MATH 4740/MSSC 5740

Chapter 10 Problem Solving # 2 (Wilcoxon Sign Rank Test), 4 (Kruskal-Wallis Test)

10.5 Summary

Sign Test: $\delta=0$, $\delta = MD_1 - MD_2$ (Paired)	H_1: $\delta>0$. If difference<0,+. If difference=0, 0. If difference>0, –. H_1: $\delta<0$. If difference<0, –. If difference=0, 0. If difference>0, +. x = number of +'s. Use Table 6.
Wilcoxon Signed Rank Test: $\delta=0$, $\delta = MD_1 - MD_2$ (Paired)	$W+$ = sum of positive ranks $W-$ = sum of negative ranks $W = \min(W+, W-)$
Kruskal-Wallis Test: $MD_1 = \ldots = MD_k$ (ANOVA)	$H = \frac{12}{N(N+1)}\sum_{j=1}^{k}\frac{R_j^2}{n_j} - 3(N+1)$

Sign Test Table (Table 6)

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
n				
1				
2				
3				
4				
5	0			
6	0	0		
7	0	0	0	
8	1	0	0	0
9	1	1	0	0
10	1	1	0	0
11	2	1	1	0
12	2	2	1	1
13	3	2	1	1
14	3	2	2	1
15	3	3	2	2
16	4	3	2	2
17	4	4	3	2
18	5	4	3	3
19	5	4	4	3
20	5	5	4	3
21	6	5	4	4
22	6	5	5	4
23	7	6	5	4
24	7	6	5	5
25	7	7	6	5

Wilcoxon Signed Rank Table (Table 7)

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
n				
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0
9	8	6	3	2
10	11	8	5	3
11	14	11	7	5
12	17	14	10	7
13	21	17	13	10
14	26	21	16	13
15	30	25	20	16
16	36	30	24	19
17	41	35	28	23
18	47	40	33	28
19	54	46	38	32
20	60	52	43	37
21	68	59	49	43
22	75	66	56	49
23	83	73	62	55
24	92	81	69	61
25	101	90	77	68
26	110	98	85	76
27	120	107	93	84
28	130	117	102	92
29	141	127	111	100
30	152	137	120	109

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Kruskal-Wallis Test Table (Table 8)

Three groups					Four groups					
n_1	n_2	n_3	$\alpha = .05$	$\alpha = .01$	n_1	n_2	n_3	n_4	$\alpha = .05$	$\alpha = .01$
2	2	2			2	2	1	1		
3	2	1			2	2	2	1	5.679	
3	2	2	4.714		2	2	2	2	6.167	6.667
3	3	1	5.143		3	1	1	1		
3	3	2	5.361		3	2	1	1		
3	3	3	5.600	7.200	3	2	2	1	5.833	
4	2	1			3	2	2	2	6.333	7.133
4	2	2	5.333		3	3	1	1	6.333	
4	3	1	5.208		3	3	2	1	6.244	7.200
4	3	2	5.444	6.444	3	3	2	2	6.527	7.636
4	3	3	5.791	6.745	3	3	3	1	6.600	7.400
4	4	1	4.967	6.667	3	3	3	2	6.727	8.015
4	4	2	5.455	7.036	3	3	3	3	7.000	8.538
4	4	3	5.598	7.144	4	1	1	1		
4	4	4	5.692	7.654	4	2	1	1	5.833	
5	2	1	5.000		4	2	2	1	6.133	7.000
5	2	2	5.160	6.533	4	2	2	2	6.545	7.391
5	3	1	4.960		4	3	1	1	6.178	7.067
5	3	2	5.251	6.909	4	3	2	1	6.309	7.455
5	3	3	5.648	7.079	4	3	2	2	6.621	7.871
5	4	1	4.985	6.955	4	3	3	1	6.545	7.758
5	4	2	5.273	7.205	4	3	3	2	6.795	8.333
5	4	3	5.656	7.445	4	3	3	3	6.984	8.659
5	4	4	5.657	7.760	4	4	1	1	5.945	7.909
5	5	1	5.127	7.309	4	4	2	1	6.386	7.909
5	5	2	5.338	7.338	4	4	2	2	6.731	8.346
5	5	3	5.705	7.578	4	4	3	1	6.635	8.231
5	5	4	5.666	7.823	4	4	3	2	6.874	8.621
5	5	5	5.780	8.000	4	4	3	3	7.038	8.876
6	1	1			4	4	4	1	6.725	8.588
6	2	1	4.822		4	4	4	2	6.957	8.871
6	2	2	5.345	6.655	4	4	4	3	7.142	9.075
6	3	1	4.855	6.873	4	4	4	4	7.235	9.287

Continued …

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10.6 Practice Problems

2. Using the data in Problem 1, assess whether there is a significant reduction in the number of sick days taken after completing the wellness program using the Wilcoxon Signed Rank Test at the 5% level of significance.

Employee	Prior	Following
1	8	7
2	6	6
3	4	5
4	12	11
5	10	7
6	8	4
7	6	3
8	2	1

Step 1. Set up hypotheses and determine level of significance.

H_0: $\delta = 0$ vs. H_1: $\delta < 0$ $\qquad \alpha = 0.05$

Step 2. Select the appropriate test statistic.

$W = \min(W+, W-)$, $W+$ = sum of + ranks, $W-$ = sum of – ranks

Step 3. Set up decision rule.

Reject H_0 if $W < W_{0.05,8}$.

Two-Sided Test α	.10	.05	.02	.01
One-Sided Test α	.05	.025	.01	.005
n				
5	1			
6	2	1		
7	4	2	0	
8	6	4	2	0

Step 4. Compute the test statistic.

P	F	d=F-B	Signs	Ranks	SgnRnk
8	7	-1	-1	3.5	-3.5
6	6	0	0	1	0
4	5	1	1	3.5	3.5
12	11	-1	-1	3.5	-3.5
10	7	-3	-1	6.5	-6.5
8	4	-4	-1	8	-8
6	3	-3	-1	6.5	-6.5
2	1	-1	-1	3.5	-3.5

$W- = 3.5$

$W+ = 31.5$

$W = 3.5$

Step 5. Conclusion.

We _____reject___ H_0 because 3.5 < 6. We _do____ have statistically significant evidence at $\alpha=0.05$ to show that the number of sick days reduced. Compare to t?

Note: $\bar{X} = -1.50$, $s = 1.69$, $t = -2.51$, $t_{0.05,14} = -2.365$

4. The following data represent the number of playground injuries occurring among children aged 5-9 over a 3-month period in 12 playgrounds in and around the neighborhoods of Boston. Playground injuries include fractures, internal injuries, lacerations, and dislocations. The question of interest is whether there are differences in the numbers of injuries at playgrounds in various locations. The data below represents the numbers of injuries recorded at randomly selected playgrounds located on school properties, at day-care centers, and residential neighborhoods.

School Properties: 39 51 42 29 (Group 1)
Day Care Centers: 28 25 30 15 (Group 2)
Residential Neighborhoods: 18 16 25 22 (Group 3)

Run the appropriate test at a 5% level of significance.

Step 1. Set up hypotheses and determine level of significance. $\alpha = 0.05$

H_0: The three population medians are equal vs.

H_1: The three population medians are not equal.

Step 2. Select the appropriate test statistic.

$$H = \frac{12}{N(N+1)} \sum_{j=1}^{k} \frac{R_j^2}{n_j} - 3(N+1),$$

R_j=sum of ranks of sample j, $N=n_1+n_2+n_3$.

Step 3. Set up decision rule.
Reject H_0 if $H \geq H_{\alpha,n_1,n_2,n_3} = H_{0.05,4,4,4} = 5.692$.

Three groups				
n_1	n_2	n_3	$\alpha = .05$	$\alpha = .01$
4	4	4	5.692	7.654

Step 4. Compute the test statistic.

Group 1	Group 2	Group 3	Ranks 1	Ranks 2	Ranks 3
	15			1	
		16			2
		18			3
		22			4
	25			5.5	
		25			5.5
	28			7	
29			8		
	30			9	
39			10		
42			11		
51			12		
			R_1=41	R_2=22.5	R_3=14.5

$$H = \frac{12}{N(N+1)} \left(\frac{41^2}{4} + \frac{22.5^2}{4} + \frac{14.5^2}{4} \right) - 3(12+1) = 7.1058$$

Step 5. Conclusion.

Because $H = 7.1058 \geq H_{0.05,4,4,4} = 5.692$, reject H_0. Evidence to show there are differences in the numbers of injuries at playgrounds in various locations at the $\alpha = 0.05$ level.