MATH 4740/MSSC 5740

Chapter 7 Problem Solving # 14, 22

7.10 Summary

Number of Groups, Outcome: Parameter	Test Statistic, $n<30$	Test Statistic, $n \geq 30$
Two independent samples, continuous: $\mu_1 = \mu_2$	$t = \dfrac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$, $df=n_1+n_2-2$ (Technically assumes populations are normal, independent within and between populations, and population variances equal.)	$z = \dfrac{\bar{X}_1 - \bar{X}_2}{S_P\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$, $S_P = \sqrt{\dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$
Two matched samples, continuous: μ_d	$t = \dfrac{\bar{X}_d}{s_d/\sqrt{n}}$, $df=n-1$ (Technically assumes data is normal, pairs independent of each other, variances equal between populations.)	$z = \dfrac{\bar{X}_d}{s_d/\sqrt{n}}$, $\bar{X}_d = \dfrac{1}{n}\sum_{i=1}^{n} d_i$, $s_d = \sqrt{\dfrac{\Sigma d^2 - (\Sigma d)^2/n}{n-1}}$
Two independent samples, dichotomous: $p_1=p_2$	Multinomial Test (Not taught in this class.)	$z = \dfrac{\hat{p}_1-\hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$, $\hat{p}_1 = \dfrac{X_1}{n_1}$, $\hat{p}_2 = \dfrac{X_2}{n_2}$, $\hat{p} = \dfrac{X_1+X_2}{n_1+n_2}$
More than two samples, continuous: $\mu_1 = \ldots = \mu_k$	Same as large sample. (Technically assumes populations are normal, independent within and between populations, and population variances equal.)	$F = \dfrac{MSB}{MSE}$, $df_1=k-1$, $df_2=N-k$, $MSB = \dfrac{1}{k-1}\sum_{j=1}^{k} n_j(\bar{X}_j - \bar{X})^2$, $MSE = \dfrac{1}{N-k}\sum_{j=1}^{k}\sum_{i=1}^{n_j} n_j(X_i - \bar{X}_j)^2$
Two or more samples, Categorical and Ordinal: $p_{11},\ldots,p_{rc}$	Multinomial Test (Not taught in this class.)	$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\dfrac{(O_{ij}-E_{ij})^2}{E_{ij}}$, $df=(r-1)(c-1)$

7.11 Practice Problems

14. Suppose a hypertension trial is mounted and 18 participants are randomly assigned to one of the comparison treatments. Each participant takes the assigned medication and his or her systolic blood pressure is recorded after 6 months on the assigned treatment. The data are shown in Table 7.58.

Standard Treatment (1)	Placebo (2)	New Treatment (3)
124	134	114
111	143	117
133	148	121
125	142	124
128	150	122
115	160	128

Is there a difference in mean systolic blood pressure among treatments? Run the appropriate test at $\alpha = 0.05$.

Answer:

Step 1. Set up hypotheses and determine level of significance.

H_0:

H_1: $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

Step 3. Set up decision rule.

	df_1													$\alpha = 0.05$
df_2	1	2	3	4	5	6	7	8	9	10	20	30	40	50
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	248.0	250.1	251.1	251.8
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.45	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.25	2.20	2.18

Reject H_0 if $F \geq$.

Step 4. Compute the test statistic.

Standard	Placebo	New Treatment
$n_1 =$	$n_2 =$	$n_3 =$
$\bar{X}_1 =$	$\bar{X}_2 =$	$\bar{X}_3 =$

If we pool all $N = 18$ observations, the overall (or grand) mean is

$\bar{X} =$

We can now compute $SSB = \sum_{j=1}^{3} n_j \, (\bar{X}_j - \bar{X})^2$

$SSB =$

Next, $SSE = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_i - \bar{X}_j)^2$

Standard	$(X-\quad)$	$(X-\quad)^2$
124		
111		
133		
125		
128		
115		
$\bar{X}_1 =$		

Thus, $\sum_{i=1}^{6} (X_i - \bar{X}_1)^2 =$

Placebo	$(X-\quad)$	$(X-\quad)^2$
134		
143		
148		
142		
150		
160		
$\bar{X}_2 =$		

Thus, $\sum_{i=1}^{6} (X_i - \bar{X}_2)^2 =$

New treatment	$(X-\quad)$	$(X-\quad)^2$
114		
117		
121		
124		
122		
128		
$\bar{X}_3 =$		

Thus, $\sum_{i=1}^{6} (X_i - \bar{X}_3)^2 =$

$SSE = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (X_i - \bar{X}_j)^2 =$

We can now construct the ANOVA table.

Source of Variation	Sums of Squares (SS)	Degrees of Freedom (df)	Mean Squares (MS)	F
Between Treatments				
Error or Residual				
Total				

Step 5. Conclusion.

We reject H_0 because $\geq$. We statistically significant evidence at $\alpha = 0.05$ to show that there is a difference in mean systolic blood pressure among treatments.

MATH 4740/MSSC 5740

Chapter 7 Problem Solving # 14, 22

22. Use the data shown in Problem 21 and test if there is an association between mother's BMI and the child's obesity status (i.e., normal versus overweight/obese)? Run the test at a 5% level of significance.

Mother:	Child: Normal	Child: Overweight/Obese	Total
Normal	40	16	56
Overweight	15	14	29
Obese	7	8	15
Total	62	38	100

Answer:

Step 1. Set up hypotheses and determine level of significance.

H_0:

H_1: $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

The condition for appropriate use of this test statistic is that each expected frequency is at least 5. In Step 4 we will compute the expected frequencies and we will ensure that the condition is met.

Step 3. Set up decision rule.

$df = (\quad - 1)(\quad - 1) =$ and the decision rule is: Reject H_0 if $\chi^2 \geq$.

Step 4. Compute the test statistic.

We now compute the expected frequencies using the formula:

Expected Frequency = (*Row Total* * *Column Total*)/*N*

The top number in each cell of the table is the observed frequency and the bottom number is the expected frequency (shown in parentheses).

Mother:	Child: Normal	Child: Overweight/Obese	Total
Normal	40 ( )	16 ( )	56
Overweight	15 ( )	14 ( )	29
Obese	7 ( )	8 ( )	15
Total	62	38	100

The test statistic is computed as follows:

$$\chi^2 = \frac{(40-\quad)^2}{\quad} + \frac{(16-\quad)^2}{\quad} + \frac{(15-\quad)^2}{\quad} + \frac{(14-\quad)^2}{\quad} + \frac{(7-\quad)^2}{\quad} + \frac{(8-\quad)^2}{\quad}$$

$\chi^2 =$.

Step 5. Conclusion.

H_0 because $<$. We have statistically significant evidence at $\alpha = 0.05$ to show that H_0 or that mother's BMI and child's obesity status are