MATH 4740/MSSC 5740

Chapter 10 Problem Solving # * (sign test), 5 (Mann-Whitney U Test)

10.5 Summary

Sign Test: $MD=MD_0$ (One Sample)	x = number of observations > MD_0 If value<MD_0, –. If value=MD_0, 0. If value>MD_0, +.
Mann-Whitney U Test: Two populations equal or not (not-Paired)	$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$ $U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$ $U = \min(U_1, U_2)$

Sign Test Table (Table 6)

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	**.05**	**.025**	**.01**	**.005**
n				
1				
2				
3				
4				
5	0			
6	0	0		
7	0	0	0	
8	1	0	0	0
9	1	1	0	0
10	1	1	0	0
11	2	1	1	0
12	2	2	1	1
13	3	2	1	1
14	3	2	2	1
15	3	3	2	2
16	4	3	2	2
17	4	4	3	2
18	5	4	3	3
19	5	4	4	3
20	5	5	4	3
21	6	5	4	4
22	6	5	5	4
23	7	6	5	4
24	7	6	5	5
25	7	7	6	5

MATH 4740/MSSC 5740
Chapter 10 Problem Solving # * (sign test), 5 (Mann-Whitney U Test)

Mann-Whitney U Test Table (Table 7)
$n_1 \le n_2$

Two-Sided Test $\alpha = 0.05$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2								0	0	0	0	1	1	1	1	1	2	2	2	2
3					0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4				0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7			1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8		0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9		0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
10		0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14		1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15		1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17		2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20		2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

One-Sided Test $\alpha = 0.05$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2					0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3			0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10	11
4			0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5		0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6		0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7		0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
8		1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
9		1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
10		1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
11		1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
12		2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
13		2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
14		2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
15		3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
16		3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
17		3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
18		4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

MATH 4740/MSSC 5740

Chapter 10 Problem Solving # * (sign test), 5 (Mann-Whitney U Test)

10.6 Practice Problems

* A group of n=15 students was surveyed about the number of times they've unlocked their phone yesterday. Unlocks: 12 13 19 20 21 21 23 23 24 25 28 29 34 38 47
Their statistics professor claims students unlock their phone more than 20 times per day.
Go through the 5 hypothesis testing steps to test whether the median number is greater than 20.
α=0.05

Step 1. Set up hypotheses and determine level of significance.

H_0: MD=20 vs. H_1: MD>20 $\alpha = 0.05$

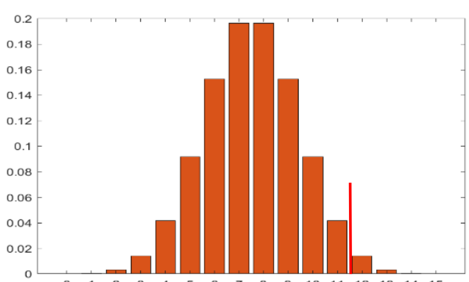
Step 2. Select the appropriate test statistic.

x = (the number of observations > MD_0) Use binomial probabilities n=5, p=1/2.

Step 3. Set up decision rule.

Reject H_0 if $P(X \geq x_\alpha) \leq \alpha$

x	P(X=x)	P(X≤x)	P(X≥x)
0	0.0000	0.0000	1.0000
1	0.0005	0.0005	1.0000
2	0.0032	0.0037	0.9995
3	0.0139	0.0176	0.9963
4	0.0417	0.0592	0.9824
5	0.0916	0.1509	0.9408
6	0.1527	0.3036	0.8491
7	0.1964	0.5000	0.6964
8	0.1964	0.6964	0.5000
9	0.1527	0.8491	0.3036
10	0.0916	0.9408	0.1509
11	0.0417	0.9824	0.0592
12	0.0139	0.9963	0.0176
13	0.0032	0.9995	0.0037
14	0.0005	1.0000	0.0005
15	0.0000	1.0000	0.0000



Reject H_0 if $x \geq 12$.

Step 4. Compute the test statistic.

x = (the number of observations > MD_0)

Sorted	Signs>20	Ranks
12	-1	1
13	-1	2
19	-1	3
20	0	4
21	+1	5.5
21	+1	5.5
23	+1	7.5
23	+1	7.5
24	+1	9
25	+1	10
28	+1	11
29	+1	12
34	+1	13
38	+1	14
47	+1	15

x=11

Step 5. Conclusion.

We do not reject H_0 because 11 < 12. We do not have statistically significant evidence at α = 0.05 to show that the statistics students look at their phone more than 20 times per day. Compare to t?

Note: $\bar{X}$ =24.933, s =9.0512, t=2.1713, $t_{0.05,14}$=1.761

5. The recommended daily allowance of Vitamin A for children between 1 and 3 years of age is 400 micrograms (mcg). Vitamin A deficiency is linked to a number of adverse health outcomes, including poor eyesight, susceptibility to infection, and dry skin. The following are Vitamin A concentrations in children with and without poor eyesight, a history of infection, and dry skin.

With: 120 420 180 345 390 430 (Group 1)
Without: 450 500 395 380 430 (Group 2)

Is there a significant difference in Vitamin A concentrations between children with and without poor eyesight, a history of infection, and dry skin? Run the appropriate test at a 5% level of significance.

Step 1. Set up hypotheses and determine level of significance.

H_0: The two populations are equal

vs.

H_1: The two populations are not equal. $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

$$U = \min(U_1, U_2), \qquad U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1, \quad U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

Step 3. Set up decision rule.

Reject H_0 if $U < U_{\alpha, n_1, n_2} = U_{0.05,6,5} = 3$.

Two-Sided Test $\alpha = 0.05$

n_2	1	2	3	4	5	6
2						
3					0	1
4				0	1	2
5			0	1	2	3
6			1	2	3	5

Step 4. Compute the test statistic.

Total Sample		Ranks	
With	Without	With	Without
120		1	
180		2	
345		3	
	380		4
390		5	
	395		6
420		7	
430	430	8.5	8.5
	450		10
	500		11
		R_1=26.5	R_2=39.5

$$U_1 = (6)(5) + \frac{6(6+1)}{2} - 26.5 = 24.5$$

$$U_2 = (6)(5) + \frac{5(5+1)}{2} - 39.5 = 5.5$$

$$U = \min(24.5, 5.5) = 5.5$$

Step 5. Conclusion.

Because $U = 5.5 \geq U_{0.05,6,5} = 3$, fail to reject H_0. No evidence to show there is a significant difference in Vitamin A concentrations between children with and without poor eyesight, a history of infection, and dry skin.