

10.5 Summary

Sign Test: $MD=MD_0$ (One Sample)	$x = \text{number of observations} > MD_0$ If value $< MD_0$, -. If value $= MD_0$, 0. If value $> MD_0$, +.
Mann-Whitney U Test: Two populations equal or not (not-Paired)	$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$ $U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$ $U = \min(U_1, U_2)$

Sign Test Table (Table 6)

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
<i>n</i>				
1				
2				
3				
4				
5	0			
6	0	0		
7	0	0	0	
8	1	0	0	0
9	1	1	0	0
10	1	1	0	0
11	2	1	1	0
12	2	2	1	1
13	3	2	1	1
14	3	2	2	1
15	3	3	2	2
16	4	3	2	2
17	4	4	3	2
18	5	4	3	3
19	5	4	4	3
20	5	5	4	3
21	6	5	4	4
22	6	5	5	4
23	7	6	5	4
24	7	6	5	5
25	7	7	6	5

Mann-Whitney U Test Table (Table 7)

$$n_1 \leq n_2$$

Two-Sided Test $\alpha = 0.05$																				
		n_1																		
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2								0	0	0	0	1	1	1	1	1	2	2	2	2
3					0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4				0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7			1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41	
9	0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48	
10	0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55	
11	0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62	
12	1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69	
13	1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76	
14	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83	
15	1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90	
16	1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98	
17	2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105	
18	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112	
19	2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119	
20	2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127	

One-Sided Test $\alpha = 0.05$

		n_1																		
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2					0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3				0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10
4				0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17
5			0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23
6			0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30
7			0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37
8			1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44
9			1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
10			1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58
11			1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65
12			2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72
13			2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80
14			2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87
15			3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94
16			3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101
17			3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109
18			4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116
19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

10.6 Practice Problems

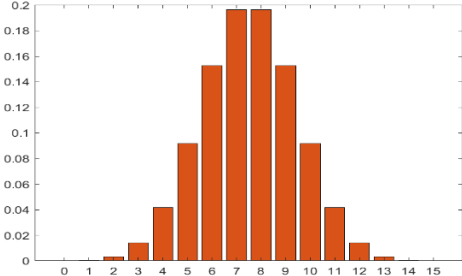
* A group of $n=15$ students was surveyed about the number of times they've unlocked their phone yesterday. Unlocks: 12 13 19 20 21 21 23 23 24 25 28 29 34 38 47
 Their statistics professor claims students unlock their phone more than 20 times per day.
 Go through the 5 hypothesis testing steps to test whether the median number is greater than 20.
 $\alpha=0.05$

Step 1. Set up hypotheses and determine level of significance.

$H_0:$ vs. $H_1:$

Step 2. Select the appropriate test statistic.

Use binomial probabilities $n=15, p=1/2$.

<p>Step 3. Set up decision rule. Reject H_0 if $P(X \geq x_\alpha) \leq \alpha$</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>x</th> <th>P(X=x)</th> <th>P(X≤x)</th> <th>P(X≥x)</th> </tr> </thead> <tbody> <tr><td>0</td><td>0.0000</td><td>0.0000</td><td>1.0000</td></tr> <tr><td>1</td><td>0.0005</td><td>0.0005</td><td>1.0000</td></tr> <tr><td>2</td><td>0.0032</td><td>0.0037</td><td>0.9995</td></tr> <tr><td>3</td><td>0.0139</td><td>0.0176</td><td>0.9963</td></tr> <tr><td>4</td><td>0.0417</td><td>0.0592</td><td>0.9824</td></tr> <tr><td>5</td><td>0.0916</td><td>0.1509</td><td>0.9408</td></tr> <tr><td>6</td><td>0.1527</td><td>0.3036</td><td>0.8491</td></tr> <tr><td>7</td><td>0.1964</td><td>0.5000</td><td>0.6964</td></tr> <tr><td>8</td><td>0.1964</td><td>0.6964</td><td>0.5000</td></tr> <tr><td>9</td><td>0.1527</td><td>0.8491</td><td>0.3036</td></tr> <tr><td>10</td><td>0.0916</td><td>0.9408</td><td>0.1509</td></tr> <tr><td>11</td><td>0.0417</td><td>0.9824</td><td>0.0592</td></tr> <tr><td>12</td><td>0.0139</td><td>0.9963</td><td>0.0176</td></tr> <tr><td>13</td><td>0.0032</td><td>0.9995</td><td>0.0037</td></tr> <tr><td>14</td><td>0.0005</td><td>1.0000</td><td>0.0005</td></tr> <tr><td>15</td><td>0.0000</td><td>1.0000</td><td>0.0000</td></tr> </tbody> </table>  <p>Reject H_0 if $x \geq$.</p>	x	P(X=x)	P(X≤x)	P(X≥x)	0	0.0000	0.0000	1.0000	1	0.0005	0.0005	1.0000	2	0.0032	0.0037	0.9995	3	0.0139	0.0176	0.9963	4	0.0417	0.0592	0.9824	5	0.0916	0.1509	0.9408	6	0.1527	0.3036	0.8491	7	0.1964	0.5000	0.6964	8	0.1964	0.6964	0.5000	9	0.1527	0.8491	0.3036	10	0.0916	0.9408	0.1509	11	0.0417	0.9824	0.0592	12	0.0139	0.9963	0.0176	13	0.0032	0.9995	0.0037	14	0.0005	1.0000	0.0005	15	0.0000	1.0000	0.0000	<p>Step 4. Compute the test statistic. $x =$ (the number of observations $> MD_0$)</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Sorted</th> <th>Signs>20</th> <th>Ranks</th> </tr> </thead> <tbody> <tr><td>12</td><td></td><td></td></tr> <tr><td>13</td><td></td><td></td></tr> <tr><td>19</td><td></td><td></td></tr> <tr><td>20</td><td></td><td></td></tr> <tr><td>21</td><td></td><td></td></tr> <tr><td>21</td><td></td><td></td></tr> <tr><td>23</td><td></td><td></td></tr> <tr><td>23</td><td></td><td></td></tr> <tr><td>24</td><td></td><td></td></tr> <tr><td>25</td><td></td><td></td></tr> <tr><td>28</td><td></td><td></td></tr> <tr><td>29</td><td></td><td></td></tr> <tr><td>34</td><td></td><td></td></tr> <tr><td>38</td><td></td><td></td></tr> <tr><td>47</td><td></td><td></td></tr> </tbody> </table> $x =$	Sorted	Signs>20	Ranks	12			13			19			20			21			21			23			23			24			25			28			29			34			38			47		
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Step 5. Conclusion.

We _____ H_0 because $<$. We _____ have statistically significant evidence at $\alpha = 0.05$ to show that the statistics students look at their phone more than 20 times per day. Compare to t ?

Note: $\bar{X} = 24.933, s = 9.0512, t = 2.1713, t_{0.05, 14} = 1.761$

5. The recommended daily allowance of Vitamin A for children between 1 and 3 years of age is 400 micrograms (mcg). Vitamin A deficiency is linked to a number of adverse health outcomes, including poor eyesight, susceptibility to infection, and dry skin. The following are Vitamin A concentrations in children with and without poor eyesight, a history of infection, and dry skin.

With: 120 420 180 345 390 430 (Group 1)

Without: 450 500 395 380 430 (Group 2)

Is there a significant difference in Vitamin A concentrations between children with and without poor eyesight, a history of infection, and dry skin? Run the appropriate test at a 5% level of significance.

Step 1. Set up hypotheses and determine level of significance.

H_0 : The two populations are equal

vs.

H_1 : The two populations are not equal. $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

$$U = \min(U_1, U_2), \quad U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1, \quad U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

Step 3. Set up decision rule.

Reject H_0 if $U < U_{\alpha, n_1, n_2}$

Step 4. Compute the test statistic.

Total Sample		Ranks	
With	Without	With	Without
120			
180			
345			
	380		
390			
	395		
420			
430	430		
	450		
	500		
		R ₁ =	R ₂ =

$U_1 =$
 $U_2 =$
 $U = \min(\quad) =$

Step 5. Conclusion.