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Chapter 5 Problem Solving # 7, 8^+

5.7 Summary

Concept	Formula
Basic probability	$P(\text{Characteristic}) = \frac{\text{Number of persons with characteristic}}{N}$
Conditional probability rule	$P(A / B) = \frac{P(A \text{ and } B)}{P(B)}$

Sensitivity	$P(screen\ \ positive \mid disease) = a/(a+c)$
Specificity	$P(screen\ negative \mid disease\ \ \ free) = d/(b+d)$
False Positive Fraction	$P(screen\ \ positive \mid disease\ \ free) = b/(b+d)$
False Negative Fraction	$P(screen\ negative \mid disease) = c/(a+c)$
Positive Predictive Value	$P(disease \mid screen\ \ positive) = a/(a+b)$
Negative Predictive Value	$P(disease\ \ \ free \mid screen\ negative) = d/(c+d)$

	Disease present	Disease Free	Total
Screen positive	*a*	*b*	*a* + *b*
Screen negative	*c*	*d*	*c* + *d*
Total	*a* + *c*	*b* + *d*	*N*

Independent events	P(A | B) = P(A) or P(B | A) = P(B)
Bayes Theorem	$P(A / B) = \frac{P(B / A) P(A)}{P(B)}$
Binomial distribution	$P(x \text{ successes}) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$\mu = np$

$\sigma^2 = np(1-p)$

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5.8 Practice Problems

7. As part of the study described in Problem 6, investigators wanted to assess the accuracy of self-reported smoking status. Participants are asked whether they currently smoke or not. In addition, laboratory tests are performed on hair samples to determine the presence or absence of nicotine. The laboratory assessment is considered the gold standard, or the truth about nicotine consumption. The data are shown in **Table 5.13**.

   a. What is the sensitivity of the self-reported smoking status?

   b. What is the specificity of the self-reported smoking status?

TABLE 5.13 Self-Reported Smoking Status

	Nicotine Absent	Nicotine Present
Self-reported nonsmoker	82	14
Self-reported smoker	12	52

a) Sensitivity = 52/(52+14) = 0.79

b) Specificity = 82/(12+82) = 0.87

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8. A recent study of cardiovascular risk factors reported that 30% of adults meet criteria for hypertension. If 15 adults are assessed:
   a. What is the probability that exactly 5 meet the criteria for hypertension?
   b. What is the probability that none meet the criteria for hypertension?
   c. How many would you expect to meet the criteria for hypertension? μ

   d. What is the standard deviation σ of those that meet the criteria?
   e. What is μ-σ to μ+σ?
   f. What is the probability that more than 12 meet the criteria?
   g. What is the probability that less than 2 meet the criteria?

a. $p(X=5)=\frac{15!}{5!(15-5)!}(.3)^5(1-.3)^{10}$

$$p(X=5)=\frac{\cancel{15}\cdot 14\cdot 13\cdot 12\cdot 11\cdot \cancel{10!}}{\cancel{5}\cdot 4\cdot \cancel{3}\cdot 2\cancel{10!}}(.3)^5(1-.3)^{10}$$

$$p(X=5)=3003\cdot 6.86\times 10^{-5}$$

$$p(X=5)=0.206$$

b. n=15, x=0, p=0.3

$$p(X=5)=\frac{15!}{0!(15-0)!}(.3)^0(1-.3)^{15}$$

$$p(X=5)=\frac{\cancel{15!}}{\cancel{0!\ (15-0)!}}0.0047$$

$$p(X=5)=0.004747561$$

0.47% chance

c. $\mu=np=15(0.3)=4.5$

d. 4.50-1.77 to 4.50+1.77

2.77 to 6.27

e. P(X>12)=P(X=13)+P(X=14)+P(X=15)

$$p(X>12)=\frac{15!}{2!(15-2)!}(.3)^{13}(1-.3)^2+\frac{15!}{1!(15-1)!}(.3)^{14}(1-.3)^1+\frac{15!}{0!(15-0)!}(.3)^{15}(1-.3)^0$$

(Finish Calculation)

f. P(X<2)=P(X=0)+P(X=1)

$$p(X<2)=\frac{15!}{0!(15-0)!}(.3)^0(1-.3)^{15}+\frac{15!}{1!(15-1)!}(.3)^1(1-.3)^{14}$$

(Finish Calculation)