.967	0.927	0.867	0.788	0.696	0.598	0.500	0.407	0.324	0.252	0.192	0.143	0.105	0.076	0.054
9								1.000	0.999	0.994	0.981	0.954	0.910	0.849	0.773	0.685	0.593	0.500	0.412	0.332	0.262	0.202	0.154	0.115
10									1.000	1.000	0.997	0.989	0.971	0.941	0.895	0.834	0.760	0.676	0.588	0.500	0.416	0.339	0.271	0.212
11										1.000	1.000	0.998	0.994	0.982	0.962	0.928	0.881	0.820	0.748	0.668	0.584	0.500	0.419	0.345
12											1.000	1.000	0.999	0.996	0.989	0.975	0.952	0.916	0.868	0.808	0.738	0.661	0.581	0.500
13												1.000	1.000	1.000	0.998	0.994	0.985	0.968	0.942	0.905	0.857	0.798	0.729	0.655
14													1.000	1.000	1.000	0.999	0.996	0.990	0.979	0.961	0.933	0.895	0.846	0.788
15														1.000	1.000	1.000	0.999	0.998	0.994	0.987	0.974	0.953	0.924	0.885
16															1.000	1.000	1.000	1.000	0.999	0.996	0.992	0.983	0.968	0.946
17																1.000	1.000	1.000	1.000	0.999	0.998	0.995	0.989	0.978
18																	1.000	1.000	1.000	1.000	1.000	0.999	0.997	0.993
19																		1.000	1.000	1.000	1.000	1.000	0.999	0.998
20																			1.000	1.000	1.000	1.000	1.000	1.000
21																				1.000	1.000	1.000	1.000	1.000
22																					1.000	1.000	1.000	1.000
23																						1.000	1.000	1.000
24																							1.000	1.000
25																								1.000

Binomial Reverse Cumulative Probabilities p=1/2

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	0.750	0.875	0.938	0.969	0.984	0.992	0.996	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.250	0.500	0.688	0.813	0.891	0.938	0.965	0.980	0.989	0.994	0.997	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.000	0.125	0.313	0.500	0.656	0.773	0.855	0.910	0.945	0.967	0.981	0.989	0.994	0.996	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4		0.000	0.063	0.187	0.344	0.500	0.637	0.746	0.828	0.887	0.927	0.954	0.971	0.982	0.989	0.994	0.996	0.998	0.999	0.999	1.000	1.000	1.000	1.000
5				0.031	0.109	0.227	0.363	0.500	0.623	0.726	0.806	0.867	0.910	0.941	0.962	0.975	0.985	0.990	0.994	0.996	0.998	0.999	0.999	1.000
6					0.016	0.063	0.145	0.254	0.377	0.500	0.613	0.709	0.788	0.849	0.895	0.928	0.952	0.968	0.979	0.987	0.992	0.995	0.997	0.998
7						0.008	0.035	0.090	0.172	0.274	0.387	0.500	0.605	0.696	0.773	0.834	0.881	0.916	0.942	0.961	0.974	0.983	0.989	0.993
8							0.004	0.020	0.055	0.113	0.194	0.291	0.395	0.500	0.598	0.685	0.760	0.820	0.868	0.905	0.933	0.953	0.968	0.978
9								0.002	0.011	0.033	0.073	0.133	0.212	0.304	0.402	0.500	0.593	0.676	0.748	0.808	0.857	0.895	0.924	0.946
10									0.001	0.006	0.019	0.046	0.090	0.151	0.227	0.315	0.407	0.500	0.588	0.668	0.738	0.798	0.846	0.885
11										0.000	0.003	0.011	0.029	0.059	0.105	0.166	0.240	0.324	0.412	0.500	0.584	0.661	0.729	0.788
12											0.000	0.002	0.006	0.018	0.038	0.072	0.119	0.180	0.252	0.332	0.416	0.500	0.581	0.655
13												0.000	0.001	0.004	0.011	0.025	0.048	0.084	0.132	0.192	0.262	0.339	0.419	0.500
14													0.000	0.000	0.002	0.006	0.015	0.032	0.058	0.095	0.143	0.202	0.271	0.345
15														0.000	0.000	0.001	0.004	0.010	0.021	0.039	0.067	0.105	0.154	0.212
16															0.000	0.000	0.001	0.002	0.006	0.013	0.026	0.047	0.076	0.115
17																0.000	0.000	0.000	0.001	0.004	0.008	0.017	0.032	0.054
18																	0.000	0.000	0.000	0.001	0.002	0.005	0.011	0.022
19																		0.000	0.000	0.000	0.000	0.001	0.003	0.007
20																			0.000	0.000	0.000	0.000	0.001	0.002
21																				0.000	0.000	0.000	0.000	0.000
22																					0.000	0.000	0.000	0.000
23																						0.000	0.000	0.000
24																							0.000	0.000
25																								0.000

Table 5: Mann-Whitney U Test

Two-Sided Test $\alpha = 0.05$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2								0	0	0	0	1	1	1	1	1	2	2	2	2
3					0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4				0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7			1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8		0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9		0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
10		0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14		1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15		1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17		2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20		2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

One-Sided Test $\alpha = 0.05$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2					0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3			0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10	11
4			0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5		0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6		0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7		0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
8		1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
9		1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
10		1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
11		1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
12		2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
13		2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
14		2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
15		3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
16		3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
17		3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
18		4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

Table 6: Sign Test

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
n				
1				
2				
3				
4				
5	0			
6	0	0		
7	0	0	0	
8	1	0	0	0
9	1	1	0	0
10	1	1	0	0
11	2	1	1	0
12	2	2	1	1
13	3	2	1	1
14	3	2	2	1
15	3	3	2	2
16	4	3	2	2
17	4	4	3	2
18	5	4	3	3
19	5	4	4	3
20	5	5	4	3
21	6	5	4	4
22	6	5	5	4
23	7	6	5	4
24	7	6	5	5
25	7	7	6	5

Tabe 7: Wilcoxon-Signed Rank Test

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
n				
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

Table 8: Kruskal-Wallis Test

Three groups					Four groups					
n_1	n_2	n_3	$\alpha = .05$	$\alpha = .01$	n_1	n_2	n_3	n_4	$\alpha = .05$	$\alpha = .01$
2	2	2			2	2	1	1		
3	2	1			2	2	2	1	5.679	
3	2	2	4.714		2	2	2	2	6.167	6.667
3	3	1	5.143		3	1	1	1		
3	3	2	5.361		3	2	1	1		
3	3	3	5.600	7.200	3	2	2	1	5.833	
4	2	1			3	2	2	2	6.333	7.133
4	2	2	5.333		3	3	1	1	6.333	
4	3	1	5.208		3	3	2	1	6.244	7.200
4	3	2	5.444	6.444	3	3	2	2	6.527	7.636
4	3	3	5.791	6.745	3	3	3	1	6.600	7.400
4	4	1	4.967	6.667	3	3	3	2	6.727	8.015
4	4	2	5.455	7.036	3	3	3	3	7.000	8.538
4	4	3	5.598	7.144	4	1	1	1		
4	4	4	5.692	7.654	4	2	1	1	5.833	
5	2	1	5.000		4	2	2	1	6.133	7.000
5	2	2	5.160	6.533	4	2	2	2	6.545	7.391
5	3	1	4.960		4	3	1	1	6.178	7.067
5	3	2	5.251	6.909	4	3	2	1	6.309	7.455
5	3	3	5.648	7.079	4	3	2	2	6.621	7.871
5	4	1	4.985	6.955	4	3	3	1	6.545	7.758
5	4	2	5.273	7.205	4	3	3	2	6.795	8.333
5	4	3	5.656	7.445	4	3	3	3	6.984	8.659
5	4	4	5.657	7.760	4	4	1	1	5.945	7.909
5	5	1	5.127	7.309	4	4	2	1	6.386	7.909
5	5	2	5.338	7.338	4	4	2	2	6.731	8.346
5	5	3	5.705	7.578	4	4	3	1	6.635	8.231
5	5	4	5.666	7.823	4	4	3	2	6.874	8.621
5	5	5	5.780	8.000	4	4	3	3	7.038	8.876
6	1	1			4	4	4	1	6.725	8.588
6	2	1	4.822		4	4	4	2	6.957	8.871
6	2	2	5.345	6.655	4	4	4	3	7.142	9.075
6	3	1	4.855	6.873	4	4	4	4	7.235	9.287

Three groups					Five groups						
n_1	n_2	n_3	$\alpha = .05$	$\alpha = .01$	n_1	n_2	n_3	n_4	n_5	$\alpha = .05$	$\alpha = .01$
6	3	2	5.348	6.970	2	2	1	1	1		
6	3	3	5.615	7.410	2	2	2	1	1	6.750	
6	4	1	4.947	7.106	2	2	2	2	1	7.133	7.533
6	4	2	5.340	7.340	2	2	2	2	2	7.418	8.291
6	4	3	5.610	7.500	3	1	1	1	1		
6	4	4	5.681	7.795	3	2	1	1	1	6.583	
6	5	1	4.990	7.182	3	2	2	1	1	6.800	7.600
6	5	2	5.338	7.376	3	2	2	2	1	7.309	8.127
6	5	3	5.602	7.590	3	2	2	2	2	7.682	8.682
6	5	4	5.661	7.936	3	3	1	1	1	7.111	
6	5	5	5.729	8.028	3	3	2	1	1	7.200	8.073
6	6	1	4.945	7.121	3	3	2	2	1	7.591	8.576
6	6	2	5.410	7.467	3	3	2	2	2	7.910	9.115
6	6	3	5.625	7.725	3	3	3	1	1	7.576	8.424
6	6	4	5.725	8.000	3	3	3	2	1	7.769	9.051
6	6	5	5.765	8.124	3	3	3	2	2	8.044	9.505
6	6	6	5.801	8.222	3	3	3	3	1	8.000	9.451
7	7	7	5.819	8.378	3	3	3	3	2	8.200	9.876
8	8	8	5.805	8.465	3	3	3	3	3	8.333	10.200