Chapter 10: Nonparametric Tests II Supplement

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10.3 Tests with Matched Samples – Sign Test

We learned the parametric matched difference hypothesis test,

H_0: $\mu_d \leq 0$ *vs*. H_1: $\mu_d > 0$ (prove greater than), $\mu_d = \mu_1 - \mu_2$

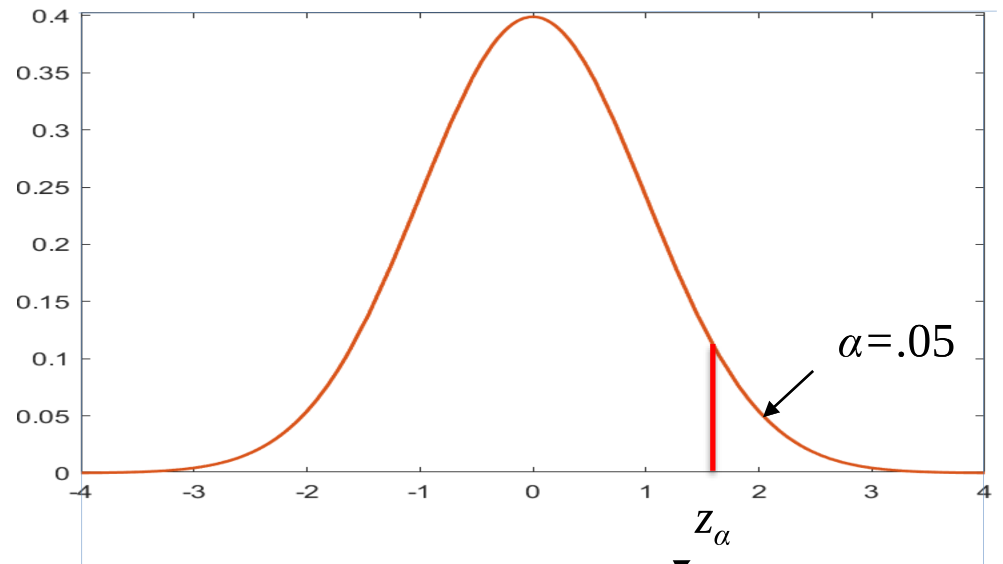
We know for the parametric test, we reject for “large" average differences

$\bar{X}_d = \frac{1}{n}\sum_{i=1}^{n} d_i$ or $z = \frac{\bar{X}_d}{s/\sqrt{n}}$, (assuming n large).

10.3 Tests with Matched Samples – Sign Test

H_0: $\mu_d \leq 0$ *vs*. H_1: $\mu_d > 0$ (prove greater than), $\mu_d = \mu_1 - \mu_2$

When we calculate $z = \dfrac{\bar{X}_d}{s/\sqrt{n}}$, we see where it lies



The z value that has an area α larger than it.

We look up the z value in the table.

Z_α = z value with area α less than

z_α = z value with area α greater than

$Z_\alpha = -1.645$

$z_\alpha = +1.645$

$z_\alpha = -Z_\alpha$

Z_i	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

Table 1.

So we need to either multiply Z_α=-1.645 by -1 to get 1.645 or turn it into H_0: $\mu_d = 0$ *vs*. H_1: $\mu_d < 0$ by using $-\bar{X}_d$ and $z = \dfrac{-\bar{X}_d}{s/\sqrt{n}}$, $\mu_d = \mu_2 - \mu_1$.

10.3 Tests with Matched Samples – Sign Test

The same thing occurs in nonparametric testing with the sign test.

$H_0: \delta=0$ *vs.* $H_1: \delta > 0$ (prove greater than), $\delta=MD_1-MD_2$

For the nonparametric sign test, we reject for a “large" number of differences greater than 0.

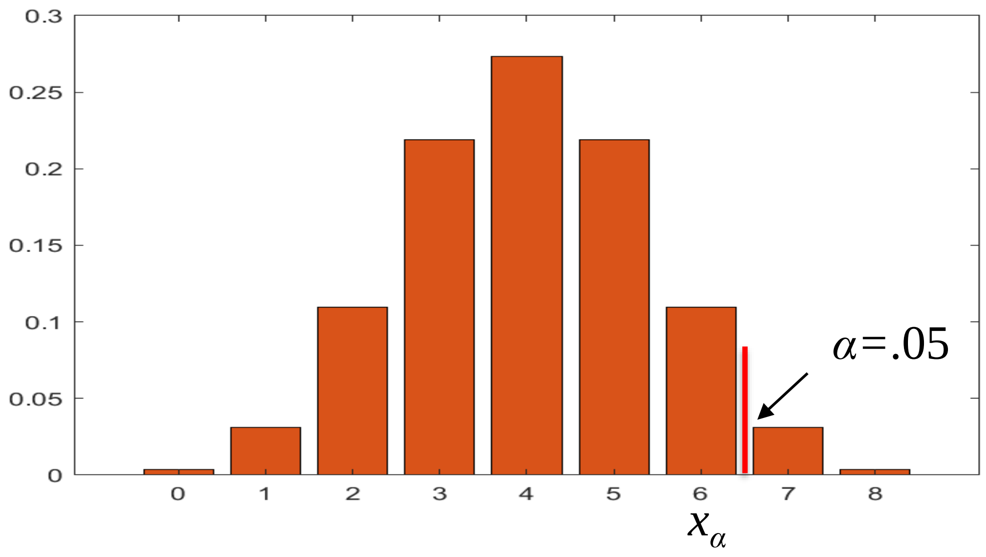
x = (the number of differences > 0) .

10.3 Tests with Matched Samples – Sign Test

x	P(X=x)	CumSum	CumSumR
0	0.004	0.004	1.000
1	0.031	0.035	0.996
2	0.109	0.145	0.965
3	0.219	0.363	0.856
4	0.273	0.637	0.637
5	0.219	0.856	0.363
6	0.109	0.965	0.145
7	0.031	0.996	0.035
8	0.004	1.000	0.004

H_0: $\mu_d \leq 0$ *vs*. H_1: $\mu_d > 0$ (prove greater than), $\delta = MD_1 - MD_2$

When we calculate $x = \#\ d\text{'s} > 0$, we see where it lies



The x value that has an area α larger than it.

We look up the x value in the table.

X_α = x value with area α less than $\quad X_\alpha = 1$

x_α = x value with area α greater than $\quad x_\alpha = 7$

$x_\alpha = n - X_\alpha$

One-Sided Test α	.05	.025	.01	.005
n				
8	1	0	0	0

Table 6.

So we need to either subtract X_α=1 from n=8 to get 7 or turn it into H_0: $\mu_d = 0$ *vs*. H_1: $\mu_d < 0$

by using $x = \#\ d\text{'s} < 0$, $\delta = MD_2 - MD_1$.

10.5 Summary

Sign Test (one sample)

x = number of observations $> MD_0$
$<$

Mann-Whitney U Test

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

$$U = \min(U_1, U_2)$$

Sign Test (two sample)

x = number of differences > 0
$<$

Wilcoxon Signed Rank Test (two sample)

$W = \min(W+, W-)$

$W+$ = sum of positive ranks

$W-$ = sum of negative ranks

Kruskal-Wallis Test (three or more samples)

$$H = \left(\frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} \right) - 3(N+1)$$

R_j = sum of ranks for sample j.

Questions?

Homework 10

Read Chapter 10.

Problems # 6 (Sign Test), 7 (Wilcoxon Signed Rank Test),
8 (Kruskal-Wallis Test) the $n_1=n_2=n_3=n_4=5$ critical value is 7.377.