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Chapter 9 Problem Solving # *, Example 9.7, Example 9.9

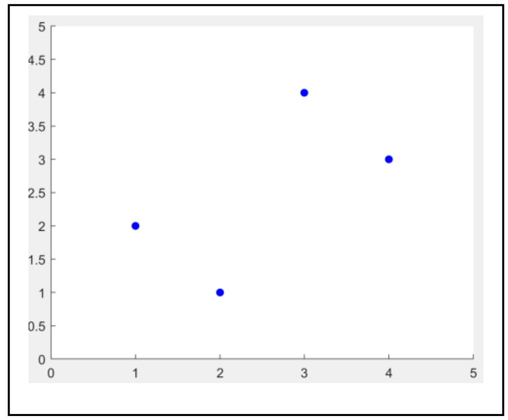
9.6 Summary

Correlation Coefficient: r	$r = \frac{\text{cov}(x,y)}{\sqrt{s_x^2 s_y^2}}$ $\text{cov}(x,y) = \frac{1}{n-1}\left[\sum XY - \frac{1}{n}\left(\sum Y\right)\left(\sum X\right)\right]$ $s_x^2 = \frac{1}{n-1}\left[\sum X^2 - \frac{1}{n}\left(\sum X\right)^2\right]$, $s_y^2 = \frac{1}{n-1}\left[\sum Y^2 - \frac{1}{n}\left(\sum Y\right)^2\right]$
Linear Regression: $\hat{y} = b_0 + b_1 x$	$b_1 = r\frac{s_y}{s_x}$, $b_0 = \overline{Y} - b_1\overline{X}$
Logistic Regression: $\hat{p} = \frac{1}{1+e^{-b_0-b_1x_1-\ldots-b_px_p}}$	$ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = b_0 + b_1x_1 + \ldots + b_px_p$ $\hat{O}R = e^{\hat{\beta}_1\Delta_1+\ldots+\hat{\beta}_p\Delta_p}$

9.6 Practice Problems

* Given (x,y) points $(1,2),(2,1),(3,4),(4,3)$,

a) Plot the points.



b) Find r, b_0 and b_1 by hand with sums.

$$\text{cov}(x,y)=\frac{1}{4-1}\left[28-(10)(10)/4\right]=1$$

$$s_x^2=\frac{1}{4-1}\left[30-(10)^2/4\right]=5/3$$

$$s_y^2=\frac{1}{n-1}\left[\sum X^2-\frac{1}{n}\left(\sum X\right)^2\right]=5/3$$

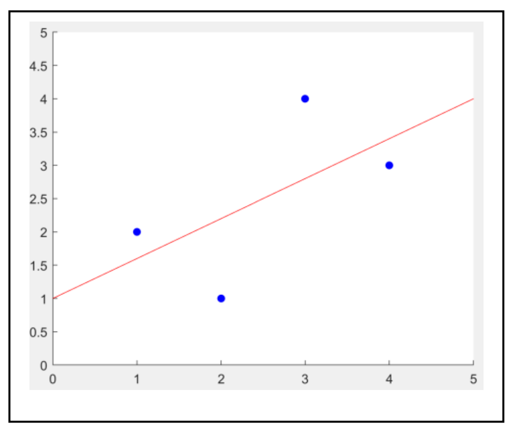
$$r=\frac{1}{\sqrt{(5/3)(5/3)}}=3/5$$

$$b_1=\frac{3}{5}\frac{\sqrt{5/3}}{\sqrt{5/3}}=\frac{3}{5}$$

$$b_0=(5/2)-(3/5)(5/2)=1$$

X	X^2	Y	Y^2	XY
1	1	2	4	2
2	4	1	1	2
3	9	4	16	12
4	19	3	9	12
10	**30**	**10**	**30**	**28**

c) Draw the fitted regression line on the same graph as points.



d) What do b_0 and b_1 mean?

b_0 =1, y reference value when x=0

b_1=3/5, expected change in y for a 1 unit change in x

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Example 9.7 (page 216)

An observational study is conducted to investigate risk factors associated with infant weight. The study involves n=832 pregnant women. Investigators wish to determine whether there are any differences in birth weight by infant sex, gestational age, mothers age, and mother's race/ethnicity. A multiple regression analysis is performed.

The model is:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + b_5x_5 + b_6x_6$$

H$_0$: β_j=0 vs. H$_0$: $\beta_j \neq 0$

y =birth weight (grams)
x_1=Infant sex (1=male, 0=female),
x_2=Gestational age (in weeks),
x_3=Mothers age (in years), and
x_4=Black race/ethnicity (1=yes, 0=no)
x_5=Hispanic race/ethnicity (1=yes, 0=no)
x_6=Other race/ethnicity (1=yes, 0=no)

$$t_j = \frac{b_j - 0}{\sqrt{\text{var}(b_j)}}$$

df=n-p-1
$t_{\alpha/2}$=$t_{0.025,825}$=1.96

	Independent Variable	Regression Coefficient	t	p-value
b_0	Intercept	−3850.92	−11.56	0.0001
b_1	Male infant	174.79	6.06	0.0001
b_2	Gestational age (weeks)	179.89	22.35	0.0001
b_3	Mother's age (years)	1.38	0.47	0.6361
b_4	Black race/ ethnicity	−138.46	−1.93	0.0535
b_5	Hispanic race/ ethnicity	−13.07	−0.37	0.7103
b_6	Other race/ ethnicity	−68.67	−1.05	0.2918

With b_0, b_1, b_2, b_3, b_4, b_5, b_6 calculated using a software program such as ***R***.

$$\hat{y} = -3850.92 + 174.79x_1 + 179.89x_2 + 1.38x_3 - 138.46x_4 - 13.07x_6 - 68.67x_6$$

a) What does b_0=-3850.92 mean?

b_0 is a reference point for when all x's are zero.

b) What does b_2=179.89 mean?

An increase of 1 week in gestational age leads to an expected 179.89 g increase in birth weight.

c) Keeping x_1, x_2, x_3, fixed, what does a change from x_4=1, x_5=0, x_6=0 to x_4=0, x_5=1, x_6=0 mean?

We would expect a 125.39 g increase in birth weight.

d) Which variables are important (β_j coefficient statistically significant from 0)?

b_0, b_1, b_2 because the absolute value of their t-statistics >1.96.

Example 9.8

Assume that a logistic regression model were fit to relate obesity to probability of CVD

$$\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = -2.367 + 0.658(Obesity) \qquad \hat{p} = \frac{1}{1+e^{2.367-0.658(Obesity)}}$$

where 1=obese and 0=not obsess.

a) What does b_1=0.685 mean?

Among obese persons, the log odds of incident CVD are 0.658 times the log odds of persons who are not obese.

If we take the antilog of the regression coefficient, exp(0.658)=1.93, we get the unadjusted odds ratio. Among obese persons, the odds of developing CVD are 1.93 times the odds of non-obese persons.

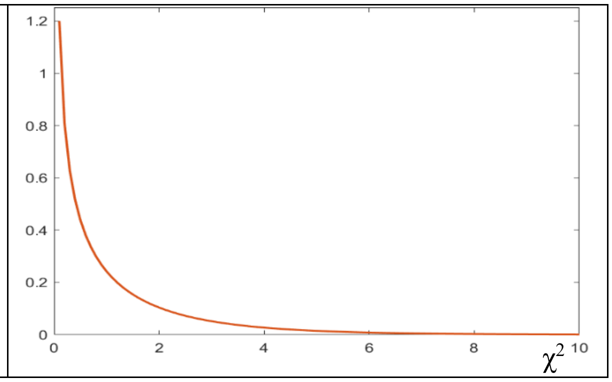
b) To look at statistical significance,

Independent Variable	Regression Coefficient	χ^2	*p*-value
Intercept	−2.367	307.38	0.0001
Obesity	0.658	9.87	0.0017

H_0: β_j=0 vs. H_0: $\beta_j \neq 0$

df=1

$\chi^2_{0.95}$=3.8415



Assume that age group is added to the model.

$$\ln\left(\frac{\hat{p}}{1-\hat{p}}\right) = -2.367 + 0.415(Obesity) + 0.655(Age\ Group)$$

$$\hat{p} = \frac{1}{1+e^{2.367-0.415(Obesity)-0.655(Age\ Group)}}$$

c) What does b_1=0.415 mean?

Among obese persons, the log odds of incident CVD are 0.415 times the log odds of persons who are not obese when adjusting (accounting for) age.

Among Obese persons, the odds of developing CVD are exp(0.415)=1.52 times the odds for non-obese persons when adjusting for age.