6.7 Summary

Number of Groups, Outcome: Parameter	Confidence Interval, $n<30$	Confidence Interval, $n\geq 30$
One sample, continuous: CI for μ	$\bar{X} \pm t_{\frac{\alpha}{2},df} \frac{s}{\sqrt{n}}$	$\bar{X} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
One sample, dichotomous: CI for p	(Not taught in this class.)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Two independent samples, continuous: CI for μ_1-μ_2	$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},df} S_P \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_P = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$ $df = n_1+n_2-2$	$(\bar{X}_1 - \bar{X}_2) \pm z_{\frac{\alpha}{2}} S_P \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ $S_P = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$
Two matched samples, continuous: CI for μ_d=μ_1-μ_2	$\bar{X}_d \pm t_{\frac{\alpha}{2},df} \frac{s_d}{\sqrt{n}}$ df=n-1	$\bar{X}_d \pm z_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$
Two independent samples, dichotomous: CI for RD=(p_1-p_2)	(Not taught in this class.)	$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$
CI for $ln(RR)=ln(p_1/p_2)$	(Not taught in this class.)	$\ln(RR) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{(n_1-X_1)/X_1}{n_1}+\frac{(n_2-X_2)/X_2}{n_2}}$
CI for $RR=p_1/p_2$	(Not taught in this class.)	$\exp(Lower\ Limit), \exp(Upper\ \ Limit)$
CI for $ln(OR)$= $ln([p_1/(1-p_1)]/[p_2/(1-p_2)])$	(Not taught in this class.)	$ln(OR) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{1}{X_1}+\frac{1}{n_1-X_1}+\frac{1}{X_2}+\frac{1}{n_2-X_2}}$
CI for $OR=[p_1/(1-p_1)]/[p_2/(1-p_2)]$	(Not taught in this class.)	$\exp(Lower\ Limit), \exp(Upper\ \ Limit)$

Number of Groups, Outcome: Parameter	**Confidence Interval[a]**
One sample, continuous: CI for μ	$\bar{X} \pm z \frac{s}{\sqrt{n}}$
Two independent samples, continuous: CI for $(\mu_1 - \mu_2)$	$(\bar{X}_1 - \bar{X}_2) \pm z S_p \sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$
Two matched samples, continuous: CI for μ_d	$\bar{X}_d \pm z \frac{s_d}{\sqrt{n}}$
Two independent samples, dichotomous: CI for RD = $(p_1 - p_2)$	$(\hat{p}_1 - \hat{p}_2) \pm z \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ exp(Lower limit), exp(Upper limit)
Two independent samples, dichotomous: CI for OR = $\frac{x_1/(n_1-x_1)}{x_2/(n_2-x_2)}$	$\ln(\hat{OR}) \pm z \sqrt{\frac{1}{x_1}+\frac{1}{(n_1-x_1)}+\frac{1}{x_2}+\frac{1}{(n_2-x_2)}}$ exp(Lower limit), exp(Upper limit)

MATH 4740/MSSC 5740
Chapter 6 Problem Solving # 2, 8, 6

MATH 4740/MSSC 5740

Chapter 6 Problem Solving # 2, 8, 6

6.8 Practice Problems

2. A clinical trial is planned to compare an experimental medication designed to lower blood pressure to a placebo. Before starting the trial, a pilot study is conducted involving 10 participants. The objective of the study is to assess how systolic blood pressure changes over time untreated. Systolic blood pressures are measured at baseline and again 4 weeks later. Compute a 95% CI for the difference in blood pressures over 4 weeks.

Answer:

Baseline:	120	145	130	160	152	143	126	121	115	135
4 Weeks:	122	142	135	158	155	140	130	120	124	130

Baseline	4 Weeks	Difference = 4 Weeks – Baseline	Difference^2
120	122	2	4
145	142	–3	9
130	135	5	25
160	158	–2	4
152	155	3	9
143	140	–3	9
126	130	4	16
121	120	–1	1
115	124	9	81
135	130	–5	25
		9	183

$$\bar{X}_d = \frac{\Sigma Difference}{n} = \frac{9}{10} = 0.9$$

$$s = \sqrt{\frac{\Sigma Differences^2 - (\Sigma Differences)^2 / n}{n-1}} = \sqrt{\frac{183-(9)^2/10}{9}} = \sqrt{19.43} = 4.4$$

$\bar{X}_d \pm t_{\frac{\alpha}{2},df} \frac{s_d}{\sqrt{n}}$, $df = n - 1 = 10 - 1 = 9$ For 95% confidence, $t = 2.262$ $0.9 \pm (2.262)\frac{4.4}{\sqrt{10}}$ 0.9 ± 3.1 (–2.2, 4.0)	

8. Table 6.26 displays descriptive statistics on participants involved in the study described in Problem 7.

TABLE 6.26 Descriptive Statistics by Treatment

	Experimental Medication (n = 100)	Placebo (n = 100)
Mean (SD) age, years	47.2 (4.3)	46.1 (5.1)
Men (%)	46%	58%
Mean (SD) educational level, years	13.1 (2.9)	14.2 (3.1)
Mean (SD) annual income	$36,560 ($1054)	$37,470 ($998)
Mean (SD) body mass index (BMI)	24.7 (2.7)	25.1 (2.4)

a. Generate a 95% CI for the mean age among participants assigned to the placebo.

Answer:

$$\bar{X} \pm t_{\frac{\alpha}{2},df} \frac{s}{\sqrt{n}}, \quad 46.1 \pm 1.96 \frac{5.1}{\sqrt{100}}, \quad 46.1 \pm 0.9996$$

45.10 to 47.10

b. Generate a 95% CI for the difference in mean ages of participants assigned to the experimental versus the placebo groups.

Answer:

$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},df} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	$S_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$	$df = n_1 + n_2 - 2$

$$S_P = \sqrt{\frac{(100-1)(4.3)^2 + (100-1)(5.1)^2}{100+100-2}} = 4.717$$

$$(47.2 - 46.1) \pm 1.96(4.717)\sqrt{\frac{1}{100} + \frac{1}{100}}, \quad 1.10 \pm 1.96(4.717)(0.141)$$

-0.204 to 2.404

c. Generate a 95% CI for the difference in mean BMI in participants assigned to the experimental versus the placebo groups.

Answer:

$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2},df} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$	$S_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$	$df = n_1 + n_2 - 2$

$$S_P = \sqrt{\frac{(100-1)(2.7)^2 + (100-1)(2.4)^2}{100+100-2}} = 2.554$$

$$(24.7 - 25.1) \pm 1.96(2.554)\sqrt{\frac{1}{100} + \frac{1}{100}}, \quad -0.40 \pm 1.96(2.554)(0.141)$$

-1.108 to 0.308

MATH 4740/MSSC 5740

Chapter 6 Problem Solving # 2, 8, 6

6. Data are collected in a clinical trial evaluating a new compound designed to improve wound healing in trauma patients. The new compound is compared against a placebo. After treatment for 5 days, with the new compound or placebo, the extent of wound healing is measured and the data are shown in Table 6.25. Suppose that clinicians feel that if the percentage reduction in the size of the wound is greater than 50%, then the treatment is a success.

TABLE 6.25 Wound Healing by Treatment

	Number of Patients with Percent Reduction in Size of Wound				
Treatment	**None**	**1–25%**	**26–50%**	**51–75%**	**76–100%**
New compound (n = 125)	4	11	37	32	41
Placebo (n = 125)	12	24	45	34	10

a. Generate a 95% CI for the percent success in patients receiving the new compound.

Answer:

	Failure	Success	Total
Compound	52	X_1=73	n_1=125
Placebo	81	X_2=44	n_2=125

$$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \quad \hat{p} = \frac{X_1}{n_1} = \frac{73}{125} = 0.584, \quad z_{0.025} = 1.96$$

$$0.585 \pm 1.96\sqrt{\frac{0.585(1-0.585)}{125}}$$

0.585±(1.96)(0.044)

0.499 to 0.671

b. Generate a 95% CI for the difference in the percent success between the new compound and placebo.

Answer:

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}, \quad \hat{p}_1 = \frac{X_1}{n_1} = \frac{73}{125} = 0.584, \quad \hat{p}_2 = \frac{X_2}{n_2} = \frac{44}{125} = 0.352$$

$$z_{0.025} = 1.96$$

$$(0.584 - 0.352) \pm 1.96\sqrt{\frac{0.584(1-0.584)}{125} + \frac{0.352(1-0.352)}{125}}$$

$$0.2320 \pm (1.96)(0.0614)$$

$$0.1117 \ to \ 0.3523$$

c. Generate a 95% CI for the RR of treatment success between treatments.

Answer:

The CI for $ln(RR)$ is:

$$\ln(RR) \pm z_{\frac{\alpha}{2}}\sqrt{\frac{(n_1 - X_1)/X_1}{n_1} + \frac{(n_2 - X_2)/X_2}{n_2}}, \quad RR = \frac{\hat{p}_1}{\hat{p}_2} = \frac{0.584}{0.352} = 1.6591$$

$$z_{0.025} = 1.96$$

$$\ln(1.6591) \pm 1.96\sqrt{\frac{(125-73)/73}{125} + \frac{(125-44)/44}{125}}$$

$$0.5063 \pm (1.96)(0.1429)$$

0.2262 to 0.7864

The CI for RR is

exp(0.2262) to exp(0.7864)

1.2538 to 2.1955

d. Generate a 95% CI for the OR of treatment success between treatments.

Answer:

$$\ln(OR) \pm z_{\frac{\alpha}{2}}\sqrt{\frac{1}{X_1} + \frac{1}{n_1 - X_1} + \frac{1}{X_2} + \frac{1}{n_2 - X_2}}, \quad OR = \frac{0.584/(1-0.584)}{0.352/(1-0.352)} = 2.5844$$

$$z_{0.025} = 1.96$$

$$\ln(2.5844) \pm 1.96\sqrt{\frac{1}{73} + \frac{1}{125-73} + \frac{1}{44} + \frac{1}{125-44}}$$

$$\ln(2.5844) \pm (1.96)(0.2608)$$

0.4383 to 1.4607

The CI for OR is:

exp(0.4383) to exp(1.4607)

1.551 to 4.3090

The odds are at least 1.6 to 1 for new treatment to be successful than placebo.