

### Chapter 3 Quantifying the Extent of Disease

$$\text{Point Prevalence (PP)} = \frac{\text{Number of persons with disease}}{\text{Number of persons examined at baseline}}$$

$$\text{Cumulative Incidence (CI)} = \frac{\text{Number of persons who develop disease during specified period}}{\text{Number of persons at risk at baseline}}$$

$$\text{Incidence Rate (IR)} = \frac{\text{Number of persons who develop disease during specified period}}{\text{Sum of lengths of time during which persons are disease free}}$$

$$\text{Risk Difference (RD)} = PP_{\text{exposed}} - PP_{\text{unexposed}} = CI_{\text{exposed}} - CI_{\text{unexposed}} = IR_{\text{exposed}} - IR_{\text{unexposed}}$$

$$\text{Population Attributable Risk (PAR)} = \frac{PP_{\text{overall}} - PP_{\text{unexposed}}}{PP_{\text{overall}}} = \frac{CI_{\text{overall}} - CI_{\text{unexposed}}}{CI_{\text{overall}}} = \frac{IR_{\text{overall}} - IR_{\text{unexposed}}}{IR_{\text{overall}}}$$

$$\text{Relative Risk (RR)} = \frac{PP_{\text{exposed}}}{PP_{\text{unexposed}}} = \frac{CI_{\text{exposed}}}{CI_{\text{unexposed}}} = \frac{IR_{\text{exposed}}}{IR_{\text{unexposed}}}$$

$$\text{Odds Ratio (OR)} = \frac{PP_{\text{exposed}} / (1 - PP_{\text{exposed}})}{PP_{\text{unexposed}} / (1 - PP_{\text{unexposed}})} = \frac{CI_{\text{exposed}} / (1 - CI_{\text{exposed}})}{CI_{\text{unexposed}} / (1 - CI_{\text{unexposed}})}$$

### Chapter 4 Summarizing Data Collected in the Sample

$$\text{Mean: } \bar{X} = \frac{\sum X}{n}$$

$$\text{Standard Deviation: } s = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}} = \sqrt{\frac{\sum X^2 - \frac{1}{n}(\sum X)^2}{n-1}}$$

Median: Middle Value in Ordered Set ( $Q_2$ )

First Quartile:  $Q_1$  = Value holding 25% below

Third Quartile:  $Q_3$  = Value holding 75% below

Interquartile Range:  $IQR = Q_3 - Q_1$

Criteria for Outliers: Values below  $Q_1 - .5 IQR$  or above  $Q_3 + 1.5 IQR$

**Chapter 5 The Role of Probability**

Basic Probability =  $P(\text{Characteristic}) = (\text{Number of persons with characteristic}) / N$

Sensitivity =  $P(\text{screen positive} | \text{disease}) = a / (a + c)$

Specificity =  $P(\text{screen negative} | \text{disease free}) = d / (b + d)$

False Positive Fraction =  $P(\text{screen positive} | \text{disease free}) = b / (b + d)$

False Negative Fraction =  $P(\text{screen negative} | \text{disease}) = c / (a + c)$

Positive Predictive Value =  $P(\text{disease} | \text{screen positive}) = a / (a + b)$

Negative Predictive Value =  $P(\text{disease free} | \text{screen negative}) = d / (c + d)$

Independent Events:  $P(A | B) = P(A)$  or  $P(B | A) = P(B)$

Binomial Distribution:  $P(x \text{ successes}) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

Standard Normal Distribution:  $z = (x - \mu) / \sigma$  (Table 1)

Percentiles of the Normal Distribution:  $X = \mu + z\sigma$  (Table 1A)

Application of Central Limit Theorem:  $z = \frac{x - \mu}{\sigma / \sqrt{n}}$  (Table 1)

**Chapter 6 Confidence Interval Estimates**

Number of Groups: Parameter	Confidence Interval, $n < 30$	Confidence Interval, $n \geq 30$
One sample, continuous: CI for $\mu$	$\bar{X} \pm t_{\frac{\alpha}{2}, df} \frac{s}{\sqrt{n}}$	$\bar{X} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
One sample, dichotomous: CI for $p$	(Not taught in this class.)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Two independent samples, continuous: CI for $\mu_1 - \mu_2$	$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, df} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ $df = n_1 + n_2 - 2$	$(\bar{X}_1 - \bar{X}_2) \pm z_{\frac{\alpha}{2}} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
Two matched samples, continuous: CI for $\mu_d = \mu_1 - \mu_2$	$\bar{X}_d \pm t_{\frac{\alpha}{2}, df} \frac{s_d}{\sqrt{n}}$ , $df = n - 1$	$\bar{X}_d \pm z_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$
One sample, continuous: CI for $\mu$	$\bar{X} \pm t_{\frac{\alpha}{2}, df} \frac{s}{\sqrt{n}}$	$\bar{X} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
One sample, dichotomous: CI for $p$	(Not taught in this class.)	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

**Chapter 6 Confidence Interval Estimates**

Number of Groups: Parameter	Confidence Interval, $n < 30$	Confidence Interval, $n \geq 30$
Two independent samples, continuous: CI for $\mu_1 - \mu_2$	$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}, df} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$ $df = n_1 + n_2 - 2$	$(\bar{X}_1 - \bar{X}_2) \pm z_{\frac{\alpha}{2}} S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ $S_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
Two matched samples, continuous: CI for $\mu_d = \mu_1 - \mu_2$	$\bar{X}_d \pm t_{\frac{\alpha}{2}, df} \frac{s_d}{\sqrt{n}}, df = n - 1$	$\bar{X}_d \pm z_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n}}$
Two independent samples, dichotomous: CI for $RD = (p_1 - p_2)$	(Not taught in this class.)	$(\hat{p}_1 - \hat{p}_2) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$
CI for $\ln(RR) = \ln(p_1/p_2)$	(Not taught in this class.)	$\ln(RR) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{(n_1 - X_1)/X_1}{n_1} + \frac{(n_2 - X_2)/X_2}{n_2}}$
CI for $RR = p_1/p_2$	(Not taught in this class.)	$\exp(\text{Lower Limit}), \exp(\text{Upper Limit})$
CI for $\ln(OR) = \ln([p_1/(1 - p_1)]/[p_2/(1 - p_2)])$	(Not taught in this class.)	$\ln(OR) \pm z_{\frac{\alpha}{2}} \sqrt{\frac{1}{X_1} + \frac{1}{n_1 - X_1} + \frac{1}{X_2} + \frac{1}{n_2 - X_2}}$
CI for $OR = [p_1/(1 - p_1)]/[p_2/(1 - p_2)]$	(Not taught in this class.)	$\exp(\text{Lower Limit}), \exp(\text{Upper Limit})$

**Chapter 7 Hypothesis Testing Procedures**

Number of Groups: Parameter	Test Statistic, $n < 30$	Test Statistic, $n \geq 30$
One sample, continuous: $\mu$	$t = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}, df = n - 1$ <p>(Technically assume normal data.)</p>	$z = \frac{\bar{X} - \mu_0}{s / \sqrt{n}}$
One sample, dichotomous: $p$	Binomial Test (Not taught in this class.)	$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$
One Sample, Categorical and Ordinal: $p_1, \dots, p_k$	Multinomial Test (Not taught in this class.)	$\chi^2 = \sum \frac{(O - E)^2}{E}, df = k - 1$

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Number of Groups, Outcome: Parameter	Test Statistic, $n < 30$	Test Statistic, $n \geq 30$
Two independent samples, continuous: $\mu_1 = \mu_2$	$t = \frac{\bar{X}_1 - \bar{X}_2}{S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, df = n_1 + n_2 - 2$ <p>(Technically assumes populations are normal, independent within and between populations, and population variances equal.)</p>	$z = \frac{\bar{X}_1 - \bar{X}_2}{S_P \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}},$ $S_P = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$
Two matched samples, continuous: $\mu_d$	$t = \frac{\bar{X}_d}{s_d / \sqrt{n}}, df = n - 1$ <p>(Technically assumes data is normal, pairs independent of each other, variances equal between populations.)</p>	$z = \frac{\bar{X}_d}{s_d / \sqrt{n}},$ $\bar{X}_d = \frac{1}{n} \sum_{i=1}^n d_i,$ $s_d = \sqrt{\frac{\sum d^2 - (\sum d)^2 / n}{n - 1}}$
Two independent samples, dichotomous: $p_1 = p_2$	<p>Multinomial Test (Not taught in this class.)</p>	$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1 - \hat{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}},$ $\hat{p}_1 = \frac{X_1}{n_1}, \hat{p}_2 = \frac{X_2}{n_2}, \hat{p} = \frac{X_1 + X_2}{n_1 + n_2}$
More than two samples, continuous: $\mu_1 = \dots = \mu_k$	<p>Same as large sample.  (Technically assumes populations are normal, independent within and between populations, and population variances equal.)</p>	$F = \frac{MSB}{MSE}, df_1 = k - 1, df_2 = N - k$ $MSB = \frac{1}{k - 1} \sum_{j=1}^k n_j (\bar{X}_j - \bar{X})^2$ $MSE = \frac{1}{N - k} \sum_{j=1}^k \sum_{i=1}^{n_j} n_j (X_i - \bar{X}_j)^2$
Two or more samples, Categorical and Ordinal: $p_{11}, \dots, p_{rc}$	<p>Multinomial Test (Not taught in this class.)</p>	$\chi^2 = \sum_{i=1}^r \sum_{j=1}^c \frac{(O_{ij} - E_{ij})^2}{E_{ij}},$ $df = (r - 1)(c - 1)$

**Chapter 9 Multivariable Models**

Correlation Coefficient: $r$	$r = \frac{\text{cov}(x, y)}{\sqrt{s_x^2 s_y^2}}$ $\text{cov}(x, y) = \frac{1}{n-1} \left[ \sum XY - \frac{1}{n} (\sum Y)(\sum X) \right]$ $s_x^2 = \frac{1}{n-1} \left[ \sum X^2 - \frac{1}{n} (\sum X)^2 \right], \quad s_y^2 = \frac{1}{n-1} \left[ \sum Y^2 - \frac{1}{n} (\sum Y)^2 \right]$
Linear Regression: $\hat{y} = b_0 + b_1 x$	$b_1 = r \frac{s_y}{s_x}, \quad b_0 = \bar{Y} - b_1 \bar{X}$
Logistic Regression: $\hat{p} = \frac{1}{1 + e^{-b_0 - b_1 x_1 - \dots - b_p x_p}}$	$\ln \left( \frac{\hat{p}}{1 - \hat{p}} \right) = b_0 + b_1 x_1 + \dots + b_p x_p$ $\hat{OR} = e^{\hat{\beta}_1 \Delta_1 + \dots + \hat{\beta}_p \Delta_p}$

**Chapter 10 Nonparametric Tests**

Sign Test: $MD=MD_0$ (One Sample)	$x$ = number of observations $> MD_0$ If value $< MD_0$ , -. If value $= MD_0$ , 0. If value $> MD_0$ , +.
Mann-Whitney U Test: $MD_1=MD_2$ (not-Paired)	$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$ $U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$ $U = \min(U_1, U_2)$
Sign Test: $\delta=0$ (Paired)	If difference $< 0$ , -. If difference $= 0$ , 0. If difference $> 0$ , +. If $H_1: \delta > 0$ , $x$ = number of observations $> 0$ . If $H_1: \delta < 0$ , $x$ = number of observations $< 0$ . Use $\min(-, +)$ and Table 6.
Wilcoxon Signed Rank Test: $\delta=0$ (Paired)	$W = \min(W+, W-)$ $W+$ = sum of positive ranks $W-$ = sum of negative ranks
Kruskal-Wallis Test: $MD_1 = \dots = MD_k$ (ANOVA)	$H = \left( \frac{12}{N(N+1)} \sum_{j=1}^k \frac{R_j^2}{n_j} \right) - 3(N+1)$

**Chapter 11 Survival Analysis**

<p>Actuarial Life Table:</p>	<p> <math>N_t</math> = # event free during interval <math>t</math>  <math>D_t</math> = # who die in interval <math>t</math>  <math>C_t</math> = # censored in interval <math>t</math>  <math>N_t^*</math> = avg. # at risk in interval <math>t</math>, <math>N_t^* = N_t - C_t/2</math>  <math>q_t</math> = prop. die in interval <math>t</math>, <math>q_t = D_t/N_t^*</math>  <math>p_t</math> = prop. survive in interval <math>t</math>, <math>p_t = 1 - q_t</math>  <math>S_t</math> = prop. survive past interval <math>t</math> </p>
<p>Kaplan-Meier Life Table:</p>	<p> <math display="block">S_{t+1} = S_t \frac{N_t - D_t}{N_t}</math> <math display="block">SE(S_t) = S_t \sqrt{\sum \frac{D_t}{N_t(N_t - D_t)}}</math> </p>
<p>Chi-Square Test:  <math>k = \#</math> groups  <math>\sum_{i=1}^T O_{ij}</math> = Obs. deaths in Group <math>j</math>  <math>\sum_{i=1}^T E_{ij}</math> = Exp. deaths in Group <math>j</math></p>	<p> <math display="block">\chi^2 = \sum_{j=1}^k \frac{\left( \sum_{i=1}^T O_{ij} - \sum_{i=1}^T E_{ij} \right)^2}{\sum_{i=1}^T E_{ij}}, \quad df = k - 1</math> </p>
<p>Cox Proportional Hazards Model:</p>	<p> <math display="block">h(t) = h_0(t) \exp(b_1 x_1 + b_2 x_2 + \dots + b_p x_p)</math> </p>

Table 1: Standard Normal z

Table entries represent  $P(Z < Z_j)$   
e.g.,  $P(Z < -1.96) = 0.0250$ ,  $P(Z < 1.96) = 0.9750$

$Z_j$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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Table entries represent $P(Z < Z_i)$ e.g., $P(Z < -1.96) = 0.0250$ , $P(Z < 1.96) = 0.9750$										
$Z_i$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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**Table 2: t Distribution**

Table entries represent values from $t$ distribution with upper-tail area equal to $\alpha$ . e.g., $P(t_{df} > t) = \alpha$ , e.g., $P(t_6 > 1.943) = 0.05$					
Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test $\alpha$	.20	.10	.05	.02	.01
One-Sided Test $\alpha$	.10	.05	.025	.01	.005
$df$					
1	3.078	6.314	12.71	31.82	63.66
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

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Table entries represent values from $t$ distribution with upper-tail area equal to $\alpha$ . e.g., $P(t_{df} > t) = \alpha$ , e.g., $P(t_5 > 1.943) = 0.05$					
Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test $\alpha$	.20	.10	.05	.02	.01
One-Sided Test $\alpha$	.10	.05	.025	.01	.005
$df$					
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690
46	1.300	1.679	2.013	2.410	2.687
47	1.300	1.678	2.012	2.408	2.685
48	1.299	1.677	2.011	2.407	2.682
49	1.299	1.677	2.010	2.405	2.680
50	1.299	1.676	2.009	2.403	2.678
51	1.298	1.675	2.008	2.402	2.676
52	1.298	1.675	2.007	2.400	2.674
53	1.298	1.674	2.006	2.399	2.672
54	1.297	1.674	2.005	2.397	2.670
55	1.297	1.673	2.004	2.396	2.668
56	1.297	1.673	2.003	2.395	2.667
57	1.297	1.672	2.002	2.394	2.665
58	1.296	1.672	2.002	2.392	2.663
59	1.296	1.671	2.001	2.391	2.662
60	1.296	1.671	2.000	2.390	2.660

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Table entries represent values from $t$ distribution with upper-tail area equal to $\alpha$ . e.g., $P(t_{df} > t) = \alpha$ , e.g., $P(t_{df} > 1.943) = 0.05$					
Confidence Level	80%	90%	95%	98%	99%
Two-Sided Test $\alpha$	.20	.10	.05	.02	.01
One-Sided Test $\alpha$	.10	.05	.025	.01	.005
$df$					
61	1.296	1.670	2.000	2.389	2.659
62	1.295	1.670	1.999	2.388	2.657
63	1.295	1.669	1.998	2.387	2.656
64	1.295	1.669	1.998	2.386	2.655
65	1.295	1.669	1.997	2.385	2.654
66	1.295	1.668	1.997	2.384	2.652
67	1.294	1.668	1.996	2.383	2.651
68	1.294	1.668	1.995	2.382	2.650
69	1.294	1.667	1.995	2.382	2.649
70	1.294	1.667	1.994	2.381	2.648
71	1.294	1.667	1.994	2.380	2.647
72	1.293	1.666	1.993	2.379	2.646
73	1.293	1.666	1.993	2.379	2.645
74	1.293	1.666	1.993	2.378	2.644
75	1.293	1.665	1.992	2.377	2.643
$\infty$	1.282	1.645	1.960	2.326	2.576

**Table 3:  $\chi^2$  Distribution**

Table entries represent values from  $\chi^2$  distribution with upper tail area equal to  $\alpha$ .  
 $P(\chi_{df}^2 > \chi^2) = \alpha$ , e.g.,  $P(\chi_3^2 > 7.81) = 0.05$

$df$	$\alpha$ .10	.05	.025	.01	.005
1	2.71	3.84	5.02	6.63	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.81	9.35	11.34	12.84
4	7.78	9.49	11.14	13.28	14.86
5	9.24	11.07	12.83	15.09	16.75
6	10.64	12.59	14.45	16.81	18.55
7	12.02	14.07	16.01	18.48	20.28
8	13.36	15.51	17.53	20.09	21.95
9	14.68	16.92	19.02	21.67	23.59
10	15.99	18.31	20.48	23.21	25.19
11	17.28	19.68	21.92	24.72	26.76
12	18.55	21.03	23.34	26.22	28.30
13	19.81	22.36	24.74	27.69	29.82
14	21.06	23.68	26.12	29.14	31.32
15	22.31	25.00	27.49	30.58	32.80
16	23.54	26.30	28.85	32.00	34.27
17	24.77	27.59	30.19	33.41	35.72
18	25.99	28.87	31.53	34.81	37.16
19	27.20	30.14	32.85	36.19	38.58
20	28.41	31.41	34.17	37.57	40.00
21	29.62	32.67	35.48	38.93	41.40
22	30.81	33.92	36.78	40.29	42.80
23	32.01	35.17	38.08	41.64	44.18
24	33.20	36.42	39.36	42.98	45.56
25	34.38	37.65	40.65	44.31	46.93
26	35.56	38.89	41.92	45.64	48.29
27	36.74	40.11	43.19	46.96	49.64
28	37.92	41.34	44.46	48.28	50.99
29	39.09	42.56	45.72	49.59	52.34
30	40.26	43.77	46.98	50.89	53.67
40	51.81	55.76	59.34	63.69	66.77
50	63.17	67.50	71.42	76.15	79.49
60	74.40	79.08	83.30	88.38	91.95
70	85.53	90.53	95.02	100.4	104.2
80	96.58	101.9	106.6	112.3	116.3
90	107.6	113.1	118.1	124.1	128.3
100	118.5	124.3	129.6	135.8	140.2

**Table 4: F Distribution**

$P(F_{df_1, df_2} > F) = 0.05,$ e.g., $P(F_{3,20} > 3.10) = 0.05$														
$df_1$														
$df_2$	1	2	3	4	5	6	7	8	9	10	20	30	40	50
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	248.0	250.1	251.1	251.8
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.45	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.66	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.80	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.56	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.87	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.44	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.15	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	2.94	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.77	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.65	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.54	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.46	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.39	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.33	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.28	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.23	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.19	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.16	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.12	2.04	1.99	1.97
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.10	2.01	1.96	1.94
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.07	1.98	1.94	1.91
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.05	1.96	1.91	1.88
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.03	1.94	1.89	1.86
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.01	1.92	1.87	1.84
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	1.99	1.90	1.85	1.82
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	1.97	1.88	1.84	1.81
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	1.96	1.87	1.82	1.79
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	1.94	1.85	1.81	1.77
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	1.93	1.84	1.79	1.76

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$P(F_{df_1, df_2} > F) = 0.05,$ e.g., $P(F_{3,20} > 3.10) = 0.05$														
$df_1$														
$df_2$	1	2	3	4	5	6	7	8	9	10	20	30	40	50
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	1.92	1.83	1.78	1.75
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	1.91	1.82	1.77	1.74
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	1.90	1.81	1.76	1.72
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	1.89	1.80	1.75	1.71
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	1.88	1.79	1.74	1.70
36	4.11	3.26	2.87	2.63	2.48	2.36	2.28	2.21	2.15	2.11	1.87	1.78	1.73	1.69
37	4.11	3.25	2.86	2.63	2.47	2.36	2.27	2.20	2.14	2.10	1.86	1.77	1.72	1.68
38	4.10	3.24	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	1.85	1.76	1.71	1.68
39	4.09	3.24	2.85	2.61	2.46	2.34	2.26	2.19	2.13	2.08	1.85	1.75	1.70	1.67
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.84	1.74	1.69	1.66
41	4.08	3.23	2.83	2.60	2.44	2.33	2.24	2.17	2.12	2.07	1.83	1.74	1.69	1.65
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	1.83	1.73	1.68	1.65
43	4.07	3.21	2.82	2.59	2.43	2.32	2.23	2.16	2.11	2.06	1.82	1.72	1.67	1.64
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	1.81	1.72	1.67	1.63
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	1.81	1.71	1.66	1.63
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.15	2.09	2.04	1.80	1.71	1.65	1.62
47	4.05	3.20	2.80	2.57	2.41	2.30	2.21	2.14	2.09	2.04	1.80	1.70	1.65	1.61
48	4.04	3.19	2.80	2.57	2.41	2.29	2.21	2.14	2.08	2.03	1.79	1.70	1.64	1.61
49	4.04	3.19	2.79	2.56	2.40	2.29	2.20	2.13	2.08	2.03	1.79	1.69	1.64	1.60
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.78	1.69	1.63	1.60
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96	1.71	1.61	1.55	1.52
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.68	1.57	1.52	1.48

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**Binomial Probabilities  $p=1/2$**

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0.250	0.125	0.063	0.031	0.016	0.008	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.500	0.375	0.250	0.156	0.094	0.055	0.031	0.018	0.010	0.005	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.250	0.375	0.375	0.313	0.234	0.164	0.109	0.070	0.044	0.027	0.016	0.010	0.006	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3		0.125	0.250	0.313	0.313	0.273	0.219	0.164	0.117	0.081	0.054	0.035	0.022	0.014	0.009	0.005	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000
4			0.063	0.156	0.234	0.273	0.273	0.246	0.205	0.161	0.121	0.087	0.061	0.042	0.028	0.018	0.012	0.007	0.005	0.003	0.002	0.001	0.001	0.000
5				0.031	0.094	0.164	0.219	0.246	0.246	0.226	0.193	0.157	0.122	0.092	0.067	0.047	0.033	0.022	0.015	0.010	0.006	0.004	0.003	0.002
6					0.016	0.055	0.109	0.164	0.205	0.226	0.226	0.209	0.183	0.153	0.122	0.094	0.071	0.052	0.037	0.026	0.018	0.012	0.008	0.005
7						0.008	0.031	0.070	0.117	0.161	0.193	0.209	0.209	0.196	0.175	0.148	0.121	0.096	0.074	0.055	0.041	0.029	0.021	0.014
8							0.004	0.018	0.044	0.081	0.121	0.157	0.183	0.196	0.196	0.185	0.167	0.144	0.120	0.097	0.076	0.058	0.044	0.032
9								0.002	0.010	0.027	0.054	0.087	0.122	0.153	0.175	0.185	0.185	0.176	0.160	0.140	0.119	0.097	0.078	0.061
10									0.001	0.005	0.016	0.035	0.061	0.092	0.122	0.148	0.167	0.176	0.176	0.168	0.154	0.136	0.117	0.097
11										0.000	0.003	0.010	0.022	0.042	0.067	0.094	0.121	0.144	0.160	0.168	0.168	0.161	0.149	0.133
12											0.000	0.002	0.006	0.014	0.028	0.047	0.071	0.096	0.120	0.140	0.154	0.161	0.161	0.155
13												0.000	0.001	0.003	0.009	0.018	0.033	0.052	0.074	0.097	0.119	0.136	0.149	0.155
14													0.000	0.000	0.002	0.005	0.012	0.022	0.037	0.055	0.076	0.097	0.117	0.133
15														0.000	0.000	0.001	0.003	0.007	0.015	0.026	0.041	0.058	0.078	0.097
16															0.000	0.000	0.001	0.002	0.005	0.010	0.018	0.029	0.044	0.061
17																0.000	0.000	0.000	0.001	0.003	0.006	0.012	0.021	0.032
18																	0.000	0.000	0.000	0.001	0.002	0.004	0.008	0.014
19																		0.000	0.000	0.000	0.001	0.001	0.003	0.005
20																			0.000	0.000	0.000	0.000	0.001	0.002
21																				0.000	0.000	0.000	0.000	0.000
22																					0.000	0.000	0.000	0.000
23																						0.000	0.000	0.000
24																							0.000	0.000
25																								0.000

**Binomial Cumulative Probabilities  $p=1/2$**

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	0.250	0.125	0.063	0.031	0.016	0.008	0.004	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.750	0.500	0.313	0.187	0.109	0.063	0.035	0.020	0.011	0.006	0.003	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	1.000	0.875	0.688	0.500	0.344	0.227	0.145	0.090	0.055	0.033	0.019	0.011	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3		1.000	0.938	0.813	0.656	0.500	0.363	0.254	0.172	0.113	0.073	0.046	0.029	0.018	0.011	0.006	0.004	0.002	0.001	0.001	0.000	0.000	0.000	0.000
4			1.000	0.969	0.891	0.773	0.637	0.500	0.377	0.274	0.194	0.133	0.090	0.059	0.038	0.025	0.015	0.010	0.006	0.004	0.002	0.001	0.001	0.000
5				1.000	0.984	0.938	0.855	0.746	0.623	0.500	0.387	0.291	0.212	0.151	0.105	0.072	0.048	0.032	0.021	0.013	0.008	0.005	0.003	0.002
6					1.000	0.992	0.965	0.910	0.828	0.726	0.613	0.500	0.395	0.304	0.227	0.166	0.119	0.084	0.058	0.039	0.026	0.017	0.011	0.007
7						1.000	0.996	0.980	0.945	0.887	0.806	0.709	0.605	0.500	0.402	0.315	0.240	0.180	0.132	0.095	0.067	0.047	0.032	0.022
8							1.000	0.998	0.989	0.967	0.927	0.867	0.788	0.696	0.598	0.500	0.407	0.324	0.252	0.192	0.143	0.105	0.076	0.054
9								1.000	0.999	0.994	0.981	0.954	0.910	0.849	0.773	0.685	0.593	0.500	0.412	0.332	0.262	0.202	0.154	0.115
10									1.000	0.997	0.989	0.971	0.941	0.895	0.834	0.760	0.676	0.588	0.500	0.416	0.339	0.271	0.212	0.161
11										1.000	0.998	0.994	0.982	0.962	0.928	0.881	0.820	0.748	0.668	0.584	0.500	0.419	0.345	0.281
12											1.000	0.999	0.996	0.989	0.975	0.952	0.916	0.868	0.808	0.738	0.661	0.581	0.500	0.428
13												1.000	0.998	0.994	0.985	0.968	0.942	0.905	0.857	0.798	0.729	0.655	0.581	0.511
14													1.000	0.999	0.996	0.990	0.979	0.961	0.933	0.895	0.846	0.788	0.724	0.658
15														1.000	0.999	0.999	0.998	0.994	0.987	0.974	0.953	0.924	0.885	0.846
16															1.000	0.999	0.999	0.999	0.996	0.992	0.983	0.968	0.946	0.911
17																1.000	0.999	0.999	0.999	0.999	0.998	0.995	0.989	0.978
18																	1.000	0.999	0.999	0.999	0.999	0.999	0.997	0.993
19																		1.000	0.999	0.999	0.999	0.999	0.999	0.998
20																			1.000	0.999	0.999	0.999	0.999	0.999
21																				1.000	0.999	0.999	0.999	0.999
22																					1.000	0.999	0.999	0.999
23																						1.000	0.999	0.999
24																							1.000	0.999
25																								1.000

**Binomial Reverse Cumulative Probabilities  $p=1/2$**

x/n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1	0.750	0.875	0.938	0.969	0.984	0.992	0.996	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.250	0.500	0.688	0.813	0.891	0.938	0.965	0.980	0.989	0.994	0.997	0.998	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.000	0.125	0.313	0.500	0.656	0.773	0.855	0.910	0.945	0.967	0.981	0.989	0.994	0.996	0.998	0.999	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4		0.000	0.063	0.187	0.344	0.500	0.637	0.746	0.828	0.887	0.927	0.954	0.971	0.982	0.989	0.994	0.996	0.998	0.999	0.999	1.000	1.000	1.000	1.000
5				0.031	0.109	0.227	0.363	0.500	0.623	0.726	0.806	0.867	0.910	0.941	0.962	0.975	0.985	0.990	0.994	0.996	0.998	0.999	0.999	1.000
6					0.016	0.063	0.145	0.254	0.377	0.500	0.613	0.709	0.788	0.849	0.895	0.928	0.952	0.9						

**Table 5: Mann-Whitney U Test**

Two-Sided Test $\alpha = 0.05$																				
		$n_1$																		
$n_2$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2								0	0	0	0	1	1	1	1	1	2	2	2	2
3					0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4				0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5			0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6			1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7			1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8		0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9		0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
10		0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11		0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12		1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13		1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14		1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15		1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17		2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20		2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

One-Sided Test $\alpha = 0.05$																				
		$n_1$																		
$n_2$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2					0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3			0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10	11
4			0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5		0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6		0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7		0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
8		1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
9		1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
10		1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
11		1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
12		2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
13		2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
14		2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
15		3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
16		3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
17		3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
18		4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138



**Table 6: Sign Test**

Two-Sided Test $\alpha$	.10	.05	.02	.01
One-Sided Test $\alpha$	.05	.025	.01	.005
$n$				
1				
2				
3				
4				
5	0			
6	0	0		
7	0	0	0	
8	1	0	0	0
9	1	1	0	0
10	1	1	0	0
11	2	1	1	0
12	2	2	1	1
13	3	2	1	1
14	3	2	2	1
15	3	3	2	2
16	4	3	2	2
17	4	4	3	2
18	5	4	3	3
19	5	4	4	3
20	5	5	4	3
21	6	5	4	4
22	6	5	5	4
23	7	6	5	4
24	7	6	5	5
25	7	7	6	5

**Table 7: Mann-Whitney U Test**

Two-Sided Test $\alpha$	.10	.05	.02	.01
One-Sided Test $\alpha$	.05	.025	.01	.005
<i>n</i>				
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

**Table 8: Kruskal-Wallis Test**

Three groups			Four groups							
$n_1$	$n_2$	$n_3$	$\alpha = .05$	$\alpha = .01$	$n_1$	$n_2$	$n_3$	$n_4$	$\alpha = .05$	$\alpha = .01$
2	2	2			2	2	1	1		
3	2	1			2	2	2	1	5.679	
3	2	2	4.714		2	2	2	2	6.167	6.667
3	3	1	5.143		3	1	1	1		
3	3	2	5.361		3	2	1	1		
3	3	3	5.600	7.200	3	2	2	1	5.833	
4	2	1			3	2	2	2	6.333	7.133
4	2	2	5.333		3	3	1	1	6.333	
4	3	1	5.208		3	3	2	1	6.244	7.200
4	3	2	5.444	6.444	3	3	2	2	6.527	7.636
4	3	3	5.791	6.745	3	3	3	1	6.600	7.400
4	4	1	4.967	6.667	3	3	3	2	6.727	8.015
4	4	2	5.455	7.036	3	3	3	3	7.000	8.538
4	4	3	5.598	7.144	4	1	1	1		
4	4	4	5.692	7.654	4	2	1	1	5.833	
5	2	1	5.000		4	2	2	1	6.133	7.000
5	2	2	5.160	6.533	4	2	2	2	6.545	7.391
5	3	1	4.960		4	3	1	1	6.178	7.067
5	3	2	5.251	6.909	4	3	2	1	6.309	7.455
5	3	3	5.648	7.079	4	3	2	2	6.621	7.871
5	4	1	4.985	6.955	4	3	3	1	6.545	7.758
5	4	2	5.273	7.205	4	3	3	2	6.795	8.333
5	4	3	5.656	7.445	4	3	3	3	6.984	8.659
5	4	4	5.657	7.760	4	4	1	1	5.945	7.909
5	5	1	5.127	7.309	4	4	2	1	6.386	7.909
5	5	2	5.338	7.338	4	4	2	2	6.731	8.346
5	5	3	5.705	7.578	4	4	3	1	6.635	8.231
5	5	4	5.666	7.823	4	4	3	2	6.874	8.621
5	5	5	5.780	8.000	4	4	3	3	7.038	8.876
6	1	1			4	4	4	1	6.725	8.588
6	2	1	4.822		4	4	4	2	6.957	8.871
6	2	2	5.345	6.655	4	4	4	3	7.142	9.075
6	3	1	4.855	6.873	4	4	4	4	7.235	9.287

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Three groups					Five groups						
$n_1$	$n_2$	$n_3$	$\alpha = .05$	$\alpha = .01$	$n_1$	$n_2$	$n_3$	$n_4$	$n_5$	$\alpha = .05$	$\alpha = .01$
6	3	2	5.348	6.970	2	2	1	1	1		
6	3	3	5.615	7.410	2	2	2	1	1	6.750	
6	4	1	4.947	7.106	2	2	2	2	1	7.133	7.533
6	4	2	5.340	7.340	2	2	2	2	2	7.418	8.291
6	4	3	5.610	7.500	3	1	1	1	1		
6	4	4	5.681	7.795	3	2	1	1	1	6.583	
6	5	1	4.990	7.182	3	2	2	1	1	6.800	7.600
6	5	2	5.338	7.376	3	2	2	2	1	7.309	8.127
6	5	3	5.602	7.590	3	2	2	2	2	7.682	8.682
6	5	4	5.661	7.936	3	3	1	1	1	7.111	
6	5	5	5.729	8.028	3	3	2	1	1	7.200	8.073
6	6	1	4.945	7.121	3	3	2	2	1	7.591	8.576
6	6	2	5.410	7.467	3	3	2	2	2	7.910	9.115
6	6	3	5.625	7.725	3	3	3	1	1	7.576	8.424
6	6	4	5.725	8.000	3	3	3	2	1	7.769	9.051
6	6	5	5.765	8.124	3	3	3	2	2	8.044	9.505
6	6	6	5.801	8.222	3	3	3	3	1	8.000	9.451
7	7	7	5.819	8.378	3	3	3	3	2	8.200	9.876
8	8	8	5.805	8.465	3	3	3	3	3	8.333	10.200