MATH 4740/MSSC 5740
Chapter 7 Problem Solving # 13, *, 2

7.10 Summary

Number of Groups, Outcome: Parameter	Test Statistic, $n<30$	Test Statistic, $n\geq 30$
One sample, continuous: μ	$t=\dfrac{\bar{X}-\mu_0}{s/\sqrt{n}}$, $df=n-1$ (Technically assume data is normal.)	$z=\dfrac{\bar{X}-\mu_0}{s/\sqrt{n}}$
One sample, dichotomous: p	Binomial Test (Not taught in this class.)	$z=\dfrac{\hat{p}-p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$
One Sample, Categorical and Ordinal: $p_1,\ldots,p_k$	Multinomial Test (Not taught in this class.)	$\chi^2=\sum\dfrac{(O-E)^2}{E}$, $df=k-1$

7.11 Practice Problems

13. A recent recommendation suggests 60 minutes of physical activity per day. A sample of 50 adults in a study of cardiovascular risk factors report exercising a mean of 38 minutes per day with a standard deviation of 19 minutes. Based on the sample data, is the physical activity significantly less than recommended? Run the appropriate test at a 5% level of significance.

Answer: n=50, $\bar{X} = 19\,\text{min}$, s=19 min

Step 1. Set up hypotheses and determine level of significance.

H_0: $\mu \geq 60$

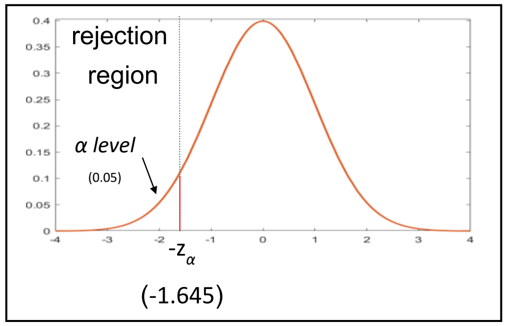
H_1: $\mu < 60$ $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

$$z = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}$$

Step 3. Set up decision rule:

Reject H_0 if $z \leq -z_\alpha$, $-z_{0.05}$=–1.645.



Step 4. Compute the test statistic.

$$z = \frac{\bar{X} - \mu_0}{s / \sqrt{n}} = \frac{38 - 60}{19 / \sqrt{50}} = -8.1876.$$

Step 5. Conclusion.

Reject H_0 because $-8.19 \leq -1.645$. We have statistically significant evidence at $\alpha = 0.05$ to show that the mean number of minutes of exercise is less than 60. The p-value is $p < 0.0001$ (actually p=1.3330×10^{-16}).

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* From Chapter 6 #6. Data are collected in a clinical trial evaluating a new compound designed to improve wound healing in trauma patients. The new compound is compared against a placebo. After treatment for 5 days, with the new compound or placebo, the extent of wound healing is measured and the data are shown in Table 6.25. Suppose that clinicians feel that if the percentage reduction in the size of the wound is greater than 50%, then the treatment is a success.

TABLE 6.25 Wound Healing by Treatment

	Number of Patients with Percent Reduction in Size of Wound				
Treatment	**None**	**1–25%**	**26–50%**	**51–75%**	**76–100%**
New compound (n = 125)	4	11	37	32	41
Placebo (n = 125)	12	24	45	34	10

Perform a hypothesis test to determine if the true percent success in patients receiving the new compound is greater than 0.5.

Step 1. Set up hypotheses and determine level of significance.

H_0: $p \le 0.5$

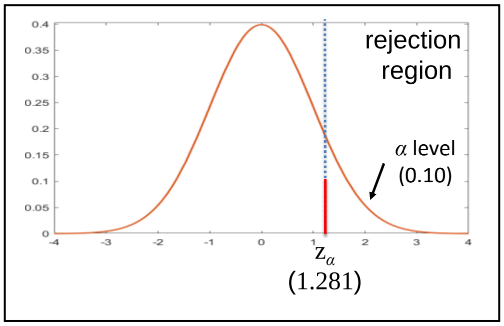
H_1: $p > 0.5$ $\alpha = 0.10$

Step 2. Select the appropriate test statistic.

$$z = \frac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$$

Step 3. Set up decision rule.

Reject H_0 if $z \ge z_\alpha$, $z_{0.10}$=1.282.



Step 4. Compute the test statistic.

	Failure	Success	Total
Compound	52	X_1=73	n_1=125
Placebo	81	X_2=44	n_2=125

$$\hat{p} = 73/125 = 0.5840$$

The test statistic is computed as follows:

$$z = \frac{0.5840 - 0.50}{\sqrt{\dfrac{0.50(1-0.50)}{125}}} = 1.878$$

Step 5. Conclusion.

Reject H_0 because 1.878 ≥ 1.282. We have statistically significant evidence at $\alpha = 0.10$ to show that H_0 is false. The p-value is $p < 0.05$ (actually p=0.0302).

2. Use the data in Problem 1 (from the book) and pool the data across the treatments into one sample of size $n = 250$. Use the pooled data to test whether the distribution of the percent wound healing is approximately normal. Specifically, use the following distribution: 30%, 40%, 20%, and 10% and $\alpha = 0.05$ to run the appropriate test.

Answer:

	Percent Wound Healing				
Treatment	0–25%	26–50%	51–75%	76–100%	Total
Number of patients	51	82	66	51	250

Step 1. Set up hypotheses and determine level of significance.

H_0: $p_1 = 0.3$, $p_2 = 0.4$, $p_3 = 0.2$, $p_4 = 0.1$

H_1: H_0 is false. $\alpha = 0.05$

Step 2. Select the appropriate test statistic.

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

We must assess whether the sample size is adequate. Specifically, we need to check $min(np_1, \ldots, np_k) \geq 5$. The sample size here is $n = 250$ and the proportions specified in the null hypothesis are 0.3, 0.4, 0.2, and 0.1. Thus, $min(250(0.3), 250(0.4), 250(0.2), 250(0.1)) = min(75, 100, 50, 25) = 25$. The sample size is more than adequate, so the formula can be used.

Step 3. Set up decision rule.

$df = k - 1 = 4 - 1 = 3$: Reject H_0 if $\chi^2 \geq \chi^2_{\alpha,df}$, $\chi^2_{0.05,3}=7.81$.

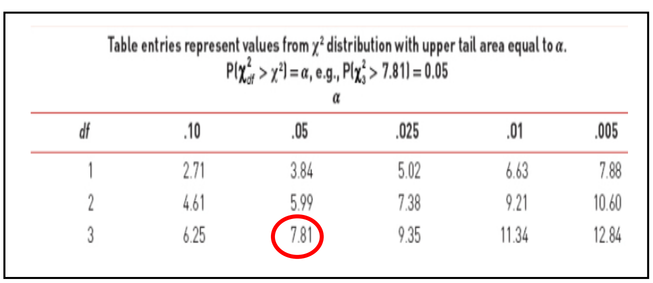
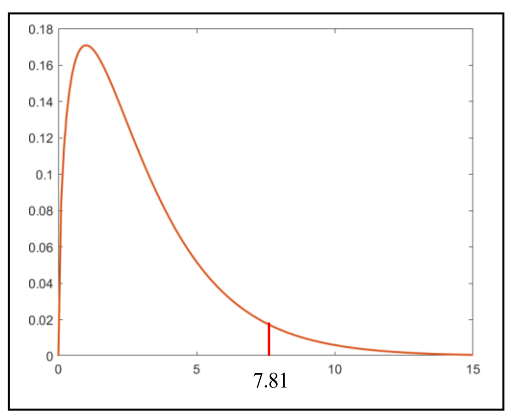


Table entries represent values from χ^2 distribution with upper tail area equal to α.
$P(\chi^2_{df} > \chi^2) = \alpha$, e.g., $P(\chi^2_3 > 7.81) = 0.05$

df	.10	.05	.025	.01	.005
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84



Step 4. Compute the test statistic.

	Percent Wound Healing				
Treatment	0–25%	26–50%	51–75%	76–100%	Total
Number of patients	51	82	66	51	250
Expected	75	100	50	25	250

The test statistic is computed as follows:

$$\chi^2 = \frac{(51-75)^2}{75} + \frac{(82-100)^2}{100} + \frac{(66-50)^2}{50} + \frac{(51-25)^2}{25}$$

$\chi^2 = 7.68 + 3.24 + 5.12 + 27.04 = 43.08.$

Step 5. Conclusion.

Reject H_0 because $43.08 \geq 7.81$. We have statistically significant evidence at $\alpha = 0.05$ to show that H_0 is false. The p-value is $p < 0.005$ (actually $2.4\text{x}10^{-9}$).